Energy Equation

5-63C It is *impossible* for the fluid temperature to decrease during steady, incompressible, adiabatic flow since this would require the entropy of an adiabatic system to decrease, which would be a violation of the 2nd law of thermodynamics.

5-64C Yes, the *frictional effects* are negligible if the fluid temperature remains constant during steady, incompressible flow since any irreversibility such as friction would cause the entropy and thus temperature of the fluid to increase during adiabatic flow.

5-65C *Head loss* is the loss of mechanical energy due to friction in piping expressed as an equivalent column height of fluid, i.e., head. It is related to the mechanical energy loss in piping by $h_L = \dfrac{e_{\text{mech loss, piping}}}{g} = \dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$.

5-66C Useful pump head is the useful power input to the pump expressed as an equivalent column height of fluid. It is related to the useful pumping power input by $h_{\text{pump}} = \dfrac{w_{\text{pump, u}}}{g} = \dfrac{\dot{W}_{\text{pump, u}}}{\dot{m}g}$

5-67C The *kinetic energy correction factor* is a correction factor to account for the fact that kinetic energy using average velocity is not the same as the actual kinetic energy using the actual velocity profile. Its effect is usually negligible (the square of a sum is not equal to the sum of the squares of its components).

5-68C By Bernoulli Equation, the maximum theoretical height to which the water stream could rise is the tank water level, which is 20 meters above the ground. Since the water rises above the tank level, the tank cover must be airtight, containing pressurized air above the water surface. Otherwise, a pump would have to pressurize the water somewhere in the hose.

5-69 Underground water is pumped to a pool at a given elevation. The maximum flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible. **3** We assume the frictional effects in piping to be negligible since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech loss, pipping}} = 0$. **4** The effect of the kinetic energy correction factors is negligible, $\alpha = 1$.

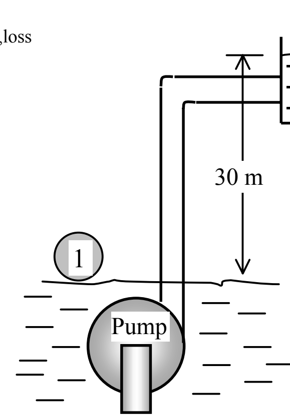
Properties We take the density of water to be 1 kg/L = 1000 kg/m^3.

Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\text{ kW}) = 2.1\text{ kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($V_1 \cong V_2 \cong 0$), and frictional losses in piping are disregarded. Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$



In the absence of a turbine, $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, piping}}$ and $\dot{W}_{\text{pump, u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech loss, pump}}$. Thus,

$$\dot{W}_{\text{pump, u}} = \dot{m}gz_2$$

Then the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump, u}}}{gz_2} = \frac{2.1\text{ kJ/s}}{(9.81\text{ m/s}^2)(30\text{ m})}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ}}\right) = 7.14\text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{7.14\text{ kg/s}}{1000\text{ kg/m}^3} = \mathbf{7.14 \times 10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$V_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{7.14\times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.07\text{ m})^2/4} = 1.86\text{ m/s}, \quad V_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{7.14\times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4} = 3.64\text{ m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$, the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \alpha_3 \frac{V_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_4}{\rho} + \alpha_4 \frac{V_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech loss, pump}}$$

$$\rightarrow P_4 - P_3 = \frac{\rho\alpha(V_3^2 - V_4^2)}{2} + \frac{\dot{W}_{\text{pump,u}}}{\dot{V}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\text{ kg/m}^3)(1.0)[(1.86\text{ m/s})^2 - (3.64\text{ m/s})^2]}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + \frac{2.1\text{ kJ/s}}{7.14\times 10^{-3}\text{ m}^3/\text{s}}\left(\frac{1\text{ kN}\cdot\text{m}}{1\text{ kJ}}\right)$$

$$= (-4.9 + 294.1)\text{ kN/m}^2 = \mathbf{289.2\ kPa}$$

Discussion In an actual system, the flow rate of water will be less because of friction in pipes. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (under 2%) and can be ignored.

5-70 Underground water is pumped to a pool at a given elevation. For a given head loss, the flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible. **3** The effect of the kinetic energy correction factors is negligible, $\alpha = 1$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m³.

Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

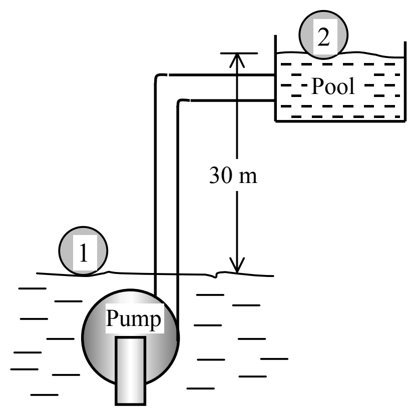
$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\text{ kW}) = 2.1\text{ kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), and the velocities are negligible at both points ($V_1 \cong V_2 \cong 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

In the absence of a turbine, $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, piping}}$ and $\dot{W}_{\text{pump, u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech loss, pump}}$ and thus

$$\dot{W}_{\text{pump, u}} = \dot{m}gz_2 + \dot{E}_{\text{mech loss, piping}}$$



Noting that $\dot{E}_{\text{mech, loss}} = \dot{m}gh_L$, the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump, u}}}{gz_2 + gh_L} = \frac{\dot{W}_{\text{pump, u}}}{g(z_2 + h_L)}$$

$$= \frac{2.1\text{ kJ/s}}{(9.81\text{ m/s}^2)(30+5\text{ m})}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ}}\right) = 6.116\text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{6.116\text{ kg/s}}{1000\text{ kg/m}^3} = 6.116 m^3/s \cong \mathbf{6.12 \times 10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$V_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{6.116\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.07\text{ m})^2/4} = 1.589\text{ m/s}, \quad V_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{6.116\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4} = 3.115\text{ m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$, the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \alpha_3 \frac{V_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_4}{\rho} + \alpha_4 \frac{V_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech loss, pump}}$$

