Chapter 6
MOMENTUM ANALYSIS OF FLOW SYSTEMS

Newton's Laws and Conservation of Momentum

6-1C Newton's first law states that "*a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero*." Therefore, a body tends to preserve its state or inertia. Newton's second law states that "*the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.*" Newton's third law states "*when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first*."

6-2C Since momentum ($m\vec{V}$) is the product of a vector (velocity) and a scalar (mass), momentum must be a vector that points in the same direction as the velocity vector.

6-3C The *conservation of momentum principle* is expressed as "the momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved". The momentum of a body remains constant if the net force acting on it is zero.

6-4C Newton's second law of motion, also called the angular momentum equation, is expressed as "*the rate of change of the angular momentum of a body is equal to the net torque acting it*." For a non-rigid body with zero net torque, the angular momentum remains constant, but the angular velocity changes in accordance with $I\omega = constant$ where *I* is the moment of inertia of the body.

6-5C No. Two rigid bodies having the same mass and angular speed will have different angular momentums unless they also have the same moment of inertia *I*.

Linear Momentum Equation

6-6C The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the Reynolds transport theorem, which provides the link between the system and control volume concepts. The linear momentum equation is obtained by setting $b = \vec{V}$ and thus $B = m\vec{V}$ in the Reynolds transport theorem.

6-7C The forces acting on the control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as the pressure forces and reaction forces at points of contact). The net force acting on a control volume is the sum of all body and surface forces. Fluid weight is a body force, and pressure is a surface force (acting per unit area).

6-8C All of these surface forces arise as the control volume is isolated from its surroundings for analysis, and the effect of any detached object is accounted for by a force at that location. We can minimize the number of surface forces exposed by choosing the control volume such that the forces that we are not interested in remain internal, and thus they do not complicate the analysis. A well-chosen control volume exposes only the forces that are to be determined (such as reaction forces) and a minimum number of other forces.

6-9C The momentum-flux correction factor β enables us to express the momentum flux in terms of the mass flow rate and mean flow velocity as $\int_{A_c} \rho\vec{V}(\vec{V}\cdot\vec{n})dA_c = \beta\dot{m}\vec{V}_{avg}$. The value of β is unity for uniform flow, such as a jet flow, nearly unity for turbulent flow (between 1.01 and 1.04), but about 1.3 for laminar flow. So it should be considered in laminar flow.

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6-10C The momentum equation for steady one-dimensional flow for the case of no external forces is

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

where the left hand side is the net force acting on the control volume, and first term on the right hand side is the incoming momentum flux and the second term is the outgoing momentum flux by mass.

6-11C In the application of the momentum equation, we can disregard the atmospheric pressure and work with gage pressures only since the atmospheric pressure acts in all directions, and its effect cancels out in every direction.

6-12C The fireman who holds the hose backwards so that the water makes a U-turn before being discharged will experience a greater reaction force since the numerical values of momentum fluxes across the nozzle are added in this case instead of being subtracted.

6-13C No, V is not the upper limit to the rocket's ultimate velocity. Without friction the rocket velocity will continue to increase as more gas outlets the nozzle.

6-14C A helicopter hovers because the strong downdraft of air, caused by the overhead propeller blades, manifests a momentum in the air stream. This momentum must be countered by the helicopter lift force.

6-15C As the air density decreases, it requires more energy for a helicopter to hover, because more air must be forced into the downdraft by the helicopter blades to provide the same lift force. Therefore, it takes more power for a helicopter to hover on the top of a high mountain than it does at sea level.

6-16C In winter the air is generally colder, and thus denser. Therefore, less air must be driven by the blades to provide the same helicopter lift, requiring less power.

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6-17C The force required to hold the plate against the horizontal water stream will increase by a factor of 4 when the velocity is doubled since

$$F = \dot{m}V = (\rho AV)V = \rho AV^2$$

and thus the force is proportional to the square of the velocity.

6-18C The acceleration will not be constant since the force is not constant. The impulse force exerted by water on the plate is $F = \dot{m}V = (\rho AV)V = \rho AV^2$, where V is the relative velocity between the water and the plate, which is moving. The plate acceleration will be **a = F/m.** But as the plate begins to move, V decreases, so the acceleration must also decrease.

6-19C The maximum velocity possible for the plate is the velocity of the water jet. As long as the plate is moving slower than the jet, the water will exert a force on the plate, which will cause it to accelerate, until terminal jet velocity is reached.

6-20 It is to be shown that the force exerted by a liquid jet of velocity V on a stationary nozzle is proportional to V^2, or alternatively, to $\dot{m}^2$. √

Assumptions **1** The flow is steady and incompressible. **2** The nozzle is given to be stationary. **3** The nozzle involves a 90° turn and thus the incoming and outgoing flow streams are normal to each other. **4** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero.

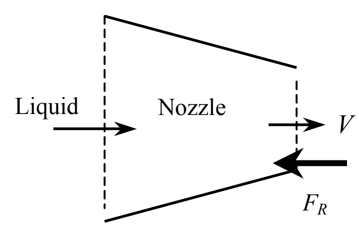
Analysis We take the nozzle as the control volume, and the flow direction at the outlet as the x axis. Note that the nozzle makes a 90° turn, and thus it does not contribute to any pressure force or momentum flux term at the inlet in the x direction. Noting that $\dot{m} = \rho AV$ where A is the nozzle outlet area and V is the average nozzle outlet velocity, the momentum equation for steady one-dimensional flow in the x direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V} \quad \rightarrow \quad F_{Rx} = \beta\dot{m}_{out}V_{out} = \beta\dot{m}V$$

where F_{Rx} is the reaction force on the nozzle due to liquid jet at the nozzle outlet. Then,

$$\dot{m} = \rho AV \quad \rightarrow \quad F_{Rx} = \beta\dot{m}V = \beta\rho AVV = \beta\rho AV^2 \quad \text{or} \quad F_{Rx} = \beta\dot{m}V = \beta\dot{m}\frac{\dot{m}}{\rho A} = \beta\frac{\dot{m}^2}{\rho A}$$

Therefore, the force exerted by a liquid jet of velocity V on this stationary nozzle is proportional to V^2, or alternatively, to $\dot{m}^2$.



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6-21 A water jet of velocity V impinges on a plate moving toward the water jet with velocity ½V. The force required to move the plate towards the jet is to be determined in terms of F acting on the stationary plate.

