

## Chapter 3

## PRESSURE AND FLUID STATICS

**Pressure, Manometer, and Barometer**

**3-1C** The pressure relative to the atmospheric pressure is called the *gage pressure*, and the pressure relative to an absolute vacuum is called *absolute pressure*.

**3-2C** The atmospheric air pressure which is the external pressure exerted on the skin decreases with increasing elevation. Therefore, the pressure is lower at higher elevations. As a result, the difference between the blood pressure in the veins and the air pressure outside increases. This pressure imbalance may cause some thin-walled veins such as the ones in the nose to burst, causing bleeding. The shortness of breath is caused by the lower air density at higher elevations, and thus lower amount of oxygen per unit volume.

**3-3C** No, the absolute pressure in a liquid of constant density does not double when the depth is doubled. It is the *gage pressure* that doubles when the depth is doubled.

**3-4C** If the lengths of the sides of the tiny cube suspended in water by a string are very small, the magnitudes of the pressures on all sides of the cube will be the same.

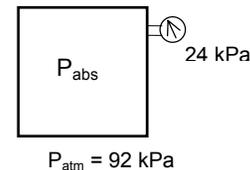
**3-5C** *Pascal's principle* states that *the pressure applied to a confined fluid increases the pressure throughout by the same amount*. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal's principle is the operation of the hydraulic car jack.

**3-6C** The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.

**3-7** The pressure in a vacuum chamber is measured by a vacuum gage. The absolute pressure in the chamber is to be determined.

**Analysis** The absolute pressure in the chamber is determined from

$$P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 92 - 24 = \mathbf{68 \text{ kPa}}$$



**3-8E** The pressure in a tank is measured with a manometer by measuring the differential height of the manometer fluid. The absolute pressure in the tank is to be determined for the cases of the manometer arm with the higher and lower fluid level being attached to the tank .

**Assumptions** The fluid in the manometer is incompressible.

**Properties** The specific gravity of the fluid is given to be  $SG = 1.25$ . The density of water at  $32^\circ\text{F}$  is  $62.4 \text{ lbm/ft}^3$ .

**Analysis** The density of the fluid is obtained by multiplying its specific gravity by the density of water,

$$\rho = SG \times \rho_{H_2O} = (1.25)(62.4 \text{ lbm/ft}^3) = 78.0 \text{ lbm/ft}^3$$

The pressure difference corresponding to a differential height of 28 in between the two arms of the manometer is

$$\Delta P = \rho g h = (78 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)(28/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = 1.26 \text{ psia}$$

Then the absolute pressures in the tank for the two cases become:

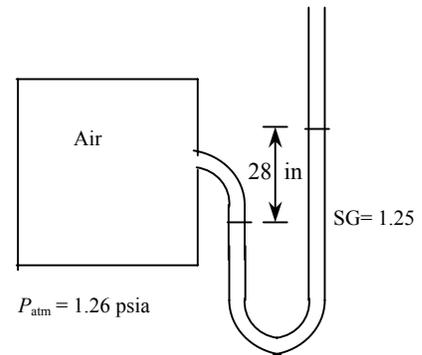
(a) The fluid level in the arm attached to the tank is higher (vacuum):

$$P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 12.7 - 1.26 = \mathbf{11.44 \text{ psia}}$$

(b) The fluid level in the arm attached to the tank is lower:

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 12.7 + 1.26 = \mathbf{13.96 \text{ psia}}$$

**Discussion** Note that we can determine whether the pressure in a tank is above or below atmospheric pressure by simply observing the side of the manometer arm with the higher fluid level.



**3-9** The pressure in a pressurized water tank is measured by a multi-fluid manometer. The gage pressure of air in the tank is to be determined.

**Assumptions** The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

**Properties** The densities of mercury, water, and oil are given to be 13,600, 1000, and 850 kg/m<sup>3</sup>, respectively.

**Analysis** Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach point 2, and setting the result equal to  $P_{atm}$  since the tube is open to the atmosphere gives

$$P_1 + \rho_{\text{water}}gh_1 + \rho_{\text{oil}}gh_2 - \rho_{\text{mercury}}gh_3 = P_{atm}$$

Solving for  $P_1$ ,

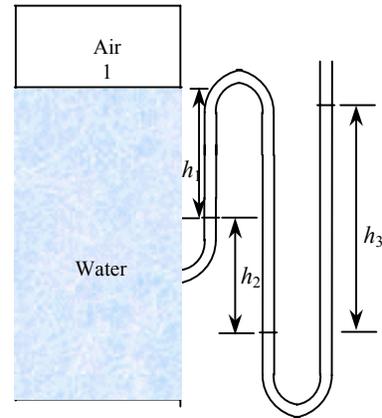
$$P_1 = P_{atm} - \rho_{\text{water}}gh_1 - \rho_{\text{oil}}gh_2 + \rho_{\text{mercury}}gh_3$$

or,

$$P_1 - P_{atm} = g(\rho_{\text{mercury}}h_3 - \rho_{\text{water}}h_1 - \rho_{\text{oil}}h_2)$$

Noting that  $P_{1,\text{gage}} = P_1 - P_{atm}$  and substituting,

$$\begin{aligned} P_{1,\text{gage}} &= (9.81 \text{ m/s}^2)[(13,600 \text{ kg/m}^3)(0.46 \text{ m}) - (1000 \text{ kg/m}^3)(0.2 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.3 \text{ m})] \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{56.9 \text{ kPa}} \end{aligned}$$



**Discussion** Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

**3-10** The barometric reading at a location is given in height of mercury column. The atmospheric pressure is to be determined.

**Properties** The density of mercury is given to be 13,600 kg/m<sup>3</sup>.

**Analysis** The atmospheric pressure is determined directly from

$$\begin{aligned} P_{atm} &= \rho gh \\ &= (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.750 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{100.1 \text{ kPa}} \end{aligned}$$

**3-11** The gage pressure in a liquid at a certain depth is given. The gage pressure in the same liquid at a different depth is to be determined.

**Assumptions** The variation of the density of the liquid with depth is negligible.

**Analysis** The gage pressure at two different depths of a liquid can be expressed as

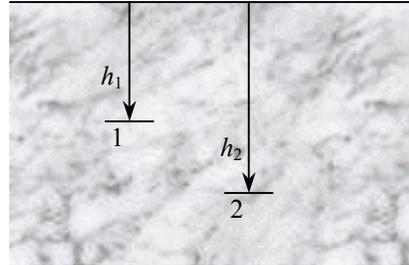
$$P_1 = \rho gh_1 \quad \text{and} \quad P_2 = \rho gh_2$$

Taking their ratio,

$$\frac{P_2}{P_1} = \frac{\rho gh_2}{\rho gh_1} = \frac{h_2}{h_1}$$

Solving for  $P_2$  and substituting gives

$$P_2 = \frac{h_2}{h_1} P_1 = \frac{12 \text{ m}}{3 \text{ m}} (28 \text{ kPa}) = \mathbf{112 \text{ kPa}}$$



**Discussion** Note that the gage pressure in a given fluid is proportional to depth.

**3-12** The absolute pressure in water at a specified depth is given. The local atmospheric pressure and the absolute pressure at the same depth in a different liquid are to be determined.

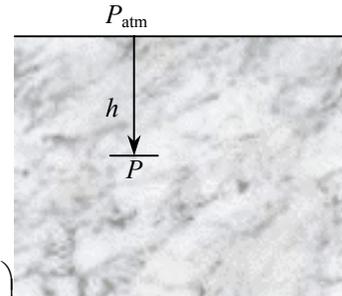
**Assumptions** The liquid and water are incompressible.

**Properties** The specific gravity of the fluid is given to be  $SG = 0.85$ . We take the density of water to be  $1000 \text{ kg/m}^3$ . Then density of the liquid is obtained by multiplying its specific gravity by the density of water,

$$\rho = SG \times \rho_{H_2O} = (0.85)(1000 \text{ kg/m}^3) = 850 \text{ kg/m}^3$$

**Analysis** (a) Knowing the absolute pressure, the atmospheric pressure can be determined from

$$\begin{aligned} P_{atm} &= P - \rho gh \\ &= (145 \text{ kPa}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{96.0 \text{ kPa}} \end{aligned}$$



(b) The absolute pressure at a depth of 5 m in the other liquid is

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (96.0 \text{ kPa}) + (850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{137.7 \text{ kPa}} \end{aligned}$$

**Discussion** Note that at a given depth, the pressure in the lighter fluid is lower, as expected.

**3-13E** It is to be shown that  $1 \text{ kgf/cm}^2 = 14.223 \text{ psi}$ .

**Analysis** Noting that  $1 \text{ kgf} = 9.80665 \text{ N}$ ,  $1 \text{ N} = 0.22481 \text{ lbf}$ , and  $1 \text{ in} = 2.54 \text{ cm}$ , we have

$$1 \text{ kgf} = 9.80665 \text{ N} = (9.80665 \text{ N}) \left( \frac{0.22481 \text{ lbf}}{1 \text{ N}} \right) = 2.20463 \text{ lbf}$$

and

$$1 \text{ kgf/cm}^2 = 2.20463 \text{ lbf/cm}^2 = (2.20463 \text{ lbf/cm}^2) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 = 14.223 \text{ lbf/in}^2 = \mathbf{14.223 \text{ psi}}$$

**3-14E** The weight and the foot imprint area of a person are given. The pressures this man exerts on the ground when he stands on one and on both feet are to be determined.

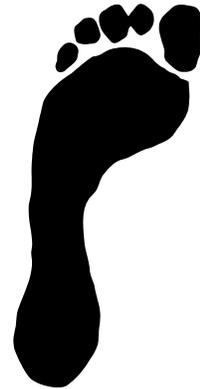
**Assumptions** The weight of the person is distributed uniformly on foot imprint area.

