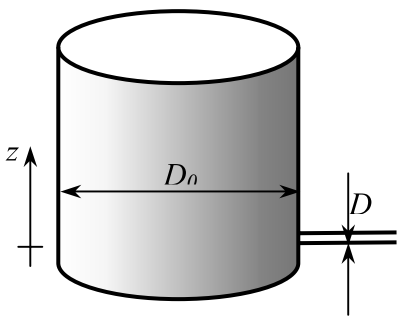
Review Problems

5-89 A water tank open to the atmosphere is initially filled with water. The tank discharges to the atmosphere through a long pipe connected to a valve. The initial discharge velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is incompressible. **2** The draining pipe is horizontal. **3** The tank is considered to be empty when the water level drops to the center of the valve.

Analysis (*a*) Substituting the known quantities, the discharge velocity can be expressed as

$$V=\sqrt{\frac{2gz}{1.5+fL/D}}=\sqrt{\frac{2gz}{1.5+0.015(100\text{ m})/(0.10\text{ m})}}=\sqrt{0.1212gz}$$



Then the initial discharge velocity becomes

$$V_1=\sqrt{0.1212gz_1}=\sqrt{0.1212(9.81\text{ m/s}^2)(2\text{ m})}=\mathbf{1.54\ m/s}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{\boldsymbol{V}}=A_{\text{pipe}}V_2=\frac{\pi D^2}{4}\sqrt{0.1212gz}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$d\boldsymbol{V}=\dot{\boldsymbol{V}}dt=\frac{\pi D^2}{4}\sqrt{0.1212gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$d\boldsymbol{V}=A_{\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{0.1212gz}\,dt=-\frac{\pi D_0^2}{4}dz \;\rightarrow\; dt=-\frac{D_0^2}{D^2}\frac{dz}{\sqrt{0.1212gz}}=-\frac{D_0^2}{D^2\sqrt{0.1212g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2\sqrt{0.1212g}}\int_{z=z_1}^{0}z^{-1/2}dz \;\rightarrow\; t_f=-\frac{D_0^2}{D^2\sqrt{0.1212g}}\left|\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2\sqrt{0.1212g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f=\frac{2D_0^2}{D^2}\sqrt{\frac{z_1}{0.1212g}}=\frac{2(10\text{ m})^2}{(0.1\text{ m})^2}\sqrt{\frac{2\text{ m}}{0.1212(9.81\text{ m/s}^2)}}=25{,}940\text{ s}=\mathbf{7.21\ h}$$

Discussion The draining time can be shortened considerably by installing a pump in the pipe.

5-90 The rate of accumulation of water in a pool and the rate of discharge are given. The rate supply of water to the pool is to be determined.

Assumptions **1** Water is supplied and discharged steadily. **2** The rate of evaporation of water is negligible. **3** No water is supplied or removed through other means.

Analysis The conservation of mass principle applied to the pool requires that the rate of increase in the amount of water in the pool be equal to the difference between the rate of supply of water and the rate of discharge. That is,

$$\frac{dm_{\text{pool}}}{dt} = \dot{m}_i - \dot{m}_e \quad \rightarrow \quad \dot{m}_i = \frac{dm_{\text{pool}}}{dt} + \dot{m}_e \quad \rightarrow \quad \dot{\boldsymbol{V}}_i = \frac{d\boldsymbol{V}_{\text{pool}}}{dt} + \dot{\boldsymbol{V}}_e$$

since the density of water is constant and thus the conservation of mass is equivalent to conservation of volume. The rate of discharge of water is

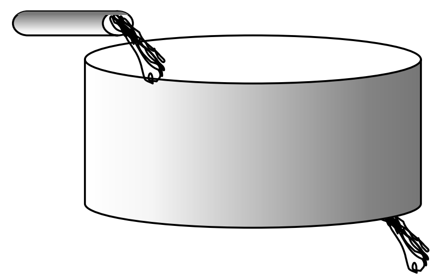
$$\dot{\boldsymbol{V}}_e = A_e V_{\text{e}} = (\pi D^2/4)V_{\text{e}} = [\pi(0.05\text{ m})^2/4](5\text{ m/s}) = 0.00982\text{ m}^3\text{/s}$$

