Angular Momentum Equation

6-44C The angular momentum equation is obtained by replacing B in the Reynolds transport theorem by the total angular momentum $\vec{H}_{sys}$, and b by the angular momentum per unit mass $\vec{r} \times \vec{V}$.

6-45C The angular momentum equation in this case is expressed as $I\vec{\alpha} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{\alpha}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

6-46C The angular momentum equation in this case is expressed as $I\vec{\alpha} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{\alpha}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

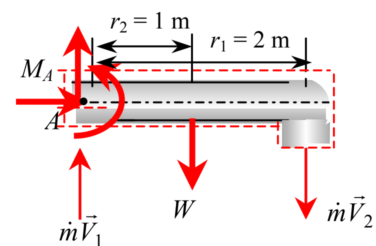
6-47 Water is pumped through a piping section. The moment acting on the elbow for the cases of downward and upward discharge is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **3** Effects of water falling down during upward discharge is disregarded. **4** Pipe outlet diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the entire pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the x and y coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $V_1 = V_2 = V$ since A_c = constant. The mass flow rate and the weight of the horizontal section of the pipe are



$$\dot{m} = \rho A_c V = (1000 \text{ kg/m}^3)[\pi(0.12 \text{ m})^2/4](4 \text{ m/s}) = 45.24 \text{ kg/s}$$

$$W = mg = (15 \text{ kg/m})(2 \text{ m})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 294.3 \text{ N/m}$$

(a) **Downward discharge**: To determine the moment acting on the pipe at point A, we need to take the moment of all forces and momentum flows about that point. This is a steady and uniform flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as $\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$ where r is the moment arm, all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative.

The free body diagram of the pipe section is given in the figure. Noting that the moments of all forces and momentum flows passing through point A are zero, the only force that will yield a moment about point A is the weight W of the horizontal pipe section, and the only momentum flow that will yield a moment is the outlet stream (both are negative since both moments are in the clockwise direction). Then the angular momentum equation about point A becomes

$$M_A - r_1 W = -r_2 \dot{m} V_2$$

Solving for M_A and substituting,

$$M_A = r_1 W - r_2 \dot{m} V_2 = (1 \text{ m})(294.3 \text{ N}) - (2 \text{ m})(45.54 \text{ kg/s})(4 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-70.0\ N\cdot m}$$

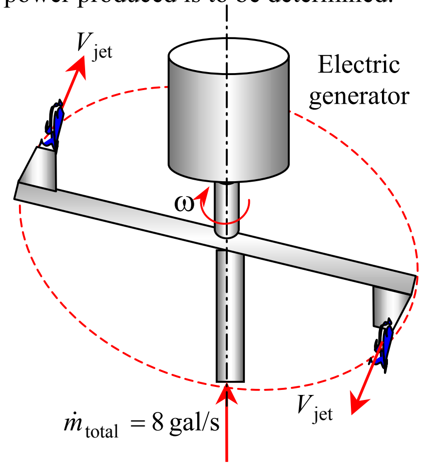
The negative sign indicates that the assumed direction for M_A is wrong, and should be reversed. Therefore, a moment of 70 N·m acts at the stem of the pipe in the clockwise direction.

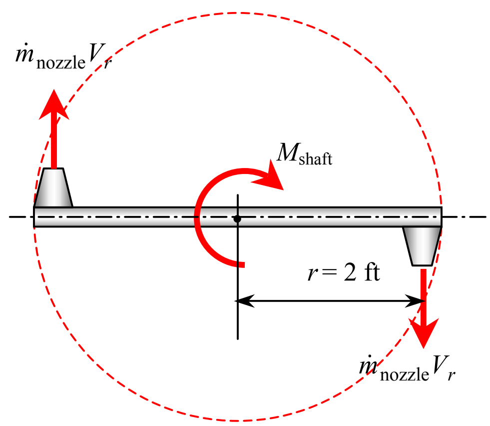
(b) **Upward discharge**: The moment due to discharge stream is positive in this case, and the moment acting on the pipe at point A is

$$M_A = r_1 W + r_2 \dot{m} V_2 = (1 \text{ m})(294.3 \text{ N}) + (2 \text{ m})(45.54 \text{ kg/s})(4 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{659\ N\cdot m}$$

Discussion Note direction of discharge can make a big difference in the moments applied on a piping system. This problem also shows the importance of accounting for the moments of momentums of flow streams when performing evaluating the stresses in pipe materials at critical cross-sections.

