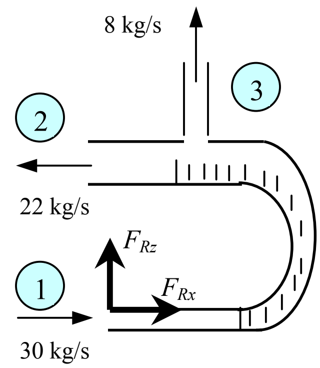
Review Problems

6-58 Water is flowing into and discharging from a pipe U-section with a secondary discharge section normal to return flow. Net x- and *z*- forces at the two flanges that connect the pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The weight of the U-turn and the water in it is negligible. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The flow velocities of the 3 streams are



$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho(\pi D_1^2/4)} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.05\ \text{m})^2/4]} = 15.3\ \text{m/s}$$

$$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho(\pi D_2^2/4)} = \frac{22\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.10\ \text{m})^2/4]} = 2.80\ \text{m/s}$$

$$V_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{\dot{m}_3}{\rho(\pi D_3^2/4)} = \frac{8\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.03\ \text{m})^2/4]} = 11.3\ \text{m/s}$$

We take the entire U-section as the control volume. We designate the horizontal coordinate by *x* with the direction of incoming flow as being the positive direction and the vertical coordinate by *z*. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the cone be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx} + P_1A_1 + P_2A_2 = \beta\dot{m}_2(-V_2) - \beta\dot{m}_1V_1 \quad \rightarrow \quad F_{Rx} = -P_1A_1 - P_2A_2 - \beta(\dot{m}_2V_2 + \dot{m}_1V_1)$$

$$F_{Rz} + 0 = \dot{m}_3V_3 - 0 \quad \rightarrow \quad F_{Rz} = \beta\dot{m}_3V_3$$

Substituting the given values,

$$\begin{aligned} F_{Rx} &= -[(200-100)\ \text{kN/m}^2]\frac{\pi(0.05\ \text{m})^2}{4} - [(150-100)\ \text{kN/m}^2]\frac{\pi(0.10\ \text{m})^2}{4} \\ &\quad -1.03\left[(22\ \text{kg/s})(2.80\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + (30\ \text{kg/s})(15.3\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\right] \\ &= -0.733\ \text{kN} = \mathbf{-733\ N} \end{aligned}$$

$$F_{Rz} = 1.03(8\ \text{kg/s})(11.3\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{93.1\ N}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 733 N acts on the flanges in the opposite direction. A vertical force of 93.1 N acts on the flange in the vertical direction.

Discussion To assess the significance of gravity forces, we estimate the weight of the weight of water in the U-turn and compare it to the vertical force. Assuming the length of the U-turn to be 0.5 m and the average diameter to be 7.5 cm, the mass of the water becomes

$$m = \rho \boldsymbol{V} = \rho AL = \rho\frac{\pi D^2}{4}L = (1000\ \text{kg/m}^3)\frac{\pi(0.075\ \text{m})^2}{4}(0.5\ \text{m}) = 2.2\ \text{kg}$$

whose weight is 2.2×9.81 = 22 N, which is much less than 93.1, but still significant. Therefore, disregarding the gravitational effects is a reasonable assumption if great accuracy is not required.

6-59 A fireman was hit by a nozzle held by a tripod with a rated holding force. The accident is to be investigated by calculating the water velocity, the flow rate, and the nozzle velocity.

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

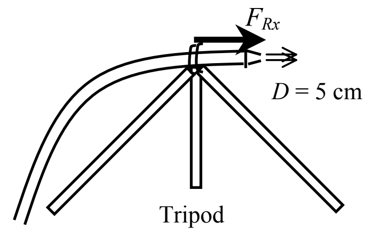
Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and outlets horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction).