Assumptions **1** The flow is steady and incompressible. **2** The plate is vertical and the jet is normal to plate. **3** The pressure on both sides of the plate is atmospheric pressure (and thus its effect cancels out). **4** Fiction during motion is negligible. **5** There is no acceleration of the plate. **6** The water splashes off the sides of the plate in a plane normal to the jet. **6** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the plate as the control volume. The relative velocity between the plate and the jet is V when the plate is stationary, and $1.5V$ when the plate is moving with a velocity ½V towards the plate. Then the momentum equation for steady one-dimensional flow in the horizontal direction reduces to

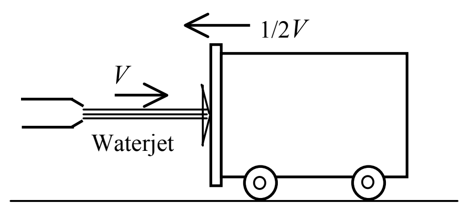
$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_R = -\dot{m}_i V_i \quad \rightarrow \quad F_R = \dot{m}_i V_i$$

Stationary plate: $(V_i = V \quad \text{and} \quad \dot{m}_i = \rho A V_i = \rho A V) \rightarrow \quad F_R = \rho A V^2 = F$

Moving plate: $(V_i = 1.5V \quad \text{and} \quad \dot{m}_i = \rho A V_i = \rho A(1.5V)) \rightarrow \quad F_R = \rho A(1.5V)^2 = 2.25 \rho A V^2 = 2.25F$

Therefore, the force required to hold the plate stationary against the oncoming water jet becomes **2.25 times** when the jet velocity becomes 1.5 times.

Discussion Note that when the plate is stationary, V is also the jet velocity. But if the plate moves toward the stream with velocity ½V, then the relative velocity is $1.5V$, and the amount of mass striking the plate (and falling off its sides) per unit time also increases by 50%.



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6-22 A 90° elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho AV$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.1\text{ m})^2/4]} = 3.18\text{ m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

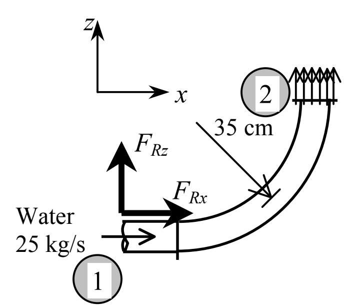
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g(z_2 - z_1) \;\rightarrow\; P_{1,\text{ gage}} = \rho g(z_2 - z_1)$$

Substituting,

$$P_{1,\text{ gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.35\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 3.434\text{ kN/m}^2 = \mathbf{3.434\ kPa}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = 0 - \beta\dot{m}(+V_1) = -\beta\dot{m}V$$
$$F_{Rz} = \beta\dot{m}(+V_2) = \beta\dot{m}V$$



Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = -\beta\dot{m}V - P_{1,\text{ gage}}A_1$$
$$= -1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (3434\text{ N/m}^2)[\pi(0.1\text{ m})^2/4]$$
$$= -109\text{ N}$$
$$F_{Ry} = \beta\dot{m}V = 1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 81.9\text{ N}$$

and $\quad F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-109)^2 + 81.9^2} = \mathbf{136\ N},\quad \theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}} = \tan^{-1}\dfrac{81.9}{-109} = -37° = \mathbf{143°}$

Discussion Note that the magnitude of the anchoring force is 136 N, and its line of action makes 143° from the positive *x* direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

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6-23 An 180° elbow forces the flow to make a U-turn and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\ \text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow P_1 - P_2 = \rho g(z_2 - z_1) \rightarrow P_{1,\text{ gage}} = \rho g(z_2 - z_1)$$

Substituting,

$$P_{1,\text{ gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.70\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 6.867\ \text{kN/m}^2 = \mathbf{6.867\ kPa}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = \beta\dot{m}(-V_2) - \beta\dot{m}(+V_1) = -2\beta\dot{m}V$$

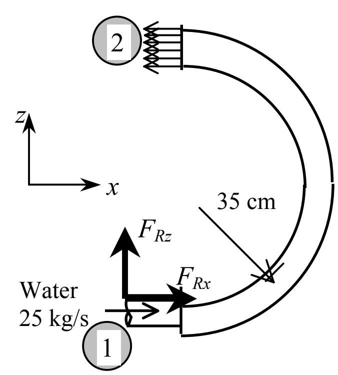
$$F_{Rz} = 0$$

Solving for F_{Rx} and substituting the given values,

$$F_{Rx} = -2\beta\dot{m}V - P_{1,\text{ gage}} A_1$$

$$= -2\times 1.03(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) - (6867\ \text{N/m}^2)[\pi(0.1\ \text{m})^2/4]$$

$$= -218\ \text{N}$$



and $F_R = F_{Rx} =$ **- 218 N** since the *y*-component of the anchoring force is zero. Therefore, the anchoring force has a magnitude of 218 N and it acts in the negative *x* direction.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

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6-24E A horizontal water jet strikes a vertical stationary plate normally at a specified velocity. For a given anchoring force needed to hold the plate in place, the flow rate of water is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.
Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V} \quad \rightarrow \quad -F_{Rx} = -\dot{m}V_1 \quad \rightarrow \quad F_R = \dot{m}V_1$$

We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Solving for $\dot{m}$ and substituting the given values,

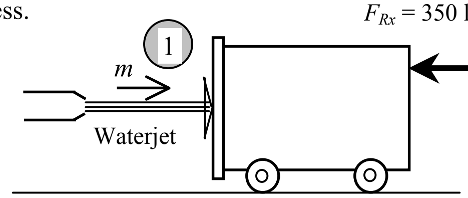
$$\dot{m} = \frac{F_{Rx}}{V_1} = \frac{350 \text{ lbf}}{30 \text{ ft/s}}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = 376 \text{ lbm/s}$$

Then the volume flow rate becomes

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{376 \text{ lbm/s}}{62.4 \text{ lbm/ft}^3} = \mathbf{6.02\ ft^3/s}$$

Therefore, the volume flow rate of water under stated assumptions must be 6.02 ft^3/s.

Discussion In reality, some water will be scattered back, and this will add to the reaction force of water. The flow rate in that case will be less.



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6-25 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m³.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot m_1 = \dot m_2 = \dot m = 30$ kg/s. Noting that $\dot m = \rho A V$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot m}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot m}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

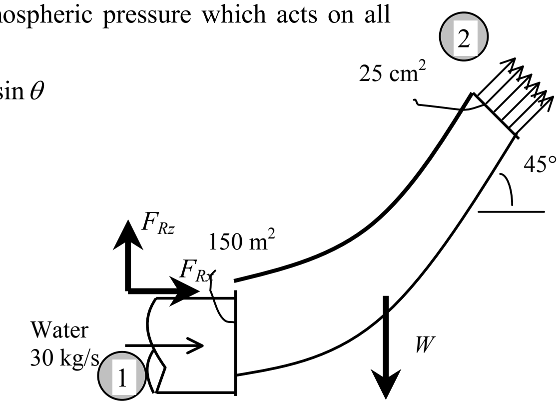
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\text{ gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

Substituting,

$$P_{1,\text{ gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec F = \sum_{\text{out}} \beta \dot m \vec V - \sum_{\text{in}} \beta \dot m \vec V$. We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = \beta \dot m V_2 \cos\theta - \beta \dot m V_1 \quad\text{and}\quad F_{Rz} - W = \beta \dot m V_2 \sin\theta$$



Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot m (V_2 \cos\theta - V_1) - P_{1,\text{ gage}} A_1$$

$$= 1.03(30\text{ kg/s})[(12\cos 45° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$-(73.9\text{ kN/m}^2)(0.0150\text{ m}^2)$$

$$= -0.908\text{ kN}$$

$$F_{Rz} = \beta \dot m V_2 \sin\theta + W = 1.03(30\text{ kg/s})(12\sin 45°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + 0.4905\text{ kN} = 0.753\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\text{ kN}}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = \mathbf{-39.7}°$$

Discussion Note that the magnitude of the anchoring force is 1.18 kN, and its line of action makes –39.7° from +x direction. Negative value for F_{Rx} indicates the assumed direction is wrong.