**Analysis** The weight of the man is given to be 200 lbf. Noting that pressure is force per unit area, the pressure this man exerts on the ground is

(a) On one foot: 
$$P = \frac{W}{A} = \frac{200 \text{ lbf}}{36 \text{ in}^2} = 5.56 \text{ lbf/in}^2 = \mathbf{5.56 \text{ psi}}$$

(a) On both feet: 
$$P = \frac{W}{2A} = \frac{200 \text{ lbf}}{2 \times 36 \text{ in}^2} = 2.78 \text{ lbf/in}^2 = \mathbf{2.78 \text{ psi}}$$

**Discussion** Note that the pressure exerted on the ground (and on the feet) is reduced by half when the person stands on both feet.



**3-15** The mass of a woman is given. The minimum imprint area per shoe needed to enable her to walk on the snow without sinking is to be determined.

**Assumptions** **1** The weight of the person is distributed uniformly on the imprint area of the shoes. **2** One foot carries the entire weight of a person during walking, and the shoe is sized for walking conditions (rather than standing). **3** The weight of the shoes is negligible.

**Analysis** The mass of the woman is given to be 70 kg. For a pressure of 0.5 kPa on the snow, the imprint area of one shoe must be

$$A = \frac{W}{P} = \frac{mg}{P} = \frac{(70 \text{ kg})(9.81 \text{ m/s}^2)}{0.5 \text{ kPa}} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = \mathbf{1.37 \text{ m}^2}$$

**Discussion** This is a very large area for a shoe, and such shoes would be impractical to use. Therefore, some sinking of the snow should be allowed to have shoes of reasonable size.



**3-16** The vacuum pressure reading of a tank is given. The absolute pressure in the tank is to be determined.

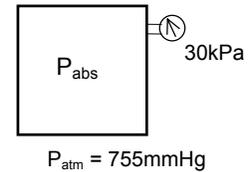
**Properties** The density of mercury is given to be  $\rho = 13,590 \text{ kg/m}^3$ .

**Analysis** The atmospheric (or barometric) pressure can be expressed as

$$\begin{aligned} P_{atm} &= \rho gh \\ &= (13,590 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= 100.6 \text{ kPa} \end{aligned}$$

Then the absolute pressure in the tank becomes

$$P_{abs} = P_{atm} - P_{vac} = 100.6 - 30 = \mathbf{70.6 \text{ kPa}}$$



**3-17E** A pressure gage connected to a tank reads 50 psi. The absolute pressure in the tank is to be determined.

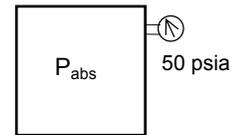
**Properties** The density of mercury is given to be  $\rho = 848.4 \text{ lbf/ft}^3$ .

**Analysis** The atmospheric (or barometric) pressure can be expressed as

$$\begin{aligned} P_{atm} &= \rho gh \\ &= (848.4 \text{ lbf/ft}^3)(32.174 \text{ ft/s}^2)(29.1/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbf} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= 14.29 \text{ psia} \end{aligned}$$

Then the absolute pressure in the tank is

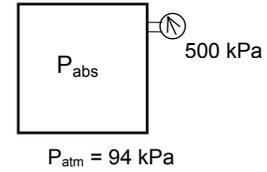
$$P_{abs} = P_{gage} + P_{atm} = 50 + 14.29 = \mathbf{64.29 \text{ psia}}$$



**3-18** A pressure gage connected to a tank reads 500 kPa. The absolute pressure in the tank is to be determined.

**Analysis** The absolute pressure in the tank is determined from

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 500 + 94 = \mathbf{594 \text{ kPa}}$$



**3-19** A mountain hiker records the barometric reading before and after a hiking trip. The vertical distance climbed is to be determined.

**Assumptions** The variation of air density and the gravitational acceleration with altitude is negligible.

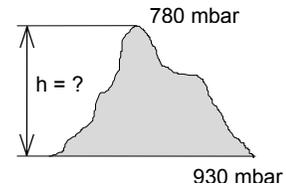
**Properties** The density of air is given to be  $\rho = 1.20 \text{ kg/m}^3$ .

**Analysis** Taking an air column between the top and the bottom of the mountain and writing a force balance per unit base area, we obtain

$$W_{\text{air}} / A = P_{\text{bottom}} - P_{\text{top}}$$

$$(\rho g h)_{\text{air}} = P_{\text{bottom}} - P_{\text{top}}$$

$$(1.20 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ bar}}{100,000 \text{ N/m}^2} \right) = (0.930 - 0.780) \text{ bar}$$



It yields  $h = \mathbf{1274 \text{ m}}$

which is also the distance climbed.

**Chapter 3 Pressure and Fluid Statics**

**3-20** A barometer is used to measure the height of a building by recording reading at the bottom and at the top of the building. The height of the building is to be determined.

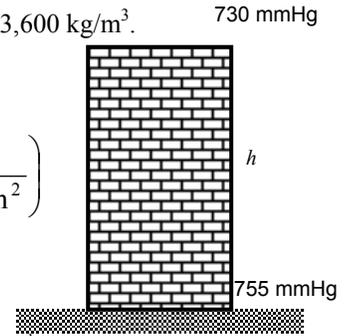
**Assumptions** The variation of air density with altitude is negligible.

**Properties** The density of air is given to be  $\rho = 1.18 \text{ kg/m}^3$ . The density of mercury is  $13,600 \text{ kg/m}^3$ .

**Analysis** Atmospheric pressures at the top and at the bottom of the building are

$$\begin{aligned} P_{\text{top}} &= (\rho g h)_{\text{top}} \\ &= (13,600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.730 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= 97.36 \text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\text{bottom}} &= (\rho g h)_{\text{bottom}} \\ &= (13,600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= 100.70 \text{ kPa} \end{aligned}$$



Taking an air column between the top and the bottom of the building and writing a force balance per unit base area, we obtain

$$\begin{aligned} W_{\text{air}} / A &= P_{\text{bottom}} - P_{\text{top}} \\ (\rho g h)_{\text{air}} &= P_{\text{bottom}} - P_{\text{top}} \\ (1.18 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(h) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) &= (100.70 - 97.36) \text{ kPa} \end{aligned}$$

It yields  $h = \mathbf{288.6 \text{ m}}$

which is also the height of the building.

**3-21** Problem 3-20 is reconsidered. The entire EES solution is to be printed out, including the numerical results with proper units."

```
P_bottom=755"[mmHg]"
P_top=730"[mmHg]"
g=9.807 "[m/s^2]" "local acceleration of gravity at sea level"
rho=1.18"[kg/m^3]"
DELTAP_abs=(P_bottom-P_top)*CONVERT('mmHg','kPa')"[kPa]" "Delta P reading from
the barometers, converted from mmHg to kPa."
DELTAP_h=rho*g*h/1000 "[kPa]" "Equ. 1-16. Delta P due to the air fluid column
height, h, between the top and bottom of the building."
"Instead of dividing by 1000 Pa/kPa we could have multiplied rho*g*h by the EES function,
CONVERT('Pa','kPa')"
DELTAP_abs=DELTAP_h
```

**SOLUTION**

Variables in Main

DELTAP\_abs=3.333 [kPa]

DELTAP\_h=3.333 [kPa]

g=9.807 [m/s^2]

h=288 [m]

P\_bottom=755 [mmHg]

P\_top=730 [mmHg]

rho=1.18 [kg/m^3]

**3-22** A diver is moving at a specified depth from the water surface. The pressure exerted on the surface of the diver by water is to be determined.

**Assumptions** The variation of the density of water with depth is negligible.

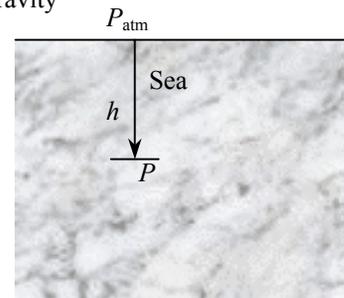
**Properties** The specific gravity of sea water is given to be  $SG = 1.03$ . We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The density of the sea water is obtained by multiplying its specific gravity by the density of water which is taken to be  $1000 \text{ kg/m}^3$ :

$$\rho = SG \times \rho_{H_2O} = (1.03)(1000 \text{ kg/m}^3) = 1030 \text{ kg/m}^3$$

The pressure exerted on a diver at 30 m below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (101 \text{ kPa}) + (1030 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(30 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{404.0 \text{ kPa}} \end{aligned}$$



**3-23E** A submarine is cruising at a specified depth from the water surface. The pressure exerted on the surface of the submarine by water is to be determined.

**Assumptions** The variation of the density of water with depth is negligible.

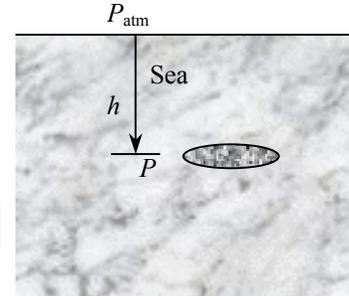
**Properties** The specific gravity of sea water is given to be  $SG = 1.03$ . The density of water at  $32^\circ\text{F}$  is  $62.4 \text{ lbm/ft}^3$ .

**Analysis** The density of the seawater is obtained by multiplying its specific gravity by the density of water,

$$\rho = SG \times \rho_{H_2O} = (1.03)(62.4 \text{ lbm/ft}^3) = 64.27 \text{ lbm/ft}^3$$

The pressure exerted on the surface of the submarine cruising 300 ft below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (14.7 \text{ psia}) + (64.27 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)(300 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= \mathbf{148.6 \text{ psia}} \end{aligned}$$



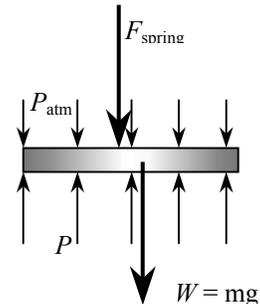
**3-24** A gas contained in a vertical piston-cylinder device is pressurized by a spring and by the weight of the piston. The pressure of the gas is to be determined.

