Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

3-53C The resultant hydrostatic force acting on a submerged surface is the resultant of the pressure forces acting on the surface. The point of application of this resultant force is called the center of pressure.

3-54C Yes, because the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface. The pressure at the centroid of the surface is $P_C = P_0 + \rho g h_C$ where h_C is the vertical distance of the centroid from the free surface of the liquid.

3-55C There will be no change on the hydrostatic force acting on the top surface of this submerged horizontal flat plate as a result of this rotation since the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface.

3-56C Dams are built much thicker at the bottom because the pressure force increases with depth, and the bottom part of dams are subjected to largest forces.

3-57C The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.

3-58C The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

3-59C The resultant hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are known, the tangent of the angle the resultant hydrostatic force makes with the horizontal is $\tan\alpha = F_V / F_H$.

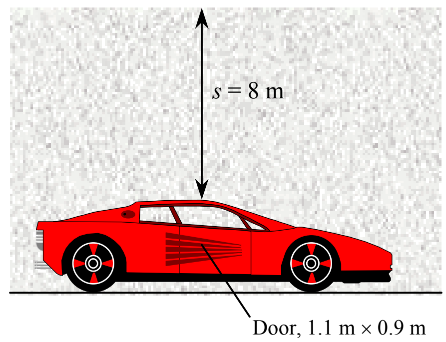
3-60 A car is submerged in water. The hydrostatic force on the door and its line of action are to be determined for the cases of the car containing atmospheric air and the car is filled with water.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The door can be approximated as a vertical rectangular plate. **3** The pressure in the car remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, we can ignore the atmospheric pressure in calculations since it acts on both sides of the door.

Properties We take the density of lake water to be 1000 kg/m^3 throughout.

Analysis (*a*) When the car is well-sealed and thus the pressure inside the car is the atmospheric pressure, the average pressure on the outer surface of the door is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(s+b/2)$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(8+1.1/2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 83.88\ \text{kN/m}^2$$



Then the resultant hydrostatic force on the door becomes

$$F_R = P_{ave}A = (83.88\ \text{kN/m}^2)(0.9\ \text{m}\times 1.1\ \text{m}) = \mathbf{83.0\ kN}$$

The pressure center is directly under the midpoint of the plate, and its distance from the surface of the lake is determined to be

$$y_P = s+\frac{b}{2}+\frac{b^2}{12(s+b/2)} = 8+\frac{1.1}{2}+\frac{1.1^2}{12(8+1.1/2)} = \mathbf{8.56\ m}$$

(*b*) When the car is filled with water, the net force normal to the surface of the door is **zero** since the pressure on both sides of the door will be the same.

Discussion Note that it is impossible for a person to open the door of the car when it is filled with atmospheric air. But it takes no effort to open the door when car is filled with water.

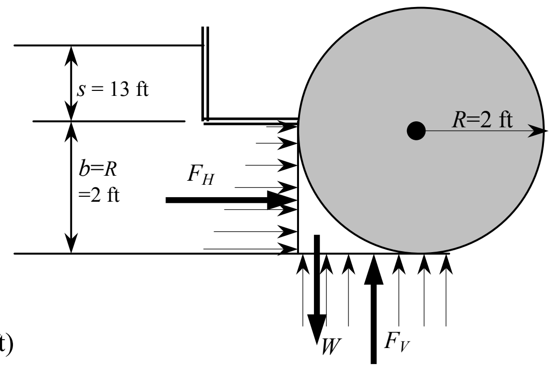
3-61E The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per ft length are to be determined. √ Feb05

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis (*a*) We consider the free body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block per ft length of the cylinder are:
Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(s + R/2)A$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(13 + 2/2 \text{ ft})(2 \text{ ft} \times 1 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 1747 \text{ lbf}$$



Vertical force on horizontal surface (*upward*):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})(2 \text{ ft} \times 1 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 1872 \text{ lbf}$$

Weight of fluid block per ft length *(downward):*

$$W = mg = \rho g \boldsymbol{V} = \rho g(R^2 - \pi R^2/4)(1 \text{ ft}) = \rho g R^2(1 - \pi/4)(1 \text{ ft})$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(2 \text{ ft})^2(1 - \pi/4)(1 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 54 \text{ lbf}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 1872 - 54 = 1818 \text{ lbf}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{1747^2 + 1818^2} = \mathbf{2521\ lbf}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{1818 \text{ lbf}}{1747 \text{ lbf}} = 1.041 \quad \rightarrow \quad \theta = 46.1°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 2521 lbf per ft length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.1° upwards from the horizontal.

(*b*) When the water level is 15-ft high, the gate opens and the reaction force at the bottom of the cylinder becomes zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about the point *A* where the hinge is and equating it to zero gives

$$F_R R \sin\theta - W_{cyl} R = 0 \quad \rightarrow \quad W_{cyl} = F_R \sin\theta = (2521 \text{ lbf})\sin 46.1° = \mathbf{1817\ lbf} \quad \text{(per ft)}$$

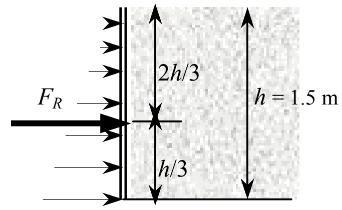
Discussion The weight of the cylinder per ft length is determined to be 1817 lbf, which corresponds to a mass of 1817 lbm, and to a density of 145 lbm/ft^3 for the material of the cylinder.

3-62 An above the ground swimming pool is filled with water. The hydrostatic force on each wall and the distance of the line of action from the ground are to be determined, and the effect of doubling the wall height on the hydrostatic force is to be assessed.

Assumptions The atmospheric pressure acts on both sides of the wall of the pool, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = \rho g(h/2)$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5/2\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 7357.5\ \text{N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = (7357.5\ \text{N/m}^2)(4\ \text{m} \times 1.5\ \text{m}) = 44{,}145\ \text{N} \cong \mathbf{44.1\ kN}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface and $h/3$ from the bottom of the pool. Therefore, the distance of the line of action from the ground is

$$y_P = \frac{h}{3} = \frac{1.5}{3} = \mathbf{0.50\ m} \quad \text{(from the bottom)}$$

If the height of the walls of the pool is doubled, the hydrostatic force **quadruples** since

$$F_R = \rho g h_C A = \rho g(h/2)(h \times w) = \rho g w h^2/2$$

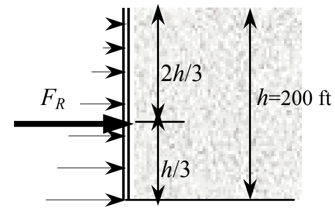
and thus the hydrostatic force is proportional to the square of the wall height, h^2.

3-63E A dam is filled to capacity. The total hydrostatic force on the dam, and the pressures at the top and the bottom are to be determined.

Assumptions The atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = \rho g h_C = \rho g(h/2)$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(200/2 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 6240 \text{ lbf/ft}^2$$

Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave}A = (6240 \text{ lbf/ft}^2)(200 \text{ ft} \times 1200 \text{ ft}) = \mathbf{1.50 \times 10^9 \text{ lbf}}$$

Resultant force per unit area is pressure, and its value at the top and the bottom of the dam becomes

$$P_{\text{top}} = \rho g h_{\text{top}} = \mathbf{0 \text{ lbf/ft}^2}$$

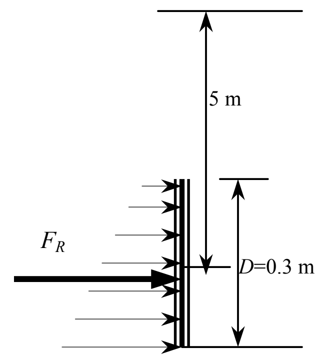
$$P_{\text{bottom}} = \rho g h_{\text{bottom}} = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(200 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{12{,}480 \text{ lbf/ft}^2}$$

3-64 A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined.

Assumptions The atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

Properties The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 50{,}276\ \text{N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = P_{ave}[\pi D^2/4] = (50{,}276\ \text{N/m}^2)[\pi(0.3\ \text{m})^2/4] = \mathbf{3554\ N}$$

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^4/4}{y_C \pi R^2} = y_C + \frac{R^2}{4y_C} = 5 + \frac{(0.15\ \text{m})^2}{4(5\ \text{m})} = \mathbf{5.0011\ m}$$

Discussion Note that for small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface.

3-65 The cross-section of a dam is a quarter-circle. The hydrostatic force on the dam and its line of action are to be determined.

Assumptions The atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m³ throughout.

