

## Review Problems

**3-110** One section of the duct of an air-conditioning system is laid underwater. The upward force the water will exert on the duct is to be determined.

**Assumptions** 1 The diameter given is the outer diameter of the duct (or, the thickness of the duct material is negligible). 2 The weight of the duct and the air in is negligible.

**Properties** The density of air is given to be  $\rho = 1.30 \text{ kg/m}^3$ . We take the density of water to be  $1000 \text{ kg/m}^3$ .

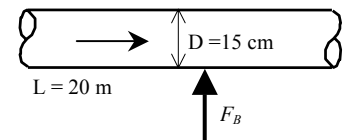
**Analysis** Noting that the weight of the duct and the air in it is negligible, the net upward force acting on the duct is the buoyancy force exerted by water. The volume of the underground section of the duct is

$$\mathcal{V} = AL = (\pi D^2 / 4)L = [\pi(0.15 \text{ m})^2 / 4](20 \text{ m}) = 0.3534 \text{ m}^3$$

Then the buoyancy force becomes

$$F_B = \rho g \mathcal{V} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3534 \text{ m}^3) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{3.47 \text{ kN}}$$

**Discussion** The upward force exerted by water on the duct is 3.47 kN, which is equivalent to the weight of a mass of 354 kg. Therefore, this force must be treated seriously.



**3-111** A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.

**Assumptions** The weight of the cage and the ropes of the balloon is negligible.

**Properties** The density of air is given to be  $\rho = 1.16 \text{ kg/m}^3$ . The density of helium gas is 1/7th of this.

**Analysis** The buoyancy force acting on the balloon is

$$V_{\text{balloon}} = 4\pi r^3/3 = 4\pi(5 \text{ m})^3/3 = 523.6 \text{ m}^3$$

$$\begin{aligned} F_B &= \rho_{\text{air}} g V_{\text{balloon}} \\ &= (1.16 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(523.6 \text{ m}^3) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 5958.4 \text{ N} \end{aligned}$$

The total mass is

$$m_{\text{He}} = \rho_{\text{He}} V = \left( \frac{1.16}{7} \text{ kg/m}^3 \right) (523.6 \text{ m}^3) = 86.8 \text{ kg}$$

$$m_{\text{total}} = m_{\text{He}} + m_{\text{people}} = 86.8 + 2 \times 70 = 226.8 \text{ kg}$$

The total weight is

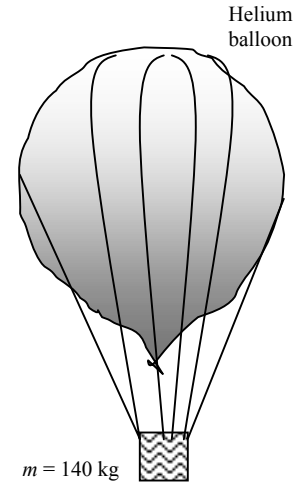
$$W = m_{\text{total}} g = (226.8 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2224.9 \text{ N}$$

Thus the net force acting on the balloon is

$$F_{\text{net}} = F_B - W = 5958.6 - 2224.9 = 3733.5 \text{ N}$$

Then the acceleration becomes

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{3733.5 \text{ N}}{226.8 \text{ kg}} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = \mathbf{16.5 \text{ m/s}^2}$$



**3-112** Problem 3-111 is reconsidered. The effect of the number of people carried in the balloon on acceleration is to be investigated. Acceleration is to be plotted against the number of people, and the results are to be discussed.

"Given Data:"

$\rho_{\text{air}} = 1.16 \text{ [kg/m}^3\text{]}$  "density of air"

$g = 9.807 \text{ [m/s}^2\text{]}$

$d_{\text{balloon}} = 10 \text{ [m]}$

$m_{\text{1person}} = 70 \text{ [kg]}$

{NoPeople = 2} "Data supplied in Parametric Table"

"Calculated values:"

$\rho_{\text{He}} = \rho_{\text{air}} / 7 \text{ [kg/m}^3\text{]}$  "density of helium"

$r_{\text{balloon}} = d_{\text{balloon}} / 2 \text{ [m]}$

$V_{\text{balloon}} = 4 \pi r_{\text{balloon}}^3 / 3 \text{ [m}^3\text{]}$

$m_{\text{people}} = \text{NoPeople} \times m_{\text{1person}} \text{ [kg]}$

$m_{\text{He}} = \rho_{\text{He}} \times V_{\text{balloon}} \text{ [kg]}$

$m_{\text{total}} = m_{\text{He}} + m_{\text{people}} \text{ [kg]}$

"The total weight of balloon and people is:"

$W_{\text{total}} = m_{\text{total}} \times g \text{ [N]}$

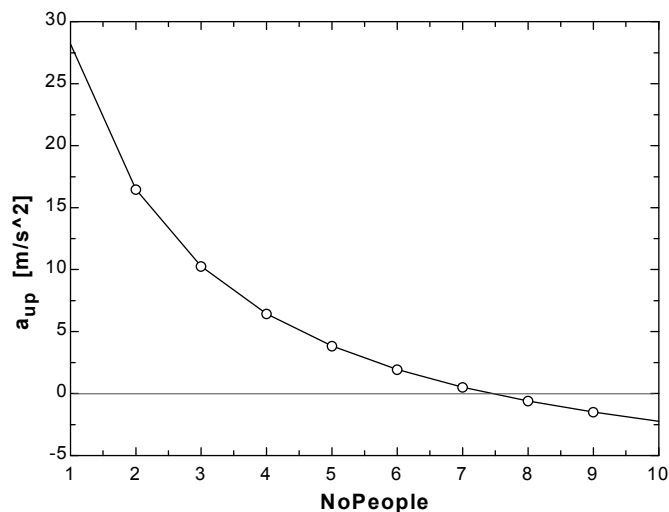
"The buoyancy force acting on the balloon,  $F_b$ , is equal to the weight of the air displaced by the balloon."

$F_b = \rho_{\text{air}} \times V_{\text{balloon}} \times g \text{ [N]}$

"From the free body diagram of the balloon, the balancing vertical forces must equal the product of the total mass and the vertical acceleration:"

$F_b - W_{\text{total}} = m_{\text{total}} \times a_{\text{up}}$

$a_{\text{up}} \text{ [m/s}^2\text{]}$	NoPeople
28.19	1
16.46	2
10.26	3
6.434	4
3.831	5
1.947	6
0.5204	7
-0.5973	8
-1.497	9
-2.236	10



**3-113** A balloon is filled with helium gas. The maximum amount of load the balloon can carry is to be determined.

**Assumptions** The weight of the cage and the ropes of the balloon is negligible.

**Properties** The density of air is given to be  $\rho = 1.16 \text{ kg/m}^3$ . The density of helium gas is 1/7th of this.

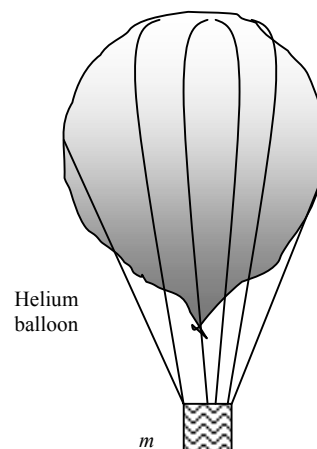
**Analysis** In the limiting case, the net force acting on the balloon will be zero. That is, the buoyancy force and the weight will balance each other:

$$W = mg = F_B$$

$$m_{\text{total}} = \frac{F_B}{g} = \frac{5958.4 \text{ N}}{9.81 \text{ m/s}^2} = 607.4 \text{ kg}$$

Thus,

$$m_{\text{people}} = m_{\text{total}} - m_{\text{He}} = 607.4 - 86.8 = \mathbf{520.6 \text{ kg}}$$



**3-114E** The pressure in a steam boiler is given in  $\text{kgf/cm}^2$ . It is to be expressed in psi, kPa, atm, and bars.

**Analysis** We note that  $1 \text{ atm} = 1.03323 \text{ kgf/cm}^2$ ,  $1 \text{ atm} = 14.696 \text{ psi}$ ,  $1 \text{ atm} = 101.325 \text{ kPa}$ , and  $1 \text{ atm} = 1.01325 \text{ bar}$  (inner cover page of text). Then the desired conversions become:

$$\text{In atm:} \quad P = (75 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) = 72.6 \text{ atm}$$

$$\text{In psi:} \quad P = (75 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{14.696 \text{ psi}}{1 \text{ atm}} \right) = \mathbf{1067 \text{ psi}}$$

$$\text{In kPa:} \quad P = (75 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{101.325 \text{ kPa}}{1 \text{ atm}} \right) = \mathbf{7355 \text{ kPa}}$$

$$\text{In bars:} \quad P = (75 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{1.01325 \text{ bar}}{1 \text{ atm}} \right) = \mathbf{73.55 \text{ bar}}$$

**Discussion** Note that the units atm,  $\text{kgf/cm}^2$ , and bar are almost identical to each other.

**3-115** A barometer is used to measure the altitude of a plane relative to the ground. The barometric readings at the ground and in the plane are given. The altitude of the plane is to be determined.

**Assumptions** The variation of air density with altitude is negligible.

**Properties** The densities of air and mercury are given to be  $\rho = 1.20 \text{ kg/m}^3$  and  $\rho = 13,600 \text{ kg/m}^3$ .

**Analysis** Atmospheric pressures at the location of the plane and the ground level are

$$\begin{aligned} P_{\text{plane}} &= (\rho g h)_{\text{plane}} \\ &= (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.690 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= 92.06 \text{ kPa} \end{aligned}$$

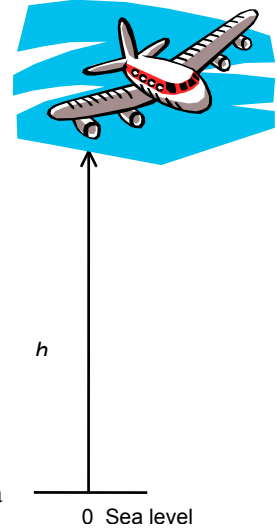
$$\begin{aligned} P_{\text{ground}} &= (\rho g h)_{\text{ground}} \\ &= (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.753 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= 100.46 \text{ kPa} \end{aligned}$$

Taking an air column between the airplane and the ground and writing a force balance per unit base area, we obtain

$$\begin{aligned} W_{\text{air}} / A &= P_{\text{ground}} - P_{\text{plane}} \\ (\rho g h)_{\text{air}} &= P_{\text{ground}} - P_{\text{plane}} \\ (1.20 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) &= (100.46 - 92.06) \text{ kPa} \end{aligned}$$

It yields  $h = 714 \text{ m}$

which is also the altitude of the airplane.



**3-116** A 10-m high cylindrical container is filled with equal volumes of water and oil. The pressure difference between the top and the bottom of the container is to be determined.

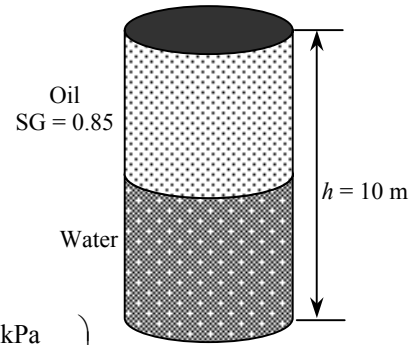
**Properties** The density of water is given to be  $\rho = 1000 \text{ kg/m}^3$ . The specific gravity of oil is given to be 0.85.