**Analysis** Drawing the free body diagram of the piston and balancing the vertical forces yield

$$PA = P_{atm}A + W + F_{spring}$$

Thus,

$$\begin{aligned} P &= P_{atm} + \frac{mg + F_{spring}}{A} \\ &= (95 \text{ kPa}) + \frac{(4 \text{ kg})(9.807 \text{ m/s}^2) + 60 \text{ N}}{35 \times 10^{-4} \text{ m}^2} \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{123.4 \text{ kPa}} \end{aligned}$$

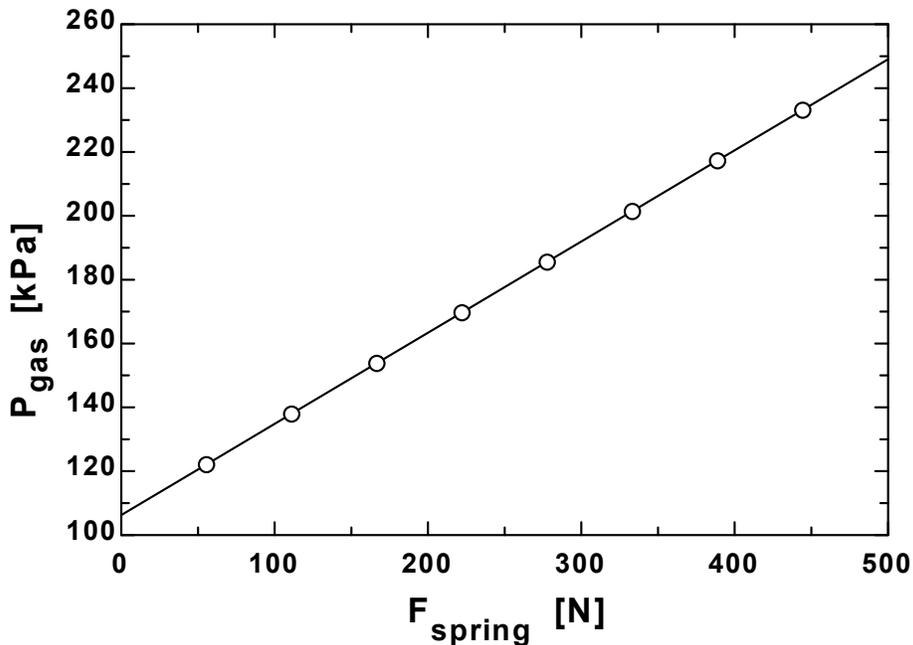


3-25 Problem 3-24 is reconsidered. The effect of the spring force in the range of 0 to 500 N on the pressure inside the cylinder is to be investigated. The pressure against the spring force is to be plotted, and results are to be discussed.

```

g=9.807"[m/s^2]"
P_atm= 95"[kPa]"
m_piston=4"[kg]"
{F_spring=60"[N]"}
A=35*CONVERT('cm^2','m^2')"[m^2]"
W_piston=m_piston*g"[N]"
F_atm=P_atm*A*CONVERT('kPa','N/m^2')"[N]"
"From the free body diagram of the piston, the balancing vertical forces yield:"
F_gas= F_atm+F_spring+W_piston"[N]"
P_gas=F_gas/A*CONVERT('N/m^2','kPa')"[kPa]"
    
```

$F_{\text{spring}}$ [N]	$P_{\text{gas}}$ [kPa]
0	106.2
55.56	122.1
111.1	138
166.7	153.8
222.2	169.7
277.8	185.6
333.3	201.4
388.9	217.3
444.4	233.2
500	249.1



**3-26** [Also solved by EES on enclosed CD] Both a gage and a manometer are attached to a gas to measure its pressure. For a specified reading of gage pressure, the difference between the fluid levels of the two arms of the manometer is to be determined for mercury and water.

**Properties** The densities of water and mercury are given to be  $\rho_{\text{water}} = 1000 \text{ kg/m}^3$  and be  $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$ .

**Analysis** The gage pressure is related to the vertical distance  $h$  between the two fluid levels by

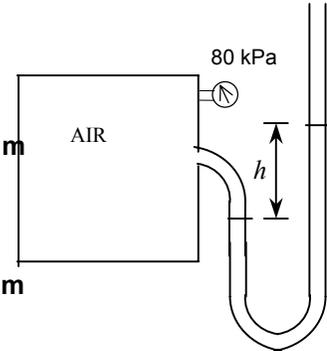
$$P_{\text{gage}} = \rho g h \longrightarrow h = \frac{P_{\text{gage}}}{\rho g}$$

(a) For mercury,

$$h = \frac{P_{\text{gage}}}{\rho_{\text{Hg}} g} = \frac{80 \text{ kPa}}{(13600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}} \right) = \mathbf{0.60 \text{ m}}$$

(b) For water,

$$h = \frac{P_{\text{gage}}}{\rho_{\text{H}_2\text{O}} g} = \frac{80 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.807 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}} \right) = \mathbf{8.16 \text{ m}}$$



3-27 Problem 3-26 is reconsidered. The effect of the manometer fluid density in the range of 800 to 13,000 kg/m<sup>3</sup> on the differential fluid height of the manometer is to be investigated. Differential fluid height against the density is to be plotted, and the results are to be discussed.

```
Function fluid_density(Fluid$)
  If fluid$='Mercury' then fluid_density=13600 else fluid_density=1000
end
```

{Input from the diagram window. If the diagram window is hidden, then all of the input must come from the equations window. Also note that brackets can also denote comments - but these comments do not appear in the formatted equations window.}

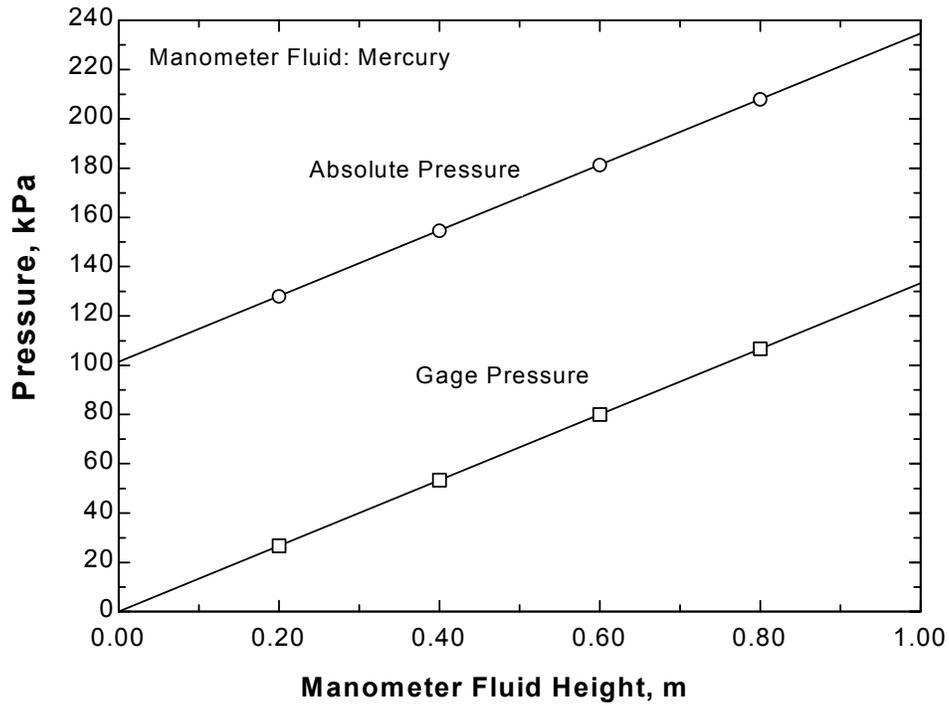
```
{Fluid$='Mercury'
P_atm = 101.325 "kpa"
DELTAP=80 "kPa Note how DELTAP is displayed on the Formatted Equations Window."}
```

```
g=9.807 "m/s2, local acceleration of gravity at sea level"
rho=Fluid_density(Fluid$) "Get the fluid density, either Hg or H2O, from the function"
"To plot fluid height against density place {} around the above equation. Then set up the parametric table and solve."
DELTAP = RHO*g*h/1000
"Instead of dividing by 1000 Pa/kPa we could have multiplied by the EES function, CONVERT('Pa','kPa')"
h_mm=h*convert('m','mm') "The fluid height in mm is found using the built-in CONVERT function."
P_abs= P_atm + DELTAP
```

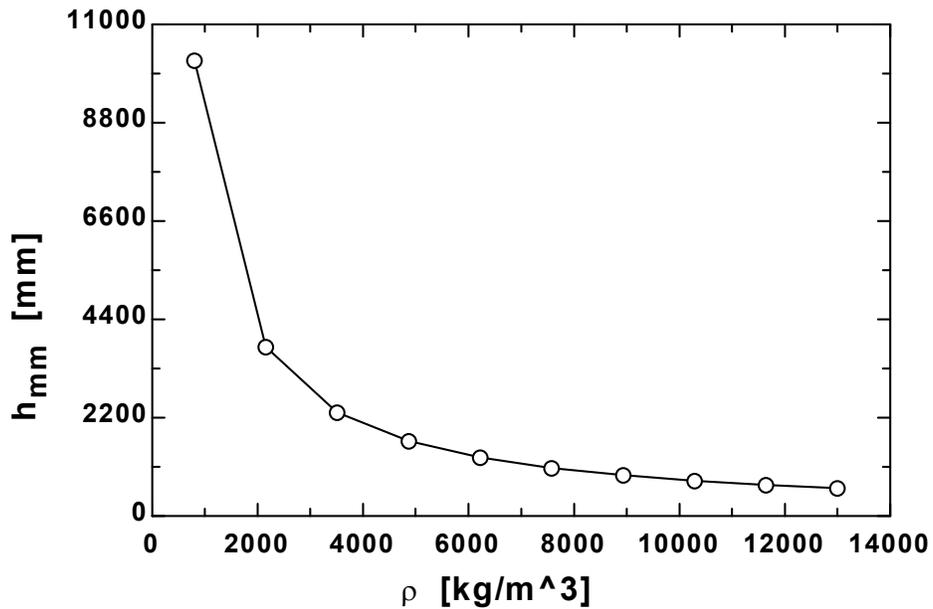
"To make the graph, hide the diagram window and remove the {}brackets from Fluid\$ and from P\_atm. Select New Parametric Table from the Tables menu. Choose P\_abs, DELTAP and h to be in the table. Choose Alter Values from the Tables menu. Set values of h to range from 0 to 1 in steps of 0.2. Choose Solve Table (or press F3) from the Calculate menu. Choose New Plot Window from the Plot menu. Choose to plot P\_abs vs h and then choose Overlay Plot from the Plot menu and plot DELTAP on the same scale."