$$\rightarrow P_4 - P_3 = \frac{\rho\alpha(V_3^2 - V_4^2)}{2} + \frac{\dot{W}_{\text{pump,u}}}{\dot{\text{V}}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\text{ kg/m}^3)(1.0)[(1.589\text{ m/s})^2 - (3.115\text{ m/s})^2]}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + \frac{2.1\text{ kJ/s}}{6.116\times10^{-3}\text{ m}^3/\text{s}}\left(\frac{1\text{ kN}\cdot\text{m}}{1\text{ kJ}}\right)$$

$$= (-3.6 + 343.4)\text{ kN/m}^2 = 339.8 kPa \cong \mathbf{340\ kPa}$$

Discussion Note that frictional losses in pipes causes the flow rate of water to decrease. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (about 1%) and can be ignored.

5-71E In a hydroelectric power plant, the elevation difference, the power generation, and the overall turbine-generator efficiency are given. The minimum flow rate required is to be determined.√

Assumptions **1** The flow is steady and incompressible. **2** The water levels at the reservoir and the discharge site remain constant. **3** We assume the flow to be *frictionless* since the *minimum* flow rate is to be determined, $\dot{E}_{\text{mech,loss}} = 0$.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge water stream, which is also taken as the reference level (z_2 = 0). Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($V_1 = V_2 = 0$), and frictional losses are disregarded. Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the turbine and the pipes reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=z_1$$

Substituting and noting that $\dot{W}_{\text{turbine, elect}} = \eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine, e}}$, the extracted turbine head and the mass and volume flow rates of water are determined to be

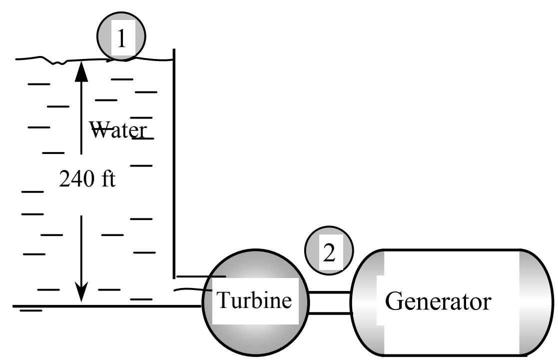
$$h_{\text{turbine, e}} = z_1 = 240\text{ ft}$$

$$\dot{m}=\frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}}\,gh_{\text{turbine}}}=\frac{100\text{ kW}}{0.83(32.2\text{ ft/s}^2)(240\text{ ft})}\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)\left(\frac{0.9478\text{ Btu/s}}{1\text{ kW}}\right)=\mathbf{370\ lbm/s}$$

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{370\text{ lbm/s}}{62.4\text{ lbm/ft}^3}=\mathbf{5.93\ ft^3/s}$$

Therefore, the flow rate of water must be at least 5.93 ft^3/s to generate the desired electric power while overcoming friction losses in pipes.

Discussion In an actual system, the flow rate of water will be more because of frictional losses in pipes.



5-72E In a hydroelectric power plant, the elevation difference, the head loss, the power generation, and the overall turbine-generator efficiency are given. The flow rate required is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge water stream, which is also taken as the reference level (z_2 = 0). Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$), the velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the turbine and the pipes reduces to

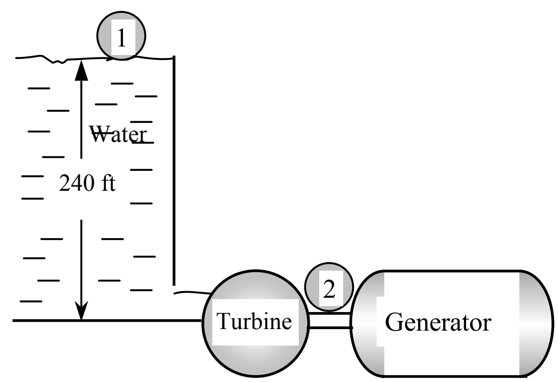
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=z_1-h_L$$

Substituting and noting that $\dot{W}_{\text{turbine, elect}}=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine, e}}$, the extracted turbine head and the mass and volume flow rates of water are determined to be

$$h_{\text{turbine, e}}=z_1-h_L=240-36=204\text{ ft}$$

$$\dot{m}=\frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}}gh_{\text{turbine}}}=\frac{100\text{ kW}}{0.83(32.2\text{ ft/s}^2)(204\text{ ft})}\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)\left(\frac{0.9478\text{ Btu/s}}{1\text{ kW}}\right)=\mathbf{435\ lbm/s}$$

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{435\text{ lbm/s}}{62.4\text{ lbm/ft}^3}=\mathbf{6.98\ ft^3/s}$$



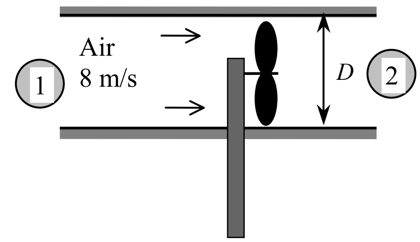
Therefore, the flow rate of water must be at least 6.98 ft^3/s to generate the desired electric power while overcoming friction losses in pipes.

Discussion Note that the effect of frictional losses in pipes is to increase the required flow rate of water to generate a specified amount of electric power.

5-73 A fan is to ventilate a bathroom by replacing the entire volume of air once every 10 minutes while air velocity remains below a specified value. The wattage of the fan-motor unit, the diameter of the fan casing, and the pressure difference across the fan are to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** Frictional losses along the flow (other than those due to the fan-motor inefficiency) are negligible. **3** The fan unit is horizontal so that z = constant along the flow (or, the elevation effects are negligible because of the low density of air). **4** The effect of the kinetic energy correction factors is negligible, $\alpha = 1$.