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6-26 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot m_1 = \dot m_2 = \dot m = 30$ kg/s. Noting that $\dot m = \rho A V$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot m}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot m}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\text{ gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

or, $$P_{1,\text{ gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec F = \sum_{\text{out}} \beta \dot m \vec V - \sum_{\text{in}} \beta \dot m \vec V$. We let the x- and y- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = \beta \dot m V_2 \cos\theta - \beta \dot m V_1 \quad \text{and} \quad F_{Ry} - W = \beta \dot m V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot m (V_2 \cos\theta - V_1) - P_{1,\text{ gage}} A_1$$

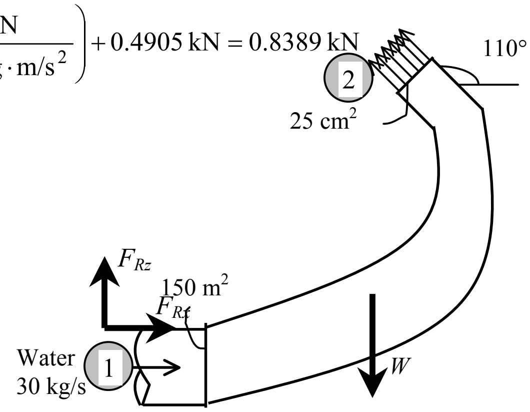
$$= 1.03(30\text{ kg/s})[(12\cos 110° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) - (73.9\text{ kN/m}^2)(0.0150\text{ m}^2) = -1.297\text{ kN}$$

$$F_{Rz} = \beta \dot m V_2 \sin\theta + W = 1.03(30\text{ kg/s})(12\sin 110°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + 0.4905\text{ kN} = 0.8389\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-1.297)^2 + 0.8389^2} = \mathbf{1.54\ kN}$$

and

$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.8389}{-1.297} = \mathbf{-32.9}°$$



Discussion Note that the magnitude of the anchoring force is 1.54 kN, and its line of action makes –32.9° from +x direction. Negative value for F_{Rx} indicates assumed direction is wrong, and should be reversed.

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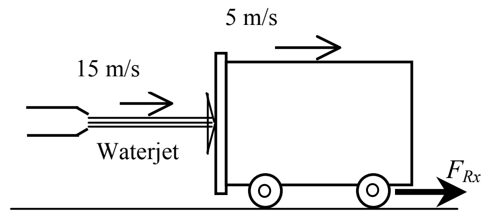
6-27 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally at a constant velocity. The braking force and the power wasted by the brakes are to be determined. .

Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** Fiction during motion is negligible. **5** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal. **6** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the cart and the jet is



$$V_r = V_{\text{jet}} - V_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$

Therefore, we can assume the cart to be stationary and the jet to move with a velocity of 10 m/s. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m} V_r$$

We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m} V_r = -(25 \text{ kg/s})(+10 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{-250 \text{ N}}$$

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} V_{\text{cart}} = (250 \text{ N})(5 \text{ m/s})\left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right) = \mathbf{1.25 \text{ kW}}$$

Discussion Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

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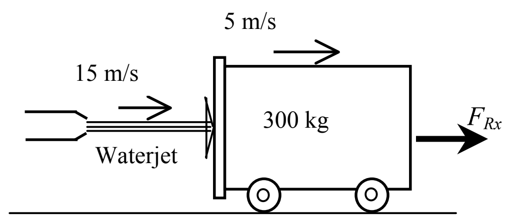
6-28 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally. The acceleration of the cart if the brakes fail is to be determined.

Analysis The braking force was determined in previous problem to be 250 N. When the brakes fail, this force will propel the cart forward, and the accelerating will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{250\text{ N}}{300\text{ kg}}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{0.833\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.



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6-29E A water jet hits a stationary splitter, such that half of the flow is diverted upward at 45°, and the other half is directed down. The force required to hold the splitter in place is to be determined. √EES

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded. **4** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis The mass flow rate of water jet is

$$\dot{m} = \rho \dot{V} = (62.4\ \text{lbm/ft}^3)(100\ \text{ft}^3/\text{s}) = 6240\ \text{lbm/s}$$

We take the splitting section of water jet, including the splitter as the control volume, and designate the entrance by 1 and the outlet of either arm by 2 (both arms have the same velocity and mass flow rate). We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by z.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the x- and y- components of the anchoring force of the splitter be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Noting that $V_2 = V_1 = V$ and $\dot{m}_2 = \frac{1}{2}\dot{m}$, the momentum equations along the x and z axes become

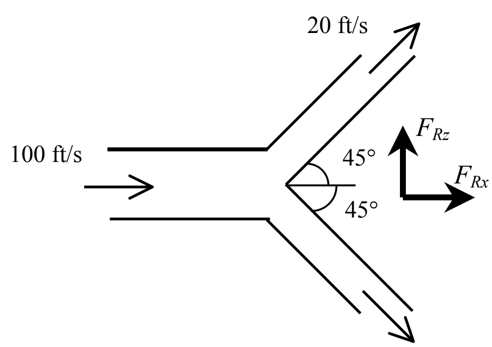
$$F_{Rx} = 2(\tfrac{1}{2}\dot{m})V_2 \cos\theta - \dot{m}V_1 = \dot{m}V(\cos\theta - 1)$$
$$F_{Rz} = \tfrac{1}{2}\dot{m}(+V_2 \sin\theta) + \tfrac{1}{2}\dot{m}(-V_2 \sin\theta) - 0 = 0$$

Substituting the given values,

$$F_{Rx} = (6240\ \text{lbm/s})(20\ \text{ft/s})(\cos 45° - 1)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2}\right) = \mathbf{-1135\ lbf}$$
$$F_{Rz} = \mathbf{0}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 1135 lbf must be applied to the splitter in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction. This can also be concluded from the symmetry.

Discussion In reality, the gravitational effects will cause the upper stream to slow down and the lower stream to speed up after the split. But for short distances, these effects are indeed negligible.