The rate of accumulation of water in the pool is equal to the cross-section of the pool times the rate at which the water level rises,

$$\frac{d\boldsymbol{V}_{\text{pool}}}{dt} = A_{\text{cross-section}} V_{\text{level}} = (3\text{ m} \times 4\text{ m})(0.015\text{ m/min}) = 0.18\text{ m}^3\text{/min} = 0.00300\text{ m}^3\text{/s}$$

Substituting, the rate at which water is supplied to the pool is determined to be

$$\dot{\boldsymbol{V}}_i = \frac{d\boldsymbol{V}_{\text{pool}}}{dt} + \dot{\boldsymbol{V}}_e = 0.003 + 0.00982 = \mathbf{0.01282\ m^3/s}$$

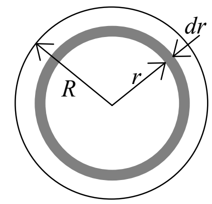


Therefore, water is supplied at a rate of 0.01282 m^3/s = 12.82 L/s.

5-91 A fluid is flowing in a circular pipe. A relation is to be obtained for the average fluid velocity in therms of $V(r)$, R, and r.

Analysis Choosing a circular ring of area $dA = 2\pi rdr$ as our differential area, the mass flow rate through a cross-sectional area can be expressed as

$$\dot{m} = \int_A \rho V(r) dA = \int_0^R \rho V(r) 2\pi r\, dr$$



Setting this equal to and solving for V_{avg},

$$V_{avg} = \frac{2}{R^2} \int_0^R V(r) r\, dr$$

5-92 Air is accelerated in a nozzle. The density of air at the nozzle exit is to be determined.

Assumptions Flow through the nozzle is steady.

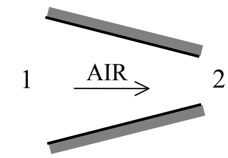
Properties The density of air is given to be 4.18 kg/m^3 at the inlet.

Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then,

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho_2 = \frac{A_1}{A_2}\frac{V_1}{V_2}\rho_1 = 2\frac{120 \text{ m/s}}{380 \text{ m/s}}(4.18 \text{ kg/m}^3) = \mathbf{2.64 \text{ kg/m}^3}$$



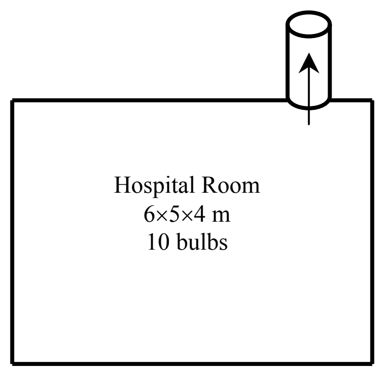
Discussion Note that the density of air decreases considerably despite a decrease in the cross-sectional area of the nozzle.

5-93 The air in a hospital room is to be replaced every 20 minutes. The minimum diameter of the duct is to be determined if the air velocity is not to exceed a certain value.

Assumptions **1** The volume occupied by the furniture etc in the room is negligible. **2** The incoming conditioned air does not mix with the air in the room.

Analysis The volume of the room is

$$\mathcal{V} = (6 \text{ m})(5 \text{ m})(4 \text{ m}) = 120 \text{ m}^3$$



To empty this air in 20 min, the volume flow rate must be

$$\dot{\mathcal{V}} = \frac{\mathcal{V}}{\Delta t} = \frac{120 \text{ m}^3}{20 \times 60 \text{ s}} = 0.10 \text{ m}^3/\text{s}$$

If the mean velocity is 5 m/s, the diameter of the duct is

$$\dot{\mathcal{V}} = AV = \frac{\pi D^2}{4}V \quad \rightarrow \quad D = \sqrt{\frac{4\dot{\mathcal{V}}}{\pi V}} = \sqrt{\frac{4(0.10 \text{ m}^3/\text{s})}{\pi(5 \text{ m/s})}} = \mathbf{0.16 \text{ m}}$$

Therefore, the diameter of the duct must be at least 0.16 m to ensure that the air in the room is exchanged completely within 20 min while the mean velocity does not exceed 5 m/s.

Discussion This problem shows that engineering systems are sized to satisfy certain constraints imposed by certain considerations.

5-94 Water discharges from the orifice at the bottom of a pressurized tank. The time it will take for half of the water in the tank to be discharged and the water level after 10 s are to be determined.