Properties The density of air is given to be 1.25 kg/m^3.



Analysis (*a*) The volume of air in the bathroom is V= 2 m × 3 m × 3 m = 18 m^3. Then the volume and mass flow rates of air through the casing must be

$$\dot{V} = \frac{V}{\Delta t} = \frac{18\text{ m}^3}{10\times 60\text{ s}} = 0.03\text{ m}^3/\text{s}$$

$$\dot{m} = \rho\dot{V} = (1.25\text{ kg/m}^3)(0.03\text{ m}^3/\text{s}) = 0.0375\text{ kg/s}$$

We take points 1 and 2 on the inlet and exit sides of the fan, respectively. Point 1 is sufficiently far from the fan so that $P_1 = P_{atm}$ and the flow velocity is negligible ($V_1 = 0$). Also, $P_2 = P_{atm}$. Then the energy equation for this control volume between the points 1 and 2 reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad\rightarrow\quad \dot{W}_{\text{fan, u}} = \dot{m}\alpha_2\frac{V_2^2}{2}$$

since $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}}$ in this case and $\dot{W}_{\text{pump, u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech loss, pump}}$. Substituting,

$$\dot{W}_{\text{fan, u}} = \dot{m}\alpha_2\frac{V_2^2}{2} = (0.0375\text{ kg/s})(1.0)\frac{(8\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = 1.2\text{ W}$$

and $$\dot{W}_{\text{fan, elect}} = \frac{\dot{W}_{\text{fan, u}}}{\eta_{\text{fan-motor}}} = \frac{1.2\text{ W}}{0.5} = \mathbf{2.4\ W}$$

Therefore, the electric power rating of the fan/motor unit must be 2.4 W.

(*b*) For air mean velocity to remain below the specified value, the diameter of the fan casing should be

$$\dot{V} = A_2V_2 = (\pi D_2^2/4)V_2 \quad\rightarrow\quad D_2 = \sqrt{\frac{4\dot{V}}{\pi V_2}} = \sqrt{\frac{4(0.03\text{ m}^3/\text{s})}{\pi(8\text{ m/s})}} = 0.069\text{ m} = \mathbf{6.9\ cm}$$

(*c*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. Noting that $z_3 = z_4$ and $V_3 = V_4$ since the fan is a narrow cross-section and neglecting flow loses (other than the loses of the fan unit, which is accounted for by the efficiency), the energy equation for the fan section reduces to

$$\dot{m}\frac{P_3}{\rho}+\dot{W}_{\text{fan, u}} = \dot{m}\frac{P_4}{\rho} \quad\rightarrow\quad P_4 - P_3 = \frac{\dot{W}_{\text{fan, u}}}{\dot{m}/\rho} = \frac{\dot{W}_{\text{fan, u}}}{\dot{V}}$$

Substituting, $$P_4 - P_3 = \frac{1.2\text{ W}}{0.03\text{ m}^3/\text{s}}\left(\frac{1\text{ N}\cdot\text{m/s}}{1\text{ W}}\right) = 40\text{ N/m}^2 = \mathbf{40\ Pa}$$

Therefore, the fan will raise the pressure of air by 40 Pa before discharging it.

Discussion Note that only half of the electric energy consumed by the fan-motor unit is converted to the mechanical energy of air while the remaining half is converted to heat because of imperfections.

5-74 Water is pumped from a large lake to a higher reservoir. The head loss of the piping system is given. The mechanical efficiency of the pump is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the lake and the reservoir is constant.

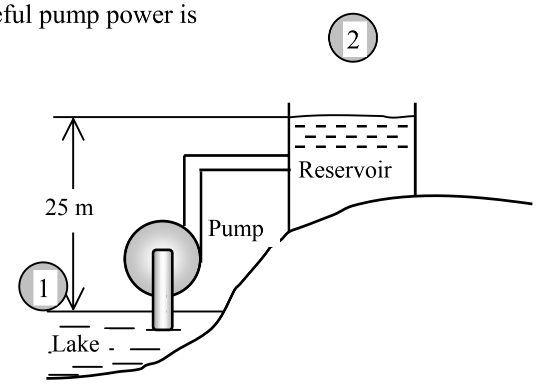
Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump}}=\dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \rightarrow$$

$$\dot{W}_{\text{pump,u}}=\dot{m}gz_2+\dot{E}_{\text{mech loss, piping}}$$

since, in the absence of a turbine, $\dot{E}_{\text{mech, loss}}=\dot{E}_{\text{mech loss, pump}}+\dot{E}_{\text{mech loss, piping}}$ and $\dot{W}_{\text{pump, u}}=\dot{W}_{\text{pump}}-\dot{E}_{\text{mech loss, pump}}$. Noting that $\dot{E}_{\text{mech loss, piping}}=\dot{m}gh_L$, the useful pump power is



$$\dot{W}_{\text{pump,u}}=\dot{m}gz_2+\dot{m}gh_L=\rho\dot{V}g(z_2+h_L)$$

$$=(1000\text{ kg/m}^3)(0.025\text{ m}^3\text{/s})(9.81 m/s^2)[(25+7)\text{ m}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$=7.85\text{ kNm/s}=7.85\text{ kW}$$