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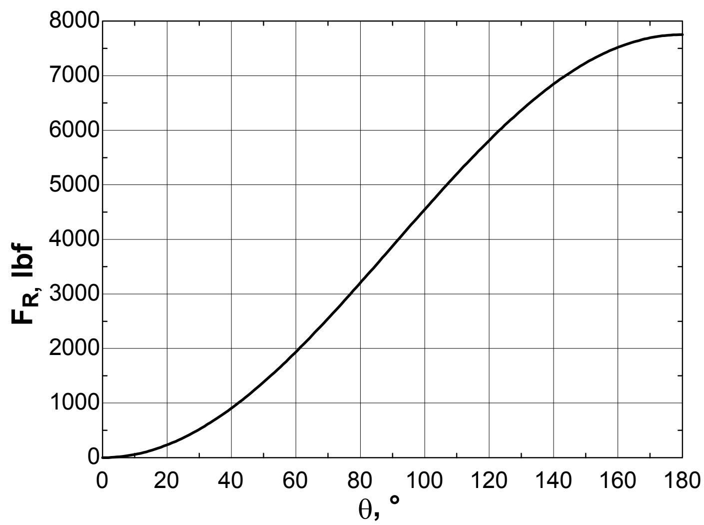
6-30E Problem 6-29E is reconsidered. The effect of splitter angle on the force exerted on the splitter as the half splitter angle varies from 0 to 180° in increments of 10° is to be investigated.

```
g=32.2 "ft/s2"
rho=62.4 "lbm/ft3"
V_dot=100 "ft3/s"
V=20 "ft/s"
m_dot=rho*V_dot
F_R=-m_dot*V*(cos(theta)-1)/g "lbf"
```

θ, °	$\dot{m}$, lbm/s	F_R, lbf
0	6240	0
10	6240	59
20	6240	234
30	6240	519
40	6240	907
50	6240	1384
60	6240	1938
70	6240	2550
80	6240	3203
90	6240	3876
100	6240	4549
110	6240	5201
120	6240	5814
130	6240	6367
140	6240	6845
150	6240	7232
160	6240	7518
170	6240	7693
180	6240	7752



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6-31 A horizontal water jet impinges normally upon a vertical plate which is held on a frictionless track and is initially stationary. The initial acceleration of the plate, the time it takes to reach a certain velocity, and the velocity at a given time are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water always splatters in the plane of the retreating plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The tract is nearly frictionless, and thus fiction during motion is negligible. **5** The motions of the water jet and the cart are horizontal. **6** The velocity of the jet relative to the plate remains constant, $V_r = V_{jet} = V$. **7** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the vertical plate on the frictionless track as the control volume, and the direction of flow as the positive direction of *x* axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho VA = (1000\ \text{kg/m}^3)(18\ \text{m/s})[\pi(0.05\ \text{m})^2/4] = 35.34\ \text{kg/s}$$

The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

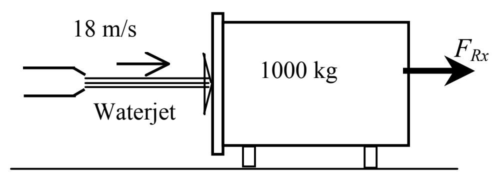
$$\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{Rx} = -\dot{m}V$$

where F_{Rx} is the reaction force required to hold the plate in place. When the plate is released, an equal and opposite impulse force acts on the plate, which is determined to

$$F_{\text{plate}} = -F_{Rx} = \dot{m}V = (35.34\ \text{kg/s})(18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 636\ \text{N}$$

Then the initial acceleration of the plate becomes

$$a = \frac{F_{\text{plate}}}{m_{\text{plate}}} = \frac{636\ \text{N}}{1000\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.636\ m/s^2}$$



This acceleration will remain constant during motion since the force acting on the plate remains constant.

(*b*) Noting that $a = dV/dt = \Delta V/\Delta t$ since the acceleration *a* is constant, the time it takes for the plate to reach a velocity of 9 m/s is

$$\Delta t = \frac{\Delta V_{\text{plate}}}{a} = \frac{(9-0)\ \text{m/s}}{0.636\ \text{m/s}^2} = \mathbf{14.2\ s}$$

(*c*) Noting that $a = dV/dt$ and thus $dV = adt$ and that the acceleration *a* is constant, the plate velocity in 20 s becomes

$$V_{\text{plate}} = V_{0,\ \text{plate}} + a\Delta t = 0 + (0.636\ \text{m/s}^2)(20\ \text{s}) = \mathbf{12.7\ m/s}$$

Discussion The assumption that the relative velocity between the water jet and the plate remains constant is valid only for the initial moments of motion when the plate velocity is low unless the water jet is moving with the plate at the same velocity as the plate.

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6-32 A 90° reducer elbow deflects water downwards into a smaller diameter pipe. The resultant force exerted on the reducer by water is to be determined.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is disregarded since the gravitational effects are negligible. **3** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.04$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 353.4$ kg/s. Noting that $\dot{m} = \rho A V$, the mass flow rate of water and its outlet velocity are

$$\dot{m} = \rho V_1 A_1 = \rho V_1 (\pi D_1^2/4) = (1000\ \text{kg/m}^3)(5\ \text{m/s})[\pi (0.3\ \text{m})^2/4] = 353.4\ \text{kg/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m}}{\rho \pi D_2^2/4} = \frac{353.4\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi (0.15\ \text{m})^2/4]} = 20\ \text{m/s}$$

The Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \to \quad P_2 = P_1 + \rho g\left(\frac{V_1^2 - V_2^2}{2g} + z_1 - z_2\right)$$

Substituting, the gage pressure at the outlet becomes

$$P_2 = (300\ \text{kPa}) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{(5\ \text{m/s})^2 - (20\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.5\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 117.4\ \text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \beta \dot{m} V_1$$
$$F_{Rz} - P_{2,\text{gage}} A_2 = \beta \dot{m}(-V_2) - 0$$

Note that we should not forget the negative sign for forces and velocities in the negative *x* or *z* direction. Solving for F_{Rx} and F_{Rz}, and substituting the given values,

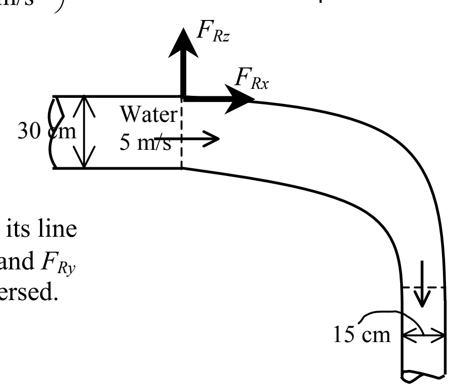
$$F_{Rx} = -\beta \dot{m} V_1 - P_{1,\text{gage}} A_1 = -1.04(353.4\ \text{kg/s})(5\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (300\ \text{kN/m}^2)\frac{\pi (0.3\ \text{m})^2}{4} = -23.0\ \text{kN}$$

$$F_{Rz} = -\beta \dot{m} V_2 + P_{2,\text{gage}} A_1 = -1.04(353.4\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + (117.4\ \text{kN/m}^2)\frac{\pi (0.15\ \text{m})^2}{4} = -5.28\ \text{kN}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-23.0)^2 + (-5.28)^2} = \mathbf{23.6\ kN}$$

$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{-5.28}{-23.0} = \mathbf{12.9}°$$



Discussion The magnitude of the anchoring force is 23.6 kN, and its line of action makes 12.9° from +*x* direction. Negative values for F_{Rx} and F_{Ry} indicate that the assumed directions are wrong, and should be reversed.