6-48E A two-armed sprinkler is used to generate electric power. For a specified flow rate and rotational speed, the power produced is to be determined.





Assumptions **1** The flow is cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Generator losses and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the two nozzles are identical, we have $\dot{m}_{\text{nozzle}} = \dot{m}/2$ or $\dot{V}_{\text{nozzle}} = \dot{V}_{total}/2$ since the density of water is constant. The average jet outlet velocity relative to the nozzle is

$$V_{\text{jet}} = \frac{\dot{V}_{\text{nozzle}}}{A_{\text{jet}}} = \frac{4\text{ gal/s}}{[\pi(0.5/12\text{ ft})^2/4]}\left(\frac{1\text{ ft}^3}{7.480\text{ gal}}\right) = 392.2\text{ ft/s}$$

The angular and tangential velocities of the nozzles are

$$\omega = 2\pi\dot{n} = 2\pi(250\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 26.18\text{ rad/s}$$

$$V_{\text{nozzle}} = r\omega = (2\text{ ft})(26.18\text{ rad/s}) = 52.36\text{ ft/s}$$

The velocity of water jet relative to the control volume (or relative to a fixed location on earth) is

$$V_r = V_{\text{jet}} - V_{\text{nozzle}} = 392.2 - 52.36 = 339.8\text{ ft/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{\text{shaft}} = -2r\dot{m}_{\text{nozzle}}V_r \qquad \text{or} \qquad M_{\text{shaft}} = r\dot{m}_{\text{total}}V_r$$

Substituting, the torque transmitted through the shaft is determined to be

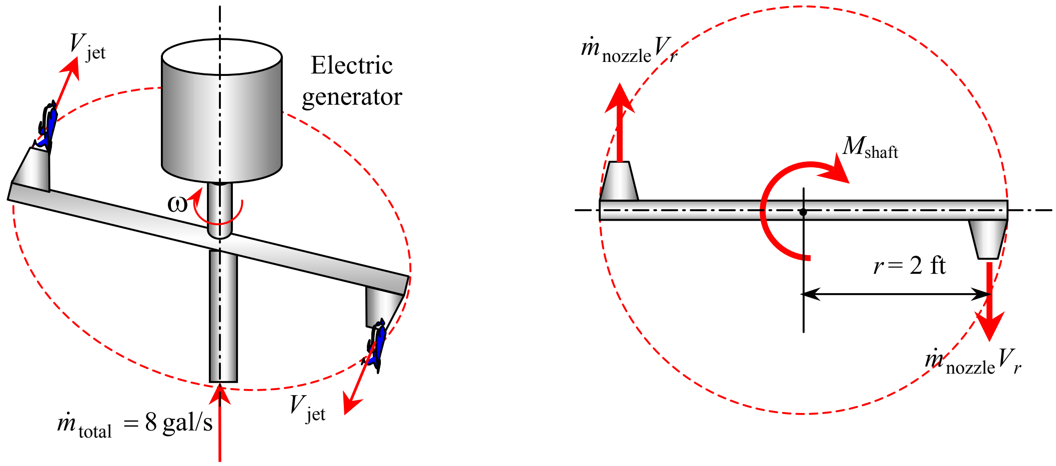
$$M_{\text{shaft}} = r\dot{m}_{\text{total}}V_r = (2\text{ ft})(66.74\text{ lbm/s})(339.8\text{ ft/s})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 1409\text{ lbf}\cdot\text{ft}$$

since $\dot{m}_{\text{total}} = \rho\dot{V}_{\text{total}} = (62.4\text{ lbm/ft}^3)(8/7.480\text{ ft}^3\text{/s}) = 66.74\text{ lbm/s}$. Then the power generated becomes

$$\dot{W} = 2\pi\dot{n}M_{\text{shaft}} = \omega M_{\text{shaft}} = (26.18\text{ rad/s})(1409\text{ lbf}\cdot\text{ft})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{50.0\ kW}$$

Therefore, this sprinkler-type turbine has the potential to produce 50 kW of power.