**Analysis** The density of the oil is obtained by multiplying its specific gravity by the density of water,

$$\rho = SG \times \rho_{H_2O} = (0.85)(1000 \text{ kg/m}^3) = 850 \text{ kg/m}^3$$

The pressure difference between the top and the bottom of the cylinder is the sum of the pressure differences across the two fluids,

$$\begin{aligned} \Delta P_{total} &= \Delta P_{oil} + \Delta P_{water} = (\rho g h)_{oil} + (\rho g h)_{water} \\ &= \left[ (850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) \right] \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{90.7 \text{ kPa}} \end{aligned}$$



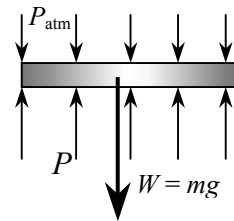
**3-117** The pressure of a gas contained in a vertical piston-cylinder device is measured to be 500 kPa. The mass of the piston is to be determined.

**Assumptions** There is no friction between the piston and the cylinder.

**Analysis** Drawing the free body diagram of the piston and balancing the vertical forces yield

$$\begin{aligned} W &= PA - P_{atm} A \\ mg &= (P - P_{atm}) A \\ (m)(9.81 \text{ m/s}^2) &= (500 - 100 \text{ kPa})(30 \times 10^{-4} \text{ m}^2) \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kPa}} \right) \end{aligned}$$

It yields  $m = \mathbf{122 \text{ kg}}$

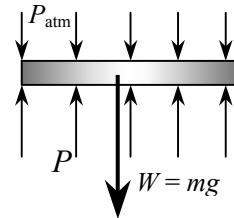


**3-118** The gage pressure in a pressure cooker is maintained constant at 100 kPa by a petcock. The mass of the petcock is to be determined.

**Assumptions** There is no blockage of the pressure release valve.

**Analysis** Atmospheric pressure is acting on all surfaces of the petcock, which balances itself out. Therefore, it can be disregarded in calculations if we use the gage pressure as the cooker pressure. A force balance on the petcock ( $\Sigma F_y = 0$ ) yields

$$\begin{aligned} W &= P_{gage} A \\ m &= \frac{P_{gage} A}{g} = \frac{(100 \text{ kPa})(4 \times 10^{-6} \text{ m}^2)}{9.81 \text{ m/s}^2} \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kPa}} \right) \\ &= \mathbf{0.0408 \text{ kg}} \end{aligned}$$



**3-119** A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube.

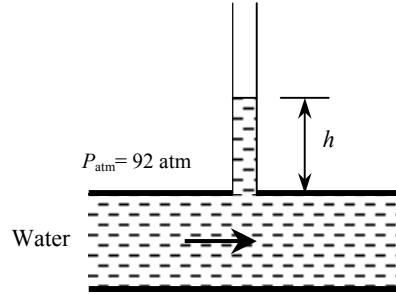
**Properties** The density of water is given to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** The pressure at the bottom of the tube can be expressed as

$$P = P_{\text{atm}} + (\rho g h)_{\text{tube}}$$

Solving for  $h$ ,

$$\begin{aligned} h &= \frac{P - P_{\text{atm}}}{\rho g} \\ &= \frac{(115 - 92) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) \left( \frac{1000 \text{ N/m}^2}{1 \text{ kPa}} \right) \\ &= \mathbf{2.34 \text{ m}} \end{aligned}$$



**3-120** The average atmospheric pressure is given as  $P_{\text{atm}} = 101.325(1 - 0.02256z)^{5.256}$  where  $z$  is the altitude in km. The atmospheric pressures at various locations are to be determined.

**Analysis** The atmospheric pressures at various locations are obtained by substituting the altitude  $z$  values in km into the relation

$$P_{\text{atm}} = 101.325(1 - 0.02256z)^{5.256}$$

Atlanta:	( $z = 0.306 \text{ km}$ ): $P_{\text{atm}} = 101.325(1 - 0.02256 \times 0.306)^{5.256} = \mathbf{97.7 \text{ kPa}}$
Denver:	( $z = 1.610 \text{ km}$ ): $P_{\text{atm}} = 101.325(1 - 0.02256 \times 1.610)^{5.256} = \mathbf{83.4 \text{ kPa}}$
M. City:	( $z = 2.309 \text{ km}$ ): $P_{\text{atm}} = 101.325(1 - 0.02256 \times 2.309)^{5.256} = \mathbf{76.5 \text{ kPa}}$
Mt. Ev.:	( $z = 8.848 \text{ km}$ ): $P_{\text{atm}} = 101.325(1 - 0.02256 \times 8.848)^{5.256} = \mathbf{31.4 \text{ kPa}}$

**3-121** The air pressure in a duct is measured by an inclined manometer. For a given vertical level difference, the gage pressure in the duct and the length of the differential fluid column are to be determined.

**Assumptions** The manometer fluid is an incompressible substance.

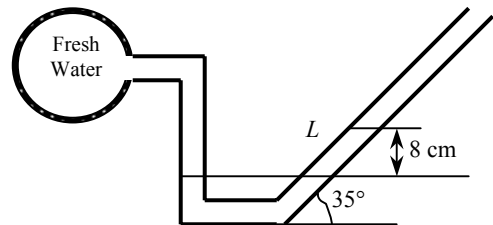
**Properties** The density of the liquid is given to be  $\rho = 0.81 \text{ kg/L} = 810 \text{ kg/m}^3$ .

**Analysis** The gage pressure in the duct is determined from

$$\begin{aligned} P_{\text{gage}} &= P_{\text{abs}} - P_{\text{atm}} = \rho g h \\ &= (810 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.08 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ Pa}}{1 \text{ N/m}^2} \right) \\ &= \mathbf{636 \text{ Pa}} \end{aligned}$$

The length of the differential fluid column is

$$L = h / \sin \theta = (8 \text{ cm}) / \sin 35^\circ = \mathbf{13.9 \text{ cm}}$$



**Discussion** Note that the length of the differential fluid column is extended considerably by inclining the manometer arm for better readability.

**3-122E** Equal volumes of water and oil are poured into a U-tube from different arms, and the oil side is pressurized until the contact surface of the two fluids moves to the bottom and the liquid levels in both arms become the same. The excess pressure applied on the oil side is to be determined.

**Assumptions** 1 Both water and oil are incompressible substances. 2 Oil does not mix with water. 3 The cross-sectional area of the U-tube is constant.

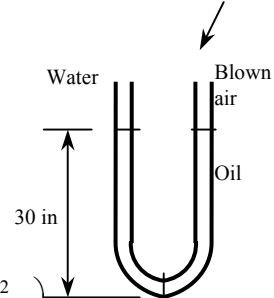
**Properties** The density of oil is given to be  $\rho_{\text{oil}} = 49.3 \text{ lbm/ft}^3$ . We take the density of water to be  $\rho_w = 62.4 \text{ lbm/ft}^3$ .

**Analysis** Noting that the pressure of both the water and the oil is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{\text{contact}} = P_{\text{blow}} + \rho_a g h_a = P_{\text{atm}} + \rho_w g h_w$$

Noting that  $h_a = h_w$  and rearranging,

$$\begin{aligned} P_{\text{gage, blow}} &= P_{\text{blow}} - P_{\text{atm}} = (\rho_w - \rho_{\text{oil}}) g h \\ &= (62.4 - 49.3 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(30/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= \mathbf{0.227 \text{ psi}} \end{aligned}$$



**Discussion** When the person stops blowing, the oil will rise and some water will flow into the right arm. It can be shown that when the curvature effects of the tube are disregarded, the differential height of water will be 23.7 in to balance 30-in of oil.



**3-123** It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height, and a certain gage pressure at the arm level is needed for sufficient flow rate. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.

**Assumptions** 1 The IV fluid is incompressible. 2 The IV bottle is open to the atmosphere.

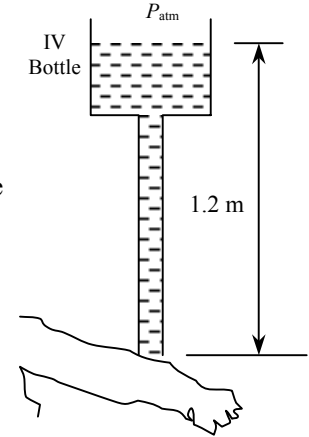
**Properties** The density of the IV fluid is given to be  $\rho = 1020 \text{ kg/m}^3$ .

**Analysis** (a) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to the gage pressure of the IV fluid at a depth of 1.2 m,

$$\begin{aligned} P_{\text{gage, arm}} &= P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{arm-bottle}} \\ &= (1020 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.20 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \\ &= \mathbf{12.0 \text{ kPa}} \end{aligned}$$

(b) To provide a gage pressure of 20 kPa at the arm level, the height of the bottle from the arm level is again determined from  $P_{\text{gage, arm}} = \rho g h_{\text{arm-bottle}}$  to be

$$\begin{aligned} h_{\text{arm-bottle}} &= \frac{P_{\text{gage, arm}}}{\rho g} \\ &= \frac{20 \text{ kPa}}{(1020 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) = \mathbf{2.0 \text{ m}} \end{aligned}$$



**Discussion** Note that the height of the reservoir can be used to control flow rates in gravity driven flows. When there is flow, the pressure drop in the tube due to friction should also be considered. This will result in raising the bottle a little higher to overcome pressure drop.

**3-124** A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

**Properties** The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the gasoline pipe, and setting the result equal to  $P_{\text{gasoline}}$  gives

$$P_{\text{gage}} - \rho_w gh_w + \rho_{\text{alcohol}} gh_{\text{alcohol}} - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{gasoline}} gh_{\text{gasoline}} = P_{\text{gasoline}}$$

Rearranging,

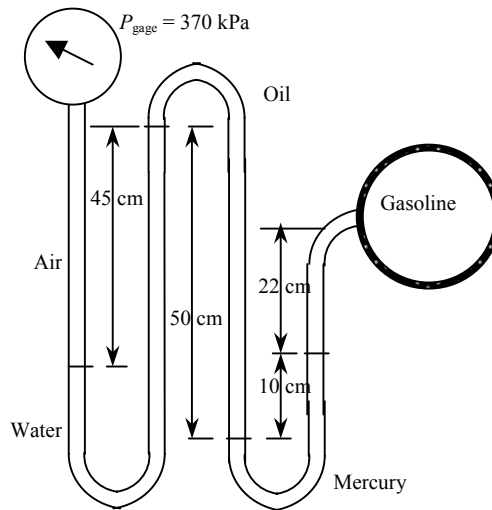
$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g(h_w - SG_{\text{alcohol}} h_{\text{alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{gasoline}} h_{\text{gasoline}})$$

Substituting,

$$\begin{aligned} P_{\text{gasoline}} &= 370 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.45 \text{ m}) - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})] \\ &\quad \times \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \\ &= \mathbf{354.6 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

**Discussion** Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.



**3-125** A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

**Properties** The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the gasoline pipe, and setting the result equal to  $P_{\text{gasoline}}$  gives

$$P_{\text{gage}} - \rho_w gh_w + \rho_{\text{alcohol}} gh_{\text{alcohol}} - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{gasoline}} gh_{\text{gasoline}} = P_{\text{gasoline}}$$

Rearranging,

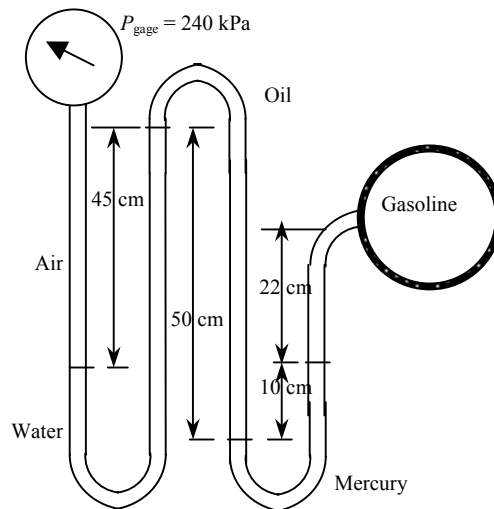
$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g(h_w - SG_{\text{alcohol}} h_{s,\text{alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{gasoline}} h_{s,\text{gasoline}})$$

Substituting,

$$\begin{aligned} P_{\text{gasoline}} &= 240 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.45 \text{ m}) - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})] \\ &\quad \times \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \\ &= \mathbf{224.6 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

**Discussion** Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.



**3-126E** A water pipe is connected to a double-U manometer whose free arm is open to the atmosphere. The absolute pressure at the center of the pipe is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The solubility of the liquids in each other is negligible.

