1. Equations in this section are referenced from *Theory of Vibration with Applications* by William T. Thomson [1]. We were asked to calculate the phase angle, φ, for the first experiment. In order to do this we must recognize the system is in harmonic motion due to the output of the sine frequency by the speakers. This can be represented by the equation,

$$m\ddot{x}+c\dot{x}+kx= F\_{o}sin⁡(ωt)$$

The recorded displacements from the vibrometer were 1.293 $mm$, 1.314 $\frac{mm}{s}$, and 1.284 $\frac{mm}{s^{2}}$ for the displacement, velocity, and acceleration respectively. The constants provided by Dr. Kassegne came from the lab manual for experiment 8 entitled “Laser Vibrometry”[2] are as follows:

 $k=63 \frac{g}{mm}\*g$ *g* = 9.81 $\frac{m}{s^{2}} $ *m* = 7 $g$ *F(t) =* $1sin⁡(2πft)$

Given these constants and the data recorded from the lab we were required to calculate the phase angle, φ which is defined as,

$$ϕ=tan^{-1}\frac{2ζ\frac{ω}{ω\_{n}}}{1-\left(\frac{ω}{ω\_{n}}\right)^{2}}$$

In order to calculate the phase angle we first need to calculate ω, $ω\_{n}$ and ζ. The equation for the frequency ratio, which is the frequency divided by the natural frequency is

$$\frac{ω}{ω\_{n}}= \frac{2πf}{\sqrt{\frac{k}{m}}}= \frac{2π1 kHz}{\sqrt{\frac{63\frac{kg}{m}\*9.81\frac{m}{s^{2}}}{7E^{-3}kg}}}= \frac{6.283 \frac{rad}{s}}{297.136\frac{rad}{s}}=0.02 $$

Knowing this the only variable left to solve for the is damping ratio, ζ. We can calculate the damping ratio via the relationship between the damping ratio ( ζ ) and the damping coefficient ( *c* ) in the equation,

$$ζ= \frac{c}{2\sqrt{km}}$$

We also have an unknown in this equation being the damping coefficient, c. To solve for c we must go back and solve the equation for harmonic motion of a system. By plugging in the values recorded for the magnitudes or the displacement, velocity, and acceleration, we can solve for the value of the damping coefficient.

$$m\ddot{x}+c\dot{x}+kx= F\_{o}sin⁡(ωt)$$

$$\left[\left(7E^{-3} kg\right)(1.284E^{-3} \frac{m}{s^{2}}\right]+ c\left[1.319E^{-3}\frac{m}{s}\right]+ \left[\left(618.03 \frac{N}{m}\right)\left(1.293 E^{-3} m\right)\right]= 1sin⁡(2π\left(1 kHz\right)\left(0.03823 s\right))$$

$$c=80 \frac{N\*s}{m}$$

Plugging *c* into the damping ratio equation,

$$ζ= \frac{80 \frac{N\*s}{m}}{2\sqrt{\left(618.03 \frac{N}{m}\right)\left(7E^{-3}m\right)}}=19.23$$

Now that we have all the unknowns needed to solve for the phase angle, we plug our values into the phase angle equation.

$$ϕ=tan^{-1}\frac{2\left(19.23\right)(0.02)}{1-\left(0.02\right)^{2}}=37.58°$$

2. Knowing that the system is critically damped (ζ = 1), we can solve for the damping coefficient, c, via the equation for the damping ratio.

$$ζ= \frac{c}{c\_{cr}}=\frac{c}{2\sqrt{km}}$$

Solving for the damping coefficient, c

$$c=2ζ\sqrt{km}$$

$$c=2\*1\sqrt{618.03\frac{N}{m}\*7E^{-3}m}$$

c = 4.15991 $\frac{N\*s}{m}$

3. The two songs chosen to analyze were 1812 Overture as the classical piece and song by Daft Punk for the electronic piece. The frequency of the two pieces differed such that the range of the classical piece was played at a higher frequency throughout and the electronic was played at a much lower frequency. The classical piece was played with high frequency instruments like violins and cellos as opposed to the lower frequency instruments such as the bass and drums used in electronic music.

4. The beat of music is related to frequency because when two different interfering sound waves approach the ear, they interfere with each other which produce noise. The two frequencies are constructive and destructive interferences, which when subtracted from each other the magnitude of the two values are equal to the frequency [3].

[1] Thomson, William T. *Theory of Vibration with Applications*. Third ed. Englewood Cliffs, N.J.: Prentice Hall, 1988.

[2] Kassegne, S. “ME495 Lab – Laser Vibrometry - Expt Number 8." Mechanical

Engineering Department. San Diego State University. Fall 2011.

[3] Nave, Rod. *Hyperphysics*. N.p., Aug. 2000. Web. 22 Sept. 2011. <http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html>.