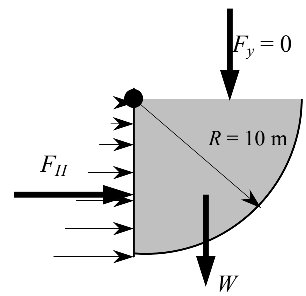
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the dam and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(10/2\ \text{m})(10\ \text{m} \times 100\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 4.905 \times 10^7\ \text{N}$$



Vertical force on horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per m length is

$$F_V = W = \rho g \boldsymbol{V} = \rho g[w \times \pi R^2/4]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(100\ \text{m})\pi(10\ \text{m})^2/4]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 7.705 \times 10^7\ \text{N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the dam become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(4.905 \times 10^7\ \text{N})^2 + (7.705 \times 10^7\ \text{N})^2} = \mathbf{9.134 \times 10^7\ N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{7.705 \times 10^7\ \text{N}}{4.905 \times 10^7\ \text{N}} = 1.571 \quad \rightarrow \quad \theta = 57.5°$$

Therefore, the line of action of the hydrostatic force passes through the center of the curvature of the dam, making 57.5° downwards from the horizontal.

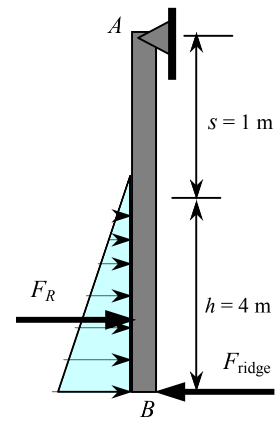
3-66 A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point *B*. The force exerted to the plate by the ridge is to be determined. √EES

Assumptions The atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(h/2)$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4/2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 19.62\ \text{kN/m}^2$$



Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = (19.62\ \text{kN/m}^2)(4\ \text{m}\times 5\ \text{m}) = \mathbf{392\ kN}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(4\ \text{m})}{3} = 2.667\ \text{m}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = F_{\text{ridge}}\overline{AB}$$

Solving for F_{ridge} and substituting, the reaction force is determined to be

$$F_{\text{ridge}} = \frac{s + y_P}{\overline{AB}}F_R = \frac{(1+2.667)\ \text{m}}{5\ \text{m}}(392\ \text{kN}) = \mathbf{288\ kN}$$

3-67 Problem 3-66 is reconsidered. The effect of water depth on the force exerted on the plate by the ridge as the water depth varies from 0 to 5 m in increments of 0.5 m is to be investigated.

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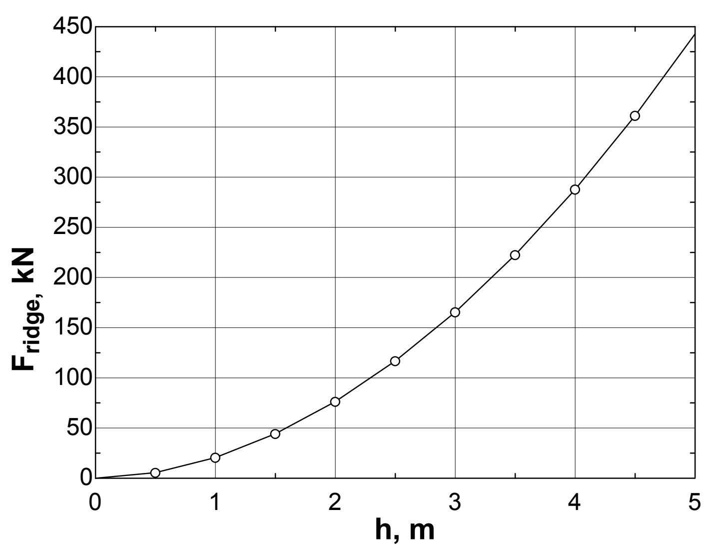
g=9.81 "m/s2"
rho=1000 "kg/m3"
s=1"m"