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. We let the horizontal force applied by the tripod to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction becomes

$$F_{Rx} = \dot{m}V_e - 0 = \dot{m}V = \rho AVV = \rho \frac{\pi D^2}{4} V^2 \quad \rightarrow \quad (1800\ \text{N})\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = (1000\ \text{kg/m}^3)\frac{\pi (0.05\ \text{m})^2}{4} V^2$$

Solving for the water outlet velocity gives V = **30.3 m/s.** Then the water flow rate becomes



$$\dot{V} = AV = \frac{\pi D^2}{4} V = \frac{\pi (0.05\ \text{m})^2}{4} (30.3\ \text{m/s}) = \mathbf{0.0595\ m^3/s}$$

When the nozzle was released, its acceleration must have been

$$a_{\text{nozzle}} = \frac{F}{m_{\text{nozzle}}} = \frac{1800\ \text{N}}{10\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 180\ \text{m/s}^2$$

Assuming the reaction force acting on the nozzle and thus its acceleration to remain constant, the time it takes for the nozzle to travel 60 cm and the nozzle velocity at that moment were (note that both the distance x and the velocity V are zero at time $t = 0$)

$$x = \tfrac{1}{2} a t^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.6\ \text{m})}{180\ \text{m/s}^2}} = 0.0816\ s$$

$$V = at = (180\ \text{m/s}^2)(0.0816\ \text{s}) = \mathbf{14.7\ m/s}$$

Thus we conclude that the nozzle hit the fireman with a velocity of 14.7 m/s.

Discussion Engineering analyses such as this one are frequently used in accident reconstruction cases, and they often form the basis for judgment in courts.

6-60 During landing of an airplane, the thrust reverser is lowered in the path of the exhaust jet, which deflects the exhaust and provides braking. The thrust of the engine and the braking force produced after the thrust reverser is deployed are to be determined. √EES

Assumptions **1** The flow of exhaust gases is steady and one-dimensional. **2** The exhaust gas stream is exposed to the atmosphere, and thus its pressure is the atmospheric pressure. **3** The velocity of exhaust gases remains constant during reversing. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis (*a*) The thrust exerted on an airplane is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = \dot{m}_{ex} V_{ex} = (18\ \text{kg/s})(250\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{4500\ N}$$

(*b*) We take the thrust reverser as the control volume such that it cuts through both exhaust streams normally and the connecting bars to the airplane, and the direction of airplane as the positive direction of *x* axis. The momentum equation for steady one-dimensional flow in the *x* direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = \dot{m}(V)\cos 20° - \dot{m}(-V) \quad \rightarrow \quad F_{Rx} = (1 + \cos 20°)\dot{m} V_i$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (1 + \cos 20°)(18\ \text{kg/s})(250\ \text{m/s}) = 8729\ \text{N}$$

The breaking force acting on the plane is equal and opposite to this force,

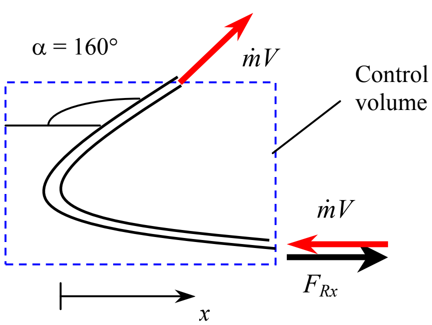
$$F_{\text{breaking}} = \mathbf{8729\ \ N}$$

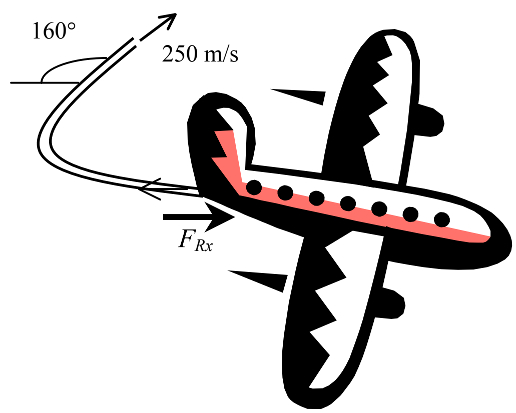
Therefore, a braking force of 8729 N develops in the opposite direction tot flight.

