Chapter 5
MASS, BERNOULLI, AND ENERGY EQUATIONS

Conservation of Mass

5-1C Mass, energy, momentum, and electric charge are conserved, and volume and entropy are not conserved during a process.

5-2C Mass flow rate is the amount of mass flowing through a cross-section per unit time whereas the volume flow rate is the amount of volume flowing through a cross-section per unit time.

5-3C The amount of mass or energy entering a control volume does not have to be equal to the amount of mass or energy leaving during an unsteady-flow process.

5-4C Flow through a control volume is steady when it involves no changes with time at any specified position.

5-5C No, a flow with the same volume flow rate at the inlet and the exit is not necessarily steady (unless the density is constant). To be steady, the mass flow rate through the device must remain constant.

5-6E A garden hose is used to fill a water bucket. The volume and mass flow rates of water, the filling time, and the discharge velocity are to be determined.

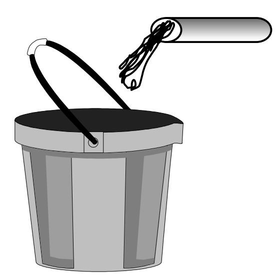
Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis (*a*) The volume and mass flow rates of water are

$$\dot{V} = AV = (\pi D^2/4)V = [\pi(1/12\text{ ft})^2/4](8\text{ ft/s}) = \mathbf{0.04363\ ft^3/s}$$

$$\dot{m} = \rho\dot{V} = (62.4\text{ lbm/ft}^3)(0.04363\text{ ft}^3\text{/s}) = \mathbf{2.72\ lbm/s}$$



(*b*) The time it takes to fill a 20-gallon bucket is

$$\Delta t = \frac{V}{\dot{V}} = \frac{20\text{ gal}}{0.04363\text{ ft}^3\text{/s}}\left(\frac{1\text{ ft}^3}{7.4804\text{ gal}}\right) = \mathbf{61.3\ s}$$

(*c*) The average discharge velocity of water at the nozzle exit is

$$V_e = \frac{\dot{V}}{A_e} = \frac{\dot{V}}{\pi D_e^2/4} = \frac{0.04363\text{ ft}^3\text{/s}}{[\pi(0.5/12\text{ ft})^2/4]} = \mathbf{32\ ft/s}$$

Discussion Note that for a given flow rate, the average velocity is inversely proportional to the square of the velocity. Therefore, when the diameter is reduced by half, the velocity quadruples.

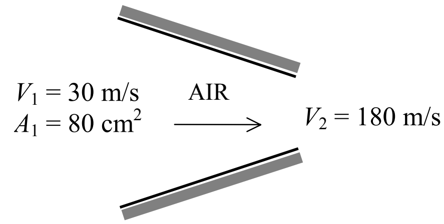
5-7 Air is accelerated in a nozzle. The mass flow rate and the exit area of the nozzle are to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 2.21 kg/m^3 at the inlet, and 0.762 kg/m^3 at the exit.

Analysis (a) The mass flow rate of air is determined from the inlet conditions to be

$$\dot{m} = \rho_1 A_1 V_1 = (2.21\ \text{kg/m}^3)(0.008\ \text{m}^2)(30\ \text{m/s}) = \mathbf{0.530\ kg/s}$$



(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then the exit area of the nozzle is determined to be

$$\dot{m} = \rho_2 A_2 V_2 \longrightarrow A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.530\ \text{kg/s}}{(0.762\ \text{kg/m}^3)(180\ \text{m/s})} = 0.00387\ \text{m}^2 = \mathbf{38.7\ cm^2}$$

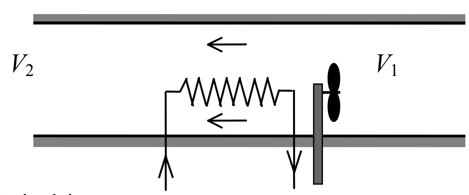
5-8 Air is expanded and is accelerated as it is heated by a hair dryer of constant diameter. The percent increase in the velocity of air as it flows through the drier is to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 1.20 kg/m^3 at the inlet, and 1.05 kg/m^3 at the exit.

Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then,

$$\dot{m}_1 = \dot{m}_2$$
$$\rho_1 A V_1 = \rho_2 A V_2$$
$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{1.20\ \text{kg/m}^3}{1.05\ \text{kg/m}^3} = 1.14 \quad \text{(or, and increase of } \mathbf{14\%}\text{)}$$



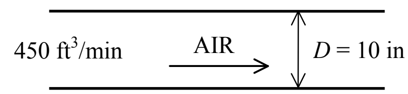
Therefore, the air velocity increases 14% as it flows through the hair drier.

5-9E The ducts of an air-conditioning system pass through an open area. The inlet velocity and the mass flow rate of air are to be determined.

Assumptions Flow through the air conditioning duct is steady.

Properties The density of air is given to be 0.078 lbm/ft^3 at the inlet.

Analysis The inlet velocity of air and the mass flow rate through the duct are



$$V_1 = \frac{\dot{\boldsymbol{V}}_1}{A_1} = \frac{\dot{\boldsymbol{V}}_1}{\pi D^2/4} = \frac{450\ \text{ft}^3/\text{min}}{\pi(10/12\ \text{ft})^2/4} = \textbf{825 ft/min} = \textbf{13.8 ft/s}$$

$$\dot{m} = \rho_1 \dot{\boldsymbol{V}}_1 = (0.078\ \text{lbm/ft}^3)(450\ \text{ft}^3/\text{min}) = 35.1\ \text{lbm/min} = \textbf{0.585 lbm/s}$$

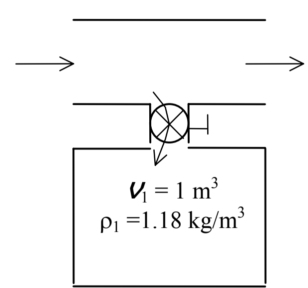
5-10 A rigid tank initially contains air at atmospheric conditions. The tank is connected to a supply line, and air is allowed to enter the tank until the density rises to a specified level. The mass of air that entered the tank is to be determined.

Properties The density of air is given to be 1.18 kg/m^3 at the beginning, and 7.20 kg/m^3 at the end.

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. The mass balance for this system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \quad \rightarrow \quad m_i = m_2 - m_1 = \rho_2 \boldsymbol{V} - \rho_1 \boldsymbol{V}$

Substituting,



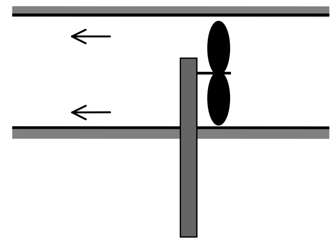
$$m_i = (\rho_2 - \rho_1)\boldsymbol{V} = [(7.20\text{-}1.18)\ \text{kg/m}^3](1\ \text{m}^3) = \textbf{6.02 kg}$$

Therefore, 6.02 kg of mass entered the tank.

5-11 The ventilating fan of the bathroom of a building runs continuously. The mass of air "vented out" per day is to be determined.

Assumptions Flow through the fan is steady.

Properties The density of air in the building is given to be 1.20 kg/m^3.



Analysis The mass flow rate of air vented out is

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.20 \text{ kg/m}^3)(0.030 \text{ m}^3/\text{s}) = 0.036 \text{ kg/s}$$

Then the mass of air vented out in 24 h becomes

$$m = \dot{m}_{\text{air}} \Delta t = (0.036 \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{3110 \text{ kg}}$$

Discussion Note that more than 3 tons of air is vented out by a bathroom fan in one day.

5-12 A desktop computer is to be cooled by a fan at a high elevation where the air density is low. The mass flow rate of air through the fan and the diameter of the casing for a given velocity are to be determined.

Assumptions Flow through the fan is steady.

Properties The density of air at a high elevation is given to be 0.7 kg/m^3.

Analysis The mass flow rate of air is

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (0.7 \text{ kg/m}^3)(0.34 \text{ m}^3/\text{min}) = 0.238 \text{ kg/min} = \mathbf{0.0040 \text{ kg/s}}$$

If the mean velocity is 110 m/min, the diameter of the casing is

$$\dot{V} = AV = \frac{\pi D^2}{4} V \quad \rightarrow \quad D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.34 \text{ m}^3/\text{min})}{\pi (110 \text{ m/min})}} = \mathbf{0.063 \text{ m}}$$

Therefore, the diameter of the casing must be at least 6.3 cm to ensure that the mean velocity does not exceed 110 m/min.

Discussion This problem shows that engineering systems are sized to satisfy certain constraints imposed by certain considerations.

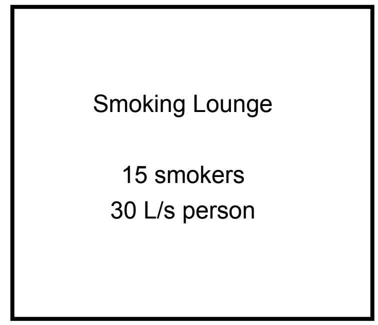
5-13 A smoking lounge that can accommodate 15 smokers is considered. The required minimum flow rate of air that needs to be supplied to the lounge and the diameter of the duct are to be determined.

Assumptions Infiltration of air into the smoking lounge is negligible.

Properties The minimum fresh air requirements for a smoking lounge is given to be 30 L/s per person.