w=5 "m"
A=w*h
P_ave=rho*g*h/2000 "kPa"
F_R=P_ave*A "kN"
y_p=2*h/3
F_ridge=(s+y_p)*F_R/(s+h)
```

Dept h, m	P_{ave}, kPa	F_R kN	y_p m	F_{ridge} kN
0.0	0	0.0	0.00	0
0.5	2.453	6.1	0.33	5
1.0	4.905	24.5	0.67	20
1.5	7.358	55.2	1.00	44
2.0	9.81	98.1	1.33	76
2.5	12.26	153.3	1.67	117
3.0	14.72	220.7	2.00	166
3.5	17.17	300.4	2.33	223
4.0	19.62	392.4	2.67	288
4.5	22.07	496.6	3.00	361
5.0	24.53	613.1	3.33	443



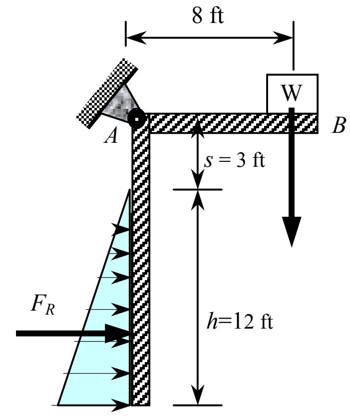
3-68E The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point *A*. The required weight *W* for the gate to open at a specified water height is to be determined. √EES

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = \rho g h_C = \rho g(h/2)$$
$$= (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(12/2\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= 374.4\ \text{lbf/ft}^2$$



Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave}A = (374.4\ \text{lbf/ft}^2)(12\ \text{ft}\times 5\ \text{ft}) = 22{,}464\ \text{lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(12\ \text{ft})}{3} = 8\ \text{ft}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = W\,\overline{AB}$$

Solving for *W* and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}}F_R = \frac{(3+8)\ \text{ft}}{8\ \text{ft}}(22{,}464\ \text{lbf}) = \mathbf{30{,}900\ \ lbf}$$

Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

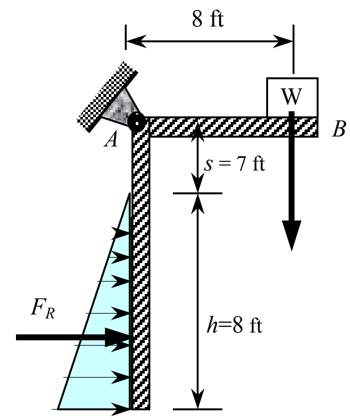
3-69E The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point A. The required weight W for the gate to open at a specified water height is to be determined. √EES

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = \rho g h_C = \rho g(h/2)$$
$$= (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(8/2\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= 249.6\ \text{lbf/ft}^2$$



Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave}A = (249.6\ \text{lbf/ft}^2)(8\ \text{ft} \times 5\ \text{ft}) = 9984\ \text{lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2 \times (8\ \text{ft})}{3} = 5.333\ \text{ft}$$

Taking the moment about point A and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = W\overline{AB}$$

Solving for W and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}} F_R = \frac{(7 + 5.333)\ \text{ft}}{8\ \text{ft}}(9984\ \text{lbf}) = \mathbf{15{,}390\ lbf}$$

Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

3-70 Two parts of a water trough of semi-circular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is full is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

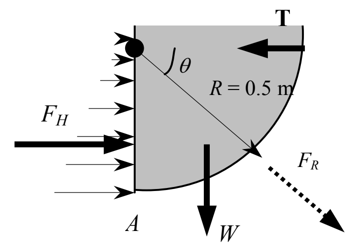
Analysis To expose the cable tension, we consider half of the trough whose cross-section is quarter-circle. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.5/2\ \text{m})(0.5\ \text{m} \times 3\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 3679\ \text{N}$$



The vertical force on the horizontal surface is zero, since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w \times \pi R^2/4]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})\pi(0.5\ \text{m})^2/4]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 5779\ \text{N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 3-m long section of the trough become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(3679\ \text{N})^2 + (5779\ \text{N})^2} = 6851\ \text{N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{5779\ \text{N}}{3679\ \text{N}} = 1.571 \quad \rightarrow \quad \theta = 57.5°$$

Therefore, the line of action passes through the center of the curvature of the trough, making 57.5° downwards from the horizontal. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R R\sin(90-57.5)° = \mathbf{T}R$$

Solving for **T** and substituting, the tension in the cable is determined to be

$$\mathbf{T} = F_R \sin(90-57.5)° = (6851\ \text{N})\sin(90-57.5)° = \mathbf{3681\ N}$$

Discussion This problem can also be solved without finding F_R by finding the lines of action of the horizontal hydrostatic force and the weight.

3-71 Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is filled to the rim is to be determined.

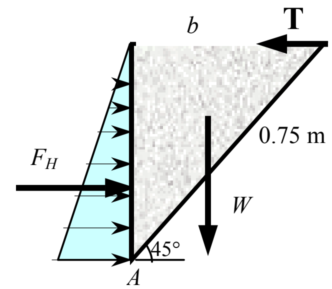
Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height *h* at the midsection of the trough and width of the free surface are

$$h = L\sin\theta = (0.75\text{ m})\sin 45° = 0.530\text{ m}$$

$$b = L\cos\theta = (0.75\text{ m})\cos 45° = 0.530\text{ m}$$



The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g (h/2)A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.530/2\text{ m})(0.530\text{ m}\times 6\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w\times bh/2]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(6\text{ m})(0.530\text{ m})(0.530\text{ m})/2]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point *A* where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H\frac{h}{3} = \mathbf{T}h$$

Solving for **T** and substituting, and noting that *h* = *b*, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_H + W}{3} = \frac{(8267 + 8267)\text{ N}}{3} = \mathbf{5511\text{ N}}$$

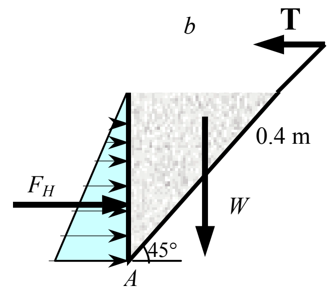
3-72 Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is filled to the rim is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m³ throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height is given to be h = 0.4 m at the midsection of the trough, which is equivalent to the width of the free surface b since tan 45° = b/h = 1.

The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:



Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.4/2\ \text{m})(0.4\ \text{m}\times 3\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 2354\ \text{N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w\times bh/2]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})(0.4\ \text{m})(0.4\ \text{m})/2]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 2354\ \text{N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H\frac{h}{3} = \mathbf{T}h$$

Solving for **T** and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_H + W}{3} = \frac{(2354 + 2354)\ \text{N}}{3} = \mathbf{1569\ N}$$

3-73 A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

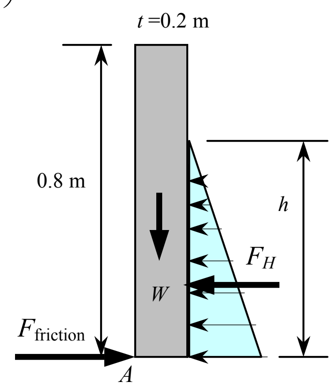
Assumptions The atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\text{ kg/m}^3)(9.81\text{ m/s}^2)[0.2\times 0.8\times 1\text{ m}^3]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 4238\text{ N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(4238\text{ N}) = 1271\text{ N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$

$$= (1800\text{ kg/m}^3)(9.81\text{ m/s}^2)(h/2)(1\times h)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\text{ N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad\to\quad 8829\,h^2 = 1271 \quad\to\quad h = \mathbf{0.38\ m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad\to\quad W_{\text{block}}(t/2) = F_H(h/3) \quad\to\quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 4238\times 0.2}{2\times 8829}\right)^{1/3} = \mathbf{0.52\ m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

3-74 A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

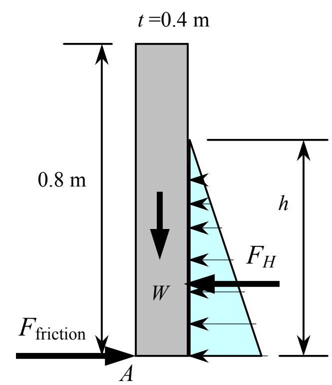
Assumptions The atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[0.4\times 0.8\times 1\ \text{m}^3]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 8476\ \text{N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(8476\ \text{N}) = 2543\ \text{N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$

$$= (1800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h/2)(1\times h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\ \text{N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad\rightarrow\quad 8829\,h^2 = 2543 \quad\rightarrow\quad h = \mathbf{0.54\ m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad\rightarrow\quad W_{\text{block}}(t/2) = F_H(h/3) \quad\rightarrow\quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 8476\times 0.3}{2\times 8829}\right)^{1/3} = \mathbf{0.76\ m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

3-75 A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at *B* where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

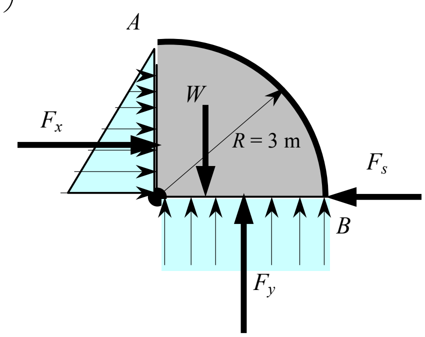