Discussion This problem can be solved more generally by measuring the reversing angle from the direction of exhaust gases (α = 0 when there is no reversing). When α < 90°, the reversed gases are discharged in the negative *x* direction, and the momentum equation reduces to

$$F_{Rx} = \dot{m}(-V)\cos\alpha - \dot{m}(-V) \quad \rightarrow \quad F_{Rx} = (1 - \cos\alpha)\dot{m} V_i$$

This equation is also valid for α >90° since cos(180°-α) = - cosα. Using α = 160°, for example, gives $F_{Rx} = (1 - \cos 160)\dot{m} V_i = (1 + \cos 20)\dot{m} V_i$, which is identical to the solution above.





6-61 Problem 6-60 reconsidered. The effect of thrust reverser angle on the braking force exerted on the airplane as the reverser angle varies from 0 (no reversing) to 180° (full reversing) in increments of 10° is to be investigated.

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V_jet=250 "m/s"

m_dot=18 "kg/s"
F_Rx=(1-cos(alpha))*m_dot*V_jet "N"
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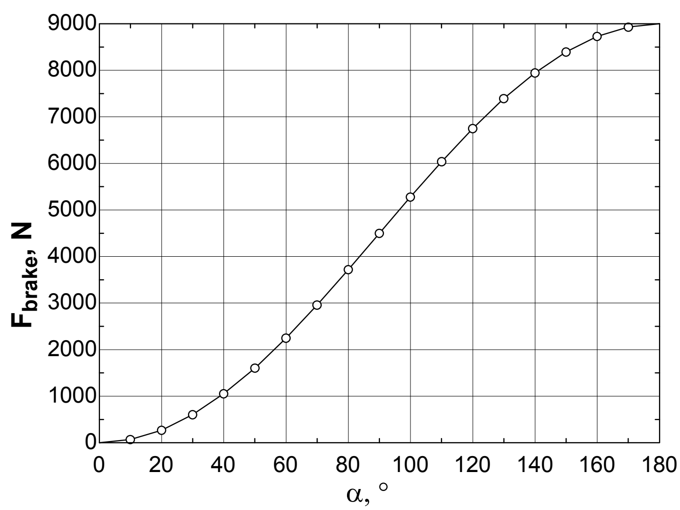
Reversing angle, α°	Braking force F_{brake}, N
0	0
10	68
20	271
30	603
40	1053
50	1607
60	2250
70	2961
80	3719
90	4500
100	5281
110	6039
120	6750
130	7393
140	7947
150	8397
160	8729
170	8932
180	9000



6-62E The rocket of a spacecraft is fired in the opposite direction to motion. The acceleration, the velocity change, and the thrust are to be determined.

Assumptions **1** The flow of combustion gases is steady and one-dimensional during firing period, but the flight of spacecraft is unsteady. **2** There are no external forces acting on the spacecraft, and the effect of pressure force at the nozzle outlet is negligible. **3** The mass of discharged fuel is negligible relative to the mass of the spacecraft, and thus the spacecraft may be treated as a solid body with a constant mass. **4** The nozzle is well-designed such that the effect of the momentum-flux correction factor is negligible, and thus $\beta \cong 1$.