Analysis The required minimum flow rate of air that needs to be supplied to the lounge is determined directly from

$$\begin{aligned}\dot{\mathbf{V}}_{\text{air}} &= \dot{\mathbf{V}}_{\text{air per person}}(\text{No. of persons})\\ &= (30\ \text{L/s}\cdot\text{person})(15\ \text{persons}) = 450\ \text{L/s} = \mathbf{0.45\ m^3/s}\end{aligned}$$



The volume flow rate of fresh air can be expressed as

$$\dot{\mathbf{V}} = VA = V(\pi D^2/4)$$

Solving for the diameter D and substituting,

$$D = \sqrt{\frac{4\dot{\mathbf{V}}}{\pi V}} = \sqrt{\frac{4(0.45\ \text{m}^3/\text{s})}{\pi(8\ \text{m/s})}} = \mathbf{0.268\ m}$$

Therefore, the diameter of the fresh air duct should be at least 26.8 cm if the velocity of air is not to exceed 8 m/s.

5-14 The minimum fresh air requirements of a residential building is specified to be 0.35 air changes per hour. The size of the fan that needs to be installed and the diameter of the duct are to be determined. √

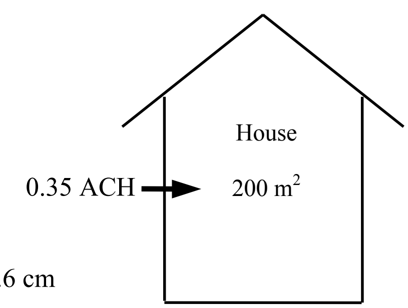
Analysis The volume of the building and the required minimum volume flow rate of fresh air are

$$\mathbf{V}_{\text{room}} = (2.7\ \text{m})(200\ \text{m}^2) = 540\ \text{m}^3$$

$$\dot{\mathbf{V}} = \mathbf{V}_{\text{room}} \times \text{ACH} = (540\ \text{m}^3)(0.35/\text{h}) = 189\ \text{m}^3/\text{h} = 189{,}000\ \text{L/h} = \mathbf{3150\ L/min}$$

The volume flow rate of fresh air can be expressed as

$$\dot{\mathbf{V}} = VA = V(\pi D^2/4)$$



Solving for the diameter D and substituting,

$$D = \sqrt{\frac{4\dot{\mathbf{V}}}{\pi V}} = \sqrt{\frac{4(189/3600\ \text{m}^3/\text{s})}{\pi(6\ \text{m/s})}} = \mathbf{0.106\ m}$$

Therefore, the diameter of the fresh air duct should be at least 10.6 cm if the velocity of air is not to exceed 6 m/s.

Mechanical Energy and Pump Efficiency

5-15C The *mechanical energy* is the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as a propeller. It differs from thermal energy in that thermal energy cannot be converted to work directly and completely. The forms of mechanical energy of a fluid stream are kinetic, potential, and flow energies.

5-16C *Mechanical efficiency* is defined as the ratio of the mechanical energy output to the mechanical energy input. A mechanical efficiency of 100% for a hydraulic turbine means that the entire mechanical energy of the fluid is converted to mechanical (shaft) work.

5-17C The *combined pump-motor efficiency* of a pump/motor system is defined as the ratio of the increase in the mechanical energy of the fluid to the electrical power consumption of the motor,

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{E}_{\text{mech,out}} - \dot{E}_{\text{mech,in}}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{elect,in}}}$$

The combined pump-motor efficiency cannot be greater than either of the pump or motor efficiency since both pump and motor efficiencies are less than 1, and the product of two numbers that are less than one is less than either of the numbers.

5-18C The turbine efficiency, generator efficiency, and *combined turbine-generator efficiency* are defined as follows:

$$\eta_{\text{turbine}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy extracted from the fluid}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

$$\eta_{\text{generator}} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generaor}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{E}_{\text{mech,in}} - \dot{E}_{\text{mech,out}}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

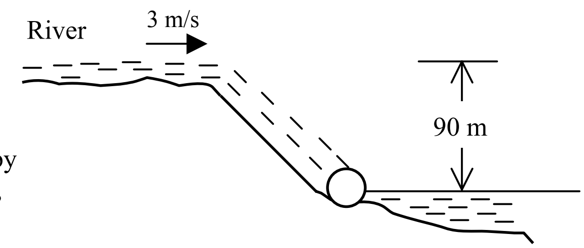
5-19 A river is flowing at a specified velocity, flow rate, and elevation. The total mechanical energy of the river water per unit mass, and the power generation potential of the entire river are to be determined. √

Assumptions **1** The elevation given is the elevation of the free surface of the river. **2** The velocity given is the average velocity. **3** The mechanical energy of water at the turbine exit is negligible.

Properties We take the density of water to be $\rho = 1000 \text{ kg/m}^3$.

Analysis Noting that the sum of the flow energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the flow energy. Then the total mechanical energy of the river water per unit mass becomes

$$\begin{aligned} e_{\text{mech}} &= pe + ke = gh + \frac{V^2}{2} \\ &= \left((9.81 \text{ m/s}^2)(90 \text{ m}) + \frac{(3 \text{ m/s})^2}{2} \right)\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= 0.887 \text{ kJ/kg} \end{aligned}$$



The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho \dot{\mathcal{V}} = (1000 \text{ kg/m}^3)(500 \text{ m}^3/\text{s}) = 500{,}000 \text{ kg/s}$$

$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m} e_{\text{mech}} = (500{,}000 \text{ kg/s})(0.887 \text{ kg/s}) = 444{,}000 \text{ kW} = \mathbf{444 \text{ MW}}$$

Therefore, 444 MW of power can be generated from this river as it discharges into the lake if its power potential can be recovered completely.

Discussion Note that the kinetic energy of water is negligible compared to the potential energy, and it can be ignored in the analysis. Also, the power output of an actual turbine will be less than 444 MW because of losses and inefficiencies.

5-20 A hydraulic turbine-generator is generating electricity from the water of a large reservoir. The combined turbine-generator efficiency and the turbine efficiency are to be determined.

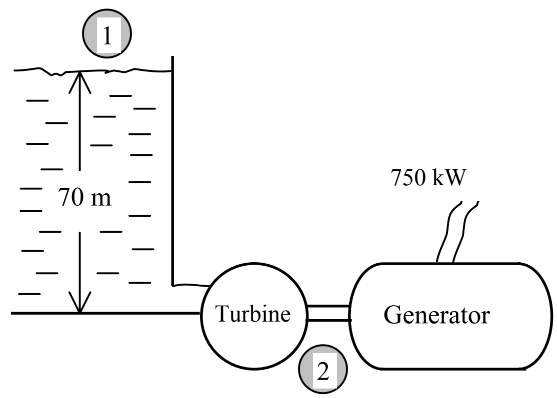
Assumptions **1** The elevation of the reservoir remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Analysis We take the free surface of the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ($z_2 = 0$), and thus the potential energy at points 1 and 2 are $\text{pe}_1 = gz_1$ and $\text{pe}_2 = 0$. The flow energy P/ρ at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($\text{ke}_1 = \text{ke}_2 = 0$) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$pe_1 = gz_1 = (9.81\ \text{m/s}^2)(70\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.687\ \text{kJ/kg}$$

Then the rate at which the mechanical energy of the fluid is supplied to the turbine become

$$\begin{aligned}\left|\Delta\dot{E}_{\text{mech,fluid}}\right| &= \dot{m}(e_{\text{mech,in}} - e_{\text{mech,out}}) = \dot{m}(pe_1 - 0) = \dot{m}pe_1 \\ &= (1500\ \text{kg/s})(0.687\ \text{kJ/kg}) \\ &= 1031\ \text{kW}\end{aligned}$$



The combined turbine-generator and the turbine efficiency are determined from their definitions,

$$\eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{750\ \text{kW}}{1031\ \text{kW}} = 0.727 \quad \text{or} \quad \mathbf{72.7\%}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{800\ \text{kW}}{1031\ \text{kW}} = 0.776 \quad \text{or} \quad \mathbf{77.6\%}$$

Therefore, the reservoir supplies 1031 kW of mechanical energy to the turbine, which converts 800 kW of it to shaft work that drives the generator, which generates 750 kW of electric power.

Discussion This problem can also be solved by taking point 1 to be at the turbine inlet, and using flow energy instead of potential energy. It would give the same result since the flow energy at the turbine inlet is equal to the potential energy at the free surface of the reservoir.

5-21 Wind is blowing steadily at a certain velocity. The mechanical energy of air per unit mass, the power generation potential, and the actual electric power generation are to be determined. √EES

Assumptions **1** The wind is blowing steadily at a constant uniform velocity. **2** The efficiency of the wind turbine is independent of the wind speed.

Properties The density of air is given to be $\rho = 1.25$ kg/m^3.

Analysis Kinetic energy is the only form of mechanical energy the wind possesses, and it can be converted to work entirely. Therefore, the power potential of the wind is its kinetic energy, which is $V^2/2$ per unit mass, and $\dot{m}V^2/2$ for a given mass flow rate:

$$e_{\text{mech}} = ke = \frac{V^2}{2} = \frac{(12\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 0.072\text{ kJ/kg}$$

$$\dot{m} = \rho VA = \rho V\frac{\pi D^2}{4} = (1.25\text{ kg/m}^3)(12\text{ m/s})\frac{\pi(50\text{ m})^2}{4} = 29{,}452\text{ kg/s}$$

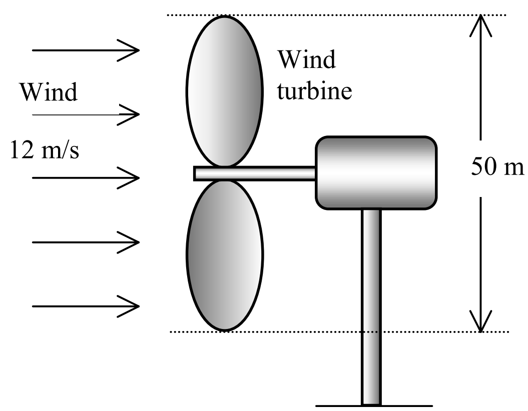
$$\dot{W}_{\max} = \dot{E}_{\text{mech}} = \dot{m}e_{\text{mech}} = (29{,}452\text{ kg/s})(0.072\text{ kJ/kg}) = \mathbf{2121\ kW}$$

The actual electric power generation is determined by multiplying the power generation potential by the efficiency,

$$\dot{W}_{\text{elect}} = \eta_{\text{wind turbine}}\dot{W}_{\max} = (0.30)(2121\text{ kW}) = \mathbf{636\ kW}$$

Therefore, 636 kW of actual power can be generated by this wind turbine at the stated conditions.