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3/2\ \text{m})(4\ \text{m}\times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 176.6\ \text{kN}$$



Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(4\ \text{m}\times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 353.2\ \text{kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g \boldsymbol{V} = \rho g[w\times \pi R^2/4]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(4\ \text{m})\pi(3\ \text{m})^2/4]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 277.4\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(176.6\ \text{kN})^2 + (75.8\ \text{kN})^2} = 192.2\ \text{kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{75.8\ \text{kN}}{176.6\ \text{kN}} = 0.429 \quad\rightarrow\quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point *A* where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad\rightarrow\quad F_R R\sin(90-\theta) - F_{\text{spring}}R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R\sin(90-\theta) = (192.2\ \text{kN})\sin(90° - 23.2°) = \mathbf{177\ kN}$$

3-76 A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at *B* where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate is to be determined.

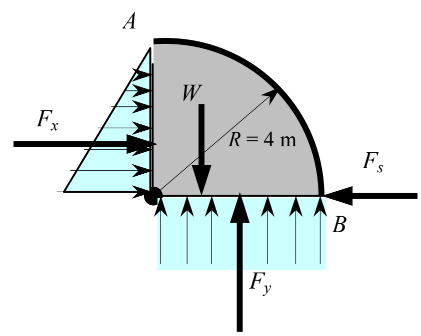
Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:
Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4/2\ \text{m})(4\ \text{m}\times 4\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 313.9\ \text{kN}$$

Vertical force on horizontal surface (upward):



$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4\ \text{m})(4\ \text{m}\times 4\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 627.8\ \text{kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g \boldsymbol{V} = \rho g[w\times \pi R^2/4]$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(4\ \text{m})\pi(4\ \text{m})^2/4]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 493.1\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 627.8 - 493.1 = 134.7\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(313.9\ \text{kN})^2 + (134.7\ \text{kN})^2} = 341.6\ \text{kN}$$
$$\tan\theta = \frac{F_V}{F_H} = \frac{134.7\text{kN}}{313.9\ \text{kN}} = 0.429 \quad\rightarrow\quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 341.6 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point *A* where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad\rightarrow\quad F_R R\sin(90-\theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90 - \theta) = (341.6\ \text{kN})\sin(90° - 23.2°) = \mathbf{314.0\ \ kN}$$

Buoyancy

3-77C The upward force a fluid exerts on an immersed body is called the *buoyant force*. The buoyant force is caused by the increase of pressure in a fluid with depth. The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$. The direction of the buoyant force is upwards, and its line of action passes through the centroid of the displaced volume.

3-78C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of depth. Therefore, the buoyant forces acting on two identical spherical balls submerged in water at different depths will be the same.

3-79C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the density of the body (ρ_f is the fluid density). Therefore, the buoyant forces acting on the 5-cm diameter aluminum and iron balls submerged in water will be the same.

3-80C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the shape of the body. Therefore, the buoyant forces acting on the cube and sphere made of copper submerged in water will be the same since they have the same volume.

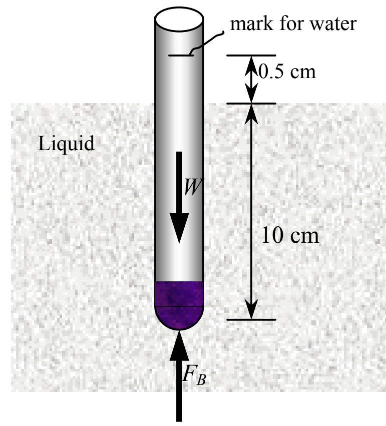
3-81C A *submerged* body whose center of gravity *G* is above the center of buoyancy *B,* which is the centroid of the displaced volume, is *unstable*. But a floating body may still be stable when *G* is above *B* since the centroid of the displaced volume shifts to the side to a point *B'* during a rotational disturbance while the center of gravity *G* of the body remains unchanged. If the point *B'* is sufficiently far, these two forces create a restoring moment, and return the body to the original position.

3-82 The density of a liquid is to be determined by a hydrometer by establishing division marks in water and in the liquid, and measuring the distance between these marks.

Properties We take the density of pure water to be 1000 kg/m^3.

Analysis A hydrometer floating in water is in static equilibrium, and the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer, $F_B = W$.

$$F_B = \rho g \boldsymbol{V}_{\text{sub}} = \rho g h A_{\text{c}}$$



where h is the height of the submerged portion of the hydrometer and A_c is the cross-sectional area which is constant.

In pure water: $W = \rho_w g h_w A_c$

In the liquid: $W = \rho_{\text{liquid}} g h_{\text{liquid}} A_c$

Setting the relations above equal to each other (since both equal the weight of the hydrometer) gives

$$\rho_w g h_w A_c = \rho_{\text{liquid}} g h_{\text{liquid}} A_c$$

Solving for the liquid density and substituting,

$$\rho_{\text{liquid}} = \frac{h_{\text{water}}}{h_{\text{liquid}}} \rho_{\text{water}} = \frac{10 \text{ cm}}{(10 - 0.5) \text{ cm}} (1000 \text{ kg/m}^3) = \mathbf{1053 \text{ kg/m}^3}$$

Discussion Note that for a given cylindrical hydrometer, the product of the fluid density and the height of the submerged portion of the hydrometer is constant in any fluid.

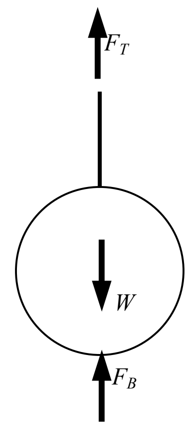
3-83E A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is immersed in water.

Assumptions **1** The buoyancy force in air is negligible. **2** The weight of the rope is negligible.

Properties The density of steel block is given to be 494 lbm/ft^3.

Analysis (*a*) The forces acting on the concrete block in air are its downward weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = 4\pi R^3 / 3 = 4\pi (1.5\text{ ft})^3 / 3 = 14.137\text{ ft}^3$$



$$F_T = W = \rho_{\text{concrete}} g V$$
$$= (494\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(14.137\text{ ft}^3)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{6984\ lbf}$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upwards. The force balance in this case gives

$$F_B = \rho_f g V = (62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(14.137\text{ ft}^3)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 882\text{ lbf}$$

$$F_{\text{T,water}} = W - F_B = 6984 - 882 = \mathbf{6102\ lbf}$$

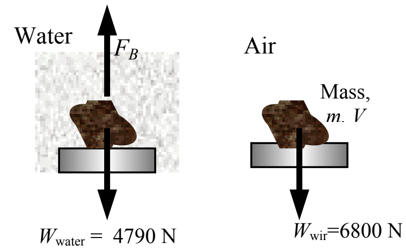
Discussion Note that the weight of the concrete block and thus the tension of the rope decreases by (6984 – 6102)/6984 = 12.6% in water.

3-84 An irregularly shaped body is weighed in air and then in water with a spring scale. The volume and the average density of the body are to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is completely submerged in water.

Analysis The mass of the body is



$$m = \frac{W_{\text{air}}}{g} = \frac{7200\text{ N}}{9.81\text{ m/s}^2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 733.9\text{ kg}$$

The difference between the weights in air and in water is due to the buoyancy force in water,

$$F_B = W_{\text{air}} - W_{\text{water}} = 7200 - 4790 = 2410\text{ N}$$

Noting that $F_B = \rho_{\text{water}} g V$, the volume of the body is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2410\text{ N}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} = \mathbf{0.2457\ m^3}$$

Then the density of the body becomes

$$\rho = \frac{m}{V} = \frac{733.9\text{ kg}}{0.2457\text{ m}^3} = \mathbf{2987\ kg/m^3}$$

Discussion The volume of the body can also be measured by observing the change in the volume of the container when the body is dropped in it (assuming the body is not porous).

3-85 The height of the portion of a cubic ice block that extends above the water surface is measured. The height of the ice block below the surface is to be determined.

Assumptions **1** The buoyancy force in air is negligible. **2** The top surface of the ice block is parallel to the surface of the sea.

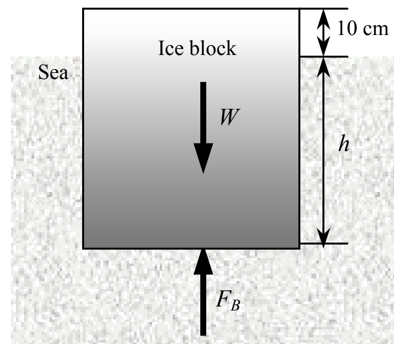
Properties The specific gravities of ice and seawater are given to be 0.92 and 1.025, respectively, and thus the corresponding densities are 920 kg/m^3 and 1025 kg/m^3.