**Properties** The specific gravities of mercury and oil are given to be 13.6 and 0.80, respectively. We take the density of water to be  $\rho_w = 62.4 \text{ lbm/ft}^3$ .

**Analysis** Starting with the pressure at the center of the water pipe, and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to  $P_{\text{atm}}$  gives

$$P_{\text{water pipe}} - \rho_{\text{water}} gh_{\text{water}} + \rho_{\text{alcohol}} gh_{\text{alcohol}} - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{oil}} gh_{\text{oil}} = P_{\text{atm}}$$

Solving for  $P_{\text{water pipe}}$ ,

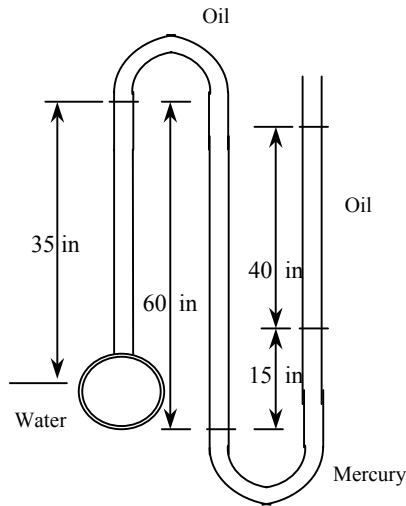
$$P_{\text{water pipe}} = P_{\text{atm}} + \rho_{\text{water}} g(h_{\text{water}} - SG_{\text{oil}} h_{\text{alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{oil}} h_{\text{oil}})$$

Substituting,

$$\begin{aligned} P_{\text{water pipe}} &= 14.2 \text{ psia} + (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)[(35/12 \text{ ft}) - 0.80(60/12 \text{ ft}) + 13.6(15/12 \text{ ft}) \\ &\quad + 0.8(40/12 \text{ ft})] \times \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= \mathbf{22.3 \text{ psia}} \end{aligned}$$

Therefore, the absolute pressure in the water pipe is 22.3 psia.

**Discussion** Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.



**3-127** The pressure of water flowing through a pipe is measured by an arrangement that involves both a pressure gage and a manometer. For the values given, the pressure in the pipe is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

**Properties** The specific gravity of gage fluid is given to be 2.4. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the water pipe, and setting the result equal to  $P_{\text{water}}$  give

$$P_{\text{gage}} + \rho_w g h_{w1} - \rho_{\text{gage}} g h_{\text{gage}} - \rho_w g h_{w2} = P_{\text{water}}$$

Rearranging,

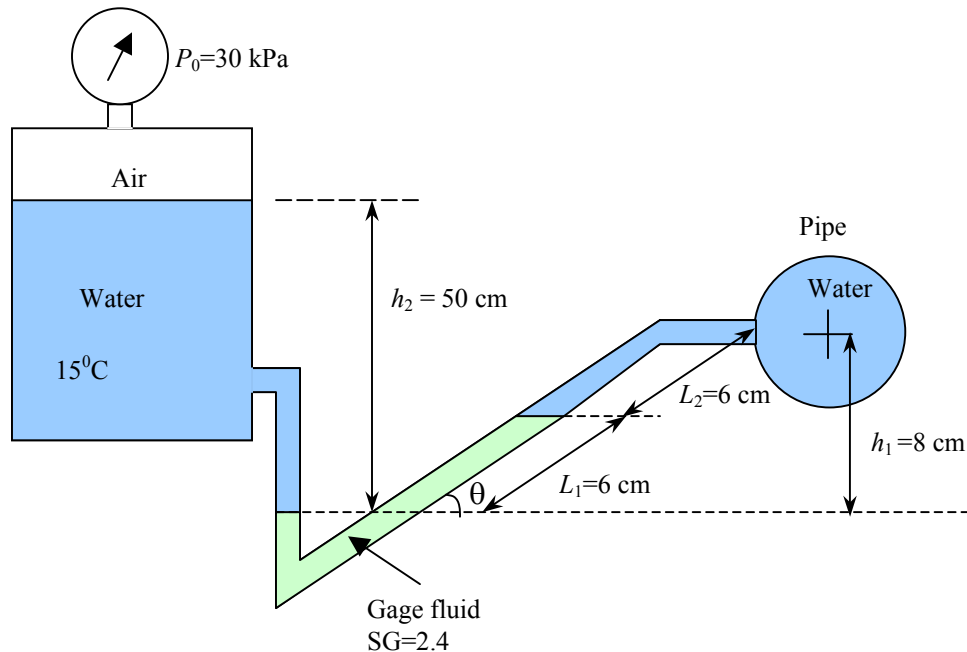
$$P_{\text{water}} = P_{\text{gage}} + \rho_w g (h_{w1} - SG_{\text{gage}} h_{\text{gage}} - h_{w2}) = P_{\text{gage}} + \rho_w g (h_2 - SG_{\text{gage}} L_1 \sin \theta - L_2 \sin \theta)$$

Noting that  $\sin \theta = 8/12 = 0.6667$  and substituting,

$$\begin{aligned} P_{\text{water}} &= 30 \text{ kPa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.50 \text{ m}) - 2.4(0.06 \text{ m})0.6667 - (0.06 \text{ m})0.6667] \\ &\quad \times \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \\ &= \mathbf{33.6 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the gasoline pipe is 3.6 kPa over the reading of the pressure gage.

**Discussion** Note that even without a manometer, the reading of a pressure gage can be in error if it is not placed at the same level as the pipe when the fluid is a liquid.



**3-128** A U-tube filled with mercury except the 18-cm high portion at the top. Oil is poured into the left arm, forcing some mercury from the left arm into the right one. The maximum amount of oil that can be added into the left arm is to be determined.

**Assumptions** 1 Both liquids are incompressible. 2 The U-tube is perfectly vertical.

**Properties** The specific gravities are given to be 2.72 for oil and 13.6 for mercury.

**Analysis** Initially, the mercury levels in both tubes are the same. When oil is poured into the left arm, it will push the mercury in the left down, which will cause the mercury level in the right arm to rise. Noting that the volume of mercury is constant, the decrease in the mercury volume in left column must be equal to the increase in the mercury volume in the right arm. Therefore, if the drop in mercury level in the left arm is  $x$ , the rise in the mercury level in the right arm  $h$  corresponding to a drop of  $x$  in the left arm is

$$V_{\text{left}} = V_{\text{right}} \rightarrow \pi(2d)^2 x = \pi d^2 h \rightarrow h = 4x$$

The pressures at points  $A$  and  $B$  are equal  $P_A = P_B$  and thus

$$P_{\text{atm}} + \rho_{\text{oil}} g(h_{\text{oil}} + x) = P_{\text{atm}} + \rho_{\text{Hg}} g h_{\text{Hg}} \rightarrow SG_{\text{oil}} \rho_w g(h_{\text{oil}} + x) = SG_{\text{Hg}} \rho_w g(5x)$$

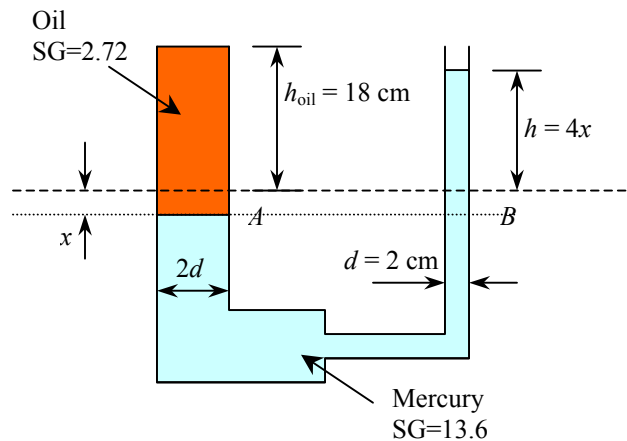
Solving for  $x$  and substituting,

$$x = \frac{SG_{\text{oil}} h_{\text{oil}}}{5SG_{\text{Hg}} - SG_{\text{oil}}} = \frac{2.72(18 \text{ cm})}{5 \times 13.6 - 2.72} = 0.75 \text{ cm}$$

Therefore, the maximum amount of oil that can be added into the left arm is

$$V_{\text{oil, max}} = \pi(2d / 2)^2 (h_{\text{oil}} + x) = \pi(2 \text{ cm})^2 (18 + 0.75 \text{ cm}) = \mathbf{236 \text{ cm}^3 = 0.236 \text{ L}}$$

**Discussion** Note that the fluid levels in the two arms of a U-tube can be different when two different fluids are involved.



**3-129** The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined.

**Assumptions** 1 Water is incompressible. 2 Thermal expansion and the amount of water in the service tube is negligible. 3 The cold water temperature is 20°C.