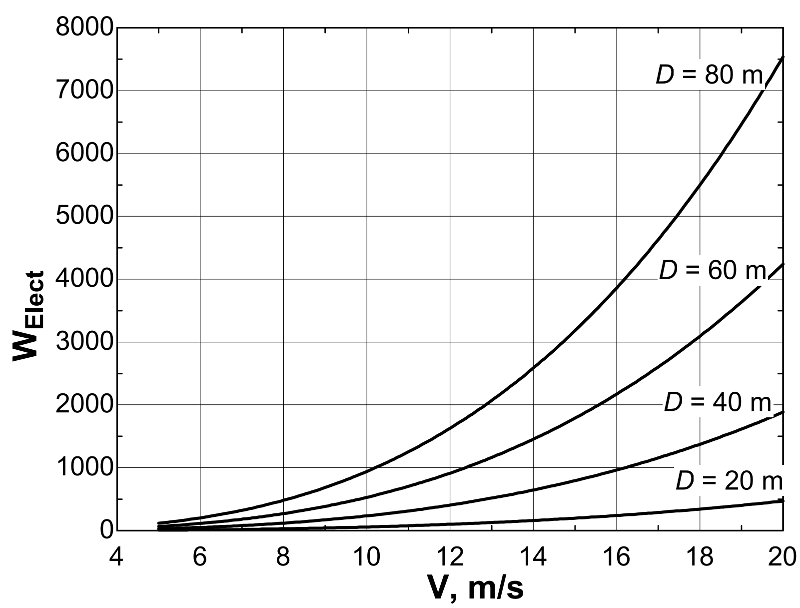
Discussion The power generation of a wind turbine is proportional to the cube of the wind velocity, and thus the power generation will change strongly with the wind conditions.



5-22 Problem 5-21 is reconsidered. The effect of wind velocity and the blade span diameter on wind power generation as the velocity varies from 5 m/s to 20 m/s in increments of 5 m/s, and the diameter varies from 20 m to 80 m in increments of 20 m is to be investigated.

```
D1=20 "m"
D2=40 "m"
D3=60 "m"
D4=80 "m"
Eta=0.30
rho=1.25 "kg/m3"
m1_dot=rho*V*(pi*D1^2/4); W1_Elect=Eta*m1_dot*(V^2/2)/1000 "kW"
m2_dot=rho*V*(pi*D2^2/4); W2_Elect=Eta*m2_dot*(V^2/2)/1000 "kW"
m3_dot=rho*V*(pi*D3^2/4); W3_Elect=Eta*m3_dot*(V^2/2)/1000 "kW"
m4_dot=rho*V*(pi*D4^2/4); W4_Elect=Eta*m4_dot*(V^2/2)/1000 "kW"
```

D, m	V, m/s	m, kg/s	W_{elect}, kW
20	5	1,963	7
	10	3,927	59
	15	5,890	199
	20	7,854	471
40	5	7,854	29
	10	15,708	236
	15	23,562	795
	20	31,416	1885
60	5	17,671	66
	10	35,343	530
	15	53,014	1789
	20	70,686	4241
80	5	31,416	118
	10	62,832	942
	15	94,248	3181
	20	125,664	7540



5-23E A differential thermocouple indicates that the temperature of water rises a certain amount as it flows through a pump at a specified rate. The mechanical efficiency of the pump is to be determined.

Assumptions **1** The pump is adiabatic so that there is no heat transfer with the surroundings, and the temperature rise of water is completely due to frictional heating. **2** Water is an incompressible substance.

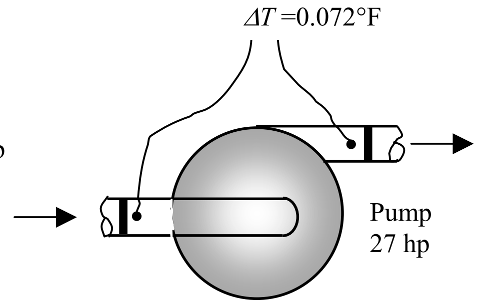
Properties We take the density of water to be ρ = 62.4 lbm/ft^3 and its specific heat to be C = 1.0 Btu/lbm·°F.

Analysis The increase in the temperature of water is due to the conversion of mechanical energy to thermal energy, and the amount of mechanical energy converted to thermal energy is equal to the increase in the internal energy of water,

$$\dot{m} = \rho \dot{\boldsymbol{V}} = (62.4 \text{ lbm/ft}^3)(1.5 \text{ ft}^3/\text{s}) = 93.6 \text{ lbm/s}$$

$$\dot{E}_{\text{mech, loss}} = \Delta \dot{U} = \dot{m} c \Delta T$$

$$= (93.6 \text{ lbm/s})(1.0 \text{ Btu/lbm}\cdot{}^\circ\text{F})(0.072^\circ\text{F})\left(\frac{1 \text{ hp}}{0.7068 \text{ Btu/s}}\right) = 9.53 \text{ hp}$$



The mechanical efficiency of the pump is determined from the general definition of mechanical efficiency,

$$\eta_{\text{pump}} = 1 - \frac{\dot{E}_{\text{mech,loss}}}{\dot{W}_{\text{mech, in}}} = 1 - \frac{9.53 \text{ hp}}{27 \text{ hp}} = 0.647 \quad \text{or} \quad \mathbf{64.7\%}$$

Discussion Note that despite the conversion of more than one-third of the mechanical power input into thermal energy, the temperature of water rises by only a small fraction of a degree. Therefore, the temperature rise of a fluid due to frictional heating is usually negligible in heat transfer analysis.

5-24 Water is pumped from a lake to a storage tank at a specified rate. The overall efficiency of the pump-motor unit and the pressure difference between the inlet and the exit of the pump are to be determined. √

Assumptions **1** The elevations of the tank and the lake remain constant. **2** Frictional losses in the pipes are negligible. **3** The changes in kinetic energy are negligible. **4** The elevation difference across the pump is negligible.

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis (*a*) We take the free surface of the lake to be point 1 and the free surfaces of the storage tank to be point 2. We also take the lake surface as the reference level ($z_1 = 0$), and thus the potential energy at points 1 and 2 are $\mathrm{pe}_1 = 0$ and $\mathrm{pe}_2 = gz_2$. The flow energy at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($\mathrm{ke}_1 = \mathrm{ke}_2 = 0$) since the water at both locations is essentially stationary. The mass flow rate of water and its potential energy at point 2 are

$$\dot{m} = \rho\dot{V} = (1000\ \mathrm{kg/m^3})(0.070\ \mathrm{m^3/s}) = 70\ \mathrm{kg/s}$$

$$pe_1 = gz_1 = (9.81\ \mathrm{m/s^2})(20\ \mathrm{m})\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 0.196\ \mathrm{kJ/kg}$$

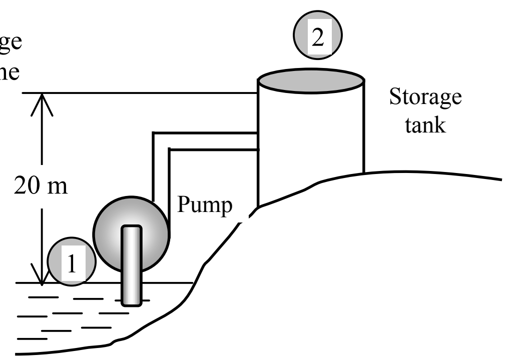
Then the rate of increase of the mechanical energy of water becomes

$$\Delta\dot{E}_{\mathrm{mech,fluid}} = \dot{m}(e_{\mathrm{mech,out}} - e_{\mathrm{mech,in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70\ \mathrm{kg/s})(0.196\ \mathrm{kJ/kg}) = 13.7\ \mathrm{kW}$$

The overall efficiency of the combined pump-motor unit is determined from its definition,

$$\eta_{\text{pump-motor}} = \frac{\Delta\dot{E}_{\mathrm{mech,fluid}}}{\dot{W}_{\mathrm{elect,in}}} = \frac{13.7\ \mathrm{kW}}{20.4\ \mathrm{kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

(*b*) Now we consider the pump. The change in the mechanical energy of water as it flows through the pump consists of the change in the flow energy only since the elevation difference across the pump and the change in the kinetic energy are negligible. Also, this change must be equal to the useful mechanical energy supplied by the pump, which is 13.7 kW:



$$\Delta\dot{E}_{\mathrm{mech,fluid}} = \dot{m}(e_{\mathrm{mech,out}} - e_{\mathrm{mech,in}}) = \dot{m}\frac{P_2 - P_1}{\rho} = \dot{V}\Delta P$$

Solving for ΔP and substituting,

$$\Delta P = \frac{\Delta\dot{E}_{\mathrm{mech,fluid}}}{\dot{V}} = \frac{13.7\ \mathrm{kJ/s}}{0.070\ \mathrm{m^3/s}}\left(\frac{1\ \mathrm{kPa\cdot m^3}}{1\ \mathrm{kJ}}\right) = \mathbf{196\ kPa}$$

Therefore, the pump must boost the pressure of water by 196 kPa in order to raise its elevation by 20 m.

Discussion Note that only two-thirds of the electric energy consumed by the pump-motor is converted to the mechanical energy of water; the remaining one-third is wasted because of the inefficiencies of the pump and the motor.