Analysis The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore, in this case the average density of the body must be equal to the density of the fluid since

$$W = F_B$$

$$\rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}}$$



The cross-sectional of a cube is constant, and thus the "volume ratio" can be replaced by "height ratio". Then,

$$\frac{h_{\text{submerged}}}{h_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} \quad \rightarrow \quad \frac{h}{h+0.10} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \quad \rightarrow \quad \frac{h}{h+0.10} = \frac{0.92}{1.025}$$

where h is the height of the ice block below the surface. Solving for h gives

$$h = 0.876 \text{ m} = \mathbf{87.6\ cm}$$

Discussion Note that the 0.92/1.025 = 90% of the volume of an ice block remains under water. For symmetrical ice blocks this also represents the fraction of height that remains under water.

3-86 A man dives into a lake and tries to lift a large rock. The force that the man needs to apply to lift it from the bottom of the lake is to be determined.

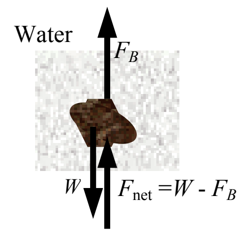
Assumptions **1** The rock is c completely submerged in water. **2** The buoyancy force in air is negligible.

Properties The density of granite rock is given to be 2700 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight and volume of the rock are

$$W = mg = (170\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 1668\text{ N}$$

$$V = \frac{m}{\rho} = \frac{170\text{ kg}}{2700\text{ kg/m}^3} = 0.06296\text{ m}^3$$



The buoyancy force acting on the rock is

$$F_B = \rho_{water}\, g V$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.06296\text{ m}^3)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 618\text{ N}$$

The weight of a body submerged in water is equal to the weigh of the body in air minus the buoyancy force,

$$W_{\text{in water}} = W_{\text{in air}} - F_B = 1079 - 618 = \mathbf{461\text{ N}}$$

Discussion This force corresponds to a mass of

$$m = \frac{W_{\text{in water}}}{g} = \frac{461\text{ N}}{9.81\text{ m/s}^2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 47.0\text{ kg}$$

Therefore, a person who can lift 47 kg on earth can lift this rock in water.

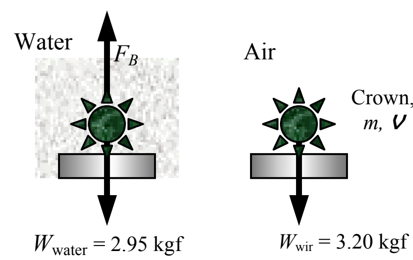
3-87 An irregularly shaped crown is weighed in air and then in water with a spring scale. It is to be determined if the crown is made of pure gold.

Assumptions **1** The buoyancy force in air is negligible. **2** The crown is completely submerged in water.

Properties We take the density of water to be 1000 kg/m^3. The density of gold is given to be 19300 kg/m^3.

Analysis The mass of the crown is

$$m = \frac{W_{\text{air}}}{g} = \frac{31.4\ \text{N}}{9.81\ \text{m/s}^2}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 3.20\ \text{kg}$$



The difference between the weights in air and in water is due to the buoyancy force in water, and thus

$$F_B = W_{\text{air}} - W_{\text{water}} = 31.4 - 28.9 = 2.50\ \text{N}$$

Noting that $F_B = \rho_{\text{water}} g V$, the volume of the crown is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2.50\ \text{N}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 2.548\times10^{-4}\ \text{m}^3$$

Then the density of the crown becomes

$$\rho = \frac{m}{V} = \frac{3.20\ \text{kg}}{2.548\times10^{-4}\ \text{m}^3} = 12{,}560\ \text{kg/m}^3$$

which is considerably less than the density of gold. Therefore, the crown is **NOT** made of pure gold.

Discussion This problem can also be solved without doing any under-water weighing as follows: We would weigh a bucket half-filled with water, and drop the crown into it. After marking the new water level, we would take the crown out, and add water to the bucket until the water level rises to the mark. We would weigh the bucket again. Dividing the weight difference by the density of water and *g* will give the volume of the crown. Knowing both the weight and the volume of the crown, the density can easily be determined.

3-88 The average density of a person is determined by weighing the person in air and then in water. A relation is to be obtained for the volume fraction of body fat in terms of densities.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is considered to consist of fat and muscle only. **3** The body is completely submerged in water, and the air volume in the lungs is negligible.

Analysis The difference between the weights of the person in air and in water is due to the buoyancy force in water. Therefore,

$$F_B = W_{\text{air}} - W_{\text{water}} \quad \rightarrow \quad \rho_{\text{water}} g V = W_{\text{air}} - W_{\text{water}}$$

Knowing the weights and the density of water, the relation above gives the volume of the person. Then the average density of the person can be determined from

$$\rho_{\text{ave}} = \frac{m}{V} = \frac{W_{\text{air}} / g}{V}$$

Under assumption #2, the total mass of a person is equal to the sum of the masses of the fat and muscle tissues, and the total volume of a person is equal to the sum of the volumes of the fat and muscle tissues. The volume fraction of body fat is the ratio of the fat volume to the total volume of the person. Therefore,

$$V = V_{\text{fat}} + V_{\text{muscle}} \quad \text{where} \quad V_{\text{fat}} = x_{\text{fat}} V \quad \text{and} \quad V_{\text{muscle}} = x_{\text{muscle}} V = (1 - x_{\text{fat}}) V$$

$$m = m_{\text{fat}} + m_{\text{muscle}}$$

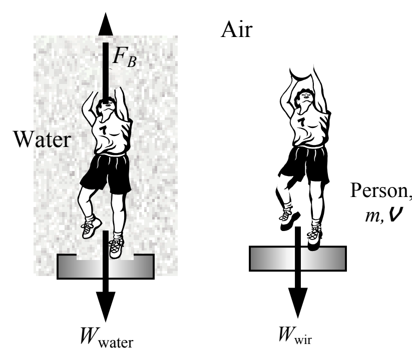
Noting that mass is density times volume, the last relation can be written as

$$\rho_{\text{ave}} V = \rho_{\text{fat}} V_{\text{fat}} + \rho_{\text{muscle}} V_{\text{muscle}}$$

$$\rho_{\text{ave}} V = \rho_{\text{fat}} x_{\text{fat}} V + \rho_{\text{muscle}} (1 - x_{\text{fat}}) V$$

Canceling the V and solving for x_{fat} gives the desired relation,

$$x_{\text{fat}} = \frac{\rho_{\text{muscle}} - \rho_{\text{ave}}}{\rho_{\text{muscle}} - \rho_{\text{fat}}}$$



Discussion Weighing a person in water in order to determine its volume is not practical. A more practical way is to use a large container, and measuring the change in volume when the person is completely submerged in it.

3-89 The volume of the hull of a boat is given. The amounts of load the boat can carry in a lake and in the sea are to be determined.

Assumptions **1** The dynamic effects of the waves are disregarded. **2** The buoyancy force in air is negligible.

Properties The density of sea water is given to be 1.03×1000 = 1030 kg/m^3. We take the density of water to be 1000 kg/m^3.

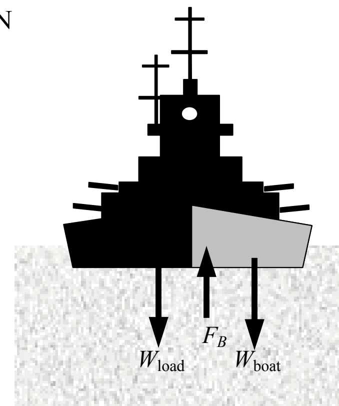
Analysis The weight of the unloaded boat is

$$W_{\text{boat}} = mg = (8560\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 84.0\text{ kN}$$

The buoyancy force becomes a maximum when the entire hull of the boat is submerged in water, and is determined to be

$$F_{B,\text{lake}} = \rho_{\text{lake}} g V = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(150\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1472\text{ kN}$$

$$F_{B,\text{sea}} = \rho_{\text{sea}} g V = (1030\text{ kg/m}^3)(9.81\text{ m/s}^2)(150\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1516\text{ kN}$$



The total weight of a floating boat (load + boat itself) is equal to the buoyancy force. Therefore, the weight of the maximum load is

$$W_{\text{load, lake}} = F_B\text{, lake} - W_{\text{boat}} = 1472 - 84 = 1388\text{ kN}$$

$$W_{\text{load, sea}} = F_{B,\text{sea}} - W_{\text{boat}} = 1516 - 84 = 1432\text{ kN}$$

The corresponding masses of load are

$$m_{\text{load,lake}} = \frac{W_{\text{load,lake}}}{g} = \frac{1388\text{ kN}}{9.81\text{ m/s}^2}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right) = 141{,}500\text{ kg}$$

$$m_{\text{load,sea}} = \frac{W_{\text{load,lsea}}}{g} = \frac{1432\text{ kN}}{9.81\text{ m/s}^2}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right) = 146.0\text{ kg}$$

Discussion Note that this boat can carry 4500 kg more load in the sea than it can in fresh water. The fully-loaded boats in sea water should expect to sink into water deeper when they enter fresh water such a river where the port may be.

Fluids in Rigid Body Motion

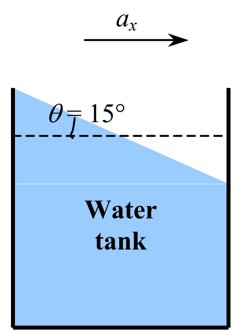
3-90C A moving body of fluid can be treated as a rigid body when there are no shear stresses (i.e., no motion between fluid layers relative to each other) in the fluid body.