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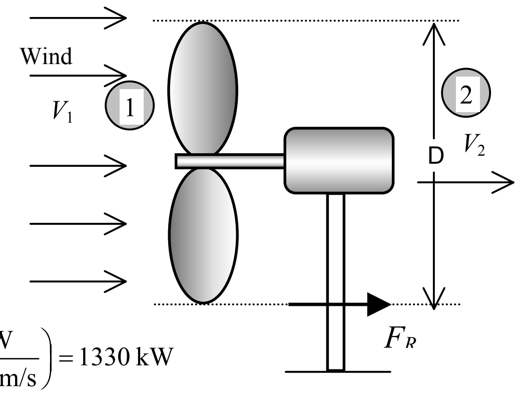
6-33 A wind turbine with a given span diameter and efficiency is subjected to steady winds. The power generated and the horizontal force on the supporting mast of the turbine are to be determined. √EES

Assumptions **1** The wind flow is steady and incompressible. **2** The efficiency of the turbine-generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy. **4** Wind flow is uniform and thus the momentum-flux correction factor is nearly unity, $\beta \cong 1$.

Properties The density of air is given to be 1.25 kg/m^3.

Analysis (*a*) The power potential of the wind is its kinetic energy, which is $V^2/2$ per unit mass, and $\dot{m}V^2/2$ for a given mass flow rate:



$$V_1 = (25 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 6.94 \text{ m/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (1.25 \text{ kg/m}^3)(6.94 \text{ m/s})\frac{\pi (90 \text{ m})^2}{4} = 55{,}200 \text{ kg/s}$$

$$\dot{W}_{\max} = \dot{m}ke_1 = \dot{m}\frac{V_1^2}{2} = (55{,}200 \text{ kg/s})\frac{(6.94 \text{ m/s})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kW}}{1 \text{ kN}\cdot\text{m/s}}\right) = 1330 \text{ kW}$$

Then the actual power produced becomes

$$\dot{W}_{\text{act}} = \eta_{\text{wind turbine}}\dot{W}_{\max} = (0.32)(1330 \text{ kW}) = \mathbf{426\ kW}$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Therefore,

$$\dot{m}ke_2 = \dot{m}ke_1(1-\eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1-\eta_{\text{wind turbine}})$$

or

$$V_2 = V_1\sqrt{1-\eta_{\text{wind turbine}}} = (6.94 \text{ m/s})\sqrt{1\text{-}0.32} = 5.72 \text{ m/s}$$

We choose the control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet, and the entire control surface is at the atmospheric pressure. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. Writing it along the x-direction (without forgetting the negative sign for forces and velocities in the negative x-direction) and assuming the flow velocity through the turbine to be equal to the wind velocity give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1) = (55{,}200 \text{ kg/s})(5.72\text{-}6.94 \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-67.3\ kN}$$

The negative sign indicates that the reaction force acts in the negative x direction, as expected.

Discussion This force acts on top of the tower where the wind turbine is installed, and the bending moment it generates at the bottom of the tower is obtained by multiplying this force by the tower height.

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6-34E A horizontal water jet strikes a curved plate, which deflects the water back to its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Friction between the plate and the surface it is on is negligible (or the friction force can be included in the required force to hold the plate). **4** There is no splashing of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor is nearly unity, $\beta \cong 1$.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction). The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho VA = \rho V[\pi D^2/4] = (62.4\ \text{lbm/ft}^3)(140\ \text{ft/s})[\pi(3/12\ \text{ft})^2/4] = 428.8\ \text{lbm/s}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. Letting the reaction force to hold the plate be F_{Rx} and assuming it to be in the positive direction, the momentum equation along the x axis becomes

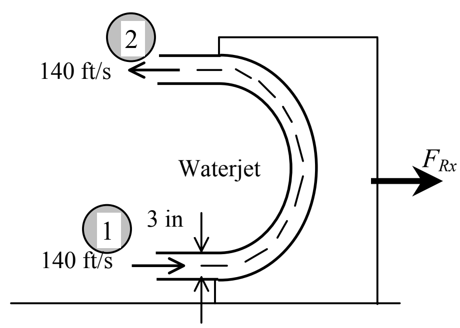
$$F_{Rx} = \dot{m}(-V_2) - \dot{m}(+V_1) = -2\dot{m}V$$

Substituting,

$$F_{Rx} = -2(428.8\ \text{lbm/s})(140\ \text{ft/s})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{-3729\ lbf}$$

Therefore, a force of 3729 lbm must be applied on the plate in the negative x direction to hold it in place.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong (as expected), and should be reversed. Also, there is no need for an analysis in the vertical direction since the fluid streams are horizontal.



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6-35E A horizontal water jet strikes a bent plate, which deflects the water by 135° from its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Frictional and gravitational effects are negligible. **4** There is no splattering of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor is nearly unity, $\beta \cong 1$.

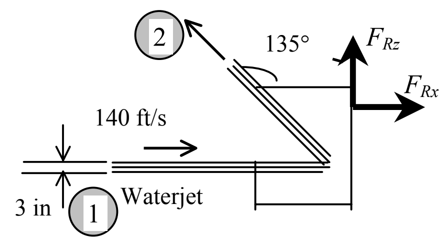
Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction), and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho VA = \rho V[\pi D^2 / 4] = (62.4 \text{ lbm/ft}^3)(140 \text{ ft/s})[\pi (3/12 \text{ ft})^2 / 4] = 428.8 \text{ lbm/s}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the x- and z- components of the anchoring force of the plate be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}(-V_2)\cos 45° - \dot{m}(+V_1) = -\dot{m}V(1+\cos 45°)$$
$$F_{Rz} = \dot{m}(+V_2)\sin 45° = \dot{m}V \sin 45°$$



Substituting the given values,

$$F_{Rx} = -2(428.8 \text{ lbm/s})(140 \text{ ft/s})(1+\cos 45°)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= -6365 \text{ lbf}$$
$$F_{Rz} = (428.8 \text{ lbm/s})(140 \text{ ft/s})\sin 45°\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = 1318 \text{ lbf}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-6365)^2 + 1318^2} = \mathbf{6500 \text{ lbf}}, \quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{1318}{-6365} = -11.7° = \mathbf{168.3}°$$