Then the mechanical efficiency of the pump becomes

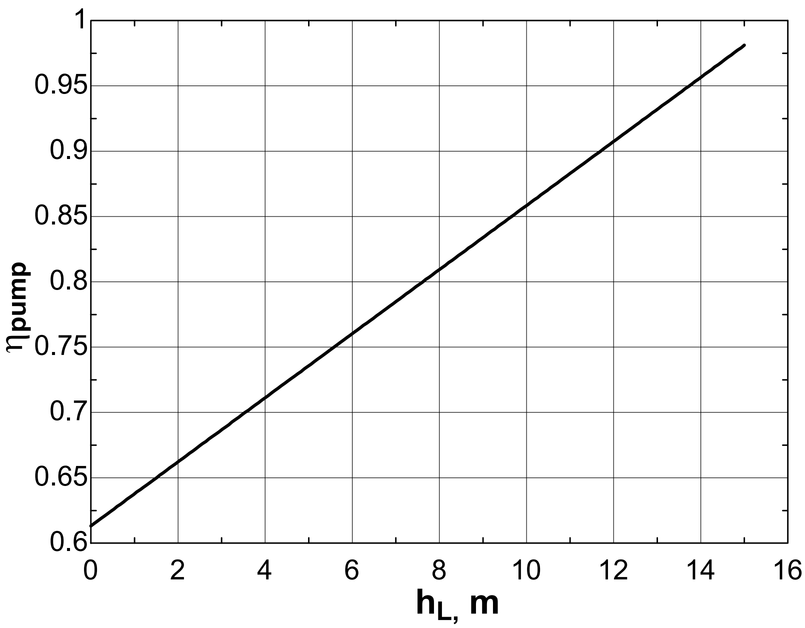
$$\eta_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{shaft}}}=\frac{7.85\text{ kW}}{10\text{ kW}}=0.785=\mathbf{78.5\%}$$

Discussion A more practical measure of performance of the pump is the overall efficiency, which can be obtained by multiplying the pump efficiency by the motor efficiency.