3-91C A glass of water is considered. The water pressure at the bottom surface will be the same since the acceleration for all four cases is zero.

3-92C The pressure at the bottom surface is constant when the glass is stationary. For a glass moving on a horizontal plane with constant acceleration, water will collect at the back but the water depth will remain constant at the center. Therefore, the pressure at the midpoint will be the same for both glasses. But the bottom pressure will be low at the front relative to the stationary glass, and high at the back (again relative to the stationary glass). Note that the pressure in all cases is the hydrostatic pressure, which is directly proportional to the fluid height.

3-93C When a vertical cylindrical container partially filled with water is rotated about its axis and rigid body motion is established, the fluid level will drop at the center and rise towards the edges. Noting that hydrostatic pressure is proportional to fluid depth, the pressure at the mid point will drop and the pressure at the edges of the bottom surface will rise due to rotation.

3-94 A water tank is being towed by a truck on a level road, and the angle the free surface makes with the horizontal is measured. The acceleration of the truck is to be determined.



Assumptions **1** The road is horizontal so that acceleration has no vertical component ($a_z = 0$). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the x-axis to be the direction of motion, the z-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

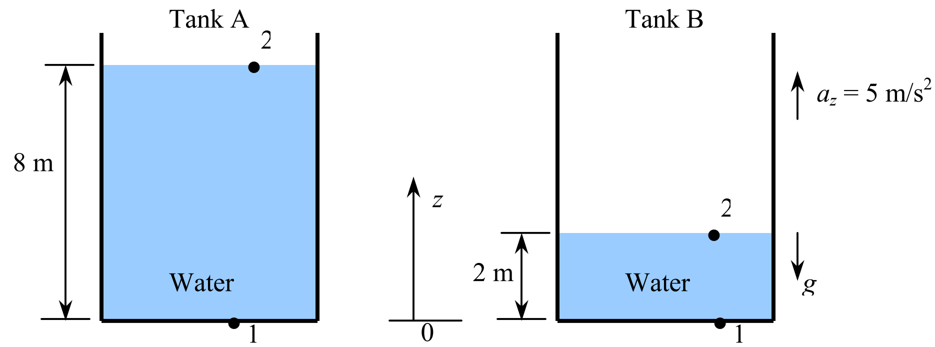
$$\tan\theta = \frac{a_x}{g + a_z}$$

Solving for a_x and substituting,

$$a_x = (g + a_z)\tan\theta = (9.81\,\text{m/s}^2 + 0)\tan 15° = \mathbf{2.63\ \ m/s^2}$$

Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-95 Two water tanks filled with water, one stationary and the other moving upwards at constant acceleration. The tank with the higher pressure at the bottom is to be determined.



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \text{or} \quad P_1 - P_2 = \rho(g + a_z)(z_2 - z_1)$$

since a_x = 0. Taking point 2 at the free surface and point 1 at the tank bottom, we have $P_2 = P_{atm}$ and $z_2 - z_1 = h$ and thus

$$P_{1,\text{ gage}} = P_{\text{bottom}} = \rho(g + a_z)h$$

Tank A: We have a_z = 0, and thus the pressure at the bottom is

$$P_{A,\text{ bottom}} = \rho g h_A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 78.5 \text{ kN/m}^2$$

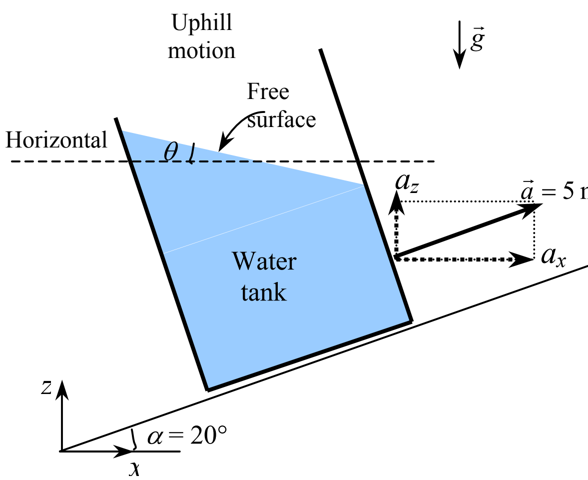
Tank B: We have $a_z = +5$ m/s^2, and thus the pressure at the bottom is

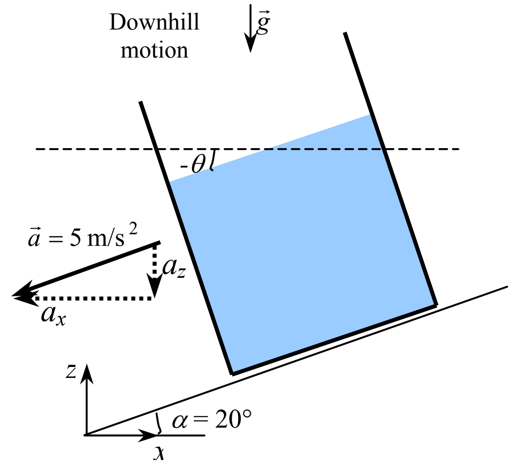
$$P_{B,\text{ bottom}} = \rho(g + a_z)h_B = (1000 \text{ kg/m}^3)(9.81 + 5 \text{ m/s}^2)(2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 29.6 \text{ kN/m}^2$$

Therefore, **tank A** has a higher pressure at the bottom.

Discussion We can also solve this problem quickly by examining the relation $P_{\text{bottom}} = \rho(g + a_z)h$. Acceleration for tank B is about 1.5 times that of Tank A (14.81 vs 9.81 m/s^2), but the fluid depth for tank A is 4 times that of tank B (8 m vs 2 m). Therefore, the tank with the larger acceleration-fluid height product (tank A in this case) will have a higher pressure at the bottom.

3-96 A water tank is being towed on an uphill road at constant acceleration. The angle the free surface of water makes with the horizontal is to be determined, and the solution is to be repeated for the downhill motion case.





Assumptions **1** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **2** The acceleration remains constant.

Analysis We take the x- and z-axes as shown in the figure. From geometrical considerations, the horizontal and vertical components of acceleration are

$$a_x = a\cos\alpha$$
$$a_z = a\sin\alpha$$

The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{a\cos\alpha}{g + a\sin\alpha} = \frac{(5\ \text{m/s}^2)\cos 20°}{9.81\ \text{m/s}^2 + (5\ \text{m/s}^2)\sin 20°} = 0.4078 \quad \rightarrow \quad \theta = \mathbf{22.2°}$$

When the direction of motion is reversed, both a_x and a_z are in negative x- and z-direction, respectively, and thus become negative quantities,

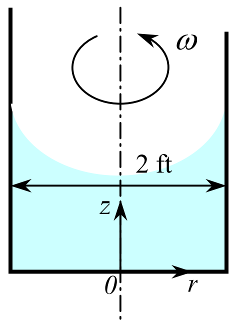
$$a_x = -a\cos\alpha$$
$$a_z = -a\sin\alpha$$

Then the tangent of the angle the free surface makes with the horizontal becomes

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{a\cos\alpha}{g + a\sin\alpha} = \frac{-(5\ \text{m/s}^2)\cos 20°}{9.81\ \text{m/s}^2 - (5\ \text{m/s}^2)\sin 20°} = -0.5801 \quad \rightarrow \quad \theta = \mathbf{-\ 30.1°}$$

Discussion Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

3-97E A vertical cylindrical tank open to the atmosphere is rotated about the centerline. The angular velocity at which the bottom of the tank will first be exposed, and the maximum water height at this moment are to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Water is an incompressible fluid.

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin ($r = 0$, $z = 0$), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 1 ft is the original height of the liquid before rotation. Just before dry spot appear at the center of bottom surface, the height of the liquid at the center equals zero, and thus $z_s(0) = 0$. Solving the equation above for ω and substituting,

$$\omega = \sqrt{\frac{4gh_0}{R^2}} = \sqrt{\frac{4(32.2 \text{ ft/s}^2)(1 \text{ ft})}{(1 \text{ ft})^2}} = \mathbf{11.35 \text{ rad/s}}$$

Noting that one complete revolution corresponds to 2π radians, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{11.35 \text{ rad/s}}{2\pi \text{ rad/rev}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{108 \text{ rpm}}$$

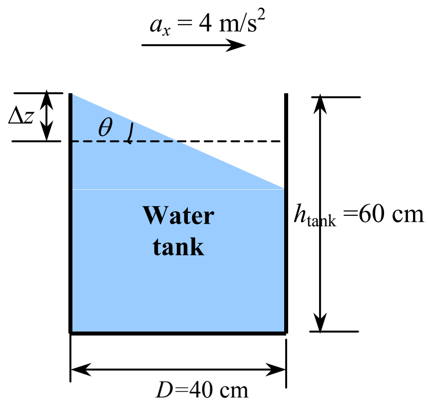
Therefore, the rotational speed of this container should be limited to 108 rpm to avoid any dry spots at the bottom surface of the tank.

The maximum vertical height of the liquid occurs a the edges of the tank ($r = R$ = 1 ft), and it is

$$z_s(R) = h_0 + \frac{\omega^2 R^2}{4g} = (1 \text{ ft}) + \frac{(11.35 \text{ rad/s})^2 (1 \text{ ft})^2}{4(32.2 \text{ ft/s}^2)} = \mathbf{2.00 \text{ ft}}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property.

3-98 A cylindrical tank is being transported on a level road at constant acceleration. The allowable water height to avoid spill of water during acceleration is to be determined



Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component ($a_z = 0$). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the x-axis to be the direction of motion, the z-axis to be the upward vertical direction, and the origin to be the midpoint of the tank bottom. The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{4}{9.81 + 0} = 0.4077 \quad \text{(and thus } \theta = 22.2°)$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the midplane is

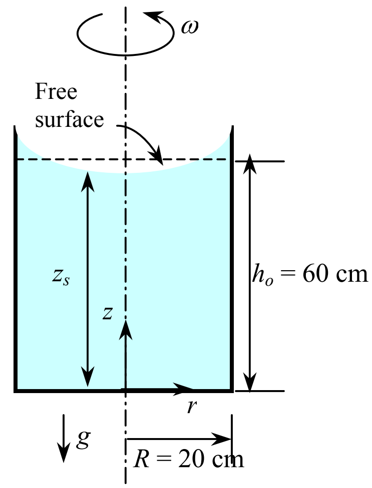
$$\Delta z_{\max} = (D/2)\tan\theta = [(0.40\text{ m})/2] \times 0.4077 = 0.082\text{ m} = 8.2\text{ cm}$$

Therefore, the maximum initial water height in the tank to avoid spilling is

$$h_{\max} = h_{\text{tank}} - \Delta z_{\max} = 60 - 8.2 = \mathbf{51.8\ cm}$$

Discussion Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

3-99 A vertical cylindrical container partially filled with a liquid is rotated at constant speed. The drop in the liquid level at the center of the cylinder is to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** The bottom surface of the container remains covered with liquid during rotation (no dry spots).

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 0.6 m is the original height of the liquid before rotation, and

$$\omega = 2\pi\dot{n} = 2\pi(120 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 12.57 \text{ rad/s}$$

Then the vertical height of the liquid at the center of the container where r = 0 becomes

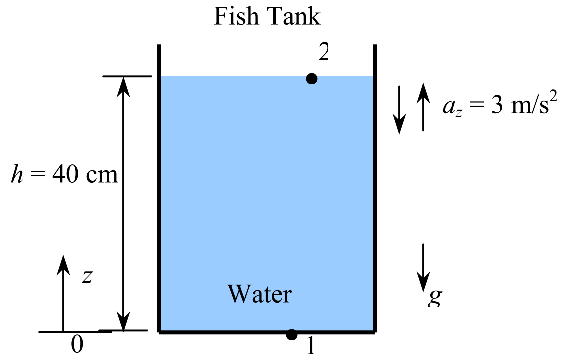
$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = (0.60 \text{ m}) - \frac{(12.57 \text{ rad/s})^2 (0.20 \text{ m})^2}{4(9.81 \text{ m/s}^2)} = 0.44 \text{ m}$$

Therefore, the drop in the liquid level at the center of the cylinder is