$h_{mm}$ [mm]	$\rho$ [kg/m <sup>3</sup> ]
10197	800
3784	2156
2323	3511
1676	4867
1311	6222
1076	7578
913.1	8933
792.8	10289
700.5	11644
627.5	13000

Tank Fluid Gage and Absolute Pressures vs Manometer Fluid Height



Manometer Fluid Height vs Manometer Fluid Density

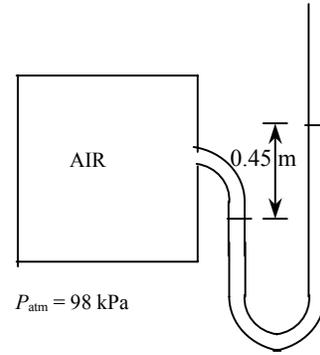


3-28 The air pressure in a tank is measured by an oil manometer. For a given oil-level difference between the two columns, the absolute pressure in the tank is to be determined.

**Properties** The density of oil is given to be  $\rho = 850 \text{ kg/m}^3$ .

**Analysis** The absolute pressure in the tank is determined from

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (98 \text{ kPa}) + (850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.45 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{101.8 \text{ kPa}} \end{aligned}$$



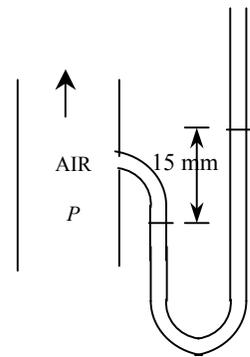
3-29 The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

**Properties** The density of mercury is given to be  $\rho = 13,600 \text{ kg/m}^3$ .

**Analysis (a)** The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(b) The absolute pressure in the duct is determined from

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (100 \text{ kPa}) + (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.015 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{102.0 \text{ kPa}} \end{aligned}$$



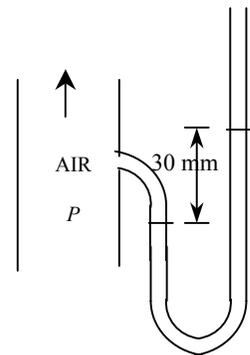
3-30 The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

**Properties** The density of mercury is given to be  $\rho = 13,600 \text{ kg/m}^3$ .

**Analysis (a)** The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(b) The absolute pressure in the duct is determined from

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (100 \text{ kPa}) + (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.030 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{104.0 \text{ kPa}} \end{aligned}$$



**3-31E** The systolic and diastolic pressures of a healthy person are given in mmHg. These pressures are to be expressed in kPa, psi, and meter water column.

**Assumptions** Both mercury and water are incompressible substances.

**Properties** We take the densities of water and mercury to be  $1000 \text{ kg/m}^3$  and  $13,600 \text{ kg/m}^3$ , respectively.

**Analysis** Using the relation  $P = \rho gh$  for gage pressure, the high and low pressures are expressed as

$$P_{\text{high}} = \rho gh_{\text{high}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.12 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = \mathbf{16.0 \text{ kPa}}$$

$$P_{\text{low}} = \rho gh_{\text{low}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.08 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = \mathbf{10.7 \text{ kPa}}$$

Noting that  $1 \text{ psi} = 6.895 \text{ kPa}$ ,

$$P_{\text{high}} = (16.0 \text{ kPa}) \left( \frac{1 \text{ psi}}{6.895 \text{ kPa}} \right) = \mathbf{2.32 \text{ psi}} \quad \text{and} \quad P_{\text{low}} = (10.7 \text{ kPa}) \left( \frac{1 \text{ psi}}{6.895 \text{ kPa}} \right) = \mathbf{1.55 \text{ psi}}$$

For a given pressure, the relation  $P = \rho gh$  can be expressed for mercury and water as  $P = \rho_{\text{water}} gh_{\text{water}}$  and  $P = \rho_{\text{mercury}} gh_{\text{mercury}}$ . Setting these two relations equal to each other and solving for water height gives

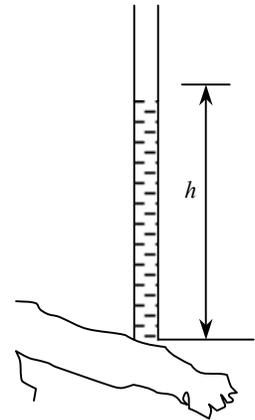
$$P = \rho_{\text{water}} gh_{\text{water}} = \rho_{\text{mercury}} gh_{\text{mercury}} \quad \rightarrow \quad h_{\text{water}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury}}$$

Therefore,

$$h_{\text{water, high}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, high}} = \frac{13,600 \text{ kg/m}^3}{1000 \text{ kg/m}^3} (0.12 \text{ m}) = \mathbf{1.63 \text{ m}}$$

$$h_{\text{water, low}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, low}} = \frac{13,600 \text{ kg/m}^3}{1000 \text{ kg/m}^3} (0.08 \text{ m}) = \mathbf{1.09 \text{ m}}$$

**Discussion** Note that measuring blood pressure with a “water” monometer would involve differential fluid heights higher than the person, and thus it is impractical. This problem shows why mercury is a suitable fluid for blood pressure measurement devices.



**3-32** A vertical tube open to the atmosphere is connected to the vein in the arm of a person. The height that the blood will rise in the tube is to be determined.

**Assumptions** 1 The density of blood is constant. 2 The gage pressure of blood is 120 mmHg.

**Properties** The density of blood is given to be  $\rho = 1050 \text{ kg/m}^3$ .

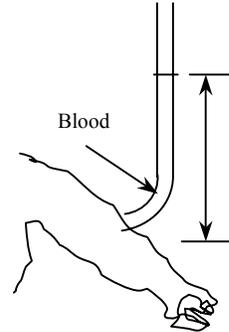
**Analysis** For a given gage pressure, the relation  $P = \rho gh$  can be expressed for mercury and blood as  $P = \rho_{\text{blood}} gh_{\text{blood}}$  and  $P = \rho_{\text{mercury}} gh_{\text{mercury}}$ . Setting these two relations equal to each other we get

$$P = \rho_{\text{blood}} gh_{\text{blood}} = \rho_{\text{mercury}} gh_{\text{mercury}}$$

Solving for blood height and substituting gives

$$h_{\text{blood}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{blood}}} h_{\text{mercury}} = \frac{13,600 \text{ kg/m}^3}{1050 \text{ kg/m}^3} (0.12 \text{ m}) = \mathbf{1.55 \text{ m}}$$

**Discussion** Note that the blood can rise about one and a half meters in a tube connected to the vein. This explains why IV tubes must be placed high to force a fluid into the vein of a patient.



**3-33** A man is standing in water vertically while being completely submerged. The difference between the pressures acting on the head and on the toes is to be determined.

**Assumptions** Water is an incompressible substance, and thus the density does not change with depth.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** The pressures at the head and toes of the person can be expressed as

$$P_{\text{head}} = P_{\text{atm}} + \rho gh_{\text{head}} \quad \text{and} \quad P_{\text{toe}} = P_{\text{atm}} + \rho gh_{\text{toe}}$$

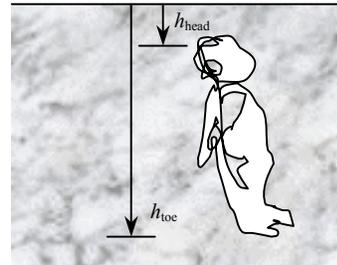
where  $h$  is the vertical distance of the location in water from the free surface. The pressure difference between the toes and the head is determined by subtracting the first relation above from the second,

$$P_{\text{toe}} - P_{\text{head}} = \rho gh_{\text{toe}} - \rho gh_{\text{head}} = \rho g(h_{\text{toe}} - h_{\text{head}})$$

Substituting,

$$P_{\text{toe}} - P_{\text{head}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.80 \text{ m} - 0) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = \mathbf{17.7 \text{ kPa}}$$

**Discussion** This problem can also be solved by noting that the atmospheric pressure (1 atm = 101.325 kPa) is equivalent to 10.3-m of water height, and finding the pressure that corresponds to a water height of 1.8 m.



**3-34** Water is poured into the U-tube from one arm and oil from the other arm. The water column height in one arm and the ratio of the heights of the two fluids in the other arm are given. The height of each fluid in that arm is to be determined.