Bernoulli Equation

5-25C The acceleration of a fluid particle along a streamline is called *streamwise acceleration*, and it is due to a change in speed along a streamline. *Normal acceleration* (or centrifugal acceleration), on the other hand, is the acceleration of a fluid particle in the direction normal to the streamline, and it is due to a change in direction.

5-26C The Bernoulli equation can be expressed in three different ways as follows:

(a) energies: $\frac{P}{\rho}+\frac{V^2}{2}+gz=\text{constant}$

(b) pressures: $P+\rho\frac{V^2}{2}+\rho gz=\text{constant}$

(c) heads: $\frac{P}{\rho g}+\frac{V^2}{2g}+z=H=\text{constant}$

12-27C The three major assumptions used in the derivation of the Bernoulli equation are that the flow is steady, frictionless, and incompressible.

5-28C The *static pressure P* is the actual pressure of the fluid. The *dynamic pressure* $\rho\mathbf{V}^2/2$ is the pressure rise when the fluid in motion is brought to a stop. The *hydrostatic pressure* ρgz is not pressure in a real sense since its value depends on the reference level selected, and it accounts for the effects of fluid weight on pressure.

The sum of static, dynamic, and hydrostatic pressures is constant when flow is steady, frictionless, and incompressible.

5-29C The sum of the static and dynamic pressures is called the stagnation pressure, and it is expressed as $P_{\text{stag}}=P+\rho V^2/2$. The *stagnation pressure* can be measured by a pitot tube whose inlet is normal to flow.

5-30C The *pressure head* $P/\rho g$ is the height of a fluid column that produces the static pressure *P*. The *velocity head* $V^2/2$ is the elevation needed for a fluid to reach the velocity **V** during frictionless free fall. The *elevation head z* is the height of a fluid relative to a reference level.

5-31C The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line*. The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the *energy line*. For stationary bodies such as reservoirs or lakes, the EL and HGL coincide with the free surface of the liquid.

5-32C For open channel flow, the hydraulic grade line (HGL) coincides with the free surface of the liquid. At the exit of a pipe discharging to the atmosphere, it coincides with the center of the pipe.

5-33C With no losses and a 100% efficient nozzle, the water stream could reach to the water level in the tank, or 20 meters. In reality, friction losses in the hose, nozzle inefficiencies, orifice losses and air drag would prevent attainment of the maximum theoretical height.

5-34C The lower density liquid can go over a higher wall, provided that cavitation pressure is not reached. Therefore, oil can go over a higher wall.

5-35C Siphoning works because of the elevation and thus pressure difference between the inlet and exit of a tube. The pressure at the tube exit and at the free surface of a liquid is the atmospheric pressure. When the tube exit is below the free surface of the liquid, the elevation head difference drives the flow through the tube. At sea level, 1 atm pressure can support about 10.3 m of cold water (cold water has a low vapor pressure). Therefore, siphoning cold water over a 7 m wall is feasible.

5-36C At sea level, a person can siphon water over a wall as high as 10.3 m. At the top of a high mountain where the pressure is about half of the atmospheric pressure at sea level, a person can siphon water over a wall that is only half as high. An atmospheric pressure of 58.5 kPa is insufficient to support a 8.5 meter high siphon.

5-37C By Bernoulli's Equation, the smaller pipe section is consistent with higher velocity and concomitant lower pressure. Thus Manometer A is correct. The fluid levels in a manometer is independent of the flow direction, and reversing the flow direction will have no effect on the manometer.

5-38C The arrangement *B* measures the total head and static head at the same location, and thus it is more accurate. The static probe in arrangement *A* will indicate $D/2$ less water head, and thus the difference between the static and stagnation pressures (the dynamic pressure) will be larger. Consequently, arrangement *A* will indicate a higher velocity. In the case of air, the static pressure difference corresponding to the elevation head of $D/2$ is negligible, and thus both arrangements will indicate the same velocity.

5-39 A water pipe bursts as a result of freezing, and water shoots up into the air a certain height. The gage pressure of water in the pipe is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The water pressure in the pipe at the burst section is equal to the water main pressure. **3** Friction between the water and air is negligible. **4** The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli Equation simplifies to

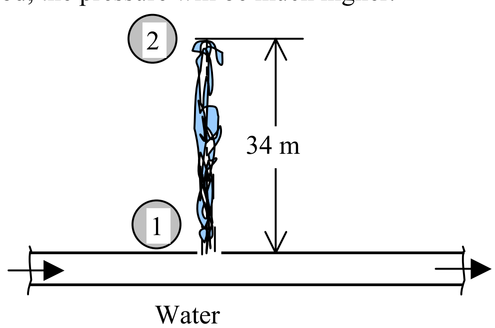
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=z_2$$

Solving for $P_{1,\text{gage}}$ and substituting,

$$P_{1,\text{gage}}=\rho g z_2=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(34\ \text{m})\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=\mathbf{334\ kPa}$$

Therefore, the pressure in the main must be at least 334 kPa above the atmospheric pressure.

Discussion The result obtained by the Bernoulli equation represents a limit, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 334 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.



5-40 The velocity of an aircraft is to be measured by a Pitot-static probe. For a given differential pressure reading, the velocity of the aircraft is to be determined. √

Assumptions **1**The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

Properties The density of the atmosphere at an elevation of 3000 m is ρ = 0.909 kg/m³.

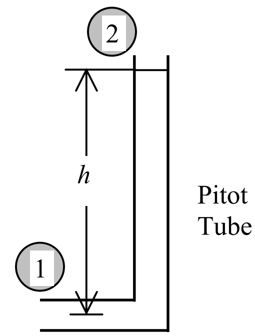
Analysis We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{V_1^2}{2}=\frac{P_{stag}-P_1}{\rho}$$

Solving for V_1 and substituting,

$$V_1=\sqrt{\frac{2(P_{stag}-P_1)}{\rho}}=\sqrt{\frac{2(3000\text{ N/m}^2)}{0.909\text{ kg/m}^3}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}=\mathbf{81.2\ m/s}=\mathbf{292\ km/h}$$

since 1 Pa = 1 N/m² and 1 m/s = 3.6 km/h.



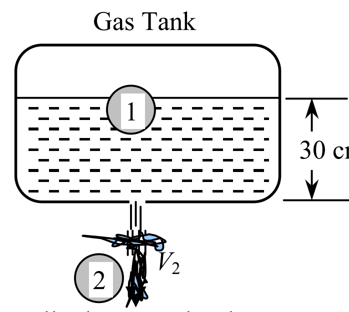
Discussion Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a Pitot-static probe.

5-41 The bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. For a given height of gasoline, the initial velocity of the gasoline out of the hole is to be determined. Also, the variation of velocity with time and the effect of the tightness of the lid on flow rate are to be discussed. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The air space in the tank is at atmospheric pressure. **3** The splashing of the gasoline in the tank during travel is not considered.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere) $V_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 0.3$ m and $z_2 = 0$ (we take the reference level at the hole. Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$



Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\,\text{m/s}^2)(0.3\,\text{m})} = \mathbf{2.43\ m/s}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s.

Discussion The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss.

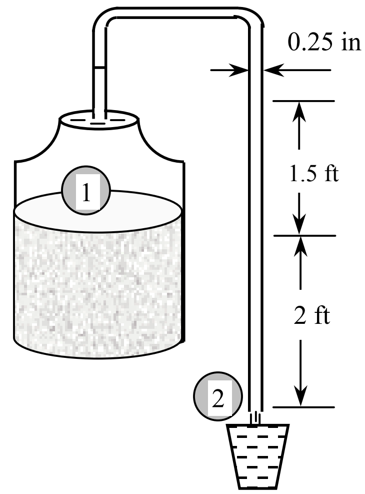
As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.

5-42E The drinking water needs of an office are met by large water bottles with a plastic hose inserted in it. The minimum filling time of an 8-oz glass is to be determined when the bottle is full and when it is near empty. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** All losses are neglected to obtain the minimum filling time.

Analysis We take point 1 to be at the free surface of water in the bottle and point 2 at the exit of the tube so that $P_1 = P_2 = P_{atm}$ (the bottle is open to the atmosphere and water discharges into the atmosphere), $V_1 \cong 0$ (the bottle is large relative to the tube diameter), and $z_2 = 0$ (we take point 2 as the reference level). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{2gz_1}$$



Substituting, the discharge velocity of water and the filling time are determined as follows:

(*a*) *Full bottle* (z_1 = 3.5 ft):

$$V_2=\sqrt{2(32.2\text{ ft/s}^2)(3.5\text{ ft})}=15.0\text{ ft/s}$$

$$A=\pi D^2/4=\pi(0.25/12\text{ ft})^2/4=3.41\times10^{-4}\text{ ft}^2$$

$$\Delta t=\frac{V}{\dot V}=\frac{V}{AV_2}=\frac{0.00835\text{ ft}^3}{(3.41\times10^{-4}\text{ ft}^2)(15\text{ ft/s})}=\mathbf{1.6\ s}$$

(*b*) *Empty bottle* (z_1 = 2 ft):

$$V_2=\sqrt{2(32.2\text{ ft/s}^2)(2\text{ ft})}=11.3\text{ ft/s}$$

$$\Delta t=\frac{V}{\dot V}=\frac{V}{AV_2}=\frac{0.00835\text{ ft}^3}{(3.41\times10^{-4}\text{ ft}^2)(11.3\text{ ft/s})}=\mathbf{2.2\ s}$$

Discussion The siphoning time is determined assuming frictionless flow, and thus this is the *minimum time* required. In reality, the time will be longer because of friction between water and the tube surface.