**Properties** The density of water at 20°C is  $\rho_w = 998.0 \text{ kg/m}^3$ .

**Analysis** From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

$$h_{\text{tip}} = 4 + 12 \cos 40^\circ = 13.2 \text{ cm}$$

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

$$P_{v, \text{gage}} = \rho_w g \Delta h_w$$

or,

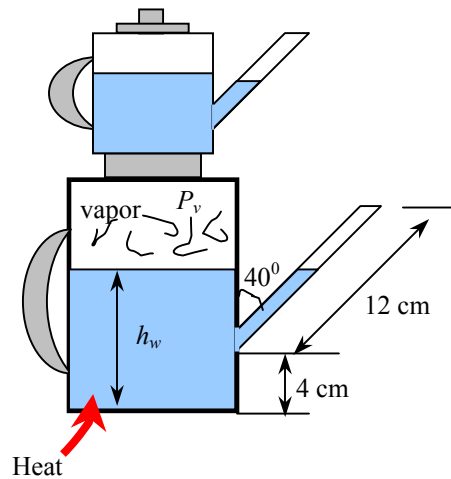
$$\Delta h_w = \frac{P_{v, \text{gage}}}{\rho_w g} = \frac{0.32 \text{ kPa}}{(998.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) = 0.033 \text{ m} = 3.3 \text{ cm}$$

Therefore, the water level inside the teapot must be 3.3 cm below the tip of the service tube. Then the maximum initial water height inside the teapot to avoid overflow becomes

$$h_{w, \text{max}} = h_{\text{tip}} - \Delta h_w = 13.2 - 3.3 = \mathbf{9.9 \text{ cm}}$$

**Discussion** We can obtain the same result formally by starting with the vapor pressure in the teapot and moving along the service tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the atmosphere, and setting the result equal to  $P_{\text{atm}}$ :

$$P_{\text{atm}} + P_{v, \text{gage}} - \rho_w g h_w = P_{\text{atm}} \rightarrow P_{v, \text{gage}} = \rho_w g h_w$$



**3-130** The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined by considering the effect of thermal expansion.

**Assumptions** 1 The amount of water in the service tube is negligible. 3 The cold water temperature is 20°C.

**Properties** The density of water is  $\rho_w = 998.0 \text{ kg/m}^3$  at 20°C, and  $\rho_w = 957.9 \text{ kg/m}^3$  at 100°C

**Analysis** From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

$$h_{\text{tip}} = 4 + 12 \cos 40^\circ = 13.2 \text{ cm}$$

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

$$P_{v, \text{gage}} = \rho_w g \Delta h_w$$

or,

$$\Delta h_w = \frac{P_{v, \text{gage}}}{\rho_w g} = \frac{0.32 \text{ kPa}}{(998.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) = 0.033 \text{ m} = 3.3 \text{ cm}$$

Therefore, the water level inside the teapot must be 3.4 cm below the tip of the service tube. Then the height of hot water inside the teapot to avoid overflow becomes

$$h_w = h_{\text{tip}} - \Delta h_w = 13.2 - 3.4 = 9.8 \text{ cm}$$

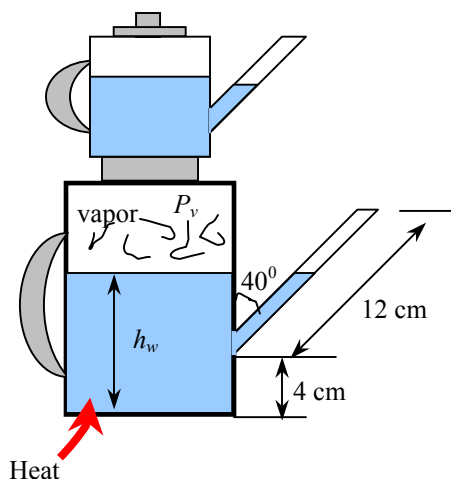
The specific volume of water is  $1/998 \text{ m}^3/\text{kg}$  at 20°C and  $1/957.9 \text{ m}^3/\text{kg}$  at 100°C. Then the percent drop in the volume of water as it cools from 100°C to 20°C is

$$\text{Volume reduction} = \frac{v_{100^\circ\text{C}} - v_{20^\circ\text{C}}}{v_{100^\circ\text{C}}} = \frac{1/957.9 - 1/998.0}{1/957.9} = 0.040 \quad \text{or } 4.0\%$$

Volume is proportional to water height, and to allow for thermal expansion, the volume of cold water should be 4% less. Therefore, the maximum initial water height to avoid overflow should be

$$h_{w, \text{max}} = (1 - 0.040)h_w = 0.96 \times 9.8 \text{ cm} = \mathbf{9.4 \text{ cm}}$$

**Discussion** Note that the effect of thermal expansion can be quite significant.





**3-131** The temperature of the atmosphere varies with altitude  $z$  as  $T = T_0 - \beta z$ , while the gravitational acceleration varies by  $g(z) = g_0 / (1 + z / 6,370,320)^2$ . Relations for the variation of pressure in atmosphere are to be obtained (a) by ignoring and (b) by considering the variation of  $g$  with altitude.

**Assumptions** The air in the troposphere behaves as an ideal gas.

**Analysis** (a) Pressure change across a differential fluid layer of thickness  $dz$  in the vertical  $z$  direction is

$$dP = -\rho g dz$$

From the ideal gas relation, the air density can be expressed as  $\rho = \frac{P}{RT} = \frac{P}{R(T_0 - \beta z)}$ . Then,

$$dP = -\frac{P}{R(T_0 - \beta z)} g dz$$

Separating variables and integrating from  $z = 0$  where  $P = P_0$  to  $z = z$  where  $P = P$ ,

$$\int_{P_0}^P \frac{dP}{P} = -\int_0^z \frac{g dz}{R(T_0 - \beta z)}$$

Performing the integrations,

$$\ln \frac{P}{P_0} = -\frac{g}{R\beta} \ln \frac{T_0 - \beta z}{T_0}$$

Rearranging, the desired relation for atmospheric pressure for the case of constant  $g$  becomes

$$P = P_0 \left( 1 - \frac{\beta z}{T_0} \right)^{\frac{g}{R\beta}}$$

(b) When the variation of  $g$  with altitude is considered, the procedure remains the same but the expressions become more complicated,

$$dP = -\frac{P}{R(T_0 - \beta z)} \frac{g_0}{(1 + z / 6,370,320)^2} dz$$

Separating variables and integrating from  $z = 0$  where  $P = P_0$  to  $z = z$  where  $P = P$ ,

$$\int_{P_0}^P \frac{dP}{P} = -\int_0^z \frac{g_0 dz}{R(T_0 - \beta z)(1 + z / 6,370,320)^2}$$

Performing the integrations,

$$\ln P \Big|_{P_0}^P = \frac{g_0}{R\beta} \left[ \frac{1}{(1 + kT_0 / \beta)(1 + kz)} - \frac{1}{(1 + kT_0 / \beta)^2} \ln \frac{1 + kz}{T_0 - \beta z} \right]_0^z$$

where  $R = 287 \text{ J/kg}\cdot\text{K} = 287 \text{ m}^2/\text{s}^2\cdot\text{K}$  is the gas constant of air. After some manipulations, we obtain

$$P = P_0 \exp \left[ -\frac{g_0}{R(\beta + kT_0)} \left( \frac{1}{1 + 1/kz} + \frac{1}{1 + kT_0 / \beta} \ln \frac{1 + kz}{1 - \beta z / T_0} \right) \right]$$

where  $T_0 = 288.15 \text{ K}$ ,  $\beta = 0.0065 \text{ K/m}$ ,  $g_0 = 9.807 \text{ m/s}^2$ ,  $k = 1/6,370,320 \text{ m}^{-1}$ , and  $z$  is the elevation in m..

**Discussion** When performing the integration in part (b), the following expression from integral tables is used, together with a transformation of variable  $x = T_0 - \beta z$ ,

$$\int \frac{dx}{x(a + bx)^2} = \frac{1}{a(a + bx)} - \frac{1}{a^2} \ln \frac{a + bx}{x}$$

Also, for  $z = 11,000 \text{ m}$ , for example, the relations in (a) and (b) give 22.62 and 22.69 kPa, respectively.

**3-132** The variation of pressure with density in a thick gas layer is given. A relation is to be obtained for pressure as a function of elevation  $z$ .

**Assumptions** The property relation  $P = C\rho^n$  is valid over the entire region considered.

**Analysis** The pressure change across a differential fluid layer of thickness  $dz$  in the vertical  $z$  direction is given as,

$$dP = -\rho g dz$$

Also, the relation  $P = C\rho^n$  can be expressed as  $C = P / \rho^n = P_0 / \rho_0^n$ , and thus

$$\rho = \rho_0 (P / P_0)^{1/n}$$

Substituting,

$$dP = -g\rho_0 (P / P_0)^{1/n} dz$$

Separating variables and integrating from  $z = 0$  where  $P = P_0 = C\rho_0^n$  to  $z = z$  where  $P = P$ ,

$$\int_{P_0}^P (P / P_0)^{-1/n} dP = -\rho_0 g \int_0^z dz$$

Performing the integrations.

$$P_0 \frac{(P / P_0)^{-1/n+1}}{-1/n+1} \Big|_{P_0}^P = -\rho_0 g z \quad \rightarrow \quad \left( \frac{P}{P_0} \right)^{(n-1)/n} - 1 = -\frac{n-1}{n} \frac{\rho_0 g z}{P_0}$$

Solving for  $P$ ,

$$P = P_0 \left( 1 - \frac{n-1}{n} \frac{\rho_0 g z}{P_0} \right)^{n/(n-1)}$$

which is the desired relation.

**Discussion** The final result could be expressed in various forms. The form given is very convenient for calculations as it facilitates unit cancellations and reduces the chance of error.

**3-133** A pressure transducer is used to measure pressure by generating analogue signals, and it is to be calibrated by measuring both the pressure and the electric current simultaneously for various settings, and the results are tabulated. A calibration curve in the form of  $P = aI + b$  is to be obtained, and the pressure corresponding to a signal of 10 mA is to be calculated.

**Assumptions** Mercury is an incompressible liquid.

**Properties** The specific gravity of mercury is given to be 13.56, and thus its density is  $13,560 \text{ kg/m}^3$ .

**Analysis** For a given differential height, the pressure can be calculated from

$$P = \rho g \Delta h$$

For  $\Delta h = 28.0 \text{ mm} = 0.0280 \text{ m}$ , for example,

$$P = 13.56(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0280 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) = 3.72 \text{ kPa}$$

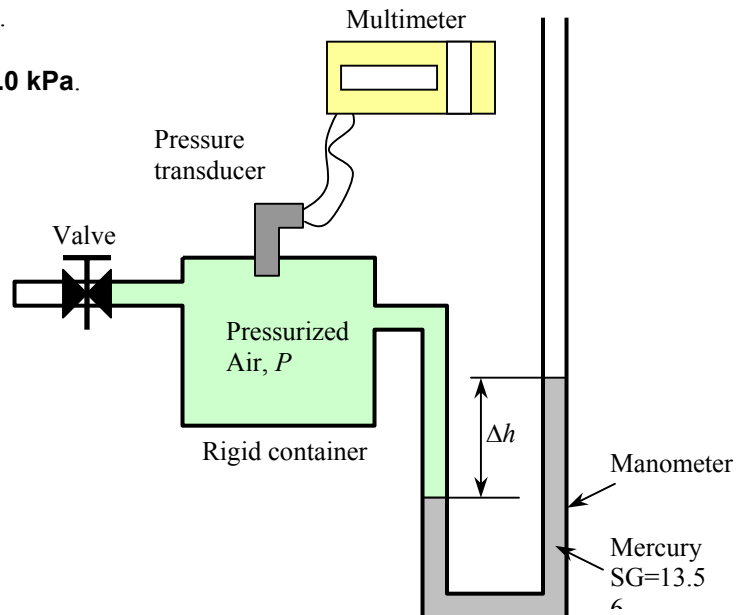
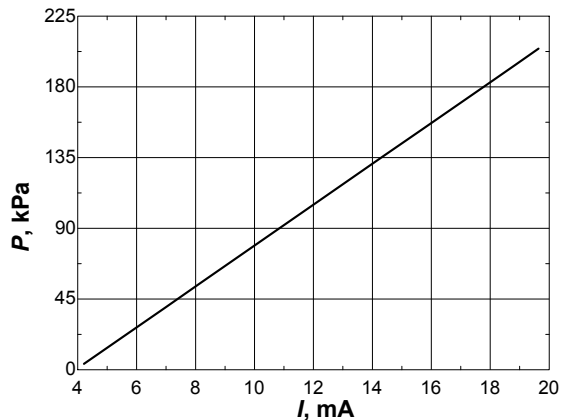
Repeating the calculations and tabulating, we have

$\Delta h(\text{mm})$	28.0	181.5	297.8	413.1	765.9	1027	1149	1362	1458	1536
<b><math>P(\text{kPa})</math></b>	<b>3.72</b>	<b>24.14</b>	<b>39.61</b>	<b>54.95</b>	<b>101.9</b>	<b>136.6</b>	<b>152.8</b>	<b>181.2</b>	<b>193.9</b>	<b>204.3</b>
<b><math>I(\text{mA})</math></b>	<b>4.21</b>	<b>5.78</b>	<b>6.97</b>	<b>8.15</b>	<b>11.76</b>	<b>14.43</b>	<b>15.68</b>	<b>17.86</b>	<b>18.84</b>	<b>19.64</b>

A plot of  $P$  versus  $I$  is given below. It is clear that the pressure varies linearly with the current, and using EES, the best curve fit is obtained to be

$$P = 13.00I - 51.00 \quad (\text{kPa}) \quad \text{for } 4.21 \leq I \leq 19.64.$$

For  $I = 10 \text{ mA}$ , for example, we would get  $P = \mathbf{79.0 \text{ kPa}}$ .