**Assumptions** Both water and oil are incompressible substances.

**Properties** The density of oil is given to be  $\rho = 790 \text{ kg/m}^3$ . We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** The height of water column in the left arm of the manometer is given to be  $h_{w1} = 0.70 \text{ m}$ . We let the height of water and oil in the right arm to be  $h_{w2}$  and  $h_a$ , respectively. Then,  $h_a = 6h_{w2}$ . Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as

$$P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w1} \quad \text{and} \quad P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w2} + \rho_a g h_a$$

Setting them equal to each other and simplifying,

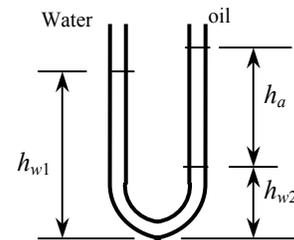
$$\rho_w g h_{w1} = \rho_w g h_{w2} + \rho_a g h_a \quad \rightarrow \quad \rho_w h_{w1} = \rho_w h_{w2} + \rho_a h_a \quad \rightarrow \quad h_{w1} = h_{w2} + (\rho_a / \rho_w) h_a$$

Noting that  $h_a = 6h_{w2}$ , the water and oil column heights in the second arm are determined to be

$$0.7 \text{ m} = h_{w2} + (790/1000)6h_{w2} \quad \rightarrow \quad h_{w2} = \mathbf{0.122 \text{ m}}$$

$$0.7 \text{ m} = 0.122 \text{ m} + (790/1000)h_a \quad \rightarrow \quad h_a = \mathbf{0.732 \text{ m}}$$

**Discussion** Note that the fluid height in the arm that contains oil is higher. This is expected since oil is lighter than water.



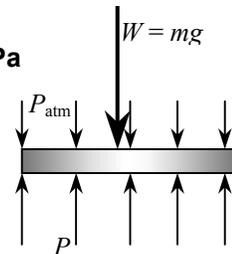
**3-35** The hydraulic lift in a car repair shop is to lift cars. The fluid gage pressure that must be maintained in the reservoir is to be determined.

**Assumptions** The weight of the piston of the lift is negligible.

**Analysis** Pressure is force per unit area, and thus the gage pressure required is simply the ratio of the weight of the car to the area of the lift,

$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2 / 4} = \frac{(2000 \text{ kg})(9.81 \text{ m/s}^2)}{\pi (0.30 \text{ m})^2 / 4} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 278 \text{ kN/m}^2 = \mathbf{278 \text{ kPa}}$$

**Discussion** Note that the pressure level in the reservoir can be reduced by using a piston with a larger area.



**3-36** Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

**Properties** The densities of seawater and mercury are given to be  $\rho_{\text{sea}} = 1035 \text{ kg/m}^3$  and  $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$ . We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the sea water pipe (point 2), and setting the result equal to  $P_2$  gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{air}} gh_{\text{air}} + \rho_{\text{sea}} gh_{\text{sea}} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

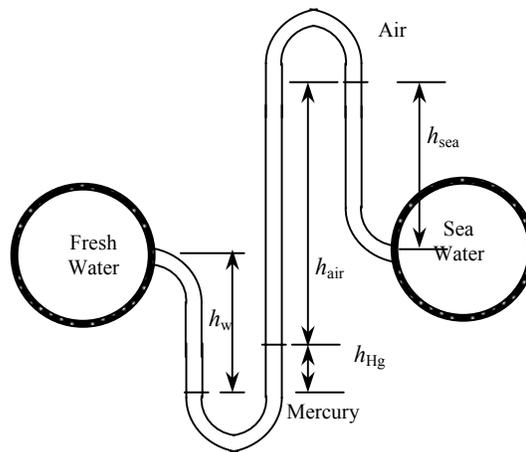
$$P_1 - P_2 = -\rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{sea}} gh_{\text{sea}} = g(\rho_{\text{Hg}} h_{\text{Hg}} - \rho_w h_w - \rho_{\text{sea}} h_{\text{sea}})$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81 \text{ m/s}^2)[(13600 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (1000 \text{ kg/m}^3)(0.6 \text{ m}) - (1035 \text{ kg/m}^3)(0.4 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 3.39 \text{ kN/m}^2 = \mathbf{3.39 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

**Discussion** A 0.70-m high air column with a density of  $1.2 \text{ kg/m}^3$  corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.



3-37 Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

**Assumptions** All the liquids are incompressible.

**Properties** The densities of seawater and mercury are given to be  $\rho_{\text{sea}} = 1035 \text{ kg/m}^3$  and  $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$ . We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ . The specific gravity of oil is given to be 0.72, and thus its density is  $720 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the sea water pipe (point 2), and setting the result equal to  $P_2$  gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{sea}} gh_{\text{sea}} = P_2$$

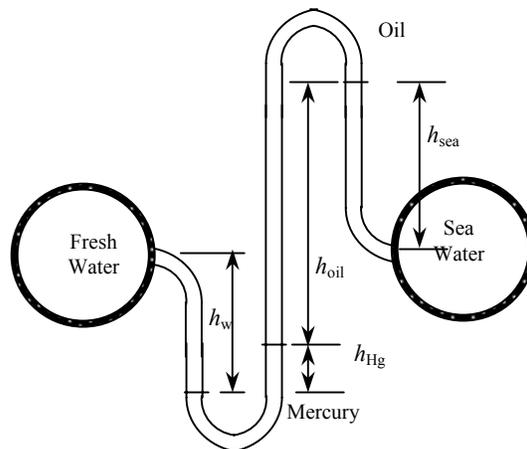
Rearranging,

$$\begin{aligned} P_1 - P_2 &= -\rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} + \rho_{\text{oil}} gh_{\text{oil}} - \rho_{\text{sea}} gh_{\text{sea}} \\ &= g(\rho_{\text{Hg}} h_{\text{Hg}} + \rho_{\text{oil}} h_{\text{oil}} - \rho_w h_w - \rho_{\text{sea}} h_{\text{sea}}) \end{aligned}$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81 \text{ m/s}^2)[(13600 \text{ kg/m}^3)(0.1 \text{ m}) + (720 \text{ kg/m}^3)(0.7 \text{ m}) - (1000 \text{ kg/m}^3)(0.6 \text{ m}) \\ &\quad - (1035 \text{ kg/m}^3)(0.4 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 8.34 \text{ kN/m}^2 = \mathbf{8.34 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 8.34 kPa higher than the pressure in the sea water pipe.



**3-38E** The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible. 3 The pressure throughout the natural gas (including the tube) is uniform since its density is low.

**Properties** We take the density of water to be  $\rho_w = 62.4 \text{ lbm/ft}^3$ . The specific gravity of mercury is given to be 13.6, and thus its density is  $\rho_{\text{Hg}} = 13.6 \times 62.4 = 848.6 \text{ lbm/ft}^3$ .

**Analysis** Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  gives

$$P_1 - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{water}} g h_{\text{water}} = P_{\text{atm}}$$

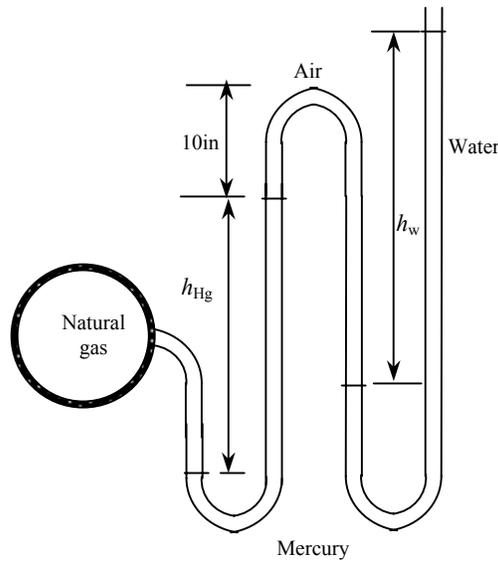
Solving for  $P_1$ ,

$$P_1 = P_{\text{atm}} + \rho_{\text{Hg}} g h_{\text{Hg}} + \rho_{\text{water}} g h_1$$

Substituting,

$$P = 14.2 \text{ psia} + (32.2 \text{ ft/s}^2) [(848.6 \text{ lbm/ft}^3)(6/12 \text{ ft}) + (62.4 \text{ lbm/ft}^3)(27/12 \text{ ft})] \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = \mathbf{18.1 \text{ psia}}$$

**Discussion** Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly. Also, it can be shown that the 15-in high air column with a density of  $0.075 \text{ lbm/ft}^3$  corresponds to a pressure difference of  $0.00065 \text{ psi}$ . Therefore, its effect on the pressure difference between the two pipes is negligible.



**3-39E** The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

**Assumptions 1** All the liquids are incompressible. **2** The pressure throughout the natural gas (including the tube) is uniform since its density is low.

**Properties** We take the density of water to be  $\rho_w = 62.4 \text{ lbf/ft}^3$ . The specific gravity of mercury is given to be 13.6, and thus its density is  $\rho_{\text{Hg}} = 13.6 \times 62.4 = 848.6 \text{ lbf/ft}^3$ . The specific gravity of oil is given to be 0.69, and thus its density is  $\rho_{\text{oil}} = 0.69 \times 62.4 = 43.1 \text{ lbf/ft}^3$ .