6-49E A two-armed sprinkler is used to generate electric power. For a specified flow rate and rotational speed, the moment acting on the rotating head when the head is stuck is to be determined.



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the two nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}/2$ or $\dot{V}_{nozzle} = \dot{V}_{total}/2$ since the density of water is constant. The average jet outlet velocity relative to the nozzle is

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{4 \text{ gal/s}}{[\pi(0.5/12 \text{ ft})^2/4]}\left(\frac{1 \text{ ft}^3}{7.480 \text{ gal}}\right) = 392.2 \text{ ft/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{shaft} = -2r\dot{m}_{nozzle}V_{jet} \quad \text{or} \quad M_{shaft} = r\dot{m}_{total}V_{jet}$$

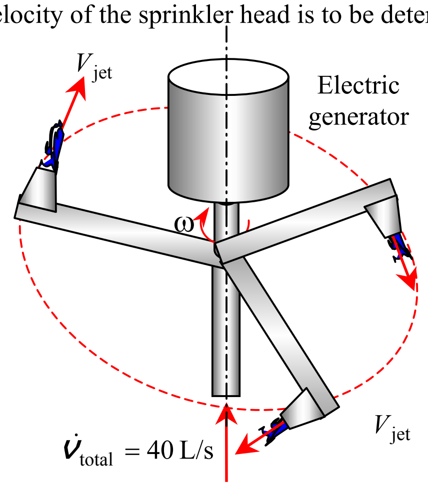
Substituting, the torque transmitted through the shaft is determined to be

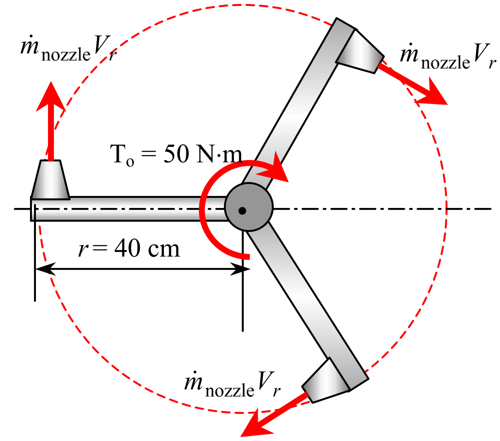
$$M_{shaft} = r\dot{m}_{total}V_{jet} = (2 \text{ ft})(66.74 \text{ lbm/s})(392.2 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{1626 \text{ lbf}\cdot\text{ft}}$$

since $\dot{m}_{total} = \rho\dot{V}_{total} = (62.4 \text{ lbm/ft}^3)(8/7.480 \text{ ft}^3\text{/s}) = 66.74 \text{ lbm/s}$.

Discussion When the sprinkler is stuck and thus the angular velocity is zero, the torque developed is maximum since $V_{nozzle} = 0$ and thus $V_r = V_{jet} = 392.2$ ft/s, giving $M_{shaft, max} = 1626 \text{ lbf}\cdot\text{ft}$. But the power generated is zero in this case since the shaft does not rotate.

6-50 A three-armed sprinkler is used to water a garden. For a specified flow rate and resistance torque, the angular velocity of the sprinkler head is to be determined.





Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the three nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}/3$ or $\dot{V}_{nozzle} = \dot{V}_{total}/3$ since the density of water is constant. The average jet outlet velocity relative to the nozzle and the mass flow rate are

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{40\ \text{L/s}}{3[\pi(0.012\ \text{m})^2/4]}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right) = 117.9\ \text{m/s}$$

$$\dot{m}_{total} = \rho\dot{V}_{total} = (1\ \text{kg/L})(40\ \text{L/s}) = 40\ \text{kg/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-T_0 = -3r\dot{m}_{nozzle}V_r \qquad \text{or} \qquad T_0 = r\dot{m}_{total}V_r$$

Solving for the relative velocity V_r and substituting,

$$V_r = \frac{T_0}{r\dot{m}_{total}} = \frac{50\ \text{N}\cdot\text{m}}{(0.40\ \text{m})(40\ \text{kg/s})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 3.1\ \text{m/s}$$