**Discussion** Note that the calibration relation is valid in the specified range of currents or pressures.

**3-134** A system is equipped with two pressure gages and a manometer. For a given differential fluid height, the pressure difference  $\Delta P = P_2 - P_1$  is to be determined.

**Assumptions** 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

**Properties** The specific gravities are given to be 2.67 for the gage fluid and 0.87 for oil. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ .

**Analysis** Starting with the pressure indicated by the pressure gage 2 and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms and ignoring the air spaces until we reach the pressure gage 1, and setting the result equal to  $P_1$  give

$$P_2 - \rho_{\text{gage}} g h_{\text{gage}} + \rho_{\text{oil}} g h_{\text{oil}} = P_1$$

Rearranging,

$$P_2 - P_1 = \rho_w g (SG_{\text{gage}} h_{\text{gage}} - SG_{\text{oil}} h_{\text{oil}})$$

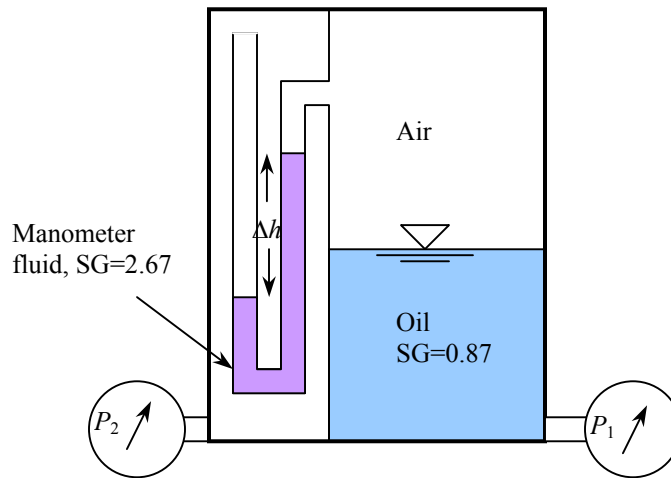
Substituting,

$$P_2 - P_1 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[2.67(0.08 \text{ m}) - 0.87(0.65 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right)$$

$$= -3.45 \text{ kPa}$$

Therefore, the pressure reading of the left gage is 3.45 kPa lower than that of the right gage.

**Discussion** The negative pressure difference indicates that the pressure differential across the oil level is greater than the pressure differential corresponding to the differential height of the manometer fluid.



**3-135** An oil pipeline and a rigid air tank are connected to each other by a manometer. The pressure in the pipeline and the change in the level of manometer fluid due to a air temperature drop are to be determined.

**Assumptions** **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible. **3** The air volume in the manometer is negligible compared with the volume of the tank.

**Properties** The specific gravities are given to be 2.68 for oil and 13.6 for mercury. We take the standard density of water to be  $\rho_w = 1000 \text{ kg/m}^3$ . The gas constant of air is  $0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ .

**Analysis** (a) Starting with the oil pipe and moving along the tube by adding (as we go down) or subtracting (as we go up) the  $\rho gh$  terms until we reach the air tank, and setting the result equal to  $P_{\text{air}}$  give

$$P_{\text{oil}} + \rho_{\text{oil}}gh_{\text{oil}} + \rho_{\text{Hg}}gh_{\text{Hg}} = P_{\text{air}}$$

The absolute pressure in the air tank is determined from the ideal-gas relation  $PV = mRT$  to be

$$P_{\text{air}} = \frac{mRT}{V} = \frac{(15 \text{ kg})(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(80 + 273)\text{K}}{1.3 \text{ m}^3} = 1169 \text{ kPa}$$

Then the absolute pressure in the oil pipe becomes

$$\begin{aligned} P_{\text{oil}} &= P_{\text{air}} - \rho_{\text{oil}}gh_{\text{oil}} - \rho_{\text{Hg}}gh_{\text{Hg}} \\ &= 1169 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[2.68(0.75 \text{ m}) + 13.6(0.20 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \\ &= \mathbf{1123 \text{ kPa}} \end{aligned}$$

(b) The pressure in the air tank when the temperature drops to  $20^\circ\text{C}$  becomes

$$P_{\text{air}} = \frac{mRT}{V} = \frac{(15 \text{ kg})(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20 + 273)\text{K}}{1.3 \text{ m}^3} = 970 \text{ kPa}$$

When the mercury level in the left arm drops a distance  $x$ , the rise in the mercury level in the right arm  $y$  becomes

$$V_{\text{left}} = V_{\text{right}} \rightarrow \pi(3d)^2x = \pi d^2y \rightarrow y = 9x \text{ and } y_{\text{vert}} = 9x \sin 50^\circ$$

and the mercury fluid height will change by  $x + 9x \sin 50^\circ$  or  $7.894x$ . Then,

$$P_{\text{oil}} + \rho_{\text{oil}}g(h_{\text{oil}} + x) + \rho_{\text{Hg}}g(h_{\text{Hg}} - 7.894x) = P_{\text{air}} \rightarrow SG_{\text{oil}}(h_{\text{oil}} + x) + SG_{\text{Hg}}(h_{\text{Hg}} - 7.894x) = \frac{P_{\text{air}} - P_{\text{oil}}}{\rho_w g}$$

Substituting,

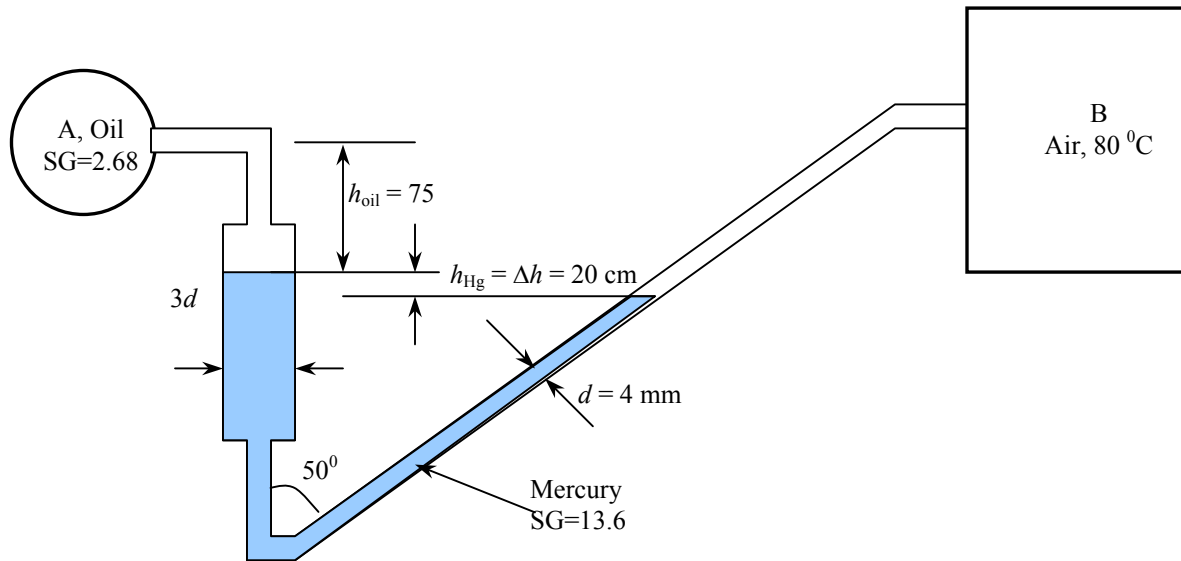
$$2.68(0.75 + x) + 13.6(0.20 - 7.894x) = \frac{(970 - 1123) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}} \right) \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right)$$

It gives

$$x = \mathbf{0.194 \text{ m} = 19.4 \text{ cm}}$$

Therefore, the oil-mercury interface will drop 19.4 cm as a result of the temperature drop of air.

**Discussion** Note that the pressure in constant-volume gas chambers is very sensitive to temperature changes.



**3-136** The density of a wood log is to be measured by tying lead weights to it until both the log and the weights are completely submerged, and then weighing them separately in air. The average density of a given log is to be determined by this approach.

**Properties** The density of lead weights is given to be  $11,300 \text{ kg/m}^3$ . We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The weight of a body is equal to the buoyant force when the body is floating in a fluid while being completely submerged in it (a consequence of vertical force balance from static equilibrium). In this case the average density of the body must be equal to the density of the fluid since

$$W = F_B \rightarrow \rho_{\text{body}} g V = \rho_{\text{fluid}} g V \rightarrow \rho_{\text{body}} = \rho_{\text{fluid}}$$

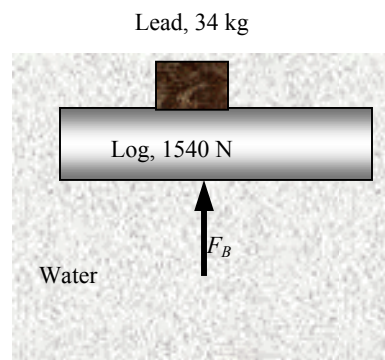
Therefore,

$$\rho_{\text{ave}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{lead}} + m_{\text{log}}}{V_{\text{lead}} + V_{\text{log}}} = \rho_{\text{water}} \rightarrow V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}}$$

where

$$V_{\text{lead}} = \frac{m_{\text{lead}}}{\rho_{\text{lead}}} = \frac{34 \text{ kg}}{11,300 \text{ kg/m}^3} = 3.01 \times 10^{-3} \text{ m}^3$$

$$m_{\text{log}} = \frac{W_{\text{log}}}{g} = \frac{1540 \text{ N}}{9.81 \text{ m/s}^2} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 157.0 \text{ kg}$$



Substituting, the volume and density of the log are determined to be

$$V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}} = 3.01 \times 10^{-3} \text{ m}^3 + \frac{(34 + 157) \text{ kg}}{1000 \text{ kg/m}^3} = \mathbf{0.194 \text{ m}^3}$$

$$\rho_{\text{log}} = \frac{m_{\text{log}}}{V_{\text{log}}} = \frac{157 \text{ kg}}{0.194 \text{ m}^3} = \mathbf{809 \text{ kg/m}^3}$$

**Discussion** Note that the log must be completely submerged for this analysis to be valid. Ideally, the lead weights must also be completely submerged, but this is not very critical because of the small volume of the lead weights.