5-75 Problem 5-74 is reconsidered. The effect of head loss on mechanical efficiency of the pump. as the head loss varies 0 to 20 m in increments of 2 m is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
z2=25 "m"
W_shaft=10 "kW"
V_dot=0.025 "m3/s"
W_pump_u=rho*V_dot*g*(z2+h_L)/1000 "kW"
Eta_pump=W_pump_u/W_shaft
```

Head Loss, h_L, m	Pumping power $W_{pump, u}$	Efficiency η_{pump}
0	6.13	0.613
1	6.38	0.638
2	6.62	0.662
3	6.87	0.687
4	7.11	0.711
5	7.36	0.736
6	7.60	0.760
7	7.85	0.785
8	8.09	0.809
9	8.34	0.834
10	8.58	0.858
11	8.83	0.883
12	9.07	0.907
13	9.32	0.932
14	9.56	0.956
15	9.81	0.981



Note that the useful pumping power is used to raise the fluid and to overcome head losses. For a given power input, the pump that overcomes more head loss is more efficient.

5-76 A pump with a specified shaft power and efficiency is used to raise water to a higher elevation. The maximum flow rate of water is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the reservoirs is constant. **3** We assume the flow in the pipes to be *frictionless* since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech loss, piping}} = 0$.

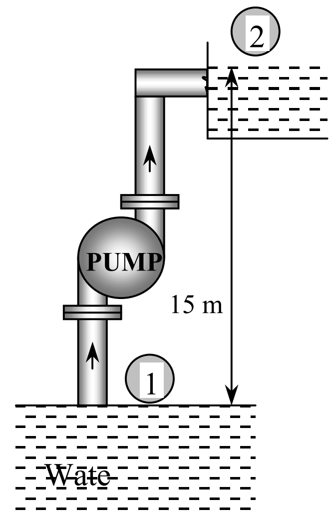
Properties We take the density of water to be ρ = 1000 kg/m³.

Analysis We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump}}=\dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \rightarrow \dot{W}_{\text{pump,u}}=\dot{m}gz_2=\rho\dot{V}gz_2$$

since $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}}$ in this case and $\dot{W}_{\text{pump, u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech loss, pump}}$.
The useful pumping power is

$$\dot{W}_{\text{pump,u}} = \eta_{\text{pump}}\dot{W}_{\text{pump, shaft}} = (0.82)(7 \text{ hp}) = 5.74 \text{ hp}$$



Substituting, the volume flow rate of water is determined to be

$$\dot{V} = \frac{\dot{W}_{\text{pump,u}}}{\rho g z_2} = \frac{5.74 \text{ hp}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m})}\left(\frac{745.7 \text{ W}}{1 \text{ hp}}\right)\left(\frac{1 \text{ N}\cdot\text{m/s}}{1 \text{ W}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$

$$= \mathbf{0.0291\ m^3/s}$$

Discussion This is the maximum flow rate since the frictional effects are ignored. In an actual system, the flow rate of water will be less because of friction in pipes.

5-77 Water flows at a specified rate in a horizontal pipe whose diameter is decreased by a reducer. The pressures are measured before and after the reducer. The head loss in the reducer is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The pipe is horizontal. **3** The kinetic energy correction factors are given to be $\alpha_1 = \alpha_2 = \alpha = 1.05$.

Properties We take the density of water to be ρ = 1000 kg/m³.

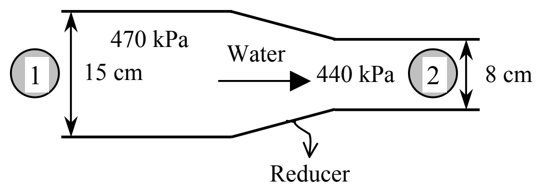
Analysis We take points 1 and 2 along the centerline of the pipe before and after the reducer, respectively. Noting that $z_1 = z_2$, the energy equation for steady incompressible flow through a control volume between these two points reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_L = \frac{P_1 - P_2}{\rho g} + \frac{\alpha(V_1^2 - V_2^2)}{2g}$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.035\ \text{m}^3/\text{s}}{\pi(0.15\ \text{m})^2/4} = 1.98\ \text{m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.035\ \text{m}^3/\text{s}}{\pi(0.08\ \text{m})^2/4} = 6.96\ \text{m/s}$$



Substituting, the head loss in the reducer is determined to be

$$h_L = \frac{(470-440)\ \text{kPa}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kN/m}^2}{1\ \text{kPa}}\right)\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) + \frac{1.05[(1.98\ \text{m/s})^2 - (6.96\ \text{m/s})^2]}{2(9.81\ \text{m/s}^2)}$$
$$= 3.06 - 2.38 = \mathbf{0.68\ m}$$

Discussion Note that the 0.79 m of the head loss is due to frictional effects and 2.27 m is due to the increase in velocity. This head loss corresponds to a power potential loss of

$$\dot{E}_{\text{mech loss, piping}} = \rho\dot{V}gh_L = (1000\ \text{kg/m}^3)(0.035\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(0.79\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{271\ W}$$

5-78 A hose connected to the bottom of a tank is equipped with a nozzle at the end pointing straight up. The water is pressurized by a pump, and the height of the water jet is measured. The minimum pressure rise supplied by the pump is to be determined.

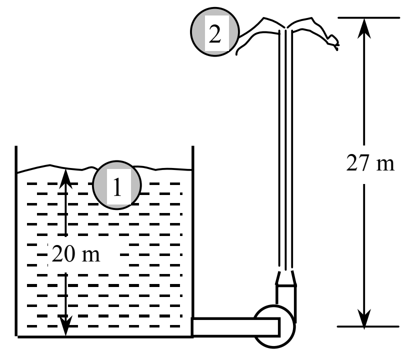
Assumptions **1** The flow is steady and incompressible. **2** Friction between the water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $V_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that z_1 = 20 m and z_2 = 27 m, h_L =0 (to get the minimum value for required pressure rise), and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the water stream reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L$$

$$\rightarrow \quad h_{\text{pump, u}}=z_2-z_1$$



Substituting,

$$h_{\text{pump, u}}=27-20=7\text{ m}$$

A water column height of 7 m corresponds to a pressure rise of

$$\Delta P_{\text{pump, min}}=\rho g h_{\text{pump, u}}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(7\text{ m})\left(\frac{1\text{ N}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$=68.7\text{ kN/m}^2=\mathbf{68.7\ kPa}$$

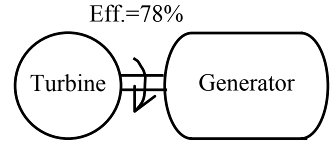
Therefore, the pump must supply a minimum pressure rise of 68.7 kPa.

Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure rise will need to be supplied to overcome friction.

5-79 The available head of a hydraulic turbine and its overall efficiency are given. The electric power output of this turbine is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The available head remains constant.

Properties We take the density of water to be ρ = 1000 kg/m^3.



Analysis When the turbine head is available, the corresponding power output is determined from