Then the tangential and angular velocity of the nozzles become

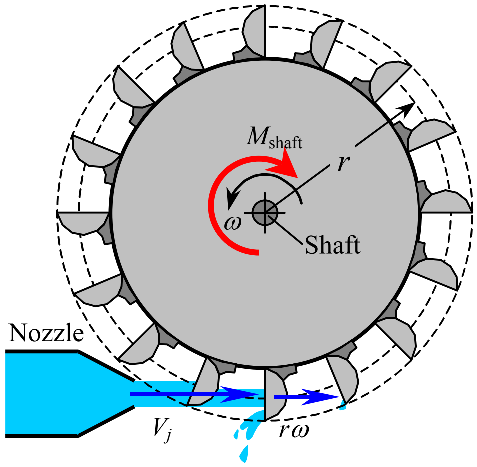
$$V_{nozzle} = V_{jet} - V_r = 117.9 - 3.1 = 114.8\ \text{m/s}$$

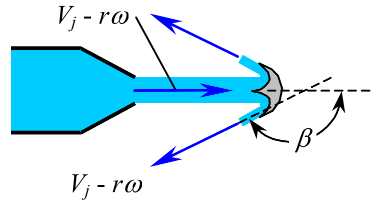
$$\omega = \frac{V_{nozzle}}{r} = \frac{114.8\ \text{m/s}}{0.4\ \text{m}} = \mathbf{287\ rad/s}$$

$$\dot{n} = \frac{\omega}{2\pi} = \frac{287\ \text{rad/s}}{2\pi}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{2741\ rpm}$$

Therefore, this sprinkler will rotate at 2741 revolutions per minute.

6-51 A Pelton wheel is considered for power generation in a hydroelectric power plant. A relation is to be obtained for power generation, and its numerical value is to be obtained.





Assumptions **1** The flow is uniform and cyclically steady. **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Generator losses and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis The tangential velocity of buckets corresponding to an angular velocity of $\omega = 2\pi\dot{n}$ is $V_{bucket} = r\omega$. Then the relative velocity of the jet (relative to the bucket) becomes

$$V_r = V_j - V_{bucket} = V_j - r\omega$$

We take the imaginary disk that contains the Pelton wheel as the control volume. The inlet velocity of the fluid into this control volume is V_r, and the component of outlet velocity normal to the moment arm is $V_r\cos\beta$. The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{shaft} = r\dot{m}V_r\cos\beta - r\dot{m}V_r \quad \text{or} \quad M_{shaft} = r\dot{m}V_r(1-\cos\beta) = r\dot{m}(V_j - r\omega)(1-\cos\beta)$$

Noting that $\dot{W}_{shaft} = 2\pi\dot{n}M_{shaft} = \omega M_{shaft}$ and $\dot{m} = \rho\dot{V}$, the power output of a Pelton turbine becomes

$$\dot{W}_{shaft} = \rho\dot{V}r\omega(V_j - r\omega)(1-\cos\beta)$$

which is the desired relation. For given values, the power output is determined to be

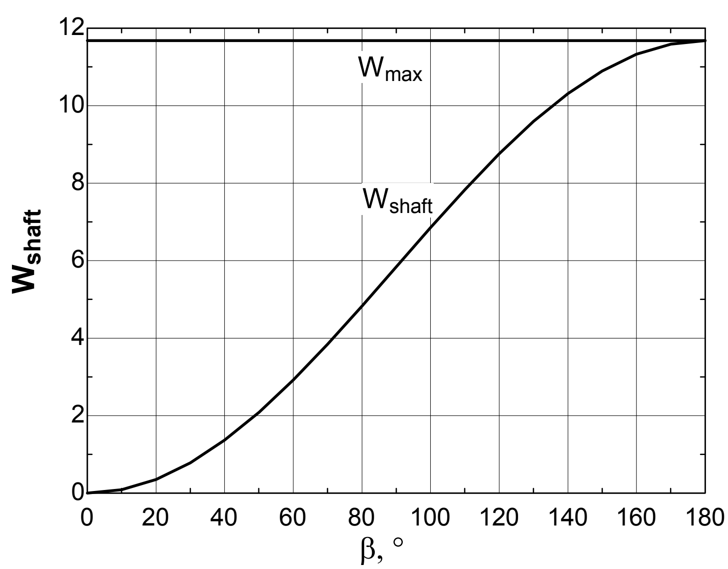
$$\dot{W}_{shaft} = (1000\ \text{kg/m}^3)(10\ \text{m}^3/\text{s})(2\ \text{m})(15.71\ \text{rad/s})(50 - 2\times 15.71\ \text{m/s})(1 - \cos 160°)\left(\frac{1\ \text{MW}}{10^6\ \text{N}\cdot\text{m/s}}\right) = \mathbf{11.3\ MW}$$