5-43 The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined. √

Assumptions The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the entrance of the Pitot-static probe (the stagnation point). This is a steady flow with straight and parallel streamlines, and thus the static pressure at any point is equal to the hydrostatic pressure at that point. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

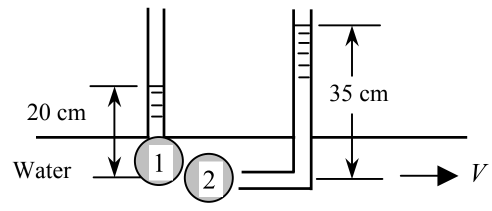
Substituting the P_1 and P_2 expressions give

$$\frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_{\text{pitot}}+R)-\rho g(h_{\text{piezo}}+R)}{\rho g}=\frac{\rho g(h_{\text{pitot}}-h_{\text{piezo}})}{\rho g}=h_{\text{pitot}}-h_{\text{piezo}}$$

Solving for V_1 and substituting,

$$V_1=\sqrt{2g(h_{\text{pitot}}-h_{\text{piezo}})}=\sqrt{2(9.81\text{ m/s}^2)[(0.35-0.20)\text{ m}]}=\mathbf{1.72\ m/s}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot-static probe.



5-44 A water tank of diameter D_o and height H open to the atmosphere is initially filled with water. An orifice of diameter D with a smooth entrance (no losses) at the bottom drains to the atmosphere. Relations are to be developed for the time required for the tank to empty completely and half-way. √

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

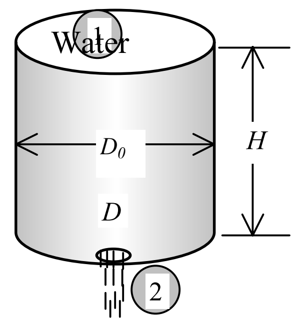
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{2gz_1}$$

For generality, we express the water height in the tank at any time t by z, and the discharge velocity by $V_2=\sqrt{2gz}$. Note that water surface in the tank moves down as the tank drains, and thus z is a variable whose value changes from H at the beginning to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V} = A_{\text{orifice}}V_2 = \frac{\pi D^2}{4}\sqrt{2gz}$$



Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt = -\frac{\pi D_0^2}{4}dz \quad\rightarrow\quad dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz}}dz = -\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_i = H$ to $t = t_f$ when $z = z_f$ gives

$$\int_{t=0}^{t_f}dt = -\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_1}^{z_f}z^{-1/2}dz \quad\rightarrow\quad t_f = -\frac{D_0^2}{D^2\sqrt{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{z_f} = \frac{2D_0^2}{D^2\sqrt{2g}}\left(\sqrt{z_i}-\sqrt{z_f}\right) = \frac{D_0^2}{D^2}\left(\sqrt{\frac{2z_i}{g}}-\sqrt{\frac{2z_f}{g}}\right)$$

Then the discharging time for the two cases becomes as follows:

(*a*) *The tank empties halfway*: $z_i = H$ and $z_f = H/2$: $\quad t_f = \dfrac{D_0^2}{D^2}\left(\sqrt{\dfrac{2H}{g}}-\sqrt{\dfrac{H}{g}}\right)$

(*b*) *The tank empties completely*: $z_i = H$ and $z_f = 0$: $\quad t_f = \dfrac{D_0^2}{D^2}\sqrt{\dfrac{2H}{g}}$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

5-45 Water discharges to the atmosphere from the orifice at the bottom of a pressurized tank. Assuming frictionless flow, the discharge rate of water from the tank is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

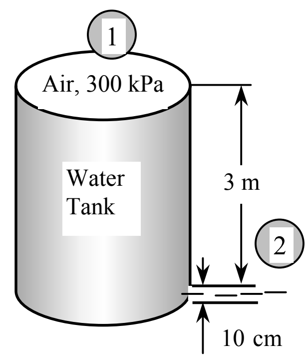
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_2^2}{2g}=\frac{P_1-P_2}{\rho g}+z_1$$

Solving for V_2 and substituting, the discharge velocity is determined to

$$V_2=\sqrt{\frac{2(P_1-P_2)}{\rho}+2gz_1}=\sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+2(9.81\text{ m/s}^2)(3\text{ m})}$$
$$=21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V}=A_{orifice}V_2=\frac{\pi D^2}{4}V_2=\frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s})=\mathbf{0.168\ m^3/s}$$



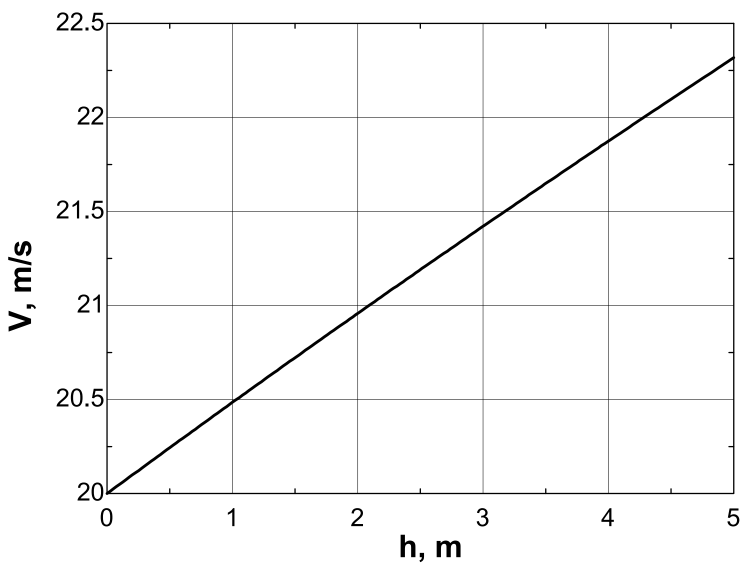
Discussion Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

5-46 Problem 5-45 is reconsidered. The effect of water height in the tank on the discharge velocity as the water height varies from 0 to 5 m in increments of 0.5 m is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
d=0.10 "m"