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine}}\dot{m}gh_{\text{turbine}} = \eta_{\text{turbine}}\rho\dot{V}gh_{\text{turbine}}$$

Substituting,

$$\dot{W}_{\text{turbine}} = 0.78(1000\text{ kg/m}^3)(0.25\text{ m}^3\text{/s})(9.81\text{ m/s}^2)(85\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = \mathbf{163\text{ kW}}$$

Discussion Note that the power output of a hydraulic turbine is proportional to the available turbine head and the flow rate.

5-80 An entrepreneur is to build a large reservoir above the lake level, and pump water from the lake to the reservoir at night using cheap power, and let the water flow from the reservoir back to the lake during the day, producing power. The potential revenue this system can generate per year is to be determined.

Assumptions **1** The flow in each direction is steady and incompressible. **2** The elevation difference between the lake and the reservoir can be taken to be constant, and the elevation change of reservoir during charging and discharging is disregarded. **3** The given unit prices remain constant. **4** The system operates every day of the year for 10 hours in each mode.

Properties We take the density of water to be $\rho = 1000 \text{ kg/m}^3$.

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level. Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the pump (or the turbine) and the pipes reduces to

Pump mode: $$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \rightarrow$$

$$h_{\text{pump, u}} = z_2 + h_L = 40 + 4 = 44 \text{ m}$$

Turbine mode: (switch points 1 and 2 so that 1 is on inlet side) $\rightarrow$ $h_{\text{turbine, e}} = z_1 - h_L = 40 - 4 = 36 \text{ m}$

The pump and turbine power corresponding to these heads are

$$\dot{W}_{\text{pump, elect}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}}$$

$$= \frac{(1000 \text{ kg/m}^3)(2 \text{ m}^3\text{/s})(9.81 \text{ m/s}^2)(44 \text{ m})}{0.75}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kW}}{1000 \text{ N}\cdot\text{m/s}}\right) = 1151 \text{ kW}$$

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine, e}} = \eta_{\text{turbine-gen}} \rho \dot{V} g h_{\text{turbine, e}}$$

$$= 0.75(1000 \text{ kg/m}^3)(2 \text{ m}^3\text{/s})(9.81 \text{ m/s}^2)(36 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kW}}{1000 \text{ N}\cdot\text{m/s}}\right) = 530 \text{ kW}$$

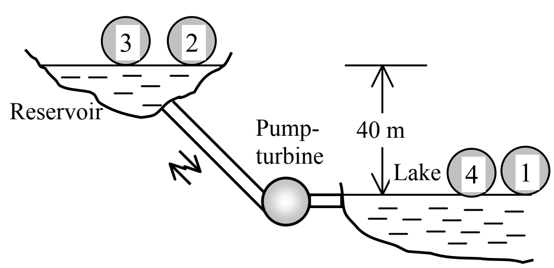
Then the power cost of the pump, the revenue generated by the turbine, and the net income (revenue minus cost) per year become

$$\text{Cost} = \dot{W}_{\text{pump, elect}} \Delta t \times \text{Unit price} = (1151 \text{ kW})(365 \times 10 \text{ h/year})(\$0.03/\text{kWh}) = \$126{,}035/\text{year}$$

$$\text{Revenue} = \dot{W}_{\text{turbine}} \Delta t \times \text{Unit price} = (530 \text{ kW})(365 \times 10 \text{ h/year})(\$0.08/\text{kWh}) = \$154{,}760/\text{year}$$

Net income = Revenue – Cost = 154,760 –126,035 = **\$28,725/year ≅ \$28,700/year**

Discussion It appears that this pump-turbine system has a potential annual income of about \$29,000. A decision on such a system will depend on the initial cost of the system, its life, the operating and maintenance costs, the interest rate, and the length of the contract period, among other things.



5-81 Water flows through a horizontal pipe at a specified rate. The pressure drop across a valve in the pipe is measured. The corresponding head loss and the power needed to overcome it are to be determined. √

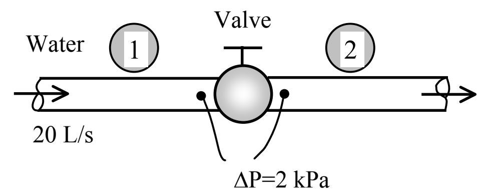
Assumptions **1** The flow is steady and incompressible. **2** The pipe is given to be horizontal (otherwise the elevation difference across the valve is negligible). **3** The mean flow velocities at the inlet and the exit of the valve are equal since the pipe diameter is constant.

Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis We take the valve as the control volume, and points 1 and 2 at the inlet and exit of the valve, respectively. Noting that $z_1 = z_2$ and $V_1 = V_2$, the energy equation for steady incompressible flow through this control volume reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \qquad \rightarrow \qquad h_L=\frac{P_1-P_2}{\rho g}$$

Substituting,



$$h_L=\frac{2\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)=\mathbf{0.204\ m}$$

The useful pumping power needed to overcome this head loss is

$$\dot{W}_{\text{pump, u}}=\dot{m}gh_L=\rho\dot{V}gh_L$$

$$=(1000\text{ kg/m}^3)(0.020\text{ m}^3\text{/s})(9.81\text{ m/s}^2)(0.204\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right)=\mathbf{40\ W}$$

Therefore, this valve would cause a head loss of 0.204 m, and it would take 40 W of useful pumping power to overcome it.

Discussion The required useful pumping power could also be determined from

$$\dot{W}_{\text{pump}}=\dot{V}\Delta P=(0.020\text{ m}^3\text{/s})(2000\text{ Pa})\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3\text{/s}}\right)=\mathbf{40\ W}$$

5-82E A hose connected to the bottom of a pressurized tank is equipped with a nozzle at the end pointing straight up. The minimum tank air pressure (gage) corresponding to a given height of water jet is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Friction between water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $V_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that z_1 = 66 ft and z_2 = 90 ft, $h_L = 0$ (to get the minimum value for the required air pressure), and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points reduces to

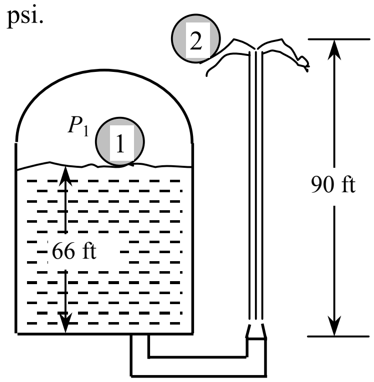
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \rightarrow$$

$$\frac{P_1-P_{atm}}{\rho g}=z_2-z_1 \rightarrow \frac{P_{1,gage}}{\rho g}=z_2-z_1$$

Rearranging and substituting, the gage pressure of pressurized air in the tank is determined to be