Assumptions **1** The flow is incompressible, and the frictional effects are negligible. **2** The tank air pressure above the water level is maintained constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the positive direction of z to be upwards with reference level at the orifice ($z_2 = 0$). Fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{\text{atm}}$) and the velocity at the free surface is very low ($V_1 \cong 0$). Then,

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\to\quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+\frac{V_2^2}{2g} \quad\to\quad V_2=\sqrt{2gz_1+2P_{1,\text{gage}}/\rho}$$

or, $V_2=\sqrt{2gz+2P_{1,\text{gage}}/\rho}$ where z is the water height in the tank at any time t. Water surface moves down as the tank drains, and the value of z changes from H initially to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V}=A_{\text{orifice}}V_2=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\text{gage}}/\rho}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\text{gage}}/\rho}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

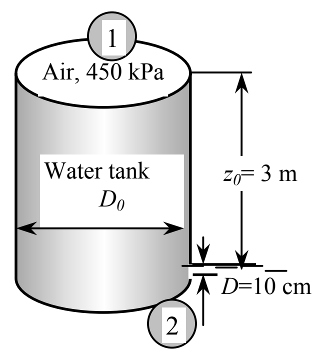
$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\text{gage}}/\rho}\,dt=-\frac{\pi D_0^2}{4}dz \quad\to\quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz+2P_{1,\text{gage}}/\rho}}\,dz$$

The last relation can be integrated since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_0$ to $t = t$ when $z = z$ gives

$$\sqrt{\frac{2z_0}{g}+\frac{2P_{1,gage}}{\rho g^2}}-\sqrt{\frac{2z}{g}+\frac{2P_{1,gage}}{\rho g^2}}=\frac{D_0^2}{D^2}t$$



where $$\frac{2P_{1,\text{gage}}}{\rho g^2}=\frac{2(450-100)\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)^2}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)=7.274\text{ s}^2$$

The time for half of the water in the tank to be discharged ($z = z_0/2$) is

$$\sqrt{\frac{2(3\text{ m})}{9.81\text{ m/s}^2}+7.274\text{ s}^2}-\sqrt{\frac{2(1.5\text{ m})}{9.81\text{ m/s}^2}+7.274\text{ s}^2}=\frac{(0.1\text{ m})^2}{(2\text{ m})^2}t \quad\to\quad t=\mathbf{22.0\ s}$$

(*b*) Water level after 10s is $$\sqrt{\frac{2(3\text{ m})}{9.81\text{ m/s}^2}+7.274\text{ s}^2}-\sqrt{\frac{2z}{9.81\text{ m/s}^2}+7.274\text{ s}^2}=\frac{(0.1\text{ m})^2}{(2\text{ m})^2}(10\text{ s})\to z=\mathbf{2.31\ m}$$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

5-95 Air flows through a pipe that consists of two sections at a specified rate. The differential height of a water manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, incompressible, and irrotational with negligible friction (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

Properties The density of air is given to be $\rho_{air} = 1.20$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3.

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

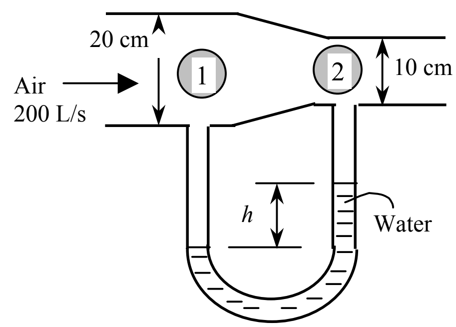
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\frac{\rho_{air}(V_2^2-V_1^2)}{2} \qquad (1)$$

We let the differential height of the water manometer be h. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1-P_2=\rho_w gh \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

$$\frac{\rho_{air}(V_2^2-V_1^2)}{2}=\rho_w gh \quad \rightarrow \quad h=\frac{\rho_{air}(V_2^2-V_1^2)}{2g\rho_w}=\frac{V_2^2-V_1^2}{2g\rho_w/\rho_{air}}$$



Calculating the velocities and substituting,

$$V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.2\text{ m}^3/\text{s}}{\pi(0.2\text{ m})^2/4}=6.37\text{ m/s}$$

$$V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.2\text{ m}^3/\text{s}}{\pi(0.1\text{ m})^2/4}=25.5\text{ m/s}$$

$$h=\frac{(25.5\text{ m/s})^2-(6.37\text{ m/s})^2}{2(9.81\text{ m/s}^2)(1000/1.20)}=0.037\text{ m}=\mathbf{3.7\text{ cm}}$$

Therefore, the differential height of the water column will be 3.7 cm.