Analysis (*a*) We choose a reference frame in which the control volume moves with the spacecraft. Then the velocities of fluid steams become simply their relative velocities relative to the moving body. We take the direction of motion of the spacecraft as the positive direction along the *x* axis. There are no external forces acting on the spacecraft, and its mass is nearly constant. Therefore, the spacecraft can be treated as a solid body with constant mass, and the momentum equation in this case is

$$0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V} \quad \rightarrow \quad m_{\text{space}} \frac{d\vec{V}_{\text{space}}}{dt} = -\dot{m}_f \vec{V}_f$$

Noting that the motion is on a straight line and the discharged gases move in the negative *x* direction, we can write the momentum equation using magnitudes as

$$m_{\text{space}} \frac{dV_{\text{space}}}{dt} = \dot{m}_f V_f \quad \rightarrow \quad \frac{dV_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} V_f$$

Substituting, the acceleration of the spacecraft during the first 5 seconds is determined to be

$$a_{\text{space}} = \frac{dV_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} V_f = \frac{150 \text{ lbm/s}}{18{,}000 \text{ lbm}} (5000 \text{ ft/s}) = \mathbf{41.7\ ft/s^2}$$

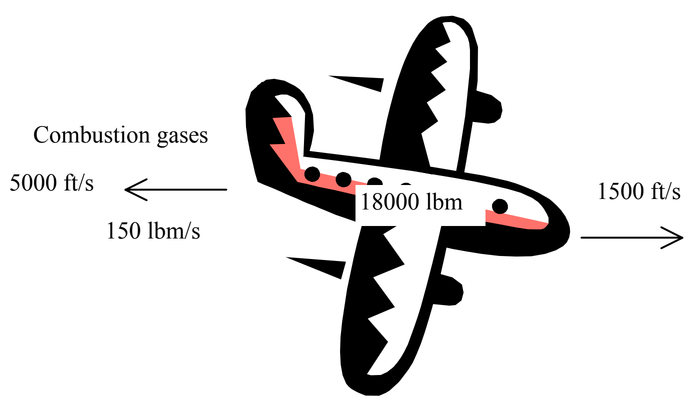
(*b*) Knowing acceleration, which is constant, the velocity change of the spacecraft during the first 5 seconds is determined from the definition of acceleration $a_{\text{space}} = dV_{\text{space}} / dt$ to be

$$dV_{\text{space}} = a_{\text{space}} dt \quad \rightarrow \quad \Delta V_{\text{space}} = a_{\text{space}} \Delta t = (41.7 \text{ ft/s}^2)(5 \text{ s}) = \mathbf{209\ ft/s}$$

(*c*) The thrust exerted on the system is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = F_R = -\dot{m}_f V_f = -(150 \text{ lbm/s})(\text{-}5000 \text{ ft/s}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = \mathbf{23{,}290\ lbf}$$

Therefore, if this spacecraft were attached somewhere, it would exert a force of 23,290 lbf (equivalent to the weight of 23,290 lbm of mass) to its support.



6-63 A horizontal water jet strikes a vertical stationary flat plate normally at a specified velocity. For a given flow velocity, the anchoring force needed to hold the plate in place is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. We take the reaction force to be in the negative x direction. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

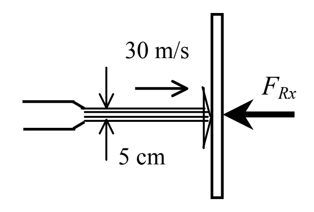
$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V} \quad \rightarrow \quad -F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{Rx} = \dot{m}V$$

We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. The mass flow rate of water is

$$\dot{m} = \rho \dot{\mathbf{V}} = \rho A V = \rho \frac{\pi D^2}{4} V = (1000\ \text{kg/m}^3)\frac{\pi (0.05\ \text{m})^2}{4}(30\ \text{m/s}) = 58.90\ \text{kg/s}$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (58.90\ \text{kg/s})(30\ \text{m/s}) = \mathbf{1767\ N}$$



Therefore, a force of 1767 N must be applied to the plate in the opposite direction to flow to hold it in place.

Discussion In reality, some water will be scattered back, and this will add to the reaction force of water.