Discussion Note that the magnitude of the anchoring force is 6500 lbf, and its line of action makes 168.3° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

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6-36 Firemen are holding a nozzle at the end of a hose while trying to extinguish a fire. The average water outlet velocity and the resistance force required of the firemen to hold the nozzle are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and outlets horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction), and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction). The average outlet velocity and the mass flow rate of water are determined from

$$V = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi D^2/4} = \frac{5\ \text{m}^3/\text{min}}{\pi(0.06\ \text{m})^2/4} = 1768\ \text{m/min} = \mathbf{29.5\ m/s}$$

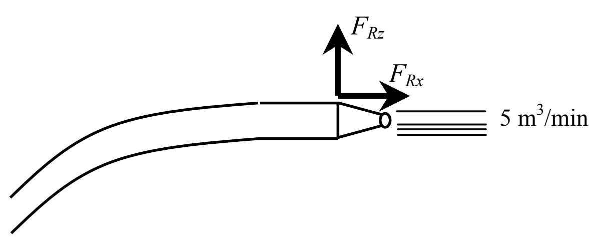
$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(5\ \text{m}^3/\text{min}) = 5000\ \text{kg/min} = 83.3\ \text{kg/s}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. We let horizontal force applied by the firemen to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction gives

$$F_{Rx} = \dot{m}V_e - 0 = \dot{m}V = (83.3\ \text{kg/s})(29.5\ \text{m/s})\left(\frac{1\ \text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{2457\ N}$$

Therefore, the firemen must be able to resist a force of 2457 N to hold the nozzle in place.

Discussion The force of 2457 N is equivalent to the weight of about 250 kg. That is, holding the nozzle requires the strength of holding a weight of 250 kg, which cannot be done by a single person. This demonstrates why several firemen are used to hold a hose with a high flow rate.



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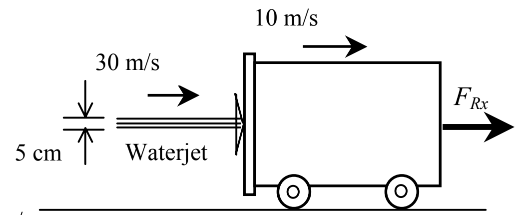
6-37 A horizontal jet of water with a given velocity strikes a flat plate that is moving in the same direction at a specified velocity. The force that the water stream exerts against the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water splatters in all directions in the plane of the plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal force exerted on the plate. **5** The velocity of the plate, and the velocity of the water jet relative to the plate, are constant. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.



Analysis We take the plate as the control volume, and the flow direction as the positive direction of x axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho V_{jet} A = \rho V_{jet} \frac{\pi D^2}{4} = (1000\ \text{kg/m}^3)(30\ \text{m/s}) \frac{\pi (0.05\ \text{m})^2}{4} = 58.9\ \text{kg/s}$$

The relative velocity between the plate and the jet is

$$V_r = V_{\text{jet}} - V_{\text{plate}} = 30 - 10 = 20\ \text{m/s}$$

Therefore, we can assume the plate to be stationary and the jet to move with a velocity of 20 m/s. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the horizontal reaction force applied to the plate in the negative x direction to counteract the impulse of the water jet be F_{Rx}. Then the momentum equation along the x direction gives

$$-F_{Rx} = 0 - \dot{m} V_i \quad \rightarrow \quad F_{Rx} = \dot{m} V_r = (58.9\ \text{kg/s})(20\ \text{m/s}) \left(\frac{1\ \text{N}}{1 \text{kg} \cdot \text{m/s}^2} \right) = \mathbf{1178\ N}$$

Therefore, the water jet applies a force of 1178 N on the plate in the direction of motion, and an equal and opposite force must be applied on the plate if its velocity is to remain constant.

Discussion Note that we used the relative velocity in the determination of the mass flow rate of water in the momentum analysis since water will enter the control volume at this rate. (In the limiting case of the plate and the water jet moving at the same velocity, the mass flow rate of water relative to the plate will be zero since no water will be able to strike the plate).

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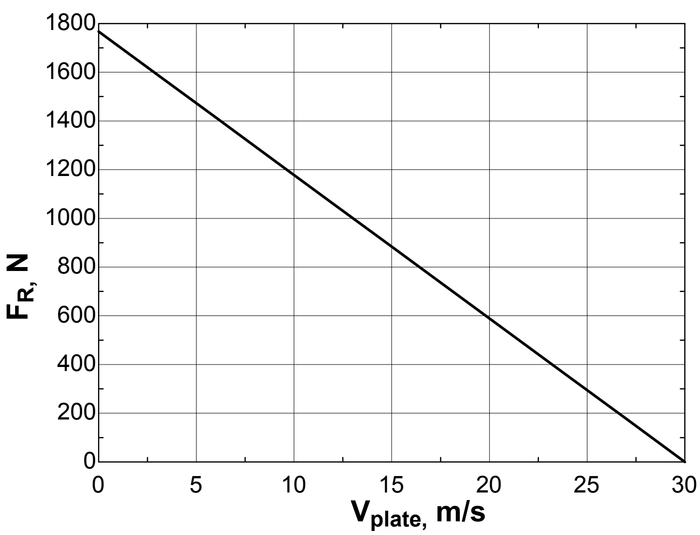
6-38 Problem 6-37 is reconsidered. The effect of the plate velocity on the force exerted on the plate as the plate velocity varies from 0 to 30 m/s in increments of 3 m/s is to be investigated.

```
rho=1000 "kg/m3"
D=0.05 "m"
V_jet=30 "m/s"

Ac=pi*D^2/4
V_r=V_jet-V_plate
m_dot=rho*Ac*V_jet
F_R=m_dot*V_r "N"
```

V_{plate}, m/s	V_r, m/s	F_R, N
0	30	1767
3	27	1590
6	24	1414
9	21	1237
12	18	1060
15	15	883.6
18	12	706.9
21	9	530.1
24	6	353.4
27	3	176.7
30	0	0



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6-39E A fan moves air at sea level at a specified rate. The force required to hold the fan and the minimum power input required for the fan are to be determined. √

Assumptions **1** The flow of air is steady and incompressible. **2** Standard atmospheric conditions exist so that the pressure at sea level is 1 atm. **3** Air leaves the fan at a uniform velocity at atmospheric pressure. **4** Air approaches the fan through a large area at atmospheric pressure with negligible velocity. **5** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **6** Wind flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties The gas constant of air is R = 0.3704 psi·ft^3/lbm·R. The standard atmospheric pressure at sea level is 1 atm = 14.7 psi.