Discussion Note that the differential height of the manometer is inversely proportional to the density of the manometer fluid. Therefore, heavy fluids such as mercury are used when measuring large pressure differences.

5-96 Air flows through a horizontal duct of variable cross-section. For a given differential height of a water manometer placed between the two pipe sections, the downstream velocity of air is to be determined, and an error analysis is to be conducted. √

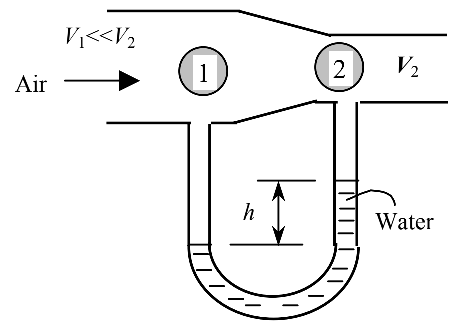
Assumptions **1**The flow through the duct is steady, incompressible, and irrotational with negligible friction (so that the Bernoulli equation is applicable). **2** The losses in this section of the duct are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K. We take the density of water to be ρ_w = 1000 kg/m^3.

Analysis We take points 1 and 2 along the centerline of the duct over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $V_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2(P_1-P_2)}{\rho_{air}}} \qquad (1)$$

where $P_1 - P_2 = \rho_w gh$



and $$\rho_{air}=\frac{P}{RT}=\frac{100\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})}=1.17\text{ kg/m}^3$$

Substituting into (1), the downstream velocity of air V_2 is determined to be

$$V_2=\sqrt{\frac{2\rho_w gh}{\rho_{air}}}=\sqrt{\frac{2(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.08\text{ m})}{1.17\text{ kg/m}^3}}=\mathbf{36.6\ m/s} \qquad (2)$$

Therefore, the velocity of air increases from a low level in the first section to 36.6 m/s in the second section.

Error Analysis We observe from Eq. (2) that the velocity is proportional to the square root of the differential height of the manometer fluid. That is, $V_2 = k\sqrt{h}$.

Taking the differential: $dV_2 = \frac{1}{2}k\frac{dh}{\sqrt{h}}$

Dividing by V_2: $$\frac{dV_2}{V_2}=\frac{1}{2}k\frac{dh}{\sqrt{h}}\frac{1}{k\sqrt{h}} \quad\rightarrow\quad \frac{dV_2}{V_2}=\frac{dh}{2h}=\frac{\pm 2\text{ mm}}{2\times 80\text{ mm}}=\pm\mathbf{0.0}13$$

Therefore, the uncertainty in the velocity corresponding to an uncertainty of 2 mm in the differential height of water is 1.3%, which corresponds to 0.013×(36.6 m/s) = 0.5m/s. Then the discharge velocity can be expressed as

$$V_2 = 36.6 \pm 0.5\text{ m/s}$$

5-97 A tap is opened on the wall of a very large tank that contains air. The maximum flow rate of air through the tap is to be determined, and the effect of a larger diameter lead section is to be assessed. √

Assumptions Flow through the tap is steady, incompressible, and irrotational with negligible friction (so that the flow rate is maximum, and the Bernoulli equation is applicable).

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K.

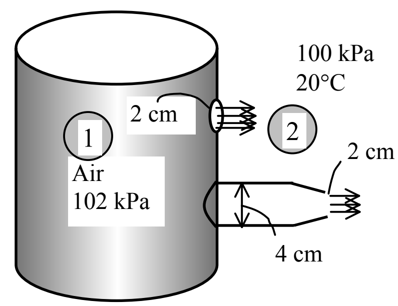
Analysis The density of air in the tank is

$$\rho_{air} = \frac{P}{RT} = \frac{102\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})} = 1.21\text{ kg/m}^3$$

We take point 1 in the tank, and point 2 at the exit of the tap along the same horizontal line. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $V_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}}$$

Substituting, the discharge velocity and the flow rate becomes



$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}} = \sqrt{\frac{2(102-100)\text{ kN/m}^2}{1.21\text{ kg/m}^3}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)} = 57.5\text{ m/s}$$

$$\dot{V} = AV_2 = \frac{\pi D_2^2}{4} V_2 = \frac{\pi(0.02\text{ m})^2}{4}(57.5\text{ m/s}) = \mathbf{0.0181\ m^3/s}$$

This is the *maximum* flow rate since it is determined by assuming frictionless flow. The actual flow rate will be less.