6-64 A water jet hits a stationary cone, such that the flow is diverted equally in all directions at 45°. The force required to hold the cone in place against the water stream is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

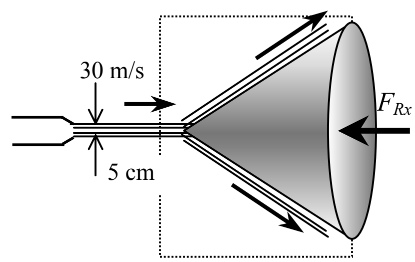
Analysis The mass flow rate of water jet is

$$\dot{m} = \rho \dot{V} = \rho A V = \rho \frac{\pi D^2}{4} V = (1000\ \text{kg/m}^3)\frac{\pi (0.05\ \text{m})^2}{4}(30\ \text{m/s}) = 58.90\ \text{kg/s}$$

We take the diverting section of water jet, including the cone as the control volume, and designate the entrance by 1 and the outlet after divergence by 2. We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.
The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $V_2 = V_1 = V$ and $\dot{m}_2 = \dot{m}_1 = \dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}V_2 \cos\theta - \dot{m}V_1 = \dot{m}V(\cos\theta - 1)$$

$$F_{Ry} = 0 \quad \text{(because of symmetry about x axis)}$$



Substituting the given values,

$$F_{Rx} = (58.90\ \text{kg/s})(30\ \text{m/s})(\cos 45° - 1)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{-518\ N}$$

$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates that the assumed direction is wrong, and should be reversed. Therefore, a force of 518 N must be applied to the cone in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction due to symmetry and neglecting gravitational effects.

Discussion In reality, the gravitational effects will cause the upper part of flow to slow down and the lower part to speed up after the split. But for short distances, these effects are negligible.

6-65 An ice skater is holding a flexible hose (essentially weightless) which directs a stream of water horizontally at a specified velocity. The velocity and the distance traveled in 5 seconds, and the time it takes to move 5 m and the velocity at that moment are to be determined.

Assumptions **1** Friction between the skates and ice is negligible. **2** The flow of water is steady and one-dimensional (but the motion of skater is unsteady). **3** The ice skating arena is level, and the water jet is discharged horizontally. **4** The mass of the hose and the water in it is negligible. **5** The skater is standing still initially at t = 0. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (a) The mass flow rate of water through the hose is

$$\dot{m} = \rho A V = \rho \frac{\pi D^2}{4} V = (1000\ \text{kg/m}^3)\frac{\pi (0.02\ \text{m})^2}{4}(10\ \text{m/s}) = 3.14\ \text{kg/s}$$

The thrust exerted on the skater by the water stream is simply the momentum flux of the water stream, and it acts in the reverse direction,

$$F = \text{Thrust} = \dot{m}V = (3.14\ \text{kg/s})(10\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 31.4\ \ \text{N}\ \ (\text{constant})$$

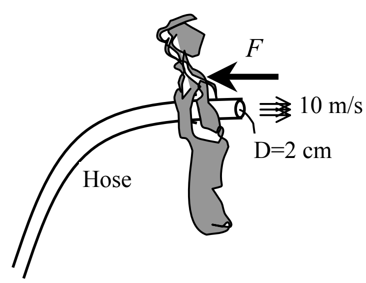
The acceleration of the skater is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of the skater,

$$a = \frac{F}{m} = \frac{31.4\ \text{N}}{60\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 0.523\ \text{m/s}^2$$

Note that thrust and thus the acceleration of the skater is constant. The velocity of the skater and the distance traveled in 5 s are

$$V_{\text{skater}} = at = (0.523\ \text{m/s}^2)(5\ \text{s}) = \mathbf{2.62\ m/s}$$

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.523\ \text{m/s}^2)(5\ \text{s})^2 = \mathbf{6.54\ m}$$



(b) The time it will take to move 5 m and the velocity at that moment are

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(5\ \text{m})}{0.523\ \text{m/s}^2}} = \mathbf{4.4\ s}$$

$$V_{\text{skater}} = at = (0.523\ \text{m/s}^2)(4.4\ \text{s}) = \mathbf{2.3\ m/s}$$

Discussion In reality, the velocity of the skater will be lower because of friction on ice and the resistance of the hose to follow the skater.