where $$\omega = 2\pi\dot{n} = 2\pi(150\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 15.71\ \text{rad/s}$$

6-52 Problem 6-51 is reconsidered. The effect of β on the power generation as β varies from 0° to 180° is to be determined, and the fraction of power loss at 160° is to be assessed.

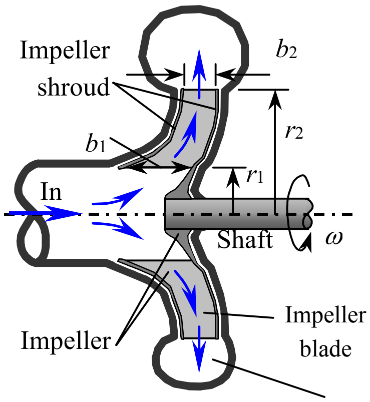
```
rho=1000 "kg/m3"
r=2 "m"
V_dot=10 "m3/s"
V_jet=50 "m/s"
n_dot=150 "rpm"
omega=2*pi*n_dot/60
V_r=V_jet-r*omega
m_dot=rho*V_dot
W_dot_shaft=m_dot*omega*r*V_r*(1-cos(Beta))/1E6 "MW"
W_dot_max=m_dot*omega*r*V_r*2/1E6 "MW"
Effectiveness=W_dot_shaft/W_dot_max
```

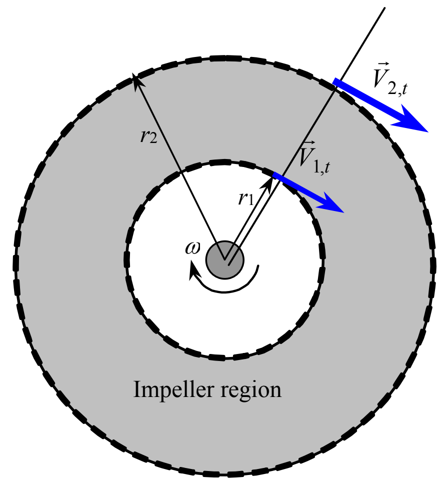
Angle, β°	Max power, $\dot{W}_{max}$, MW	Actual power, $\dot{W}_{shaft}$, MW	Effectiveness, η
0	11.7	0.00	0.000
10	11.7	0.09	0.008
20	11.7	0.35	0.030
30	11.7	0.78	0.067
40	11.7	1.37	0.117
50	11.7	2.09	0.179
60	11.7	2.92	0.250
70	11.7	3.84	0.329
80	11.7	4.82	0.413
90	11.7	5.84	0.500
100	11.7	6.85	0.587
110	11.7	7.84	0.671
120	11.7	8.76	0.750
130	11.7	9.59	0.821
140	11.7	10.31	0.883
150	11.7	10.89	0.933
160	11.7	11.32	0.970
170	11.7	11.59	0.992
180	11.7	11.68	1.000



The effectiveness of Pelton wheel for β=160° is 0.97. Therefore, at this angle, only 3% of power is lost.

6-53 A centrifugal blower is used to deliver atmospheric air. For a given angular speed and power input, the volume flow rate of air is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible. **3** The tangential components of air velocity at the inlet and the outlet are said to be equal to the impeller velocity at respective locations.