Adding a 2-m long larger diameter lead section will have **no effect** on the flow rate since the flow is frictionless (by using the Bernoulli equation, it can be shown that the velocity in this section increases, but the pressure decreases, and there is a smaller pressure difference to drive the flow through the tab, with zero net effect on the discharge rate).

Discussion If the pressure in the tank were 300 kPa, the flow is no longer incompressible, and thus the problem in that case should be analyzed using compressible flow theory.

5-98 Water is flowing through a venturi meter with known diameters and measured pressures. The flow rate of water is to be determined for the case of frictionless flow.

Assumptions **1** The flow through the venturi is steady, incompressible, and irrotational with negligible friction (so that the Bernoulli equation is applicable). **2** The flow is horizontal so that elevation along the centerline is constant. **3** The pressure is uniform at a given cross-section of the venturi meter (or the elevation effects on pressure measurement are negligible).

Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

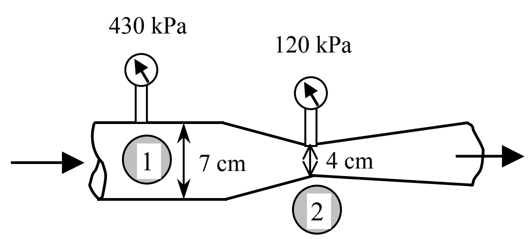
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho\frac{V_2^2-V_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad\rightarrow\quad A_1V_1=A_2V_2=\dot{V} \quad\rightarrow\quad V_1=\frac{\dot{V}}{A_1} \quad \text{and} \quad V_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(0.04\text{ m})^2}{4}\sqrt{\frac{2(430-120)\text{ kN/m}^2}{(1000\text{ kg/m}^3)[1\text{-}(4/7)^4]}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)}=\mathbf{0.0331\ m^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_dA_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_d is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

5-99E A hose is connected to the bottom of a water tank open to the atmosphere. The hose is equipped with a pump and a nozzle at the end. The maximum height to which the water stream could rise is to be determined.

Assumptions **1** The flow is incompressible with negligible friction. **2** The friction between the water and air is negligible. **3** We take the head loss to be zero ($h_L = 0$) to determine the maximum rise of water jet.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the top of the water trajectory where $V_2 = 0$. We take the reference level at the bottom of the tank. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump, u}}=z_2$$

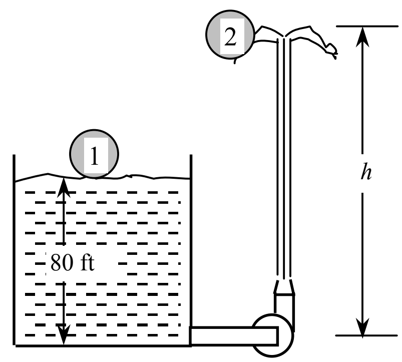
where the useful pump head is

$$h_{\text{pump, u}}=\frac{\Delta P_{\text{pump}}}{\rho g}=\frac{10\text{ psi}}{(62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=23.1\text{ ft}$$

Substituting, the maximum height rise of water jet from the ground level is determined to be

$$z_2=z_1+h_{\text{pump, u}}=80+23.1=\mathbf{103.1\ ft}$$

Discussion The actual rise of water will be less because of the frictional effects between the water and the hose walls and between the water jet and air.



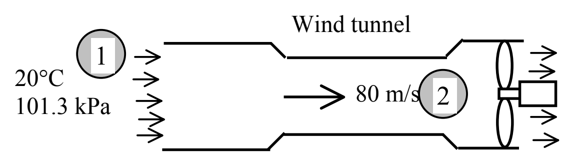
5-100 A wind tunnel draws atmospheric air by a large fan. For a given air velocity, the pressure in the tunnel is to be determined. √

Assumptions **1**The flow through the pipe is steady, incompressible, and irrotational with negligible friction (so that the Bernoulli equation is applicable). **2** Air is and ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K.