6-66 Indiana Jones is to ascend a building by building a platform, and mounting four water nozzles pointing down at each corner. The minimum water jet velocity needed to raise the system, the time it will take to rise to the top of the building and the velocity of the system at that moment, the additional rise when the water is shut off, and the time he has to jump from the platform to the roof are to be determined.

Assumptions **1** The air resistance is negligible. **2** The flow of water is steady and one-dimensional (but the motion of platform is unsteady). **3** The platform is still initially at $t = 0$. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m³.

Analysis (*a*) The total mass flow rate of water through the 4 hoses and the total weight of the platform are

$$\dot{m} = \rho A V = 4\rho \frac{\pi D^2}{4} V = 4(1000 \text{ kg/m}^3)\frac{\pi (0.05 \text{ m})^2}{4}(15 \text{ m/s}) = 118 \text{ kg/s}$$

$$W = mg = (150 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 1472 \text{ N}$$

We take the platform as the system. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. The minimum water jet velocity needed to raise the platform is determined by setting the net force acting on the platform equal to zero,

$$-W = \dot{m}(-V_{\min}) - 0 \quad \rightarrow \quad W = \dot{m}V_{\min} = \rho A V_{\min} V_{\min} = 4\rho \frac{\pi D^2}{4} V_{\min}^2$$

Solving for $V_{\min}$ and substituting,

$$V_{\min} = \sqrt{\frac{W}{\rho \pi D^2}} = \sqrt{\frac{1472 \text{ N}}{(1000 \text{ kg/m}^3)\pi(0.05 \text{ m})^2}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right)} = \mathbf{13.7\ m/s}$$

(*b*) We let the vertical reaction force (assumed upwards) acting on the platform be F_{Rz}. Then the momentum equation in the vertical direction becomes

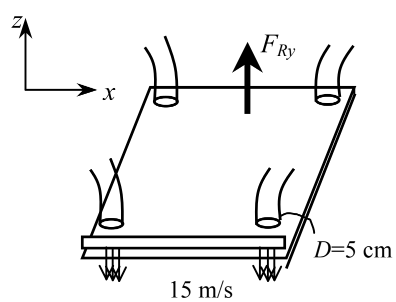
$$F_{Rz} - W = \dot{m}(-V) - 0 = \dot{m}V \quad \rightarrow \quad F_{Rz} = W - \dot{m}V = (1472 \text{ N}) - (118 \text{ kg/s})(15 \text{ m/s})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = -298 \text{ N}$$

The upward thrust acting on the platform is equal and opposite to this reaction force, and thus $F = 298$ N. Then the acceleration and the ascending time to rise 10 m and the velocity at that moment become

$$a = \frac{F}{m} = \frac{298 \text{ N}}{150 \text{ kg}}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = 2.0 \text{ m/s}^2$$

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10 \text{ m})}{2 \text{ m/s}^2}} = \mathbf{3.2\ s}$$

$$V = at = (2 \text{ m/s}^2)(3.2 \text{ s}) = \mathbf{6.4\ m/s}$$



(*c*) When water is shut off at 10 m height (where the velocity is 6.4 m/s), the platform will decelerate under the influence of gravity, and the time it takes to come to a stop and the additional rise above 10 m become

$$V = V_0 - gt = 0 \quad \rightarrow \quad t = \frac{V_0}{g} = \frac{6.4 \text{ m/s}}{9.81 \text{ m/s}^2} = \mathbf{0.65\ s}$$

$$z = V_0 t - \tfrac{1}{2}gt^2 = (6.4 \text{ m/s})(0.65 \text{ s}) - \tfrac{1}{2}(9.81 \text{ m/s}^2)(0.65 \text{ s})^2 = \mathbf{2.1\,m}$$

Therefore, Jones has 2×0.65 = 1.3 s to jump off from the platform to the roof since it takes another 0.65 s for the platform to descend to the 10 m level.