Properties The gas constant of air is 0.287 kPa·m³/kg·K. The density of air at 20°C and 95 kPa is

$$\rho = \frac{P}{RT} = \frac{95 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})} = 1.130 \text{ kg/m}^3$$

Analysis In the idealized case of the tangential fluid velocity being equal to the blade angular velocity both at the inlet and the outlet, we have $V_{1,t} = \omega r_1$ and $V_{2,t} = \omega r_2$, and the torque is expressed as

$$\mathrm{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = \dot{m}\omega(r_2^2 - r_1^2) = \rho\dot{V}\omega(r_2^2 - r_1^2)$$

where the angular velocity is

$$\omega = 2\pi\dot{n} = 2\pi(800 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 83.78 \text{ rad/s}$$

Then the shaft power becomes

$$\dot{W}_{\text{shaft}} = \omega\mathrm{T}_{\text{shaft}} = \rho\dot{V}\omega^2(r_2^2 - r_1^2)$$

Solving for $\dot{V}$ and substituting, the volume flow rate of air is determined to

$$\dot{V} = \frac{\dot{W}_{\text{shaft}}}{\rho\omega^2(r_2^2 - r_1^2)} = \frac{120 \text{ N}\cdot\text{m/s}}{(1.130 \text{ kg/m}^3)(83.78 \text{ rad/s})^2[(0.30 \text{ m})^2 - (0.15 \text{ m})^2]}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = \mathbf{0.224 \text{ m}^3/\text{s}}$$

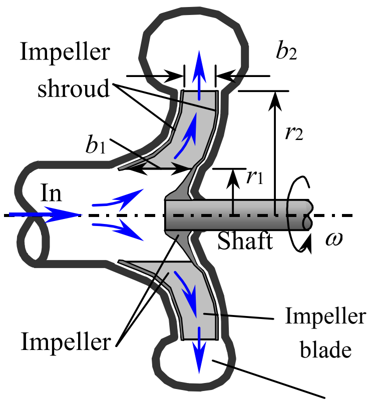
The normal velocity components at the inlet and the outlet are

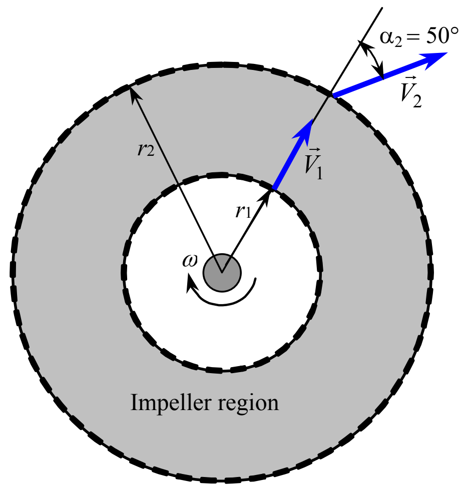
$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.224 \text{ m}^3/\text{s}}{2\pi(0.15 \text{ m})(0.061 \text{ m})} = \mathbf{3.90 \text{ m/s}}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.224 \text{ m}^3/\text{s}}{2\pi(0.30 \text{ m})(0.034 \text{ m})} = \mathbf{3.50 \text{ m/s}}$$

Discussion Note that the irreversible losses are not considered in analysis. In reality, the flow rate and the normal components of velocities will be smaller.

6-54 A centrifugal blower is used to deliver atmospheric air at a specified rate and angular speed. The minimum power consumption of the blower is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties The density of air is given to be 1.25 kg/m^3.

Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.20\ \text{m})(0.082\ \text{m})} = 6.793\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.45\ \text{m})(0.056\ \text{m})} = 4.421\ \text{m/s}$$

The tangential components of absolute velocity are:

$\alpha_1 = 0°$: $\quad V_{1,t} = V_{1,n}\tan\alpha_1 = 0$

$\alpha_2 = 60°$: $\quad V_{2,t} = V_{2,n}\tan\alpha_1 = (4.421\ \text{m/s})\tan 50° = 5.269\ \text{m/s}$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(700\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 73.30\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1.25\ \text{kg/m}^3)(0.7\ \text{m}^3/\text{s}) = 0.875\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$\text{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (0.875\ \text{kg/s})[(0.45\ \text{m})(5.269\ \text{m/s}) - 0]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 2.075\ \text{N}\cdot\text{m}$$