**3-137** A rectangular gate that leans against the floor with an angle of  $45^\circ$  with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force  $F$  required to open the water gate is to be determined.

**Assumptions** 1 The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. 2 Friction at the hinge is negligible.

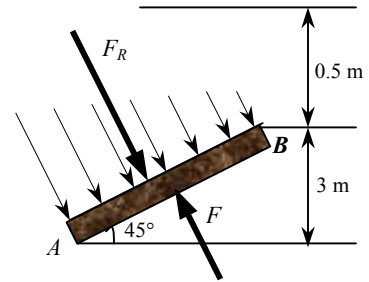
**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** The length of the gate and the distance of the upper edge of the gate (point  $B$ ) from the free surface in the plane of the gate are

$$b = \frac{3 \text{ m}}{\sin 45^\circ} = 4.243 \text{ m} \quad \text{and} \quad s = \frac{0.5 \text{ m}}{\sin 45^\circ} = 0.7071 \text{ m}$$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$\begin{aligned} F_R &= P_{ave} A = \rho g h_C A \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})[5 \times 4.243 \text{ m}^2] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 416 \text{ kN} \end{aligned}$$



The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 0.7071 + \frac{4.243}{2} + \frac{4.243^2}{12(0.7071 + 4.243/2)} = 3.359 \text{ m}$$

The distance of the pressure center from the hinge at point  $B$  is

$$L_P = y_P - s = 3.359 - 0.7071 = 2.652 \text{ m}$$

Taking the moment about point  $B$  and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = F b / 2$$

Solving for  $F$  and substituting, the required force is determined to be

$$F = \frac{2F_R L_P}{b} = \frac{2(416 \text{ kN})(2.652 \text{ m})}{4.243 \text{ m}} = \mathbf{520 \text{ kN}}$$

**Discussion** The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.



**3-138** A rectangular gate that leans against the floor with an angle of  $45^\circ$  with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force  $F$  required to open the water gate is to be determined.

**Assumptions** 1 The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. 2 Friction at the hinge is negligible.

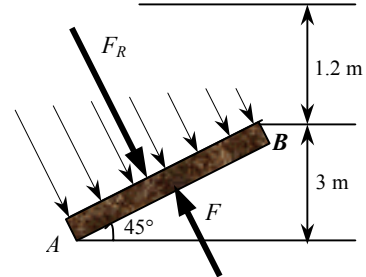
**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** The length of the gate and the distance of the upper edge of the gate (point  $B$ ) from the free surface in the plane of the gate are

$$b = \frac{3 \text{ m}}{\sin 45^\circ} = 4.243 \text{ m} \quad \text{and} \quad s = \frac{1.2 \text{ m}}{\sin 45^\circ} = 1.697 \text{ m}$$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$\begin{aligned} F_R &= P_{ave} A = \rho g h_C A \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.7 \text{ m})[5 \times 4.243 \text{ m}^2] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 562 \text{ kN} \end{aligned}$$



The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 1.697 + \frac{4.243}{2} + \frac{4.243^2}{12(1.697 + 4.243/2)} = 4.211 \text{ m}$$

The distance of the pressure center from the hinge at point  $B$  is

$$L_P = y_P - s = 4.211 - 1.697 = 2.514 \text{ m}$$

Taking the moment about point  $B$  and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = F b / 2$$

Solving for  $F$  and substituting, the required force is determined to be

$$F = \frac{2F_R L_P}{b} = \frac{2(562 \text{ N})(2.514 \text{ m})}{4.243 \text{ m}} = \mathbf{666 \text{ kN}}$$

**Discussion** The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.

**3-139** A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

**Assumptions** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

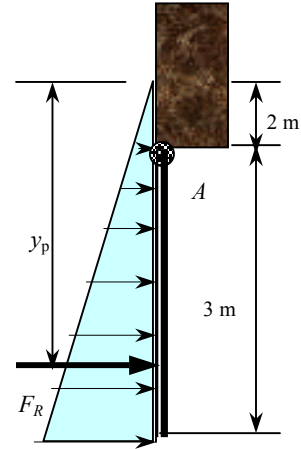
**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the gate,

$$\begin{aligned} F_R &= P_{ave} A = \rho g h_C A \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m})[3 \times 6 \text{ m}^2] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= \mathbf{618 \text{ kN}} \end{aligned}$$

The vertical distance of the pressure center from the free surface of water is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 2 + \frac{3}{2} + \frac{3^2}{12(2 + 3/2)} = \mathbf{3.71 \text{ m}}$$



**3-140** A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

**Assumptions** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

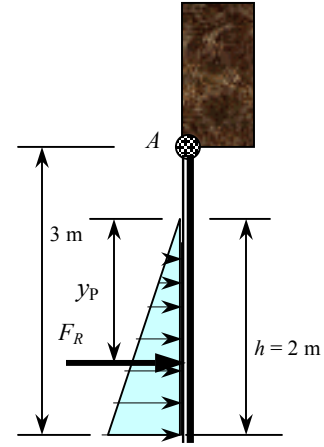
**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the wetted plate area gives the resultant hydrostatic force on the gate,

$$\begin{aligned} F_R &= P_{ave} A = \rho g h_C A \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})[2 \times 6 \text{ m}^2] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= \mathbf{118 \text{ kN}} \end{aligned}$$

The vertical distance of the pressure center from the free surface of water is

$$y_P = \frac{2h}{3} = \frac{2(2 \text{ m})}{3} = \mathbf{1.33 \text{ m}}$$



**3-141E** A semicircular tunnel is to be built under a lake. The total hydrostatic force acting on the roof of the tunnel is to be determined.

**Assumptions** The atmospheric pressure acts on both sides of the tunnel, and thus it can be ignored in calculations for convenience.

**Properties** We take the density of water to be  $62.4 \text{ lbf/ft}^3$  throughout.

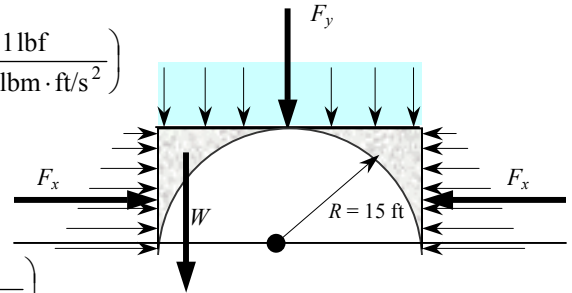
**Analysis** We consider the free body diagram of the liquid block enclosed by the circular surface of the tunnel and its vertical (on both sides) and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface (each side):

$$\begin{aligned} F_H = F_x &= P_{ave} A = \rho g h_C A = \rho g (s + R/2) A \\ &= (62.4 \text{ lbf/ft}^3)(32.2 \text{ ft/s}^2)(135 + 15/2 \text{ ft})(15 \text{ ft} \times 800 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) \\ &= 1.067 \times 10^8 \text{ lbf} \quad (\text{on each side of the tunnel}) \end{aligned}$$

Vertical force on horizontal surface (downward):

$$\begin{aligned} F_y &= P_{ave} A = \rho g h_C A = \rho g h_{top} A \\ &= (62.4 \text{ lbf/ft}^3)(32.2 \text{ ft/s}^2)(135 \text{ ft})(30 \text{ ft} \times 800 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) \\ &= 2.022 \times 10^8 \text{ lbf} \end{aligned}$$



Weight of fluid block on each side within the control volume (downward):

$$\begin{aligned} W &= mg = \rho g V = \rho g (R^2 - \pi R^2 / 4)(2000 \text{ ft}) \\ &= (62.4 \text{ lbf/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})^2 (1 - \pi/4)(800 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) \\ &= 2.410 \times 10^6 \text{ lbf} \quad (\text{on each side}) \end{aligned}$$

Therefore, the net downward vertical force is

$$F_V = F_y + 2W = 2.022 \times 10^8 + 2 \times 0.02410 \times 10^8 = \mathbf{2.07 \times 10^8 \text{ lbf}}$$

This is also the **net force** acting on the tunnel since the horizontal forces acting on the right and left side of the tunnel cancel each other since they are equal and opposite.

**3-142** A hemispherical dome on a level surface filled with water is to be lifted by attaching a long tube to the top and filling it with water. The required height of water in the tube to lift the dome is to be determined.

**Assumptions** 1 The atmospheric pressure acts on both sides of the dome, and thus it can be ignored in calculations for convenience. 2 The weight of the tube and the water in it is negligible.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** We take the dome and the water in it as the system. When the dome is about to rise, the reaction force between the dome and the ground becomes zero. Then the free body diagram of this system involves the weights of the dome and the water, balanced by the hydrostatic pressure force from below. Setting these forces equal to each other gives

$$\sum F_y = 0: \quad F_V = W_{\text{dome}} + W_{\text{water}}$$

$$\rho g (h + R) \pi R^2 = m_{\text{dome}} g + m_{\text{water}} g$$

Solving for  $h$  gives

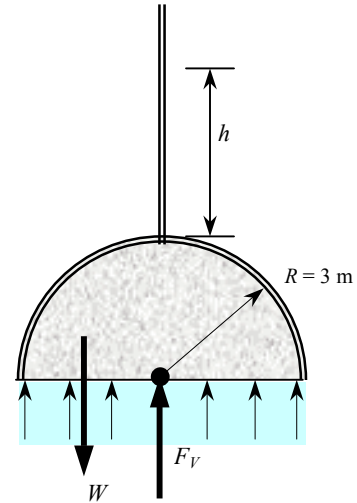
$$h = \frac{m_{\text{dome}} + m_{\text{water}}}{\rho \pi R^2} - R = \frac{m_{\text{dome}} + \rho [4\pi R^3 / 6]}{\rho \pi R^2} - R$$

Substituting,

$$h = \frac{(50,000 \text{ kg}) + 4\pi(1000 \text{ kg/m}^3)(3 \text{ m})^3 / 6}{(1000 \text{ kg/m}^3)\pi(3 \text{ m})^2} - (3 \text{ m}) = \mathbf{0.77 \text{ m}}$$

Therefore, this dome can be lifted by attaching a tube which is 77 cm long.

**Discussion** Note that the water pressure in the dome can be changed greatly by a small amount of water in the vertical tube.



**3-143** The water in a reservoir is restrained by a triangular wall. The total force (hydrostatic + atmospheric) acting on the inner surface of the wall and the horizontal component of this force are to be determined.