**Analysis** Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  gives

$$P_1 - \rho_{\text{Hg}}gh_{\text{Hg}} + \rho_{\text{oil}}gh_{\text{oil}} - \rho_{\text{water}}gh_{\text{water}} = P_{\text{atm}}$$

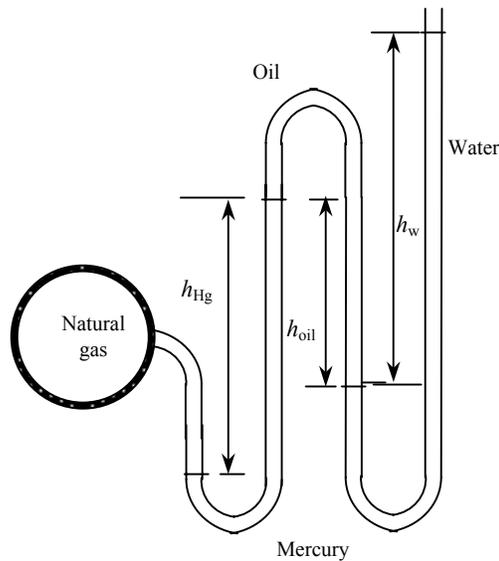
Solving for  $P_1$ ,

$$P_1 = P_{\text{atm}} + \rho_{\text{Hg}}gh_{\text{Hg}} + \rho_{\text{water}}gh_1 - \rho_{\text{oil}}gh_{\text{oil}}$$

Substituting,

$$\begin{aligned} P_1 &= 14.2 \text{ psia} + (32.2 \text{ ft/s}^2)[(848.6 \text{ lbf/ft}^3)(6/12 \text{ ft}) + (62.4 \text{ lbf/ft}^3)(27/12 \text{ ft}) \\ &\quad - (43.1 \text{ lbf/ft}^3)(15/12 \text{ ft})] \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= \mathbf{17.7 \text{ psia}} \end{aligned}$$

**Discussion** Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.



**3-40** The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height  $h$  of the mercury column is to be determined.

**Assumptions** The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

**Properties** We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ . The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

**Analysis** Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{oil}} gh_{\text{oil}} = P_{\text{atm}}$$

Rearranging,

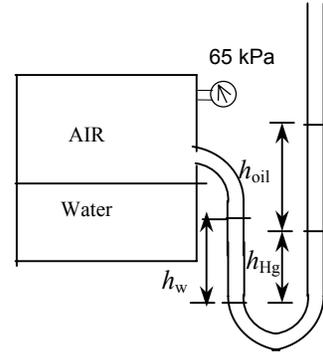
$$P_1 - P_{\text{atm}} = \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_w gh_w$$

or,

$$\frac{P_{1,\text{gage}}}{\rho_w g} = \rho_{s,\text{oil}} h_{\text{oil}} + \rho_{s,\text{Hg}} h_{\text{Hg}} - h_w$$

Substituting,

$$\left( \frac{65 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \right) \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kPa} \cdot \text{m}^2} \right) = 0.72 \times (0.75 \text{ m}) + 13.6 \times h_{\text{Hg}} - 0.3 \text{ m}$$



Solving for  $h_{\text{Hg}}$  gives  $h_{\text{Hg}} = \mathbf{0.47 \text{ m}}$ . Therefore, the differential height of the mercury column must be 47 cm.

**Discussion** Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

**3-41** The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height  $h$  of the mercury column is to be determined.

**Assumptions** The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

**Properties** We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ . The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

**Analysis** Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{oil}} gh_{\text{oil}} = P_{\text{atm}}$$

Rearranging,

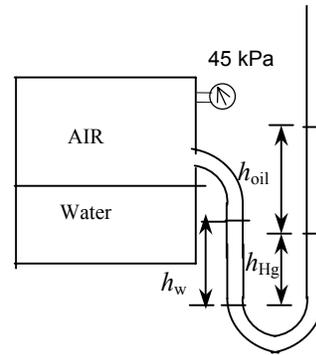
$$P_1 - P_{\text{atm}} = \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_w gh_w$$

or,

$$\frac{P_{1,\text{gage}}}{\rho_w g} = SG_{\text{oil}} h_{\text{oil}} + SG_{\text{Hg}} h_{\text{Hg}} - h_w$$

Substituting,

$$\frac{45 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kPa} \cdot \text{m}^2} \right) = 0.72 \times (0.75 \text{ m}) + 13.6 \times h_{\text{Hg}} - 0.3 \text{ m}$$



Solving for  $h_{\text{Hg}}$  gives  $h_{\text{Hg}} = \mathbf{0.32 \text{ m}}$ . Therefore, the differential height of the mercury column must be 32 cm.

**Discussion** Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

**3-42** The top part of a water tank is divided into two compartments, and a fluid with an unknown density is poured into one side. The levels of the water and the liquid are measured. The density of the fluid is to be determined.

**Assumptions** 1 Both water and the added liquid are incompressible substances. 2 The added liquid does not mix with water.

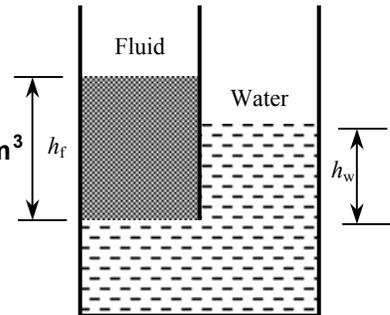
**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** Both fluids are open to the atmosphere. Noting that the pressure of both water and the added fluid is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{\text{contact}} = P_{\text{atm}} + \rho_f g h_f = P_{\text{atm}} + \rho_w g h_w$$

Simplifying and solving for  $\rho_f$  gives

$$\rho_f g h_f = \rho_w g h_w \rightarrow \rho_f = \frac{h_w}{h_f} \rho_w = \frac{45 \text{ cm}}{80 \text{ cm}} (1000 \text{ kg/m}^3) = \mathbf{562.5 \text{ kg/m}^3}$$



**Discussion** Note that the added fluid is lighter than water as expected (a heavier fluid would sink in water).

**3-43** A load on a hydraulic lift is to be raised by pouring oil from a thin tube. The height of oil in the tube required in order to raise that weight is to be determined.

**Assumptions** 1 The cylinders of the lift are vertical. 2 There are no leaks. 3 Atmospheric pressure act on both sides, and thus it can be disregarded.

**Properties** The density of oil is given to be  $\rho = 780 \text{ kg/m}^3$ .

**Analysis** Noting that pressure is force per unit area, the gage pressure in the fluid under the load is simply the ratio of the weight to the area of the lift,

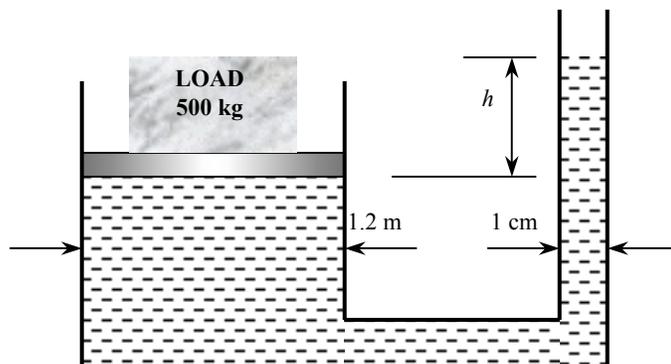
$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2 / 4} = \frac{(500 \text{ kg})(9.81 \text{ m/s}^2)}{\pi (1.20 \text{ m})^2 / 4} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 4.34 \text{ kN/m}^2 = 4.34 \text{ kPa}$$

The required oil height that will cause 4.34 kPa of pressure rise is

$$P_{\text{gage}} = \rho g h \rightarrow h = \frac{P_{\text{gage}}}{\rho g} = \frac{4.34 \text{ kN/m}^2}{(780 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN/m}^2} \right) = \mathbf{0.567 \text{ m}}$$

Therefore, a 500 kg load can be raised by this hydraulic lift by simply raising the oil level in the tube by 56.7 cm.

**Discussion** Note that large weights can be raised by little effort in hydraulic lift by making use of Pascal's principle.



**3-44E** Two oil tanks are connected to each other through a mercury manometer. For a given differential height, the pressure difference between the two tanks is to be determined.

**Assumptions** 1 Both the oil and mercury are incompressible fluids. 2 The oils in both tanks have the same density.

**Properties** The densities of oil and mercury are given to be  $\rho_{oil} = 45 \text{ lbm/ft}^3$  and  $\rho_{Hg} = 848 \text{ lbm/ft}^3$ .

**Analysis** Starting with the pressure at the bottom of tank 1 (where pressure is  $P_1$ ) and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the bottom of tank 2 (where pressure is  $P_2$ ) gives

$$P_1 + \rho_{oil}g(h_1 + h_2) - \rho_{Hg}gh_2 - \rho_{oil}gh_1 = P_2$$

where  $h_1 = 10 \text{ in}$  and  $h_2 = 32 \text{ in}$ . Rearranging and simplifying,

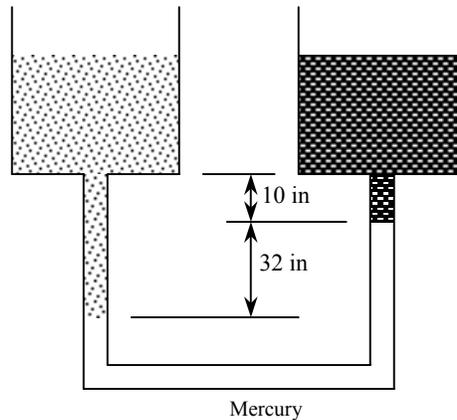
$$P_1 - P_2 = \rho_{Hg}gh_2 - \rho_{oil}gh_2 = (\rho_{Hg} - \rho_{oil})gh_2$$

Substituting,

$$\Delta P = P_1 - P_2 = (848 - 45 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(32/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = \mathbf{14.9 \text{ psia}}$$

Therefore, the pressure in the left oil tank is 14.9 psia higher than the pressure in the right oil tank.

**Discussion** Note that large pressure differences can be measured conveniently by mercury manometers. If a water manometer were used in this case, the differential height would be over 30 ft.



**3-45** The standard atmospheric pressure is expressed in terms of mercury, water, and glycerin columns.  $\surd$   
**Assumptions** The densities of fluids are constant.