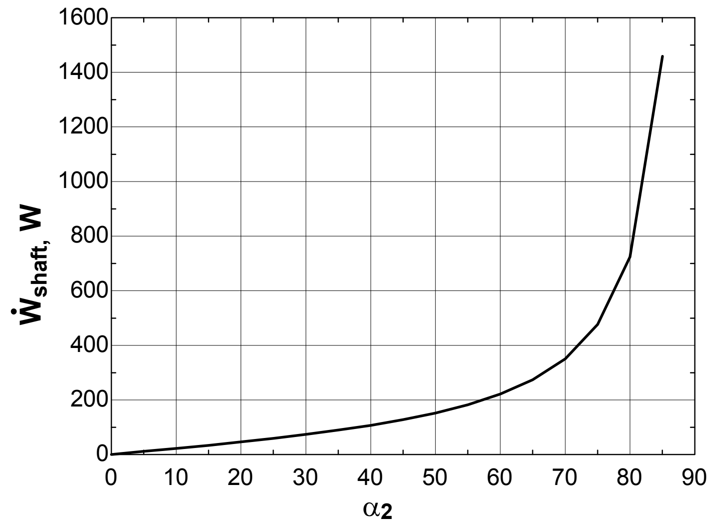
Then the shaft power becomes

$$\dot{W} = \omega\text{T}_{\text{shaft}} = (73.30\ \text{rad/s})(2.075\ \text{N}\cdot\text{m})\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{152\ W}$$

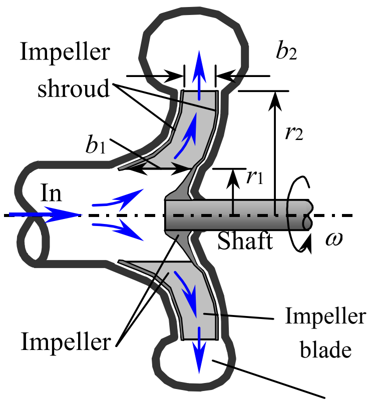
6-55 Problem 6-54 is reconsidered. The effect of discharge angle α_2 on the minimum power input requirements as α_2 varies from 0° to 85° in increments of 5° is to be investigated.

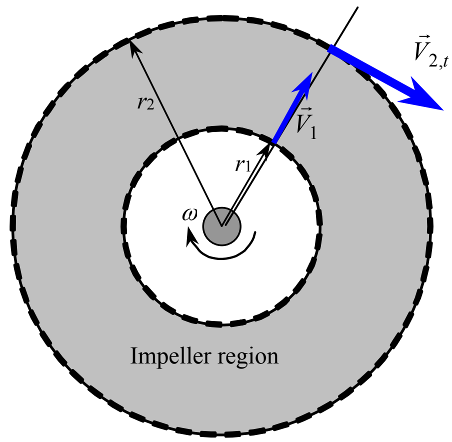
```
rho=1.25 "kg/m3"
r1=0.20 "m"
b1=0.082 "m"
r2=0.45 "m"
b2=0.056 "m"
V_dot=0.70 "m3/s"
V1n=V_dot/(2*pi*r1*b1) "m/s"
V2n=V_dot/(2*pi*r2*b2) "m/s"
Alpha1=0
V1t=V1n*tan(Alpha1) "m/s"
V2t=V2n*tan(Alpha2) "m/s"
n_dot=700 "rpm"
omega=2*pi*n_dot/60 "rad/s"
m_dot=rho*V_dot "kg/s"
T_shaft=m_dot*(r2*V2t-r1*V1t) "Nm"
W_dot_shaft=omega*T_shaft "W"
```

Angle, α_2°	$V_{2,t}$, m/s	Torque, T_{shaft}, Nm	Shaft power, $\dot{W}_{shaft}$, W
0	0.00	0.00	0
5	0.39	0.15	11
10	0.78	0.31	23
15	1.18	0.47	34
20	1.61	0.63	46
25	2.06	0.81	60
30	2.55	1.01	74
35	3.10	1.22	89
40	3.71	1.46	107
45	4.42	1.74	128
50	5.27	2.07	152
55	6.31	2.49	182
60	7.66	3.02	221
65	9.48	3.73	274
70	12.15	4.78	351
75	16.50	6.50	476
80	25.07	9.87	724
85	50.53	19.90	1459