$$P_{1,\text{gage}}=\rho g(z_2-z_1)=(62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(90-66\text{ ft})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ psi}}{144\text{ lbf/ft}^2}\right)=10.4\text{ psi}$$

Therefore, the gage air pressure on top of the water tank must be at least 10.4 psi.



Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure will be needed to overcome friction.

5-83 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial discharge velocity from the tank is to be determined. √

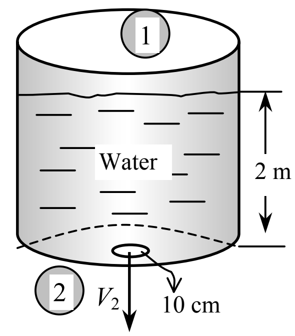
Assumptions **1** The flow is steady and incompressible. **2** The tank is open to the atmosphere. **3** The kinetic energy correction factor at the orifice is given to be $\alpha_2 = \alpha = 1.2$.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \qquad \rightarrow \qquad z_1 + \alpha_2 \frac{V_2^2}{2g} = z_2 + h_L$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2g(z_1 - z_2 - h_L)/\alpha} = \sqrt{2(9.81\,\text{m/s}^2)(2 - 0.3\,\text{m})/1.2} = \mathbf{5.27\ m/s}$$



Discussion This is the velocity that will prevail at the beginning. The mean flow velocity will decrease as the water level in the tank decreases.

5-84 Water enters a hydraulic turbine-generator system with a known flow rate, pressure drop, and efficiency. The net electric power output is to be determined. √

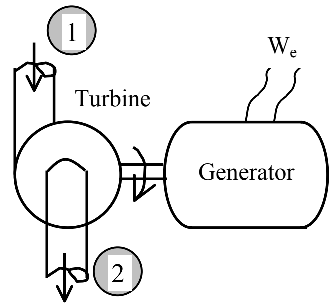
Assumptions **1** The flow is steady and incompressible. **2** All losses in the turbine are accounted for by turbine efficiency and thus $h_L = 0$. **3** The elevation difference across the turbine is negligible. **4** The effect of the kinetic energy correction factors is negligible, $\alpha_1 = \alpha_2 = \alpha = 1$.

Properties We take the density of water to be 1000 kg/m^3 and the density of mercury to be 13,560 kg/m^3.

Analysis We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=\frac{P_1-P_2}{\rho_{\text{water}}g}+\frac{\alpha(V_1^2-V_2^2)}{2g} \qquad (1)$$

where



$$V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}\frac{0.6\text{ m}^3/\text{s}}{\pi(0.30\text{ m})^2/4}=8.49\text{ m/s}$$

$$V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.6\text{ m}^3/\text{s}}{\pi(0.25\text{ m})^2/4}=12.2\text{ m/s}$$

The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$\begin{aligned}P_1-P_2&=(\rho_{\text{Hg}}-\rho_{\text{water}})gh\\&=[(13{,}560-1000)\text{ kg/m}^3](9.81\text{ m/s}^2)(1.2\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\\&=148\text{ kN/m}^2=148\text{ kPa}\end{aligned}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine, e}}=\frac{148\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)+(1.0)\frac{(8.49\text{ m/s})^2-(12.2\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=15.1-3.9=11.2\text{ m}$$

Then the net electric power output of this hydroelectric turbine becomes

$$\begin{aligned}\dot{W}_{\text{turbine}}&=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine, e}}=\eta_{\text{turbine-gen}}\rho\dot{V}gh_{\text{turbine, e}}\\&=0.83(1000\text{ kg/m}^3)(0.6\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(11.2\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right)=\mathbf{55\text{ kW}}\end{aligned}$$

Discussion It appears that this hydroelectric turbine will generate 55 kW of electric power under given conditions. Note that almost half of the available pressure head is discarded as kinetic energy. This demonstrates the need for a larger turbine exit area and better recovery. For example, the power output can be increased to 74 kW by redesigning the turbine and making the exit diameter of the pipe equal to the inlet diameter, $D_2 = D_1$. Further, if a much larger exit diameter is used and the exit velocity is reduced to a very low level, the power generation can increase to as much as 92 kW.

5-85 The velocity profile for turbulent flow in a circular pipe is given. The kinetic energy correction factor for this flow is to be determined.

Analysis The velocity profile is given by $u(r) = u_{\max}(1 - r/R)^{1/n}$ with n = 7 The kinetic energy correction factor is then expressed as

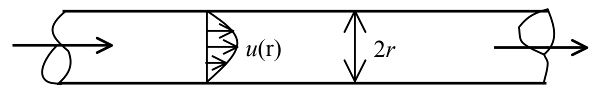
$$\alpha = \frac{1}{A}\int_A \left(\frac{u(r)}{V_{avg}}\right)^3 dA = \frac{1}{AV_{avg}^3}\int_A u(r)^3 dA = \frac{1}{\pi R^2 V_{avg}^3}\int_{r=0}^{R} u_{\max}^3 \left(1-\frac{r}{R}\right)^{\frac{3}{n}}(2\pi r)dr = \frac{2u_{\max}^3}{R^2 V_{avg}^3}\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{\frac{3}{n}} rdr$$

where the average velocity is

$$V_{avg} = \frac{1}{A}\int_A u(r)dA = \frac{1}{\pi R^2}\int_{r=0}^{R} u_{\max}\left(1-\frac{r}{R}\right)^{1/n}(2\pi r)dr = \frac{2u_{\max}}{R^2}\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{1/n} rdr$$

From integral tables,

$$\int (a+bx)^n xdx = \frac{(a+bx)^{n+2}}{b^2(n+2)} - \frac{a(a+bx)^{n+1}}{b^2(n+1)}$$



Then,

$$\int_{r=0}^{R} u(r)rdr = \int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{1/n} rdr = \left.\frac{(1-r/R)^{\frac{1}{n}+2}}{\frac{1}{R^2}(\frac{1}{n}+2)} - \frac{(1-r/R)^{\frac{1}{n}+1}}{\frac{1}{R^2}(\frac{1}{n}+1)}\right|_{r=0}^{R} = \frac{n^2R^2}{(n+1)(2n+1)}$$

$$\int_{r=0}^{R} u(r)^3 rdr = \int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{3/n} rdr = \left.\frac{(1-r/R)^{\frac{3}{n}+2}}{\frac{1}{R^2}(\frac{3}{n}+2)} - \frac{(1-r/R)^{\frac{3}{n}+1}}{\frac{1}{R^2}(\frac{3}{n}+1)}\right|_{r=0}^{R} = \frac{n^2R^2}{(n+3)(2n+3)}$$

Substituting,

$$V_{avg} = \frac{2u_{\max}}{R^2}\frac{n^2R^2}{(n+1)(2n+1)} = \frac{2n^2u_{\max}}{(n+1)(2n+1)} = 0.8167u_{\max}$$

and

$$\alpha = \frac{2u_{\max}^3}{R^2}\left(\frac{2n^2u_{\max}}{(n+1)(2n+1)}\right)^{-3}\frac{n^2R^2}{(n+3)(2n+3)} = \frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)} = \frac{(7+1)^3(2\times 7+1)^3}{4\times 7^4(7+3)(2\times 7+3)} = \mathbf{1.06}$$

Discussion Note that ignoring the kinetic energy correction factor results in an error of just 6% in this case in the kinetic energy term (which may be small itself). Considering that the uncertainties in some terms are usually more that 6%, we can usually ignore this correction factor in analysis.