**Properties** The specific gravities are given to be  $SG = 13.6$  for mercury,  $SG = 1.0$  for water, and  $SG = 1.26$  for glycerin. The standard density of water is  $1000 \text{ kg/m}^3$ , and the standard atmospheric pressure is  $101,325 \text{ Pa}$ .

**Analysis** The atmospheric pressure is expressed in terms of a fluid column height as

$$P_{atm} = \rho gh = SG\rho_w gh \quad \rightarrow \quad h = \frac{P_{atm}}{SG\rho_w g}$$

Substituting,

$$(a) \text{ Mercury: } h = \frac{P_{atm}}{SG\rho_w g} = \frac{101,325 \text{ N/m}^2}{13.6(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N/m}^2} \right) = \mathbf{0.759 \text{ m}}$$

$$(b) \text{ Water: } h = \frac{P_{atm}}{SG\rho_w g} = \frac{101,325 \text{ N/m}^2}{1(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N/m}^2} \right) = \mathbf{10.3 \text{ m}}$$

$$(c) \text{ Glycerin: } h = \frac{P_{atm}}{SG\rho_w g} = \frac{101,325 \text{ N/m}^2}{1.26(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N/m}^2} \right) = \mathbf{8.20 \text{ m}}$$

**Discussion** Using water or glycerin to measure atmospheric pressure requires very long vertical tubes (over 10 m for water), which is not practical. This explains why mercury is used instead of water or a light fluid..

**3-46** A glass filled with water and covered with a thin paper is inverted. The pressure at the bottom of the glass is to be determined.

**Assumptions** 1 Water is an incompressible substance. 2 The weight of the paper is negligible. 3 The atmospheric pressure is 100 kPa.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** The paper is in equilibrium, and thus the net force acting on the paper must be zero. A vertical force balance on the paper involves the pressure forces on both sides, and yields

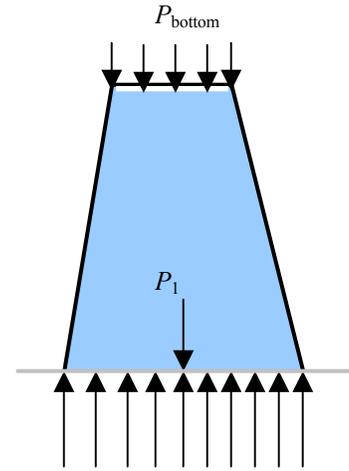
$$P_1 A_{\text{glass}} = P_{\text{atm}} A_{\text{glass}} \quad \rightarrow \quad P_1 = P_{\text{atm}}$$

That is, the pressures on both sides of the paper must be the same. The pressure at the bottom of the glass is determined from the hydrostatic pressure relation to be

$$P_{\text{atm}} = P_{\text{bottom}} + \rho g h_{\text{glass}} \quad \rightarrow \quad P_{\text{bottom}} = P_{\text{atm}} - \rho g h_{\text{glass}}$$

Substituting,

$$P_{\text{bottom}} = (100 \text{ kPa}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = \mathbf{99.0 \text{ kPa}} \quad P_{\text{atm}}$$



**Discussion** Note that there is a vacuum of 1 kPa at the bottom of the glass, and thus there is an upward pressure force acting on the water body, which is balanced by the weight of water. As a result, the net downward force on water is zero, and thus water does not flow down.

3-47 Two chambers with the same fluid at their base are separated by a piston. The gage pressure in each air chamber is to be determined.

**Assumptions** 1 Water is an incompressible substance. 2 The variation of pressure with elevation in each air chamber is negligible because of the low density of air.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** The piston is in equilibrium, and thus the net force acting on the piston must be zero. A vertical force balance on the piston involves the pressure force exerted by water on the piston face, the atmospheric pressure force, and the piston weight, and yields

$$P_C A_{\text{piston}} = P_{\text{atm}} A_{\text{piston}} + W_{\text{piston}} \rightarrow P_C = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}}$$

The pressure at the bottom of each air chamber is determined from the hydrostatic pressure relation to be

$$P_{\text{air A}} = P_E = P_C + \rho g \overline{CE} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE} \rightarrow P_{\text{air A, gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE}$$

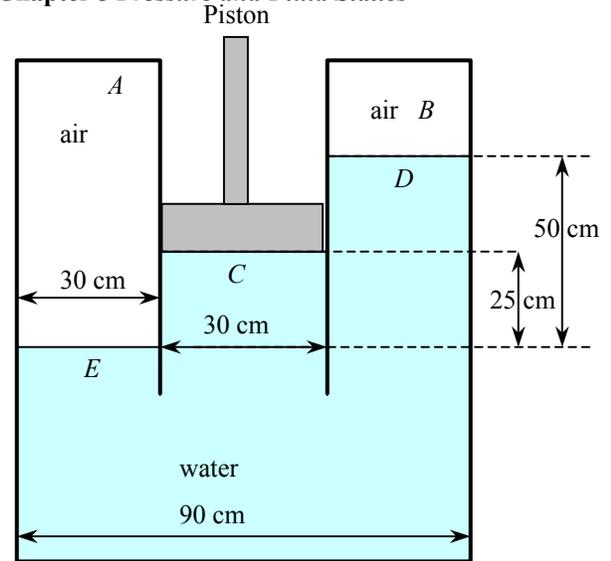
$$P_{\text{air B}} = P_D = P_C - \rho g \overline{CD} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD} \rightarrow P_{\text{air B, gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD}$$

Substituting,

$$P_{\text{air A, gage}} = \frac{25 \text{ N}}{\pi(0.3 \text{ m})^2 / 4} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2806 \text{ N/m}^2 = \mathbf{2.806 \text{ kPa}}$$

$$P_{\text{air B, gage}} = \frac{25 \text{ N}}{\pi(0.3 \text{ m})^2 / 4} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = -2099 \text{ N/m}^2 = \mathbf{-2.099 \text{ kPa}}$$

**Discussion** Note that there is a vacuum of about 2 kPa in tank B which pulls the water up.



**3-48** A double-fluid manometer attached to an air pipe is considered. The specific gravity of one fluid is known, and the specific gravity of the other fluid is to be determined.

**Assumptions** 1 Densities of liquids are constant. 2 The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

**Properties** The specific gravity of one fluid is given to be 13.55. We take the standard density of water to be  $1000 \text{ kg/m}^3$ .

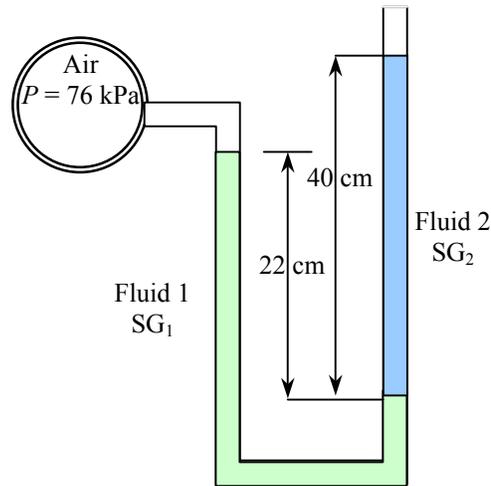
**Analysis** Starting with the pressure of air in the tank, and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  give

$$P_{\text{air}} + \rho_1 gh_1 - \rho_2 gh_2 = P_{\text{atm}} \quad \rightarrow \quad P_{\text{air}} - P_{\text{atm}} = SG_2 \rho_w gh_2 - SG_1 \rho_w gh_1$$

Rearranging and solving for  $SG_2$ ,

$$SG_2 = SG_1 \frac{h_1}{h_2} + \frac{P_{\text{air}} - P_{\text{atm}}}{\rho_w gh_2} = 13.55 \frac{0.22 \text{ m}}{0.40 \text{ m}} + \left( \frac{76 - 100 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \right) \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kPa} \cdot \text{m}^2} \right) = \mathbf{5.0}$$

**Discussion** Note that the right fluid column is higher than the left, and this would imply above atmospheric pressure in the pipe for a single-fluid manometer.



**3-49** The pressure difference between two pipes is measured by a double-fluid manometer. For given fluid heights and specific gravities, the pressure difference between the pipes is to be calculated.

**Assumptions** All the liquids are incompressible.

**Properties** The specific gravities are given to be 13.5 for mercury, 1.26 for glycerin, and 0.88 for oil. We take the standard density of water to be  $\rho_w=1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the oil pipe (point B), and setting the result equal to  $P_B$  give

$$P_A + \rho_w gh_w + \rho_{Hg} gh_{Hg} - \rho_{gly} gh_{gly} + \rho_{oil} gh_{oil} = P_B$$

Rearranging and using the definition of specific gravity,

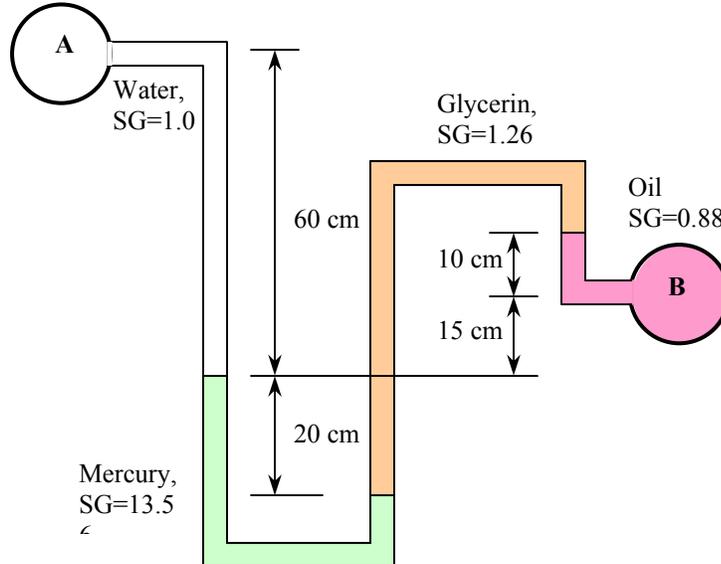
$$\begin{aligned} P_B - P_A &= SG_w \rho_w gh_w + SG_{Hg} \rho_w gh_{Hg} - SG_{gly} \rho_w gh_{gly} + SG_{oil} \rho_w gh_{oil} \\ &= g \rho_w (SG_w h_w + SG_{Hg} h_{Hg} - SG_{gly} h_{gly} + SG_{oil} h_{oil}) \end{aligned}$$

Substituting,

$$\begin{aligned} P_B - P_A &= (9.81 \text{ m/s}^2)(1000 \text{ kg/m}^3)[1(0.6 \text{ m}) + 13.5(0.2 \text{ m}) - 1.26(0.45 \text{ m}) + 0.88(0.1 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 27.7 \text{ kN/m}^2 = \mathbf{27.7 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the oil pipe is 27.7 kPa higher than the pressure in the water pipe.