**Assumptions** **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** The length of the wall surface underwater is

$$b = \frac{25 \text{ m}}{\sin 60^\circ} = 28.87 \text{ m}$$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the surface,

$$\begin{aligned} F_R &= P_{ave} A = (P_{atm} + \rho g h_C) A \\ &= [100,000 \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(12.5 \text{ m})](150 \times 28.87 \text{ m}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= \mathbf{9.64 \times 10^8 \text{ N}} \end{aligned}$$

Noting that

$$\frac{P_0}{\rho g \sin 60^\circ} = \frac{100,000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \sin 60^\circ} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 11.77 \text{ m}$$

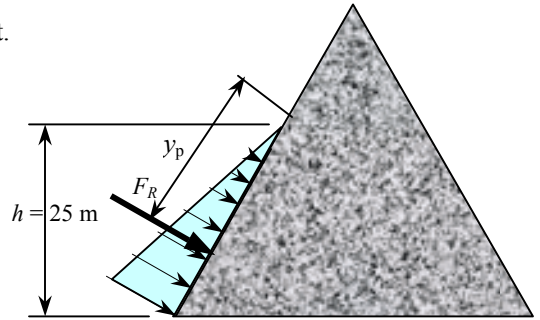
the distance of the pressure center from the free surface of water along the wall surface is

$$y_p = s + \frac{b}{2} + \frac{b^2}{12 \left( s + \frac{b}{2} + \frac{P_0}{\rho g \sin \theta} \right)} = 0 + \frac{28.87 \text{ m}}{2} + \frac{(28.87 \text{ m})^2}{12 \left( 0 + \frac{28.87 \text{ m}}{2} + 11.77 \text{ m} \right)} = \mathbf{17.1 \text{ m}}$$

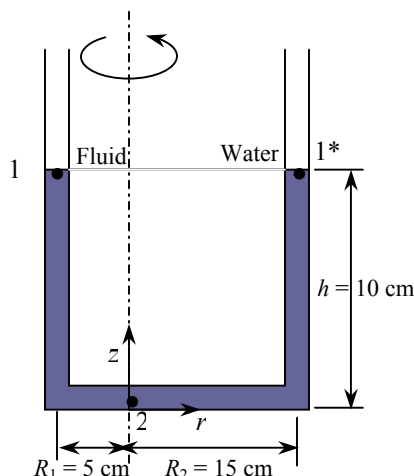
The magnitude of the horizontal component of the hydrostatic force is simply  $F_R \sin \theta$ ,

$$F_H = F_R \sin \theta = (9.64 \times 10^8 \text{ N}) \sin 60^\circ = \mathbf{8.35 \times 10^8 \text{ N}}$$

**Discussion** The atmospheric pressure is usually ignored in the analysis for convenience since it acts on both sides of the walls.



**3-144** A U-tube that contains water in right arm and another liquid in the left is rotated about an axis closer to the left arm. For a known rotation rate at which the liquid levels in both arms are the same, the density of the fluid in the left arm is to be determined.



**Assumptions** **1** Both the fluid and the water are incompressible fluids. **2** The two fluids meet at the axis of rotation, and thus there is only water to the right of the axis of rotation.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion (the *same* fluid) is given by

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

where  $\omega = 2\pi n = 2\pi(30 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 3.14 \text{ rad/s}$  (for both arms of the U-tube).

Pressure at point 2 is the same for both fluids, so are the pressures at points 1 and 1\* ( $P_1 = P_{1^*} = P_{\text{atm}}$ ). Therefore,  $P_2 - P_1$  is the same for both fluids. Noting that  $z_2 - z_1 = -h$  for both fluids and expressing  $P_2 - P_1$  for each fluid,

$$\text{Water: } P_2 - P_{1^*} = \frac{\rho_w\omega^2}{2}(0 - R_2^2) - \rho_w g(-h) = \rho_w(-\omega^2 R_2^2 / 2 + gh)$$

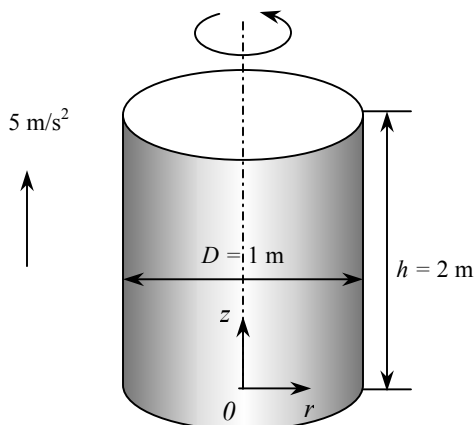
$$\text{Fluid: } P_2 - P_1 = \frac{\rho_f\omega^2}{2}(0 - R_1^2) - \rho_f g(-h) = \rho_f(-\omega^2 R_1^2 / 2 + gh)$$

Setting them equal to each other and solving for  $\rho_f$  gives

$$\rho_f = \frac{-\omega^2 R_2^2 / 2 + gh}{-\omega^2 R_1^2 / 2 + gh} \rho_w = \frac{-(3.14 \text{ rad/s})^2 (0.15 \text{ m})^2 + (9.81 \text{ m/s}^2)(0.10 \text{ m})}{-(3.14 \text{ rad/s})^2 (0.05 \text{ m})^2 + (9.81 \text{ m/s}^2)(0.10 \text{ m})} (1000 \text{ kg/m}^3) = 794 \text{ kg/m}^3$$

**Discussion** Note that this device can be used to determine relative densities, though it wouldn't be a very practical.

**3-145** A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate while being accelerated upward. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined. ✓EES



**Assumptions** 1 The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. 2 Gasoline is an incompressible substance.

**Properties** The density of the gasoline is given to be  $740 \text{ kg/m}^3$ .

**Analysis** The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by  $P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$ . The effect of linear acceleration in the vertical direction is accounted for by replacing  $g$  by  $g + a_z$ . Then,

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho(g + a_z)(z_2 - z_1)$$

where  $R = 0.50 \text{ m}$  is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(90 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 9.425 \text{ rad/s}$$

(a) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have  $r_1 = r_2 = 0$  and  $z_2 - z_1 = h = 2 \text{ m}$ . Then,

$$\begin{aligned} P_{\text{center, top}} - P_{\text{center, bottom}} &= 0 - \rho(g + a_z)(z_2 - z_1) = -\rho(g + a_z)h \\ &= -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2 + 5)(2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 21.8 \text{ kN/m}^2 = \mathbf{21.9 \text{ kPa}} \end{aligned}$$

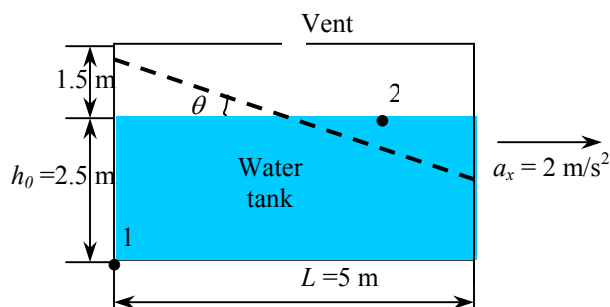
(b) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have  $r_1 = 0$ ,  $r_2 = R$ , and  $z_2 = z_1 = 0$ . Then,

$$\begin{aligned} P_{\text{edge, bottom}} - P_{\text{center, bottom}} &= \frac{\rho\omega^2}{2}(R_2^2 - 0) - 0 = \frac{\rho\omega^2 R^2}{2} \\ &= \frac{(740 \text{ kg/m}^3)(9.425 \text{ rad/s})^2 (0.50 \text{ m})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 8.22 \text{ kN/m}^2 = \mathbf{8.22 \text{ kPa}} \end{aligned}$$

**Discussion** Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. Likewise, the vertical acceleration does not affect the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane).



**3-146** A rectangular water tank open to the atmosphere is accelerated to the right on a level surface at a specified rate. The maximum pressure in the tank above the atmospheric level is to be determined. ✓EES



**Assumptions** 1 The road is horizontal during acceleration so that acceleration has no vertical component ( $a_z = 0$ ). 2 Effects of splashing, breaking and driving over bumps are assumed to be secondary, and are not considered. 3 The vent is never blocked, and thus the minimum pressure is the atmospheric pressure.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** We take the  $x$ -axis to be the direction of motion, the  $z$ -axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

$$\tan \theta = \frac{a_x}{g + a_z} = \frac{2}{9.81 + 0} = 0.2039 \quad (\text{and thus } \theta = 11.5^\circ)$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midsection experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the neutral midplane is

$$\Delta z_{\max} = (L/2) \tan \theta = [(5 \text{ m})/2] \times 0.2039 = 0.510 \text{ m}$$

which is less than 1.5 m high air space. Therefore, water never reaches the ceiling, and the maximum water height and the corresponding maximum pressure are

$$h_{\max} = h_0 + \Delta z_{\max} = 2.50 + 0.510 = 3.01 \text{ m}$$

$$P_{\max} = P_1 = \rho g h_{\max} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.01 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 29.5 \text{ kN/m}^2 = \mathbf{29.5 \text{ kPa}}$$

**Discussion** It can be shown that the gage pressure at the bottom of the tank varies from 29.5 kPa at the back of the tank to 24.5 kPa at the midsection and 19.5 kPa at the front of the tank.

3-147 Problem 3-146 is reconsidered. The effect of acceleration on the slope of the free surface of water in the tank as the acceleration varies from 0 to 5 m/s<sup>2</sup> in increments of 0.5 m/s<sup>2</sup> is to be investigated.

$$g = 9.81 \text{ m/s}^2$$

$$\rho = 1000 \text{ kg/m}^3$$

$$L = 5 \text{ m}$$

$$h_0 = 2.5 \text{ m}$$

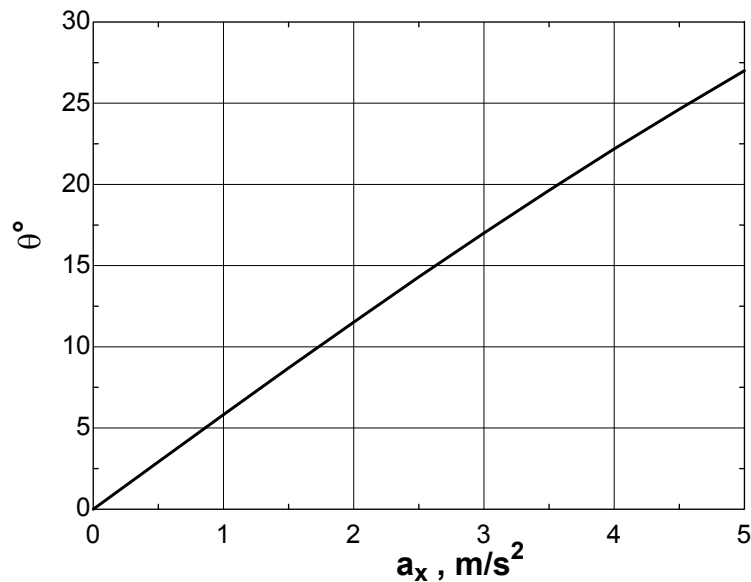
$$a_z = 0$$

$$\tan(\theta) = a_x / (g + a_z)$$

$$h_{\max} = h_0 + (L/2) \tan(\theta)$$

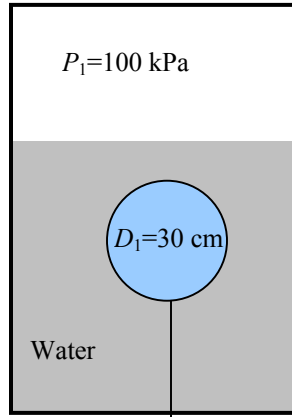
$$P_{\max} = \rho g h_{\max} / 1000 \text{ kPa}$$

Acceleration $a_x, \text{ m/s}^2$	Free surface angle, $\theta^\circ$	Maximum height $h_{\max}, \text{ m}$	Maximum pressure $P_{\max}, \text{ kPa}$
0.0	0.0	2.50	24.5
0.5	2.9	2.63	25.8
1.0	5.8	2.75	27.0
1.5	8.7	2.88	28.3
2.0	11.5	3.01	29.5
2.5	14.3	3.14	30.8
3.0	17.0	3.26	32.0
3.5	19.6	3.39	33.3
4.0	22.2	3.52	34.5
4.5	24.6	3.65	35.8
5.0	27.0	3.77	37.0



Note that water never reaches the ceiling, and a full free surface is formed in the tank.