5-86 A pump is pumping oil at a specified rate. The pressure rise of oil in the pump is measured, and the motor efficiency is specified. The mechanical efficiency of the pump is to be determined.

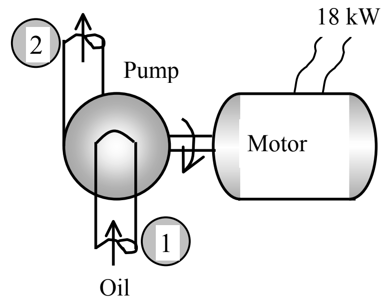
Assumptions **1** The flow is steady and incompressible. **2** The elevation difference across the pump is negligible. **3** All the losses in the pump are accounted for by the pump efficiency and thus $h_L = 0$. **4** The kinetic energy correction factors are given to be $\alpha_1 = \alpha_2 = \alpha = 1.05$.

Properties The density of oil is given to be $\rho = 860\ \text{kg/m}^3$.

Analysis We take points 1 and 2 at the inlet and the exit of the pump, respectively. Noting that $z_1 = z_2$, the energy equation for the pump reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=\frac{P_2-P_1}{\rho g}+\frac{\alpha(V_2^2-V_1^2)}{2g}$$

where



$$V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.1\ \text{m}^3\text{/s}}{\pi(0.08\ \text{m})^2/4}=19.9\ \text{m/s}$$

$$V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.1\ \text{m}^3\text{/s}}{\pi(0.12\ \text{m})^2/4}=8.84\ \text{m/s}$$

Substituting, the useful pump head and the corresponding useful pumping power are determined to be

$$h_{\text{pump, u}}=\frac{400{,}000\ \text{N/m}^2}{(860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)+\frac{1.05[(8.84\ \text{m/s})^2-(19.9\ \text{m/s})^2]}{2(9.81\ \text{m/s}^2)}=47.4-17.0=30.4\ \text{m}$$

$$\dot{W}_{\text{pump,u}}=\rho\dot{V}gh_{\text{pump, u}}=(860\ \text{kg/m}^3)(0.1\ \text{m}^3\text{/s})(9.81\ \text{m/s}^2)(30.4\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=25.6\ \text{kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot{W}_{\text{pump,shaft}}=\eta_{\text{motor}}\dot{W}_{\text{electric}}=(0.90)(35\ \text{kW})=31.5\ \text{kW}$$

$$\eta_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}}=\frac{25.6\ \text{kW}}{31.5\ \text{kW}}=0.813=81.3\%$$

Discussion The overall efficiency of this pump/motor unit is the product of the mechanical and motor efficiencies, which is 0.9×0.813 = 0.73.

5-87E Water is pumped from a lake to a nearby pool by a pump with specified power and efficiency. The head loss of the piping system and the mechanical power used to overcome it are to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the lake and the free surface of the pool is constant. **3** All the losses in the pump are accounted for by the pump efficiency and thus h_L represents the losses in piping.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis The useful pumping power and the corresponding useful pumping head are

$$\dot{W}_{\text{pump,u}} = \eta_{\text{pump}}\dot{W}_{\text{pump}} = (0.73)(12 \text{ hp}) = 8.76 \text{ hp}$$

$$h_{\text{pump, u}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g} = \frac{\dot{W}_{\text{pump, u}}}{\rho\dot{V}g}$$

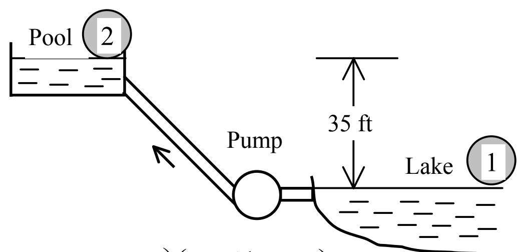
$$= \frac{8.76 \text{ hp}}{(62.4 \text{ lbm/ft}^3)(1.2 \text{ ft}^3/\text{s})(32.2 \text{ ft/s}^2)}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{1 \text{ hp}}\right) = 64.3 \text{ ft}$$

We choose points 1 and 2 at the free surfaces of the lake and the pool, respectively. Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_L = h_{\text{pump, u}} + z_1 - z_2$$

Substituting, the head loss is determined to be

$$h_L = h_{\text{pump, u}} - (z_2 - z_1) = 64.3 - 35 = \mathbf{29.3 \text{ ft}}$$



Then the power used to overcome it becomes

$$\dot{E}_{\text{mech loss, piping}} = \rho\dot{V}gh_L$$

$$= (62.4 \text{ lbm/ft}^3)(1.2 \text{ ft}^3/\text{s})(32.2 \text{ ft/s}^2)(29.3 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1 \text{ hp}}{550 \text{ lbf}\cdot\text{ft/s}}\right)$$

$$= \mathbf{4.0 \text{ hp}}$$

Discussion Note that the pump must raise the water an additional height of 29.3 ft to overcome the frictional losses in pipes, which requires an additional useful pumping power of about 4 hp.

5-88 A fireboat is fighting fires by drawing sea water and discharging it through a nozzle. The head loss of the system and the elevation of the nozzle are given. The shaft power input to the pump and the water discharge velocity are to be determined. √

Assumptions 1 The flow is steady and incompressible. **2** The effect of the kinetic energy correction factors is negligible, $\alpha = 1$.

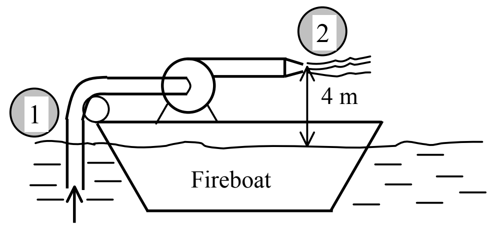
Properties The density of sea water is given to be ρ =1030 kg/m^3.

Analysis We take point 1 at the free surface of the sea and point 2 at the nozzle exit. Noting that $P_1 = P_2 = P_{atm}$ and $V_1 \cong 0$ (point 1 is at the free surface; not at the pipe inlet), the energy equation for the control volume between 1 and 2 that includes the pump and the piping system reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = z_2 - z_1 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

where the water discharge velocity is

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.1\ \text{m}^3/\text{s}}{\pi (0.05\ \text{m})^2/4} = \mathbf{50.93\ m/s}$$



Substituting, the useful pump head and the corresponding useful pump power are determined to be

$$h_{\text{pump, u}} = (4\ \text{m}) + (1)\frac{(50.93\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + (3\ \text{m}) = 139.2\ \text{m}$$

$$\dot{W}_{\text{pump,u}} = \rho \dot{V} g h_{\text{pump, u}} = (1030\ \text{kg/m}^3)(0.1\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(139.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)$$

$$= 140.7\ \text{kW}$$

Then the required shaft power input to the pump becomes

$$\dot{W}_{\text{pump, shaft}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump}}} = \frac{140.7\ \text{kW}}{0.70} = \mathbf{201\ kW}$$

Discussion Note that the pump power is used primarily to increase the kinetic energy of water.