**Discussion** Using a manometer between two pipes is not recommended unless the pressures in the two pipes are relatively constant. Otherwise, an over-rise of pressure in one pipe can push the manometer fluid into the other pipe, creating a short circuit.



**3-50** The fluid levels in a multi-fluid U-tube manometer change as a result of a pressure drop in the trapped air space. For a given pressure drop and brine level change, the area ratio is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 Pressure in the brine pipe remains constant. 3 The variation of pressure in the trapped air space is negligible.

**Properties** The specific gravities are given to be 13.56 for mercury and 1.1 for brine. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** It is clear from the problem statement and the figure that the brine pressure is much higher than the air pressure, and when the air pressure drops by 0.7 kPa, the pressure difference between the brine and the air space increases also by the same amount. Starting with the air pressure (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the brine pipe (point B), and setting the result equal to  $P_B$  before and after the pressure change of air give

Before:  $P_{A1} + \rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg},1} - \rho_{\text{br}} gh_{\text{br},1} = P_B$

After:  $P_{A2} + \rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg},2} - \rho_{\text{br}} gh_{\text{br},2} = P_B$

Subtracting,

$$P_{A2} - P_{A1} + \rho_{\text{Hg}} g \Delta h_{\text{Hg}} - \rho_{\text{br}} g \Delta h_{\text{br}} = 0 \rightarrow \frac{P_{A1} - P_{A2}}{\rho_w g} = SG_{\text{Hg}} \Delta h_{\text{Hg}} - SG_{\text{br}} \Delta h_{\text{br}} = 0 \quad (1)$$

where  $\Delta h_{\text{Hg}}$  and  $\Delta h_{\text{br}}$  are the changes in the differential mercury and brine column heights, respectively, due to the drop in air pressure. Both of these are positive quantities since as the mercury-brine interface drops, the differential fluid heights for both mercury and brine increase. Noting also that the volume of mercury is constant, we have  $A_1 \Delta h_{\text{Hg, left}} = A_2 \Delta h_{\text{Hg, right}}$  and

$$P_{A2} - P_{A1} = -0.7 \text{ kPa} = -700 \text{ N/m}^2 = -700 \text{ kg/m} \cdot \text{s}^2$$

$$\Delta h_{\text{br}} = 0.005 \text{ m}$$

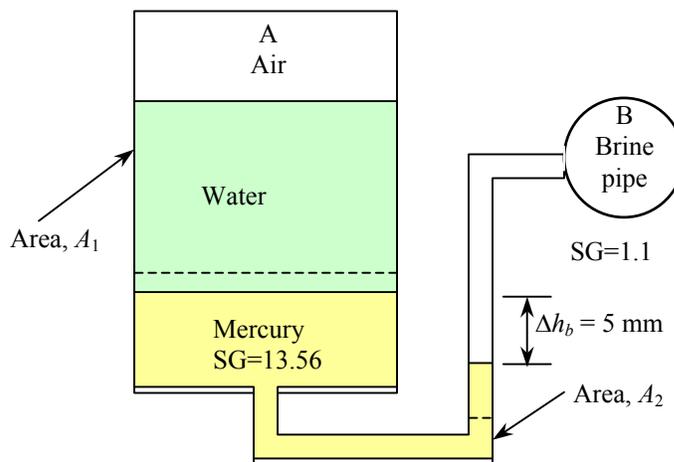
$$\Delta h_{\text{Hg}} = \Delta h_{\text{Hg, right}} + \Delta h_{\text{Hg, left}} = \Delta h_{\text{br}} + \Delta h_{\text{br}} A_2/A_1 = \Delta h_{\text{br}} (1 + A_2/A_1)$$

Substituting,

$$\frac{700 \text{ kg/m} \cdot \text{s}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = [13.56 \times 0.005(1 + A_2/A_1) - 1.1 \times 0.005] \text{ m}$$

It gives

$$A_2/A_1 = \mathbf{0.134}$$



**3-51** Two water tanks are connected to each other through a mercury manometer with inclined tubes. For a given pressure difference between the two tanks, the parameters  $a$  and  $\theta$  are to be determined.

**Assumptions** Both water and mercury are incompressible.

**Properties** The specific gravity of mercury is given to be 13.6. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure in the tank A and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach tank B, and setting the result equal to  $P_B$  give

$$P_A + \rho_w ga + \rho_{\text{Hg}} g 2a - \rho_w ga = P_B \quad \rightarrow \quad 2\rho_{\text{Hg}} ga = P_B - P_A$$

Rearranging and substituting the known values,

$$a = \frac{P_B - P_A}{2\rho_{\text{Hg}} g} = \frac{20 \text{ kN/m}^2}{2(13.6)(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) = 0.0750 \text{ m} = \mathbf{7.50 \text{ cm}}$$

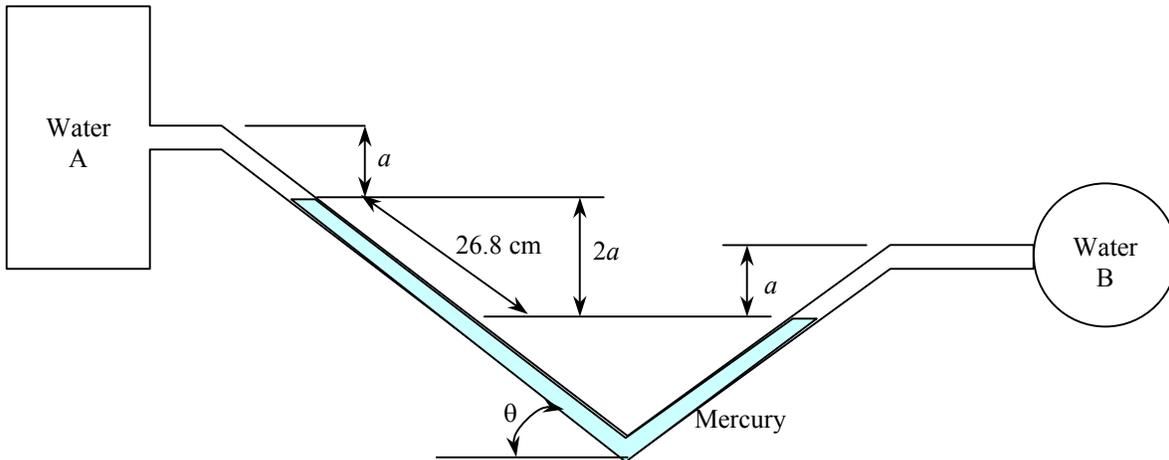
From geometric considerations,

$$26.8 \sin \theta = 2a \quad (\text{cm})$$

Therefore,

$$\sin \theta = \frac{2a}{26.8} = \frac{2 \times 7.50}{26.8} = 0.560 \quad \rightarrow \quad \theta = \mathbf{34.0^\circ}$$

**Discussion** Note that vertical distances are used in manometer analysis.



**3-52** A multi-fluid container is connected to a U-tube. For the given specific gravities and fluid column heights, the gage pressure at A and the height of a mercury column that would create the same pressure at A are to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The multi-fluid container is open to the atmosphere.

**Properties** The specific gravities are given to be 1.26 for glycerin and 0.90 for oil. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ , and the specific gravity of mercury to be 13.6.

**Analysis** Starting with the atmospheric pressure on the top surface of the container and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach point A, and setting the result equal to  $P_A$  give

$$P_{atm} + \rho_{oil}gh_{oil} + \rho_w gh_w - \rho_{gly}gh_{gly} = P_A$$

Rearranging and using the definition of specific gravity,

$$P_A - P_{atm} = SG_{oil}\rho_w gh_{oil} + SG_w\rho_w gh_w - SG_{gly}\rho_w gh_{gly}$$

or

$$P_{A,gage} = g\rho_w(SG_{oil}h_{oil} + SG_w h_w - SG_{gly}h_{gly})$$

Substituting,

$$P_{A,gage} = (9.81 \text{ m/s}^2)(1000 \text{ kg/m}^3)[0.90(0.70 \text{ m}) + 1(0.3 \text{ m}) - 1.26(0.70 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 0.471 \text{ kN/m}^2 = \mathbf{0.471 \text{ kPa}}$$

The equivalent mercury column height is

$$h_{Hg} = \frac{P_{A,gage}}{\rho_{Hg}g} = \frac{0.471 \text{ kN/m}^2}{(13.6)(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) = 0.00353 \text{ m} = \mathbf{0.353 \text{ cm}}$$

**Discussion** Note that the high density of mercury makes it a very suitable fluid for measuring high pressures in manometers.