Analysis We take point 1 in atmospheric air before it enters the wind tunnel (and thus $P_1 = P_{atm}$ and $V_1 \cong 0$), and point 2 in the wind tunnel. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_2 = P_1-\frac{\rho V_2^2}{2} \qquad (1)$$



where

$$\rho=\frac{P}{RT}=\frac{101.3\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}=1.205\text{ kg/m}^3$$

Substituting, the pressure in the wind tunnel is determined to be

$$P_2=(101.3\text{ kPa})-(1.205\text{ kg/m}^3)\frac{(80\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right)=\mathbf{97.4\ kPa}$$

Discussion Note that the velocity in a wind tunnel increases at the expense of pressure. In reality, the pressure will be even lower because of losses.

5-101 Water flows through the enlargement section of a horizontal pipe at a specified rate. For a given head loss, the pressure change across the enlargement section is to be determined. √EES

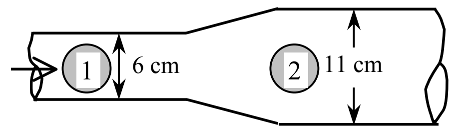
Assumptions **1** The flow through the pipe is steady and incompressible. **2** The pipe is horizontal. **3** The kinetic energy correction factors are given to be $\alpha_1 = \alpha_2 = \alpha = 1.05$.

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis We take points 1 and 2 at the inlet and exit of the enlargement section along the centerline of the pipe. Noting that $z_1 = z_2$, the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad P_2 - P_1 = \rho \frac{\alpha(V_1^2 - V_2^2)}{2} - \rho g h_L$$

where the inlet and exit velocities are



$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.025\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4} = 8.84\ \text{m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.025\ \text{m}^3/\text{s}}{\pi(0.11\ \text{m})^2/4} = 2.63\ \text{m/s}$$

Substituting, the change in static pressure across the enlargement section is determined to be

$$P_2 - P_1 = (1000\ \text{kg/m}^3)\left(\frac{1.05[(8.84\ \text{m/s})^2 - (2.63\ \text{m/s})^2]}{2} - (9.81\ \text{m/s}^2)(0.45\ \text{m})\right)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= \mathbf{33.0\ kPa}$$

Therefore, the water pressure increases by 33 kPa across the enlargement section.

Discussion Note that the pressure increases despite the head loss in the enlargement section. This is due to dynamic pressure being converted to static pressure. But the total pressure (static + dynamic) decreases by 0.45 m (or 4.41 kPa) as a result of frictional effects.

5-102 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe with a specified head loss. The initial discharge velocity is to be determined. √EES

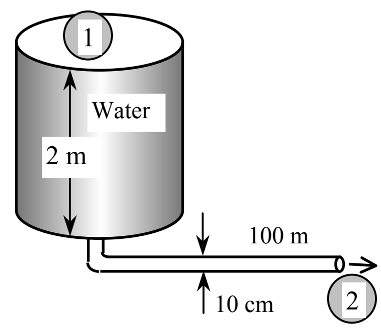
Assumptions **1** The flow is incompressible. **2** The draining pipe is horizontal. **3** There are no pumps or turbines in the system. **4** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and the head loss is given to be $h_L = 1.5$ m. Solving for V_2 and substituting, the discharge velocity of water is determined to be

$$V_2=\sqrt{2g(z_1-h_L)}=\sqrt{2(9.81\text{ m/s}^2)(2-1.5)\text{ m}}=\mathbf{3.13\ m/s}$$