P1=300 "kPa"
P_atm=100 "kPa"
V=SQRT(2*(P1-P_atm)*1000/rho+2*g*h)
Ac=pi*D^2/4
 V_dot=Ac*V
```

h, m	*V*, m/s	$\dot{V}$, m^3/s
0.00	20.0	0.157
0.50	20.2	0.159
1.00	20.5	0.161
1.50	20.7	0.163
2.00	21.0	0.165
2.50	21.2	0.166
3.00	21.4	0.168
3.50	21.6	0.170
4.00	21.9	0.172
4.50	22.1	0.174
5.00	22.3	0.175



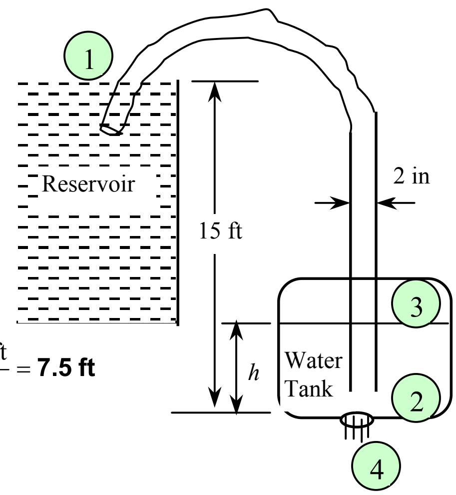
5-47E A siphon pumps water from a large reservoir to a lower tank which is initially empty. Water leaves the tank through an orifice. The height the water will rise in the tank at equilibrium is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Both the tank and the reservoir are open to the atmosphere. **3** The water level of the reservoir remains constant.

Analysis We take the reference level to be at the bottom of the tank, and the water height in the tank at any time to be h. We take point 1 to be at the free surface of reservoir, point 2 at the exit of the siphon, which is placed at the bottom of the tank, and point 3 at the free surface of the tank, and point 4 at the exit of the orifice at the bottom of the tank. Then z_1 = 20 ft, $z_2 = z_4 = 0$, $z_3 = h$, $P_1 = P_3 = P_4 = P_{atm}$ (the reservoir is open to the atmosphere and water discharges into the atmosphere) $P_2 = P_{atm}+\rho gh$ (the hydrostatic pressure at the bottom of the tank where the siphon discharges), and $V_1 \cong V_3 \cong 0$ (the free surfaces of reservoir and the tank are large relative to the tube diameter). Then the Bernoulli Equation between 1-2 and 3-4 simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_{atm}}{\rho g}+z_1=\frac{P_{atm}+\rho gh}{\rho g}+\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{2gz_1-2gh}=\sqrt{2g(z_1-h)}$$

$$\frac{P_3}{\rho g}+\frac{V_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{V_4^2}{2g}+z_4 \quad\rightarrow\quad h=\frac{V_4^2}{2g} \quad\rightarrow\quad V_4=\sqrt{2gh}$$



Noting that the diameters of the tube and the orifice are the same, the flow rates of water into and out of the tank will be the same when the water velocities in the tube and the orifice are equal since

$$\dot{V}_2=\dot{V}_4 \quad\rightarrow\quad AV_2=AV_4 \quad\rightarrow\quad V_2=V_4$$

Setting the two velocities equal to each other gives

$$V_2=V_4 \quad\rightarrow\quad \sqrt{2g(z_1-h)}=\sqrt{2gh} \quad\rightarrow\quad z_1-h=h \quad\rightarrow\quad h=\frac{z_1}{2}=\frac{15\text{ ft}}{2}=\mathbf{7.5\ ft}$$

Therefore, the water level in the tank will stabilize when the water level rises to 7.5 ft.

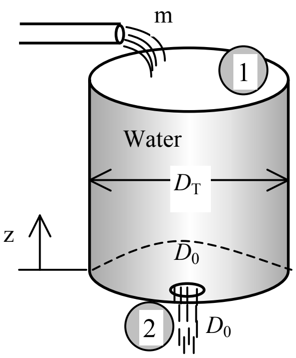
Discussion This result is obtained assuming negligible friction. The result would be somewhat different if the friction in the pipe and orifice were considered.

5-48 Water enters an empty tank steadily at a specified rate. An orifice at the bottom allows water to escape. The maximum water level in the tank is to be determined, and a relation for water height z as a function of time is to be obtained.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow through the orifice is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$) (it becomes zero when the water in the tank reaches its maximum level), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{2gz_1}$$



Then the mass flow rate through the orifice for a water height of z becomes

$$\dot{m}_{out}=\rho\dot{\mathbf{V}}_{out}=\rho A_{orifice}V_2=\rho\frac{\pi D_0^2}{4}\sqrt{2gz} \quad\rightarrow\quad z=\frac{1}{2g}\left(\frac{4\dot{m}_{out}}{\rho\pi D_0^2}\right)^2$$

Setting $z = h_{max}$ and $\dot{m}_{out}=\dot{m}_{in}$ (the incoming flow rate) gives the desired relation for the maximum height the water will reach in the tank,

$$h_{max}=\frac{1}{2g}\left(\frac{4\dot{m}_{in}}{\rho\pi D_0^2}\right)^2$$

(*b*) The amount of water that flows through the orifice and the increase in the amount of water in the tank during a differential time interval dt are

$$dm_{out}=\dot{m}_{out}dt=\rho\frac{\pi D_0^2}{4}\sqrt{2gz}\,dt$$

$$dm_{tank}=\rho A_{tank}dz=\rho\frac{\pi D_T^2}{4}dz$$

The amount of water that enters the tank during dt is $dm_{in}=\dot{m}_{in}dt$ (Recall that $\dot{m}_{in}=$ constant). Substituting them into the conservation of mass relation $dm_{tank}=dm_{in}-dm_{out}$ gives

$$dm_{tank}=\dot{m}_{in}dt-\dot{m}_{out}dt \quad\rightarrow\quad \rho\frac{\pi D_T^2}{4}dz=\left(\dot{m}_{in}-\rho\frac{\pi D_0^2}{4}\sqrt{2gz}\right)dt$$

Separating the variables, and integrating it from $z = 0$ at $t = 0$ to $z = z$ at time $t = t$ gives

$$\frac{\frac{1}{4}\rho\pi D_T^2dz}{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}=dt \quad\rightarrow\quad \int_{z=0}^{z}\frac{\frac{1}{4}\rho\pi D_T^2dz}{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}=\int_{t=0}^{t}dt=t$$

Performing the integration, the desired relation between the water height z and time t is obtained to be

$$\frac{\frac{1}{2}\rho\pi D_T^2}{(\frac{1}{4}\rho\pi D_0^2\sqrt{2g})^2}\left(\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}-\dot{m}_{in}\ln\frac{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}{\dot{m}_{in}}\right)=t$$

Discussion Note that this relation is implicit in z, and thus we can't obtain a relation in the form $z = f(t)$. Substituting a z value in the left side gives the time it takes for the fluid level in the tank to reach that level.

5-49E Water flows through a horizontal pipe that consists of two sections at a specified rate. The differential height of a mercury manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible.

Properties The densities of mercury and water are $\rho_{Hg} = 847$ lbm/ft^3 and $\rho_w = 62.4$ lbm/ft^3.

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \frac{\rho_w (V_2^2 - V_1^2)}{2} \qquad (1)$$

We let the differential height of the mercury manometer be h and the distance between the centerline and the mercury level in the tube where mercury is raised be s. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1 + \rho_w g(s+h) = P_2 + \rho_w gs + \rho_{Hg} gh \quad \rightarrow \quad P_1 - P_2 = (\rho_{Hg} - \rho_w) gh \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

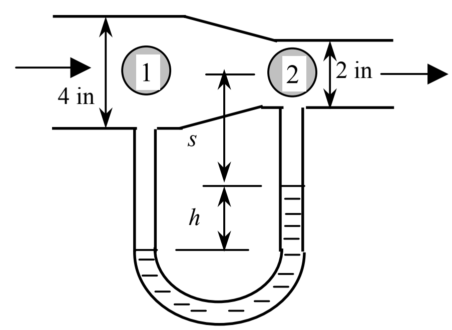
$$\frac{\rho_w (V_2^2 - V_1^2)}{2} = (\rho_{Hg} - \rho_w) gh \quad \rightarrow \quad h = \frac{\rho_w (V_2^2 - V_1^2)}{2g(\rho_{Hg} - \rho_w)} = \frac{V_2^2 - V_1^2}{2g(\rho_{Hg}/\rho_w - 1)}$$

Calculating the velocities and substituting,

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{1 \text{ gal/s}}{\pi (4/12 \text{ ft})^2/4}\left(\frac{0.13368 \text{ ft}^3}{1 \text{ gal}}\right) = 1.53 \text{ ft/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{1 \text{ gal/s}}{\pi (2/12 \text{ ft})^2/4}\left(\frac{0.13368 \text{ ft}^3}{1 \text{ gal}}\right) = 6.13 \text{ ft/s}$$

$$h = \frac{(6.13 \text{ ft/s})^2 - (1.53 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)(847/62.4 - 1)} = 0.0435 \text{ ft} = \mathbf{0.52 \text{ in}}$$



Therefore, the differential height of the mercury column will be 0.52 in.

5-50 An airplane is flying at a certain altitude at a given speed. The pressure on the stagnation point on the nose of the plane is to be determined, and the approach to be used at high velocities is to be discussed. √

Assumptions **1**The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

Properties The density of the atmospheric air at an elevation of 12,000 m is $\rho = 0.312\ \text{kg/m}^3$.

Analysis We take point 1 well ahead of the plane at the level of the nose, and point 2 at the nose where the flow comes to a stop. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

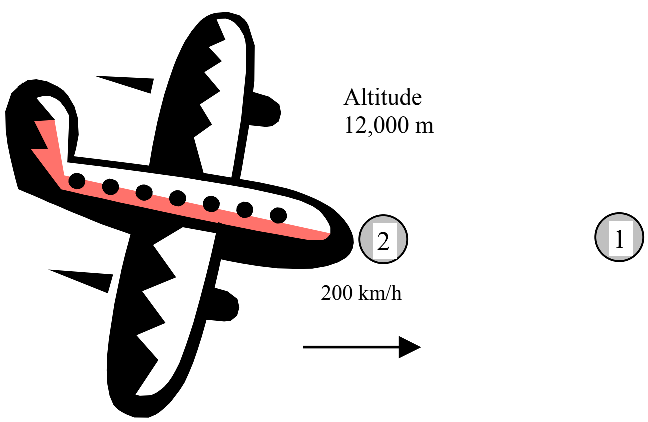
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{V_1^2}{2}=\frac{P_{stag}-P_{\text{atm}}}{\rho}=\frac{P_{\text{stag, gage}}}{\rho}$$

Solving for $P_{\text{stag, gage}}$ and substituting,

$$P_{\text{stag, gage}}=\frac{\rho V_1^2}{2}=\frac{(0.312\ \text{kg/m}^3)(200/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)=481\ \text{N/m}^2=\mathbf{481\ Pa}$$

since 1 Pa = 1 N/m^2 and 1 m/s = 3.6 km/h.

Discussion A flight velocity of 1050 km/h = 292 m/s corresponds to a Mach number much greater than 0.3 (the speed of sound is about 340 m/s at room conditions, and lower at higher altitudes, and thus a Mach number of 292/340 = 0.86). Therefore, the flow can no longer be assumed to be incompressible, and the Bernoulli equation given above cannot be used. This problem can be solved using the modified Bernoulli equation that accounts for the effects of compressibility, assuming isentropic flow.



5-51 A Pitot-static probe is inserted into the duct of an air heating system parallel to flow, and the differential height of the water column is measured. The flow velocity and the pressure rise at the tip of the Pitot-static probe are to be determined. √

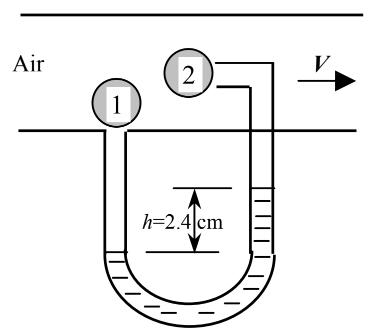
Assumptions **1**The flow through the duct is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas.

Properties We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m^3/kg·K.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad V=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}}$$

where the pressure rise at the tip of the Pitot-static probe is



$$P_2-P_1=\rho_w gh=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.024\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$=235\text{ N/m}^2=\mathbf{235\ Pa}$$

Also, $$\rho_{air}=\frac{P}{RT}=\frac{98\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(45+273\text{ K})}=1.074\text{ kg/m}^3$$

Substituting,

$$V_1=\sqrt{\frac{2(235\text{ N/m}^2)}{1.074\text{ kg/m}^3}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}=\mathbf{20.9\ m/s}$$

Discussion Note that the flow velocity in a pipe or duct can be measured easily by a Pitot-static probe by inserting the probe into the pipe or duct parallel to flow, and reading the differential pressure height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.

5-52 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a horizontal pipe attached to the bottom of the pool. The maximum discharge rate of water is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

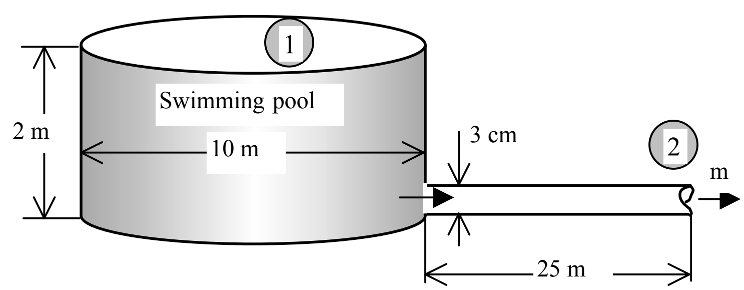
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{V_2^2}{2g} \quad \rightarrow \quad V_2=\sqrt{2gz_1}$$

The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$V_{2,\max}=\sqrt{2gh}=\sqrt{2(9.81\text{ m/s}^2)(2\text{ m})}=6.26\text{ m/s}$$

$$\dot{V}_{\max}=A_{pipe}V_{2,\max}=\frac{\pi D^2}{4}V_{2,\max}=\frac{\pi(0.03\text{ m})^2}{4}(6.26\text{ m/s})=0.00443\text{ m}^3\text{/s}=\mathbf{4.43\ L/s}$$

Discussion The result above is obtained by disregarding all frictional effects. The actual flow rate will be less because of frictional effects during flow and the resulting pressure drop. Also, the flow rate will gradually decrease as the water level in the pipe decreases.



5-53 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a horizontal pipe attached to the bottom of the pool. The time it will take to empty the tank is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of water in the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{2gz_1}$$

For generality, we express the water height in the pool at any time t by z, and the discharge velocity by $V_2=\sqrt{2gz}$. Note that water surface in the pool moves down as the pool drains, and thus z is a variable whose value changes from h at the beginning to *0* when the pool is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the pool by D_o. The flow rate of water from the pool is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V}=A_{\text{orifice}}V_2=\frac{\pi D^2}{4}\sqrt{2gz}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt=-\frac{\pi D_0^2}{4}dz \quad\rightarrow\quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz}}dz=-\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

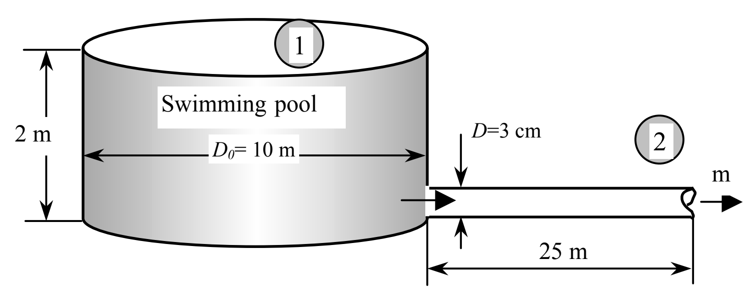
The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = h$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_1}^{0}z^{-1/2}dz \quad\rightarrow\quad t_f=-\frac{D_0^2}{D^2\sqrt{2g}}\left|\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2\sqrt{2g}}\sqrt{h}=\frac{D_0^2}{D^2}\sqrt{\frac{2h}{g}}$$

Substituting, the draining time of the pool will be

$$t_f=\frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})}{9.81\text{ m/s}^2}}=70{,}950\text{ s}=19.7\text{ h}$$

Discussion This is the minimum discharging time since it is obtained by neglecting all friction; the actual discharging time will be longer. Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

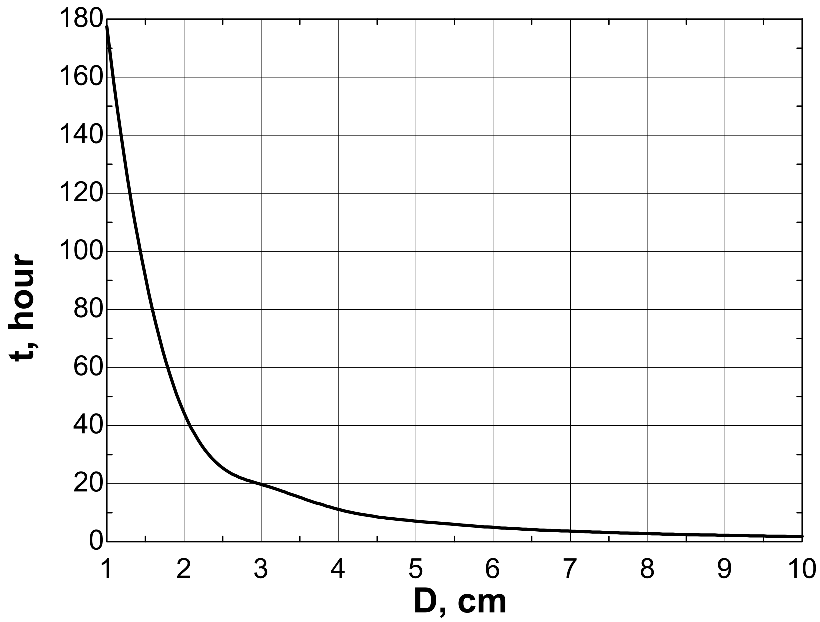


5-54 Problem 5-53 is reconsidered. The effect of the discharge pipe diameter on the time required to empty the pool completely as the diameter varies from 1 to 10 cm in increments of 1 cm is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
h=2 "m"
D=d_pipe/100 "m"
D_pool=10 "m"

V_initial=SQRT(2*g*h) "m/s"
Ac=pi*D^2/4
V_dot=Ac*V_initial*1000 "m3/s"
t=(D_pool/D)^2*SQRT(2*h/g)/3600 "hour"
```