6-56E Water enters the impeller of a centrifugal pump radially at a specified flow rate and angular speed. The torque applied to the impeller is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis Water enters the impeller normally, and thus $V_{1,t} = 0$. The tangential component of fluid velocity at the outlet is given to be $V_{2,t} = 180$ ft/s. The inlet radius r_1 is unknown, but the outlet radius is given to be r_2 = 1 ft. The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(500 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 52.36 \text{ rad/s}$$

The mass flow rate is

$$\dot{m} = \rho\dot{V} = (62.4 \text{ lbm/ft}^3)(80/60 \text{ ft}^3\text{/s}) = 83.2 \text{ lbm/s}$$

Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

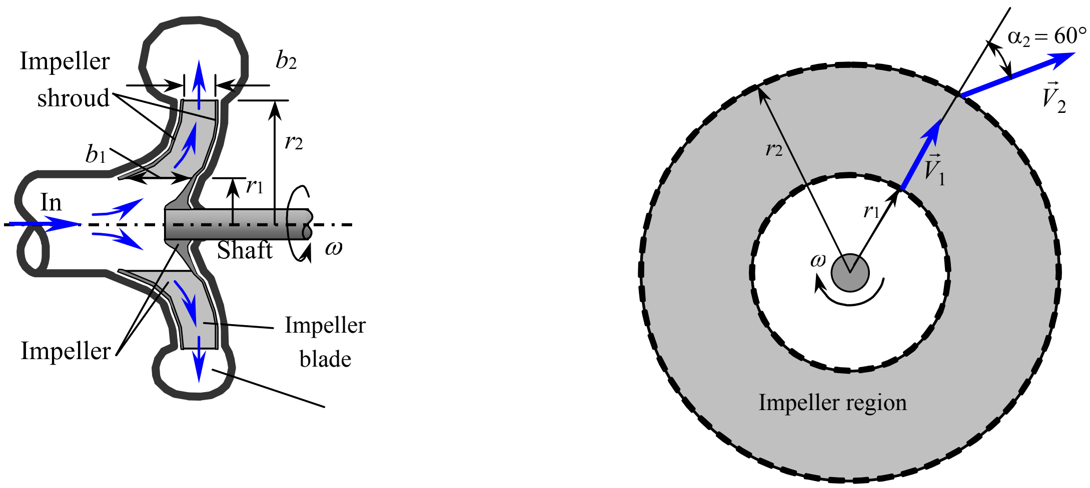
$$\text{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (83.2 \text{ lbm/s})[(1 \text{ ft})(180 \text{ ft/s}) - 0]\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{465\ lbf\cdot ft}$$

Discussion This shaft power input corresponding to this torque is

$$\dot{W} = 2\pi\dot{n}T_{\text{shaft}} = \omega T_{\text{shaft}} = (52.36 \text{ rad/s})(465 \text{ lbf}\cdot\text{ft})\left(\frac{1 \text{ kW}}{737.56 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{33.0\ kW}$$

Therefore, the minimum power input to this pump should be 33 kW.

6-57 A centrifugal pump is used to supply water at a specified rate and angular speed. The minimum power consumption of the pump is to be determined.



Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.13\ \text{m})(0.080\ \text{m})} = 2.296\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.30\ \text{m})(0.035\ \text{m})} = 2.274\ \text{m/s}$$

The tangential components of absolute velocity are:

$\alpha_1 = 0°$: $\quad V_{1,t} = V_{1,n}\tan\alpha_1 = 0$

$\alpha_2 = 60°$: $\quad V_{2,t} = V_{2,n}\tan\alpha_1 = (2.274\ \text{m/s})\tan 60° = 3.938\ \text{m/s}$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(1200\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 125.7\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 150\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$\text{T}_{\text{shaft}} = \dot{m}(r_2V_{2,t} - r_1V_{1,t}) = (150\ \text{kg/s})[(0.30\ \text{m})(3.938\ \text{m/s}) - 0]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 177.2\ \text{kN}\cdot\text{m}$$

Then the shaft power becomes

$$\dot{W} = \omega\text{T}_{\text{shaft}} = (125.7\ \text{rad/s})(177.2\ \text{kN}\cdot\text{m})\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{22.3\ kW}$$

Discussion Note that the irreversible losses are not considered in analysis. In reality, the required power input will be larger.