Analysis (*a*) We take the control volume to be a horizontal hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) and the fan located at the narrow cross-section at the end (section 2), and let its centerline be the *x* axis. The density, mass flow rate, and discharge velocity of air are

$$\rho = \frac{P}{RT} = \frac{14.7\text{ psi}}{(0.3704\text{ psi}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\text{ R})} = 0.0749\text{ lbm/ft}^3$$

$$\dot{m} = \rho\dot{V} = (0.0749\text{ lbm/ft}^3)(2000\text{ ft}^3/\text{min}) = 149.8\text{ lbm/min} = 2.50\text{ lbm/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{2000\text{ ft}^3/\text{min}}{\pi(2\text{ ft})^2/4} = 636.6\text{ ft/min} = 10.6\text{ ft/s}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. Letting the reaction force to hold the fan be F_{Rx} and assuming it to be in the positive *x* (i.e., the flow) direction, the momentum equation along the *x* axis becomes

$$F_{Rx} = \dot{m}(V_2) - 0 = \dot{m}V = (2.50\text{ lbm/s})(10.6\text{ ft/s})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.82\ lbf}$$

Therefore, a force of 0.82 lbf must be applied (through friction at the base, for example) to prevent the fan from moving in the horizontal direction under the influence of this force.

(*b*) Noting that $P_1 = P_2 = P_{\text{atm}}$ and $V_1 \cong 0$, the energy equation for the selected control volume reduces to

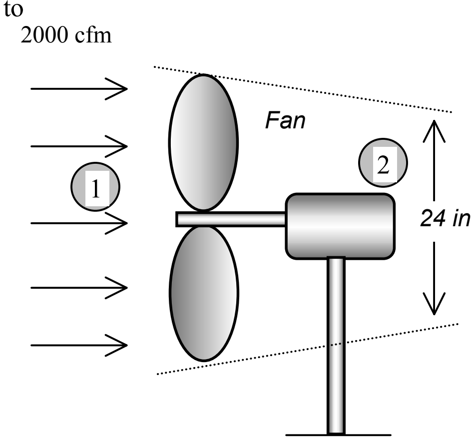
$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{V_2^2}{2} = (2.50\text{ lbm/s})\frac{(10.6\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.73756\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{5.91\ W}$$

Therefore, a useful mechanical power of 5.91 W must be supplied to air. This is the *minimum* required power input required for the fan.

Discussion The actual power input to the fan will be larger than 5.91 W because of the fan inefficiency in converting mechanical power to kinetic energy.



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6-40 A helicopter hovers at sea level while being loaded. The volumetric air flow rate and the required power input during unloaded hover, and the rpm and the required power input during loaded hover are to be determined. √

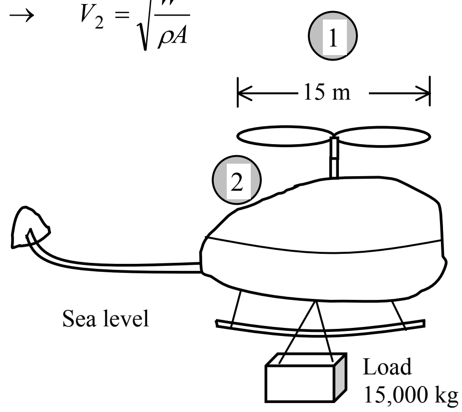
Assumptions **1** The flow of air is steady and incompressible. **2** Air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight. **7** Air flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties The density of air is given to be 1.18 kg/m^3.

Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$



where A is the blade span area,

$$A = \pi D^2/4 = \pi(15\text{ m})^2/4 = 176.7\text{ m}^2$$

Then the discharge velocity, volume flow rate, and the mass flow rate of air in the unloaded mode become

$$V_{2,\text{unloaded}} = \sqrt{\frac{m_{\text{unloaded}}\, g}{\rho A}} = \sqrt{\frac{(10{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.18\text{ kg/m}^3)(176.7\text{ m}^2)}} = 21.7\text{ m/s}$$

$$\dot{V}_{\text{unloaded}} = AV_{2,\text{unloaded}} = (176.7\text{ m}^2)(21.7\text{ m/s}) = \mathbf{3834\ m^3/s}$$

$$\dot{m}_{\text{unloaded}} = \rho\dot{V}_{\text{unloaded}} = (1.18\text{ kg/m}^3)(3834\text{ m}^3\text{/s}) = 4524\text{ kg/s}$$

Noting that $P_1 = P_2 = P_{\text{atm}}$, $V_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{unloaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{unloaded}} = (4524\text{ kg/s})\frac{(21.7\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1065\ kW}$$

(*b*) We now repeat the calculations for the *loaded* helicopter, whose mass is 10,000+15,000 = 25,000 kg:

$$V_{2,\text{loaded}} = \sqrt{\frac{m_{\text{loaded}}\, g}{\rho A}} = \sqrt{\frac{(25{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.18\text{ kg/m}^3)(176.7\text{ m}^2)}} = 34.3\text{ m/s}$$

$$\dot{m}_{\text{loaded}} = \rho\dot{V}_{\text{loaded}} = \rho A V_{2,\text{ loaded}} = (1.18\text{ kg/m}^3)(176.7\text{ m}^2)(34.3\text{ m/s}) = 7152\text{ kg/s}$$

$$\dot{W}_{\text{loaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{loaded}} = (7152\text{ kg/s})\frac{(34.3\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{4207\ kW}$$

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Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the loaded helicopter blades becomes

$$V_2 = k\dot{n} \quad \rightarrow \quad \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}} = \frac{\dot{n}_{\text{loaded}}}{\dot{n}_{\text{unloaded}}} \quad \rightarrow \quad \dot{n}_{\text{loaded}} = \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}}\dot{n}_{\text{unloaded}} = \frac{34.3}{21.7}(400 \text{ rpm}) = \mathbf{632\ rpm}$$

Discussion The actual power input to the helicopter blades will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical power to kinetic energy.

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6-41 A helicopter hovers on top of a high mountain where the air density considerably lower than that at sea level. The blade rotational velocity to hover at the higher altitude and the percent increase in the required power input to hover at high altitude relative to that at sea level are to be determined. √

Assumptions **1** The flow of air is steady and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air. **5** The change in air pressure with elevation while hovering at a given location is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight. **7** Air flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties The density of air is given to be 1.18 kg/m^3 at sea level, and 0.79 kg/m^3 on top of the mountain.

Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the *z* axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. Noting that the only force acting on the control volume is the total weight *W* and it acts in the negative *z* direction, the momentum equation along the *z* axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$

where *A* is the blade span area. Then for a given weight *W,* the ratio of discharge velocities becomes

$$\frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}} = \frac{\sqrt{W/\rho_{\text{mountain}} A}}{\sqrt{W/\rho_{\text{sea}} A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

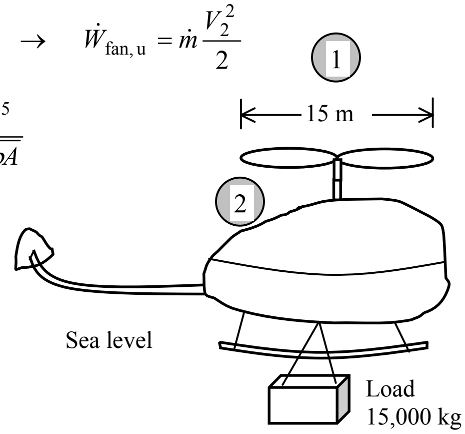
Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the helicopter blades on top of the mountain becomes

$$\dot{n} = kV_2 \quad \rightarrow \quad \frac{\dot{n}_{\text{mountain}}}{\dot{n}_{\text{sea}}} = \frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}} \quad \rightarrow \quad \dot{n}_{\text{mountain}} = \frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}}\dot{n}_{\text{sea}} = 1.222(400\ \text{rpm}) = \mathbf{489\ rpm}$$

Noting that $P_1 = P_2 = P_{\text{atm}}$, $V_1 \cong 0$, the elevation effect are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

or $$\dot{W}_{\text{fan,u}} = \dot{m}\frac{V_2^2}{2} = \rho A V_2 \frac{V_2^2}{2} = \rho A \frac{V_2^3}{2} = \tfrac{1}{2}\rho A\left(\sqrt{\frac{W}{\rho A}}\right)^3 = \tfrac{1}{2}\rho A\left(\frac{W}{\rho A}\right)^{1.5} = \frac{W^{1.5}}{2\sqrt{\rho A}}$$



Then the ratio of the required power input on top of the mountain to that at sea level becomes

$$\frac{\dot{W}_{\text{mountain fan,u}}}{\dot{W}_{\text{sea fan,u}}} = \frac{0.5W^{1.5}/\sqrt{\rho_{\text{mountain}} A}}{0.5W^{1.5}/\sqrt{\rho_{\text{sea}} A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

Therefore, the required power input will increase by **22.2%** on top of the mountain relative to the sea level.

Discussion Note that both the rpm and the required power input to the helicopter are inversely proportional to the square root of air density. Therefore, more power is required at higher elevations for the helicopter to operate because air is less dense, and more air must be forced by the blades into the downdraft.

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6-42 The flow rate in a channel is controlled by a sluice gate by raising or lowering a vertical plate. A relation for the force acting on a sluice gate of width w for steady and uniform flow is to be developed.

Assumptions **1** The flow is steady, incompressible, frictionless, and uniform (and thus the Bernoulli equation is applicable.) **2** Wall shear forces at surfaces are negligible. **3** The channel is exposed to the atmosphere, and thus the pressure at free surfaces is the atmospheric pressure. **4** The flow is horizontal. **5** Water flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis We take point 1 at the free surface of the upstream flow before the gate and point 2 at the free surface of the downstream flow after the gate. We also take the bottom surface of the channel as the reference level so that the elevations of points 1 and 2 are y_1 and y_2, respectively. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+y_2 \quad\rightarrow\quad V_2^2-V_1^2=2\,\mathrm{g}(y_1-y_2) \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad\rightarrow\quad A_1V_1=A_2V_2=\dot{V} \quad\rightarrow\quad V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{wy_1} \quad \text{and} \quad V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{wy_2} \qquad (2)$$

Substituting into Eq. (1),

$$\left(\frac{\dot{V}}{wy_2}\right)^2-\left(\frac{\dot{V}}{wy_1}\right)^2=2g(y_1-y_2) \rightarrow \dot{V}=w\sqrt{\frac{2g(y_1-y_2)}{1/y_2^2-1/y_1^2}} \quad\rightarrow \dot{V}=wy_2\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (3)$$

Substituting Eq. (3) into Eqs. (2) gives the following relations for velocities,

$$V_1=\frac{y_2}{y_1}\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad \text{and} \qquad V_2=\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (4)$$

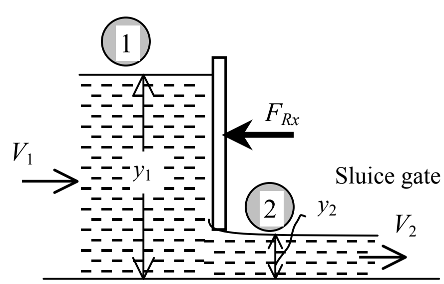
We choose the control volume as the water body surrounded by the vertical cross-sections of the upstream and downstream flows, free surfaces of water, the inner surface of the sluice gate, and the bottom surface of the channel. The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{\text{out}} \beta\dot{m}\vec{V}-\sum_{\text{in}} \beta\dot{m}\vec{V}$. The force acting on the sluice gate F_{Rx} is horizontal since the wall shear at the surfaces is negligible, and it is equal and opposite to the force applied on water by the sluice gate. Noting that the pressure force acting on a vertical surface is equal to the product of the pressure at the centroid of the surface and the surface area, the momentum equation along the x direction gives

$$-F_{Rx}+P_1A_1-P_2A_2=\dot{m}V_2-\dot{m}V_1 \quad\rightarrow\quad -F_{Rx}+\left(\rho g\frac{y_1}{2}\right)(wy_1)-\left(\rho g\frac{y_2}{2}\right)(wy_2)=\dot{m}(V_2-V_1)$$

Rearranging, the force acting on the sluice gate is determined to be

$$F_{Rx}=\dot{m}(V_1-V_2)+\frac{w}{2}\rho g(y_1^2-y_2^2) \qquad (5)$$

where V_1 and V_2 are given in Eq. (4).



Discussion Note that for $y_1 >> y_2$, Eq. (3) simplifies to $\dot{V}=y_2w\sqrt{2gy_1}$ or $V_2=\sqrt{2gy_1}$ which is the Toricelli equation for frictionless flow from a tank through a hole a distance y_1 below the free surface.

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6-43 Water enters a centrifugal pump axially at a specified rate and velocity, and leaves in the normal direction along the pump casing. The force acting on the shaft in the axial direction is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The flow is steady and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces.

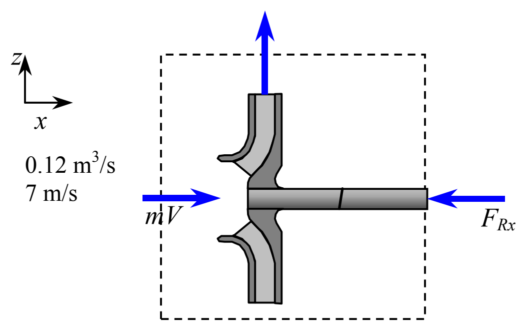
Analysis We take the pump as the control volume, and the inlet direction of flow as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_{Rx} = -\dot{m} V_i \quad \rightarrow \quad F_{Rx} = \dot{m} V_i = \rho \dot{V} V_i$$

Note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = (1000\ \text{kg/m}^3)(0.12\ \text{m}^3/\text{s})(7\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{840\ N}$$

Discussion To find the total force acting on the shaft, we also need to do a force balance for the vertical direction, and find the vertical component of the reaction force.



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