Pipe diameter *D*, m	Discharge time *t*, h
1	177.4
2	44.3
3	19.7
4	11.1
5	7.1
6	4.9
7	3.6
8	2.8
9	2.2
10	1.8



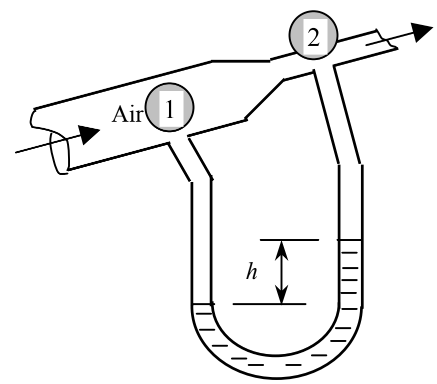
5-55 Air flows upward at a specified rate through an inclined pipe whose diameter is reduced through a reducer. The differential height between fluid levels of the two arms of a water manometer attached across the reducer is to be determined. √ Mod Feb'05.

Assumptions **1**The flow through the duct is steady, incompressible and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The effect of air column on the pressure change is negligible because of its low density. **3** The air flow is parallel to the entrance of each arm of the manometer, and thus no dynamic effects are involved.

Properties We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m^3/kg·K.

Analysis We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho_{\text{air}}\frac{V_2^2-V_1^2}{2}$$



where $$\rho_{air}=\frac{P}{RT}=\frac{110\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50+273\text{ K})}=1.19\text{ kg/m}^3$$

$$V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.045\text{ m}^3/\text{s}}{\pi(0.06\text{ m})^2/4}=15.9\text{ m/s}$$

$$V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.045\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4}=35.8\text{ m/s}$$

Substituting,

$$P_1-P_2=(1.19\text{ kg/m}^3)\frac{(35.8\text{ m/s})^2-(15.9\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)=612\text{ N/m}^2=612\text{ Pa}$$ The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P=\rho_{water}gh$ to be

$$h=\frac{P_1-P_2}{\rho_{water}g}=\frac{612\text{ N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)=0.0624\text{ m}=\mathbf{6.24\text{ cm}}$$

Discussion When the effect of air column on pressure change is considered, the pressure change becomes

$$\begin{aligned}P_1-P_2&=\frac{\rho_{air}(V_2^2-V_1^2)}{2}+\rho_{air}g(z_2-z_1)\\&=(1.19\text{ kg/m}^3)\left[\frac{(35.8\text{ m/s})^2-(15.9\text{ m/s})^2}{2}+(9.81\text{ m/s}^2)(0.2\text{ m})\right]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\\&=(612+2)\text{ N/m}^2=614\text{ N/m}^2=614\text{ Pa}\end{aligned}$$

This difference between the two results (612 and 614 Pa) is less than 1%. Therefore, the effect of air column on pressure change is, indeed, negligible as assumed. In other words, the pressure change of air in the duct is almost entirely due to velocity change, and the effect of elevation change is negligible.

Also, if we were to account for the Δz of air flow, then it would be more proper to account for the Δz of air in the manometer by using ρ_{water} - ρ_{air} instead of ρ_{water} when calculating h. The additional air column in the manometer tends to cancel out the change in pressure due to the elevation difference in the flow in this case.

5-56E Air is flowing through a venturi meter with known diameters and measured pressures. A relation for the flow rate is to be obtained, and its numerical value is to be determined.

Assumptions **1**The flow through the venturi is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the pressure can be assumed to be uniform at a given cross-section of the venturi meter (independent of elevation change). **3** The flow is horizontal (this assumption is usually unnecessary for gas flow.).

Properties The density of air is given to be $\rho = 0.075$ lbm/ft^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

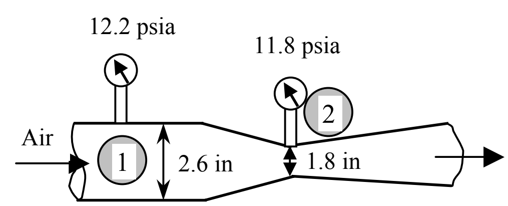
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\rho\frac{V_2^2-V_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad \rightarrow \quad A_1V_1=A_2V_2=\dot{V} \quad \rightarrow \quad V_1=\frac{\dot{V}}{A_1} \quad \text{and} \quad V_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(1.8/12\text{ ft})^2}{4}\sqrt{\frac{2(12.2-11.8)\text{ psi}}{(0.075\text{ lbm/ft}^3)[1-(1.8/2.6)^4]}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)}$$

$$= \mathbf{4.48\ ft^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_cA_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

5-57 The gage pressure in the water mains of a city at a particular location is given. It is to be determined if this main can serve water to neighborhoods that are at a given elevation relative to this location. √

Assumptions Water is incompressible and thus its density is constant.