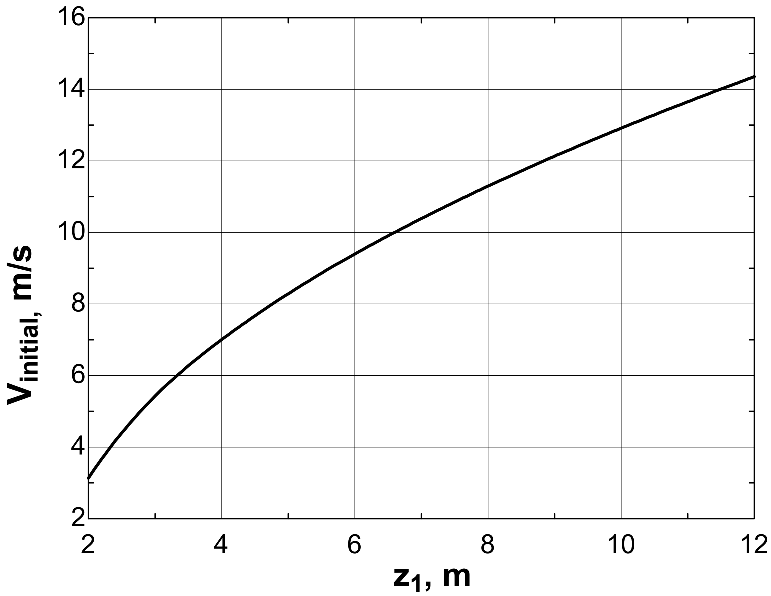
Discussion Note that this is the discharge velocity at the beginning, and the velocity will decrease as the water level in the tank drops. The head loss in that case will change since it depends on velocity.

5-103 Problem 5-102 is reconsidered. The effect of the tank height on the initial discharge velocity of water from the completely filled tank as the tank height varies from 2 to 20 m in increments of 2 m at constant heat loss is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"

h_L=1.5 "m"
D=0.10 "m"
V_initial=SQRT(2*g*(z1-h_L)) "m/s"
```

Tank height, $z1$, m	Head Loss, h_L, m	Initial velocity V_{initial}, m/s
2	1.5	3.13
3	1.5	5.42
4	1.5	7.00
5	1.5	8.29
6	1.5	9.40
7	1.5	10.39
8	1.5	11.29
9	1.5	12.13
10	1.5	12.91
11	1.5	13.65
12	1.5	14.35



5-104 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump with a specified head loss. The required pump head to assure a certain velocity is to be determined.

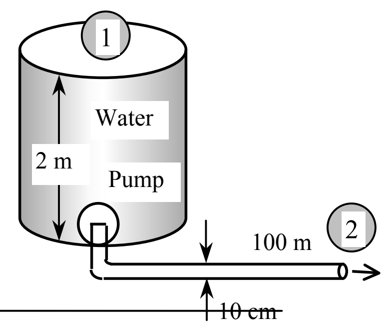
Assumptions **1** The flow is incompressible. **2** The draining pipe is horizontal. **3** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and the head loss is given to be $h_L = 1.5$ m. Solving for $h_{\text{pump, u}}$ and substituting, the required useful pump head is determined to be

$$h_{\text{pump, u}}=\sqrt{\frac{V_2^2}{2g}-z_1+h_L}=\sqrt{\frac{(6\text{ m/s})^2}{2(9.81\text{ m/s}^2)}-(2\text{ m})+(1.5\text{ m})}=\mathbf{1.15\ m}$$

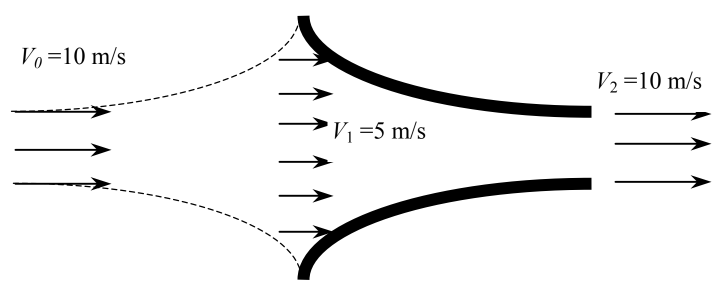


Discussion Note that this is the required useful pump head at the beginning, and it will need to be increased as the water level in the tank drops to make up for the lost elevation head to maintain the constant discharge velocity.

5-105 … 5-110 Design and Essay Problems

5-110 Using the mass and the Bernoulli equations, it can be shown that this is a bad idea – the velocity at the exit of nozzle is equal to the wind velocity. (The velocity at nozzle inlet is much lower). Sample calculation using EES using a wind velocity of 10 m/s:

```
V0=10 "m/s"
rho=1.2 "kg/m3"
g=9.81 "m/s2"
A1=2 "m2"
A2=1 "m2"
A1*V1=A2*V2
P1/rho+V1^2/2=V2^2/2
m=rho*A1*V1
m*V0^2/2=m*V2^2/2
```



Results: $V_1 = 5$ m/s, $V_2 = 10$ m/s, $m = 12$ kg/s