**3-148** An elastic air balloon submerged in water is attached to the base of the tank. The change in the tension force of the cable is to be determined when the tank pressure is increased and the balloon diameter is decreased in accordance with the relation  $P = CD^{-2}$ .



**Assumptions** 1 The atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. 2 Water is an incompressible fluid. 3 The weight of the balloon and the air in it is negligible.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The tension force on the cable holding the balloon is determined from a force balance on the balloon to be

$$F_{\text{cable}} = F_B - W_{\text{balloon}} \cong F_B$$

The buoyancy force acting on the balloon initially is

$$F_{B,1} = \rho_w g V_{\text{balloon},1} = \rho_w g \frac{\pi D_1^3}{6} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \frac{\pi (0.30 \text{ m})^3}{6} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 138.7 \text{ N}$$

The variation of pressure with diameter is given as  $P = CD^{-2}$ , which is equivalent to  $D = \sqrt{C/P}$ . Then the final diameter of the ball becomes

$$\frac{D_2}{D_1} = \frac{\sqrt{C/P_2}}{\sqrt{C/P_1}} = \sqrt{\frac{P_1}{P_2}} \rightarrow D_2 = D_1 \sqrt{\frac{P_1}{P_2}} = (0.30 \text{ m}) \sqrt{\frac{0.1 \text{ MPa}}{1.6 \text{ MPa}}} = 0.075 \text{ m}$$

The buoyancy force acting on the balloon in this case is

$$F_{B,2} = \rho_w g V_{\text{balloon},2} = \rho_w g \frac{\pi D_2^3}{6} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \frac{\pi (0.075 \text{ m})^3}{6} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2.2 \text{ N}$$

Then the percent change in the cable force becomes

$$\text{Change}\% = \frac{F_{\text{cable},1} - F_{\text{cable},2}}{F_{\text{cable},1}} * 100 = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{138.7 - 2.2}{138.7} * 100 = \mathbf{98.4\%}$$

Therefore, increasing the tank pressure in this case results in 98.4% reduction in cable tension.

**Discussion** We can obtain a relation for the change in cable tension as follows:

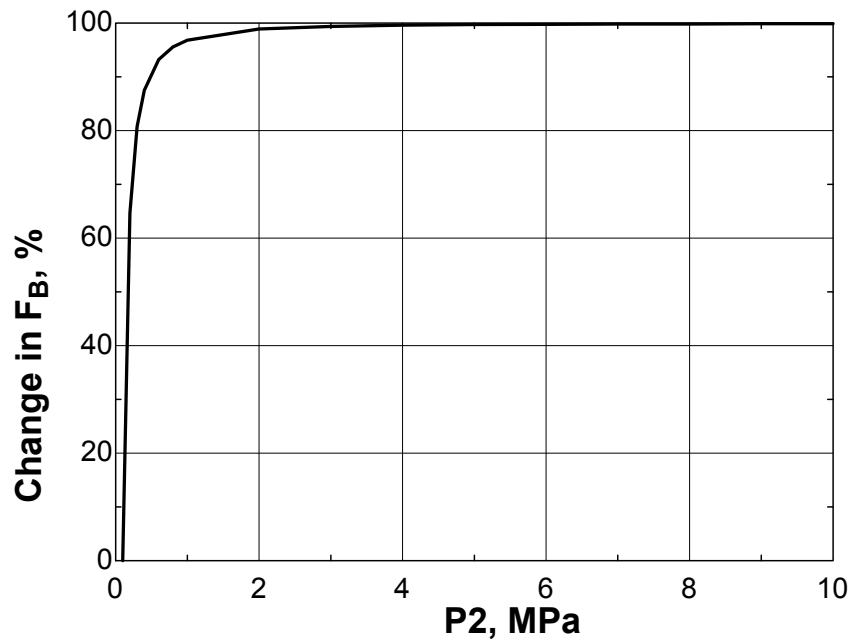
$$\begin{aligned} \text{Change}\% &= \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{\rho_w g V_{\text{balloon},1} - \rho_w g V_{\text{balloon},2}}{\rho_w g V_{\text{balloon},1}} * 100 \\ &= 100 \left( 1 - \frac{V_{\text{balloon},2}}{V_{\text{balloon},1}} \right) = 100 \left( 1 - \frac{D_2^3}{D_1^3} \right) = 100 \left( 1 - \left( \frac{P_1}{P_2} \right)^{3/2} \right) \end{aligned}$$

**3-149** Problem 3-148 is reconsidered. The effect of air pressure above water on the cable force as the pressure varies from 0.1 MPa to 10 MPa is to be investigated.

$P_1 = 0.1$  "MPa"

$$\text{Change} = 100 * (1 - (P_1/P_2)^{1.5})$$

Tank pressure $P_2$ , MPa	%Change in cable tension
0.1	0.0
0.2	64.6
0.3	80.8
0.4	87.5
0.6	93.2
0.8	95.6
1	96.8
2	98.9
3	99.4
4	99.6
5	99.7
6	99.8
7	99.8
8	99.9
9	99.9
10	99.9



**3-150** An iceberg floating in seawater is considered. The volume fraction of iceberg submerged in seawater is to be determined, and the reason for their turnover is to be explained.

**Assumptions** **1** The buoyancy force in air is negligible. **2** The density of iceberg and seawater are uniform.

**Properties** The densities of iceberg and seawater are given to be  $917 \text{ kg/m}^3$  and  $1042 \text{ kg/m}^3$ , respectively.

**Analysis** (a) The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore,

$$W = F_B$$

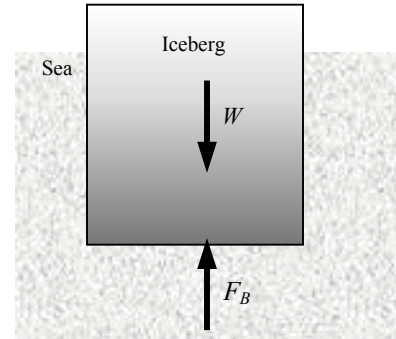
$$\rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} = \frac{\rho_{\text{iceberg}}}{\rho_{\text{seawater}}} = \frac{917}{1042} = 0.880 \text{ or } \mathbf{88\%}$$

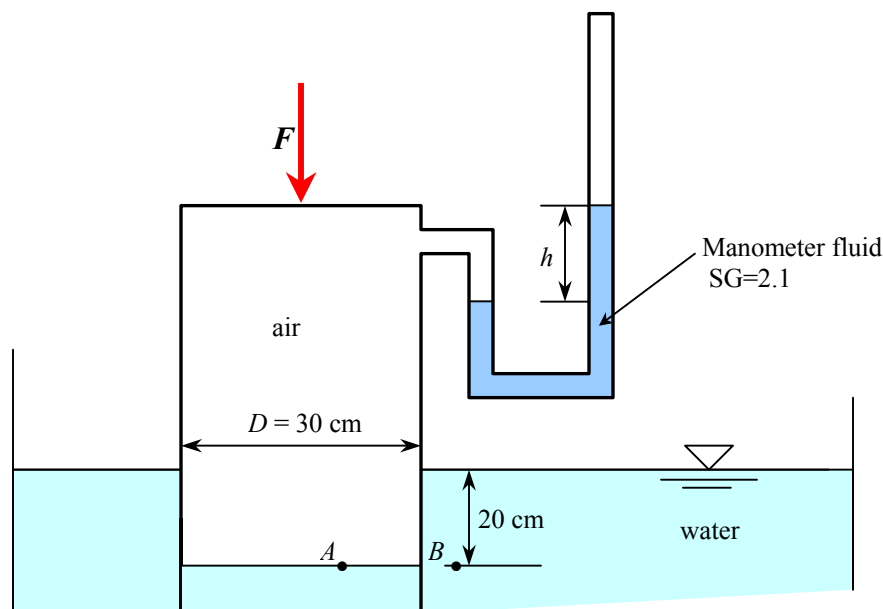
Therefore, 88% of the volume of the iceberg is submerged in this case.

(b) Heat transfer to the iceberg due to the temperature difference between the seawater and an iceberg causes uneven melting of the irregularly shaped iceberg. The resulting shift in the center of mass causes turn over.

**Discussion** Note that the submerged fraction depends on the density of seawater, and this fraction can differ in different seas.



**3-151** A cylindrical container equipped with a manometer is inverted and pressed into water. The differential height of the manometer and the force needed to hold the container in place are to be determined.  $\surd$



**Assumptions** 1 The atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. 2 The variation of air pressure inside cylinder is negligible.

**Properties** We take the density of water to be  $1000\text{ kg/m}^3$ . The density of the manometer fluid is

$$\rho_{mano} = SG \times \rho_w = 2.1(1000\text{ kg/m}^3) = 2100\text{ kg/m}^3$$

**Analysis** The pressures at point  $A$  and  $B$  must be the same since they are on the same horizontal line in the same fluid. Then the gage pressure in the cylinder becomes

$$P_{air,gage} = \rho_w g h_w = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.20\text{ m}) \left( \frac{1\text{ N}}{1\text{ kg} \cdot \text{m/s}^2} \right) = 1962\text{ N/m}^2 = 1962\text{ Pa}$$

The manometer also indicates the gage pressure in the cylinder. Therefore,

$$P_{air,gage} = (\rho g h)_{mano} \rightarrow h = \frac{P_{air,gage}}{\rho_{mano} g} = \frac{1962\text{ N/m}^2}{(2100\text{ kg/m}^3)(9.81\text{ m/s}^2)} \left( \frac{1\text{ kg} \cdot \text{m/s}^2}{1\text{ kN/m}^2} \right) = 0.095\text{ m} = \mathbf{9.5\text{ cm}}$$

A force balance on the cylinder in the vertical direction yields

$$F + W = P_{aie,gage} A_c$$

Solving for  $F$  and substituting,

$$F = P_{aie,gage} \frac{\pi D^2}{4} - W = (1962\text{ N/m}^2) \frac{\pi (0.30\text{ m})^2}{4} - 79\text{ N} = \mathbf{59.7\text{ N}}$$

**Discussion** We could also solve this problem by considering the atmospheric pressure, but we would obtain the same result since atmospheric pressure would cancel out.

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**3-152 ... 3-153 Design and Essay Problems**

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**3-153** The volume of a rock can be determined without using any volume measurement devices as follows: We weigh the rock in the air and then in the water. The difference between the two weights is due to the buoyancy force, which is equal to  $F_B = \rho_{\text{water}} g V_{\text{body}}$ . Solving this relation for  $V_{\text{body}}$  gives the volume of the rock.