Properties We take the density of water to be ρ = 1000 kg/m³.

Analysis Noting that the gage pressure at a dept of *h* in a fluid is given by $P_{gage} = \rho_{water} gh$, the height of a fluid column corresponding to a gage pressure of 400 kPa is determined to be

$$h = \frac{P_{gage}}{\rho_{water} g} = \frac{400{,}000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = 40.8 \text{ m}$$

which is less than 50 m. Therefore, this main **cannot** serve water to neighborhoods that are 50 m above this location.

Discussion Note that *h* must be much greater than 50 m for water to have enough pressure to serve the water needs of the neighborhood.

5-58 A hand-held bicycle pump with a liquid reservoir is used as an atomizer by forcing air at a high velocity through a small hole. The minimum speed that the piston must be moved in the cylinder to initiate the atomizing effect is to be determined. √

Assumptions **1**The flows of air and water are steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The liquid reservoir is open to the atmosphere. **4** The device is held horizontal. **5** The water velocity through the tube is low.

Properties We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m^3/kg·K.

Analysis We take point 1 at the exit of the hole, point 2 in air far from the hole on a horizontal line, point 3 at the exit of the tube in air stream (so that points 1 and 3 coincide), and point 4 at the free surface of the liquid in the reservoir ($P_2 = P_4 = P_{\text{atm}}$ and $P_1 = P_3$). We also take the level of the hole to be the reference level (so that $z_1 = z_2 = z_3$ =0 and z_4 = -h). Noting that $V_2 \cong V_3 \cong V_4 \cong 0$, the Bernoulli equation for the air and water streams becomes

Water (3-4): $$\frac{P_3}{\rho g}+\frac{V_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{V_4^2}{2g}+z_4 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+(-h) \quad\rightarrow\quad P_1-P_{atm}=-\rho_{\text{water}}gh \qquad (1)$$

Air (1-2): $$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_{atm}}{\rho g} \quad\rightarrow\quad V_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}} \qquad (2)$$

where

$$\rho_{air}=\frac{P}{RT}=\frac{95\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\text{ K})}=1.13\text{ kg/m}^3$$

Combining Eqs. (1) and (2) and substituting the numerical values,

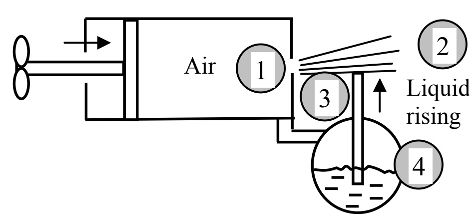
$$V_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}}=\sqrt{\frac{2\rho_{\text{water}}gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.1\text{ m})}{1.13\text{ kg/m}^3}}=41.7\text{ m/s}$$

Taking the flow of air to be steady and incompressible, the conservation of mass for air can be expressed as

$$\dot{V}_{\text{piston}}=\dot{V}_{\text{hole}} \rightarrow V_{\text{piston}}A_{\text{piston}}=V_{\text{hole}}A_{\text{hole}} \rightarrow V_{\text{piston}}=\frac{A_{\text{hole}}}{A_{\text{piston}}}V_{\text{hole}}=\frac{\pi D_{\text{hole}}^2/4}{\pi D_{\text{piston}}^2/4}V_1$$

Simplifying and substituting, the piston velocity is determined to be

$$V_{\text{piston}}=\left(\frac{D_{\text{hole}}}{D_{\text{piston}}}\right)^2V_1=\left(\frac{0.3\text{ cm}}{5\text{ cm}}\right)^2(41.7\text{ m/s})=\mathbf{0.15\ m/s}$$



Discussion In reality, the piston velocity must be higher to overcome the losses. Also, a lower piston velocity will do the job if the diameter of the hole is reduced.

5-59 The water height in an airtight pressurized tank is given. A hose pointing straight up is connected to the bottom of the tank. The maximum height to which the water stream could rise is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The friction between the water and air is negligible.

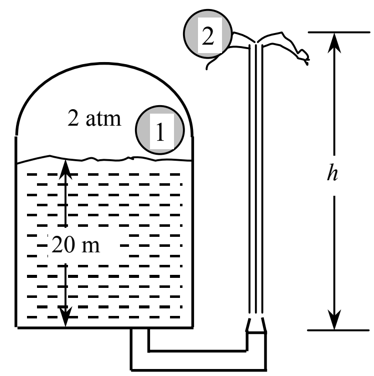
Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory $\mathbf{V}_2 = 0$, and atmospheric pressure pertains. Noting that $z_1 = 20$ m, $P_{1,\text{gage}} = 2$ atm, $P_2 = P_{\text{atm}}$, and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad z_2=\frac{P_1-P_{atm}}{\rho g}+z_1=\frac{P_{1,\text{gage}}}{\rho g}+z_1$$

Substituting,

$$z_2=\frac{2\text{ atm}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{101{,}325\text{ N/m}^2}{1\text{ atm}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+20=\mathbf{40.7\text{ m}}$$



Therefore, the water jet can rise as high as 40.7 m into the sky from the ground.

Discussion The result obtained by the Bernoulli equation represents the upper limit, and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.7 m (giving us an upper limit), and in all likelihood, the rise will be much less because of frictional losses.

5-60 A Pitot-static probe equipped with a water manometer is held parallel to air flow, and the differential height of the water column is measured. The flow velocity of air is to be determined. √

Assumptions **1**The flow of air is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the air column in the manometer can be ignored.

Properties We take the density of water to be ρ = 1000 kg/m³. The density of air is given to be 1.25 kg/m³.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad V_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}} \qquad (1)$$

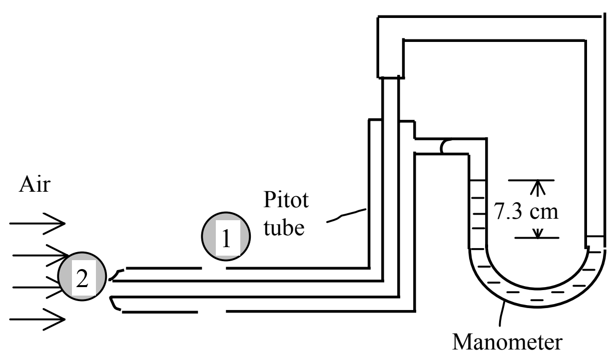
The pressure rise at the tip of the Pitot-static probe is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}}\, gh \qquad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$V_1=\sqrt{\frac{2\rho_{\text{water}}\,gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.073\text{ m})}{1.25\text{ kg/m}^3}}=\mathbf{33.8\ m/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a Pitot-static probe by inserting the probe into the pipe or duct parallel to flow, and reading the differential height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



5-61E A Pitot-static probe equipped with a differential pressure gage is used to measure the air velocity in a duct. For a given differential pressure reading, the flow velocity of air is to be determined. √

Assumptions The flow of air is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties The gas constant of air is R = 0.3704 psia·ft³/lbm·R.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad V_1=\sqrt{\frac{2(P_2-P_1)}{\rho}}$$

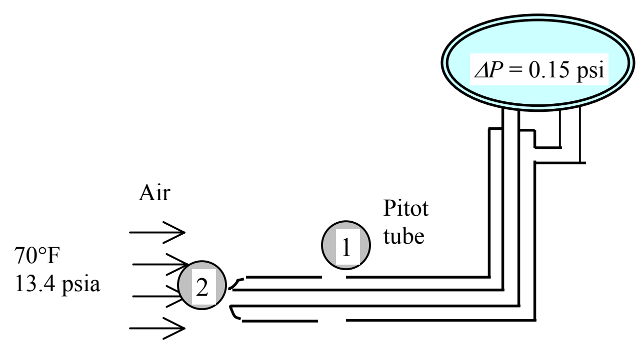
where

$$\rho=\frac{P}{RT}=\frac{13.4\ \text{psia}}{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(70+460\ \text{R})}=0.0683\ \text{lbm/ft}^3$$

Substituting the given values, the flow velocity is determined to be

$$V_1=\sqrt{\frac{2(0.15\ \text{psi})}{0.0683\ \text{lbm/ft}^3}\left(\frac{144\ \text{lbf/ft}^2}{1\ \text{psi}}\right)\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)}=\mathbf{143\ ft/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a Pitot-static probe by inserting the probe into the pipe or duct parallel to flow, and reading the pressure differential. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



5-62 In a power plant, water enters the nozzles of a hydraulic turbine at a specified pressure. The maximum velocity water can be accelerated to by the nozzles is to be determined. √

Assumptions **1**The flow of water is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Water enters the nozzle with a low velocity.

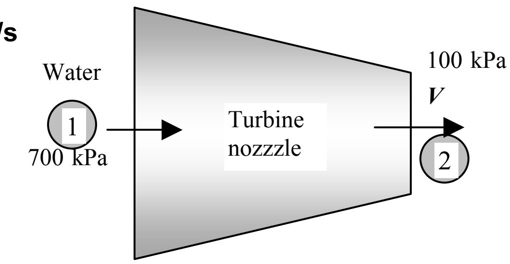
Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis We take points 1 and 2 at the inlet and exit of the nozzle, respectively. Noting that $V_1 \cong 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+\frac{V_2^2}{2g} \quad \rightarrow \quad V_2=\sqrt{\frac{2(P_1-P_{atm})}{\rho}}$$

Substituting the given values, the nozzle exit velocity is determined to be

$$V_1=\sqrt{\frac{2(700-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}=\mathbf{34.6\ m/s}$$



Discussion This is the maximum nozzle exit velocity, and the actual velocity will be less because of friction between water and the walls of the nozzle.