$$\Delta h_{\text{drop, center}} = h_0 - z_s(0) = 0.60 - 0.44 = \mathbf{0.16\ m}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. Also, our assumption of no dry spots is validated since $z_0(0)$ is positive.

3-100 The motion of a fish tank in the cabin of an elevator is considered. The pressure at the bottom of the tank when the elevator is stationary, moving up with a specified acceleration, and moving down with a specified acceleration is to be determined.



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid is given by

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \text{or} \quad P_1 - P_2 = \rho(g + a_z)(z_2 - z_1)$$

since a_x = 0. Taking point 2 at the free surface and point 1 at the tank bottom, we have $P_2 = P_{atm}$ and $z_2 - z_1 = h$ and thus

$$P_{1,\text{ gage}} = P_{\text{bottom}} = \rho(g + a_z)h$$

(***a***) **Tank stationary**: We have a_z = 0, and thus the gage pressure at the tank bottom is

$$P_{\text{bottom}} = \rho g h = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 3.92\text{ kN/m}^2 = \mathbf{3.92\ kPa}$$

(***b***) **Tank moving up**: We have a_z = +3 m/s^2, and thus the gage pressure at the tank bottom is

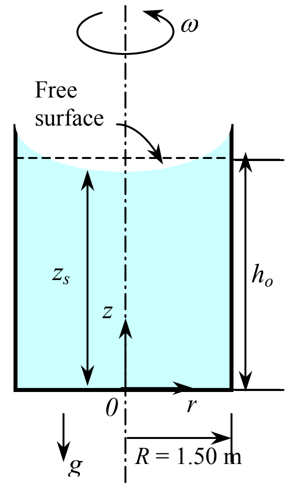
$$P_{\text{bottom}} = \rho(g + a_z)h_B = (1000\text{ kg/m}^3)(9.81 + 3\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 5.12\text{ kN/m}^2 = \mathbf{5.12\ kPa}$$

(***c***) **Tank moving down**: We have a_z = -3 m/s^2, and thus the gage pressure at the tank bottom is

$$P_{\text{bottom}} = \rho(g + a_z)h_B = (1000\text{ kg/m}^3)(9.81 - 3\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 2.72\text{ kN/m}^2 = \mathbf{2.72\ kPa}$$

Discussion Note that the pressure at the tank bottom while moving up in an elevator is almost twice that while moving down, and thus the tank is under much greater stress during upward acceleration.

3-101 vertical cylindrical milk tank is rotated at constant speed, and the pressure at the center of the bottom surface is measured. The pressure at the edge of the bottom surface is to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1030 kg/m^3.

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where R = 1.5 m is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(12\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 1.2566\text{ rad/s}$$

The fluid rise at the edge relative to the center of the tank is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 + \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(1.2566\text{ rad/s})^2(1.50\text{ m})^2}{2(9.81\text{ m/s}^2)} = 1.1811\text{ m}$$

The pressure difference corresponding to this fluid height difference is

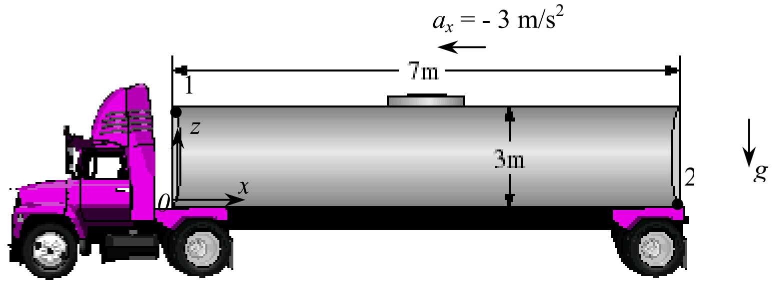
$$\Delta P_{\text{bottom}} = \rho g\Delta h = (1030\text{ kg/m}^3)(9.81\text{ m/s}^2)(1.1811\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1.83\text{ kN/m}^2 = 1.83\text{ kPa}$$

Then the pressure at the edge of the bottom surface becomes

$$P_{\text{bottom, edge}} = P_{\text{bottom, center}} + \Delta P_{\text{bottom}} = 130 + 1.83 = 131.83\text{ kPa} \cong \mathbf{131.8\ kPa}$$

Discussion Note that the pressure is 1.4% higher at the edge relative to the center of the tank, and there is a fluid level difference of 1.18 m between the edge and center of the tank, and these differences should be considered when designing rotating fluid tanks.

3-102 Milk is transported in a completely filled horizontal cylindrical tank accelerating at a specified rate. The maximum pressure difference in the tanker is to be determined. √EES



Assumptions **1** The acceleration remains constant. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1020 kg/m^3.

Analysis We take the x- and z- axes as shown. The horizontal acceleration is in the negative x direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus $a_z = 0$. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1)$$

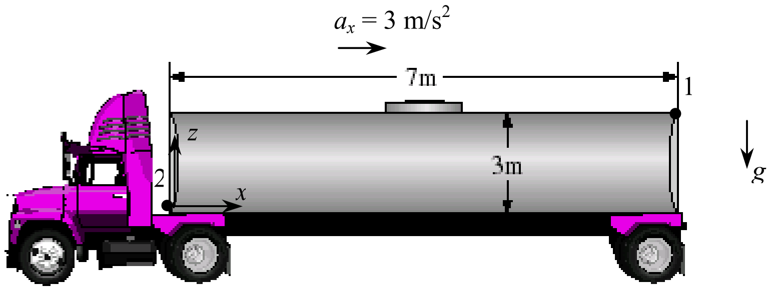
The first term is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

$$\begin{aligned}\Delta P_{\max} &= P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1) = -[a_x (x_2 - x_1) + g(z_2 - z_1)] \\ &= -(1020\ \text{kg/m}^3)\left[(-2.5\ \text{m/s}^2)(7\ \text{m}) + (9.81\ \text{m/s}^2)(-3\ \text{m})\right]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) \\ &= (17.9 + 30.0)\ \text{kN/m}^2 = \mathbf{47.9\ kPa}\end{aligned}$$

since $x_1 = 0$, $x_2 = 7$ m, $z_1 = 3$ m, and $z_2 = 0$.

Discussion Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).

3-103 Milk is transported in a completely filled horizontal cylindrical tank decelerating at a specified rate. The maximum pressure difference in the tanker is to be determined. √EES



Assumptions **1** The acceleration remains constant. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1020 kg/m^3.

Analysis We take the *x*- and *z*- axes as shown. The horizontal deceleration is in the *x* direction, and thus a_x is positive. Also, there is no acceleration in the vertical direction, and thus $a_z = 0$. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1)$$

The first term is due to deceleration in the horizontal direction and the resulting compression effect towards the front of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

$$\Delta P_{\max} = P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1) = -[a_x (x_2 - x_1) + g(z_2 - z_1)]$$

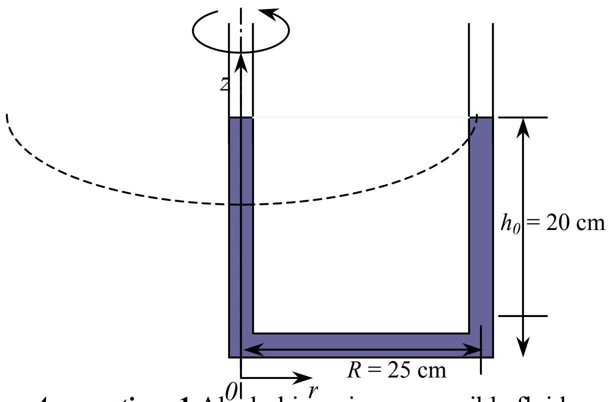
$$= -(1020 \text{ kg/m}^3)\left[(2.5 \text{ m/s}^2)(-7 \text{ m}) + (9.81 \text{ m/s}^2)(-3 \text{ m})\right]\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= (17.9 + 30.0) \text{ kN/m}^2 = \mathbf{47.9\ kPa}$$

since $x_1 = 7$ m, $x_2 = 0$, $z_1 = 3$ m, and $z_2 = 0$.

Discussion Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).

3-104 A vertical U-tube partially filled with alcohol is rotated at a specified rate about one of its arms. The elevation difference between the fluid levels in the two arms is to be determined.



Assumptions **1** Alcohol is an incompressible fluid.

Analysis Taking the base of the left arm of the U-tube as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

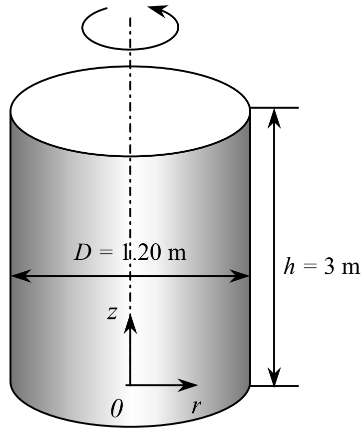
$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 0.20 m is the original height of the liquid before rotation, and ω = 4.2 rad/s. The fluid rise at the right arm relative to the fluid level in the left arm (the center of rotation) is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 + \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(4.2\ \text{rad/s})^2 (0.25\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{0.056\ m}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property.

3-105 A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined. √EES



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Gasoline is an incompressible substance.

Properties The density of the gasoline is given to be 740 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

where R = 0.60 m is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(70 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 7.330 \text{ rad/s}$$

(a) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have $r_1 = r_2 = 0$ and $z_2 - z_1 = h = 3 \text{ m}$. Then,

$$P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho g(z_2 - z_1) = -\rho gh$$
$$= -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 21.8 \text{ kN/m}^2 = \mathbf{21.8 \text{ kPa}}$$

(b) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have $r_1 = 0$, $r_2 = R$, and $z_2 = z_1 = 0$. Then,

$$P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho\omega^2}{2}(R_2^2 - 0) - 0 = \frac{\rho\omega^2 R^2}{2}$$
$$= \frac{(740 \text{ kg/m}^3)(7.33 \text{ rad/s})^2(0.60 \text{ m})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 7.16 \text{ kN/m}^2 = \mathbf{7.16 \text{ kPa}}$$

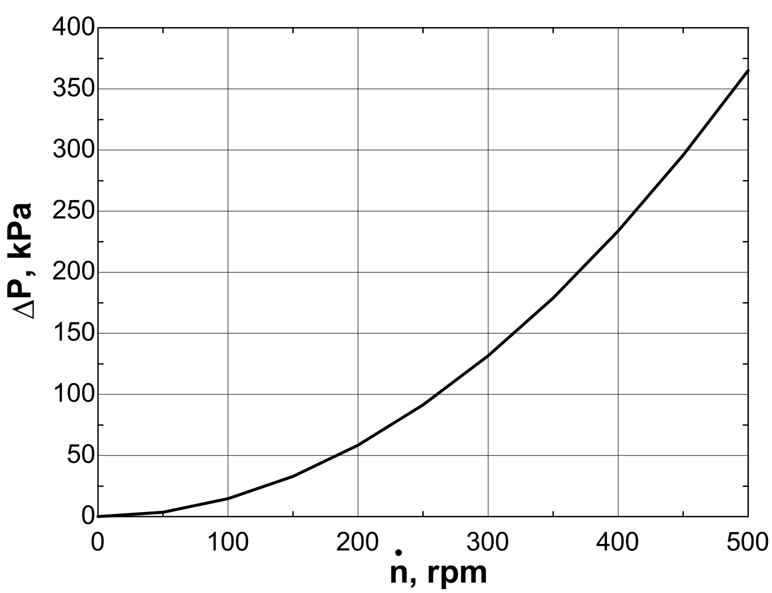
Discussion Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. But the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane) is due entirely to the rotation of the tank.

3-106 Problem 3-105 is reconsidered. The effect of rotational speed on the pressure difference between the center and the edge of the bottom surface of the cylinder as the rotational speed varies from 0 to 500 rpm in increments of 50 rpm is to be investigated.

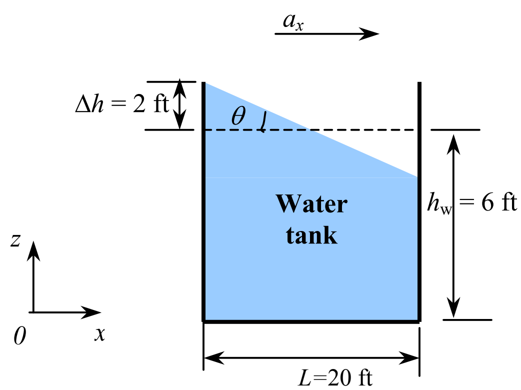
```
g=9.81 "m/s2"
rho=740 "kg/m3"
R=0.6 "m"
h=3 "m"

