**1. Determine the phase angle for the data from the first experiment.**

Equations in this section are referenced from *Theory of Vibration with Applications* by William T. Thomson [1]. We were asked to calculate the phase angle, φ, for the first experiment in which we pointed a laser vibrometer at the middle of an ArtDio speaker. When the speaker vibrates while playing a song, the laser vibrometer calculates the velocity of the speaker cone, where the displacement and acceleration can then be calculated as well. In order to do this we must recognize the system is in harmonic motion due to the output of the sine frequency by the speakers. This can be represented by the equation,

$$m\ddot{x}+c\dot{x}+kx= F\_{o}sin⁡(ωt)$$

The recorded data from the vibrometer were 1.293 $mm$, 1.314 $\frac{mm}{s}$, and 1.284 $\frac{mm}{s^{2}}$ for the displacement, velocity, and acceleration respectively. The constants provided by Dr. Kassegne came from the lab manual for experiment 8 entitled “Laser Vibrometry”[2] are as follows:

 $k=63 \frac{g}{mm}\*g$ *g* = 9.81 $\frac{m}{s^{2}} $ *m* = 7 $g$ *F(t) =* $1sin⁡(2πft)$

Given these constants and the data recorded from the lab we were required to calculate the phase angle, φ which is defined as,

$$ϕ=tan^{-1}\frac{2ζ\frac{ω}{ω\_{n}}}{1-\left(\frac{ω}{ω\_{n}}\right)^{2}}$$

In order to calculate the phase angle we first need to calculate ω, $ω\_{n}$ and ζ. The equation for the frequency ratio, which is the frequency (ω), divided by the natural frequency ($ω\_{n})$ is

$$\frac{ω}{ω\_{n}}= \frac{2πf}{\sqrt{\frac{k}{m}}}= \frac{2π1 kHz}{\sqrt{\frac{63\frac{kg}{m}\*9.81\frac{m}{s^{2}}}{7E^{-3}kg}}}= \frac{6.283 \frac{rad}{s}}{297.136\frac{rad}{s}}=0.02 $$

We can calculate the damping ratio via the relationship between the damping ratio ( ζ ) and the damping coefficient ( *c* ) in the equation,

$$ζ= \frac{c}{2\sqrt{km}}$$

We also have an unknown in this equation being the damping coefficient ( *c* ). To solve for *c* we must go back and solve the equation for harmonic motion of a system. By plugging in the values recorded for the magnitudes of the displacement, velocity, and acceleration.

$$m\ddot{x}+c\dot{x}+kx= F\_{o}sin⁡(ωt)$$

$$\left[\left(7E^{-3} kg\right)(1.284E^{-3} \frac{m}{s^{2}}\right]+ c\left[1.319E^{-3}\frac{m}{s}\right]+ \left[\left(618.03 \frac{N}{m}\right)\left(1.293 E^{-3} m\right)\right]= 1sin⁡(2π\left(1 kHz\right)\left(0.03823 s\right))$$

$$c=80 \frac{N\*s}{m}$$

Plugging *c* into the damping ratio equation,

$$ζ= \frac{80 \frac{N\*s}{m}}{2\sqrt{\left(618.03 \frac{N}{m}\right)\left(7E^{-3}m\right)}}=19.23$$

Now that we have all the unknowns needed to solve for the phase angle, so plugging in our values yield,

$$ϕ=tan^{-1}\frac{2\left(19.23\right)(0.02)}{1-\left(0.02\right)^{2}}=37.58°$$

**2. Find c when the system is critically damped.**

Knowing that the system is critically damped (ζ = 1), we can solve for the damping coefficient, c, via the equation for the damping ratio.

$$ζ= \frac{c}{c\_{cr}}=\frac{c}{2\sqrt{km}}$$

Solving for the damping coefficient, c

$$c=2ζ\sqrt{km}$$

$$c=2\*1\sqrt{618.03\frac{N}{m}\*7E^{-3}kg}$$

c = 4.15991 $\frac{N\*s}{m}$

**3. What can you say about the frequency contents of the two music pieces in Experiment 2?**

The two songs chosen to analyze were 1812 Overture as the classical piece and song by Daft Punk for the electronic piece. The frequency of the two pieces differed such that the range of the classical piece was played at a higher frequency throughout and the electronic was played at a much lower frequency. The classical piece was played with high frequency instruments like violins and cellos as opposed to the lower frequency instruments such as the bass and drums used in electronic music.

**4. How is 'beat' of music related to frequency?**

 The beat of music is related to frequency because when two different interfering sound waves approach the ear, they interfere with each other which produces noise. The two frequencies are constructive and destructive interferences, which when subtracted from each other, the magnitude of the two values is equal to the frequency [3].

[1] Thomson, William T. *Theory of Vibration with Applications*. Third ed. Englewood Cliffs, N.J.: Prentice Hall, 1988.

[2] Kassegne, S. “ME495 Lab – Laser Vibrometry - Expt Number 8." Mechanical

Engineering Department. San Diego State University. Fall 2011.

[3] Nave, Rod. *Hyperphysics*. N.p., Aug. 2000. Web. 22 Sept. 2011. <http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html>.