omega=2*pi*n_dot/60 "rad/s"
DeltaP_axis=rho*g*h/1000 "kPa"
DeltaP_bottom=rho*omega^2*R^2/2000 "kPa"
```

Rotation rate $\dot{n}$, rpm	Angular speed ω, rad/s	$\Delta P_{center\text{-}edge}$ kPa
0	0.0	0.0
50	5.2	3.7
100	10.5	14.6
150	15.7	32.9
200	20.9	58.4
250	26.2	91.3
300	31.4	131.5
350	36.7	178.9
400	41.9	233.7
450	47.1	295.8
500	52.4	365.2



3-107E A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.



Assumptions **1** The road is horizontal so that acceleration has no vertical component ($a_z = 0$). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the x-axis to be the direction of motion, the z-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

$$\tan\theta = \frac{a_x}{g + a_z} \qquad \rightarrow \qquad a_x = g\tan\theta$$

where $a_z = 0$ and, from geometric considerations, $\tan\theta$ is

$$\tan\theta = \frac{\Delta h}{L/2}$$

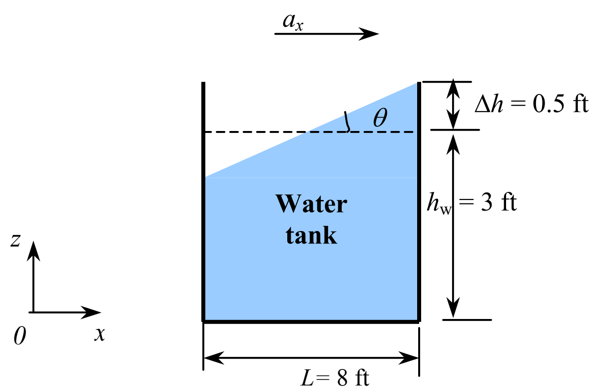
Substituting,

$$a_x = g\tan\theta = g\frac{\Delta h}{L/2} = (32.2\text{ ft/s}^2)\frac{2\text{ ft}}{(20\text{ ft})/2} = \mathbf{6.44\ m/s^2}$$

The solution can be repeated for deceleration by replacing a_x by $-a_x$. We obtain a_x **= -6.44 m/s²**.

Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-108E A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.



Assumptions **1** The road is horizontal so that deceleration has no vertical component (a_z = 0). **2** Effects of splashing and driving over bumps are assumed to be secondary, and are not considered. **3** The deceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

$$\tan\theta = \frac{-a_x}{g + a_z} \qquad \rightarrow \qquad a_x = -g\tan\theta$$

where a_z = 0 and, from geometric considerations, $\tan\theta$ is

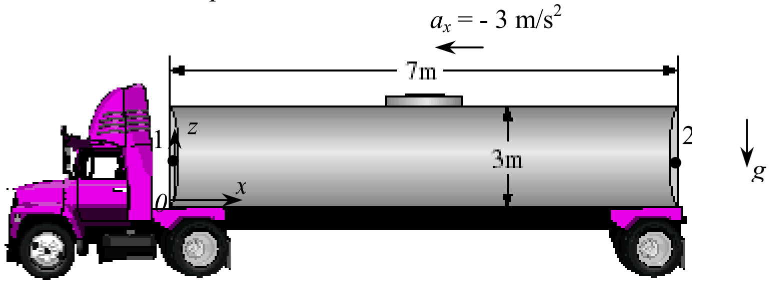
$$\tan\theta = \frac{\Delta h}{L/2}$$

Substituting,

$$a_x = -g\tan\theta = -g\frac{\Delta h}{L/2} = -(32.2\ \text{ft/s}^2)\frac{0.5\ \text{ft}}{(8\ \text{ft})/2} = \mathbf{-4.08\ ft/s^2}$$

Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-109 Water is transported in a completely filled horizontal cylindrical tanker accelerating at a specified rate. The pressure difference between the front and back ends of the tank along a horizontal line when the truck accelerates and decelerates at specified rates. √EES



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of the water to be 1000 kg/m^3.

Analysis (*a*) We take the *x*- and *z*- axes as shown. The horizontal acceleration is in the negative *x* direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus $a_z = 0$. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1)$$

since $z_2 - z_1 = 0$ along a horizontal line. Therefore, the pressure difference between the front and back of the tank is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tank. Then the pressure difference along a horizontal line becomes

$$\Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(-3 \text{ m/s}^2)(7 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 21 \text{ kN/m}^2 = \mathbf{21 \text{ kPa}}$$

since $x_1 = 0$ and $x_2 = 7$ m.

(*b*) The pressure difference during deceleration is determined the way, but $a_x = 4$ m/s^2 in this case,

$$\Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(4 \text{ m/s}^2)(7 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = -28 \text{ kN/m}^2 = \mathbf{-28 \text{ kPa}}$$

Discussion Note that the pressure is higher at the back end of the tank during acceleration, but at the front end during deceleration (during breaking, for example) as expected.