6-67E A box-enclosed fan is faced down so the air blast is directed downwards, and it is to be hovered by increasing the blade rpm. The required blade rpm, air outlet velocity, the volumetric flow rate, and the minimum mechanical power are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure, and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the fan, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 0.078 lbm/ft^3.

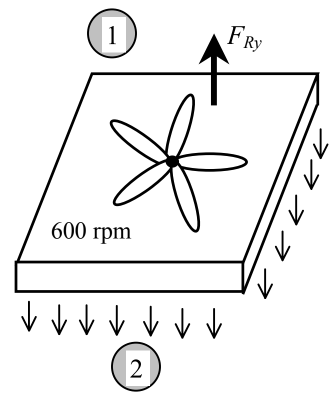
Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the *z* axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$. Noting that the only force acting on the control volume is the total weight *W* and it acts in the negative *z* direction, the momentum equation along the *z* axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$

where *A* is the blade span area,

$$A = \pi D^2/4 = \pi(3\text{ ft})^2/4 = 7.069\text{ ft}^2$$



Then the discharge velocity to produce 5 lbf of upward force becomes

$$V_2 = \sqrt{\frac{5\text{ lbf}}{(0.078\text{ lbm/ft}^3)(7.069\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = \mathbf{17.1\ ft/s}$$

(*b*) The volume flow rate and the mass flow rate of air are determined from their definitions,

$$\dot{V} = AV_2 = (7.069\text{ ft}^2)(17.1\text{ ft/s}) = \mathbf{121\ ft^3/s}$$

$$\dot{m} = \rho\dot{V} = (0.078\text{ lbm/ft}^3)(121\text{ ft}^3/\text{s}) = 9.43\text{ lbm/s}$$

(*c*) Noting that $P_1 = P_2 = P_{\text{atm}}$, $V_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{V_2^2}{2} = (9.43\text{ lbm/s})\frac{(18.0\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.73756\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{64.3\ W}$$

Therefore, the minimum mechanical power that must be supplied to the air stream is 64.3 W.

Discussion The actual power input to the fan will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical work to kinetic energy.

6-68 A parachute slows a soldier from his terminal velocity V_T to his landing velocity of V_F. A relation is to be developed for the soldier's velocity after he opens the parachute at time $t = 0$.

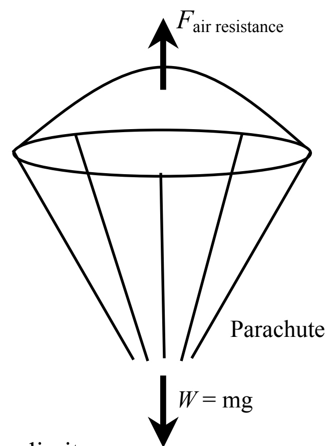
Assumptions **1** The air resistance is proportional to the velocity squared (i.e. $F = -kV^2$). **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low density of air. **4** The final velocity of the soldier is equal to its terminal velocity with his parachute open.

Analysis The terminal velocity of a free falling object is reached when the air resistance (or air drag) equals the weight of the object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_{\text{air resistance}} = W \quad \to \quad kV_F^2 = mg \quad \to \quad k = \frac{mg}{V_F^2}$$

This is the desired relation for the constant of proportionality k. When the parachute is deployed and the soldier starts to decelerate, the net downward force acting on him is his weight less the air resistance,

$$F_{\text{net}} = W - F_{\text{air resistance}} = mg - kV^2 = mg - \frac{mg}{V_F^2}V^2 = mg\left(1 - \frac{V^2}{V_F^2}\right)$$



Substituting it into Newton's 2nd law relation $F_{\text{net}} = ma = m\dfrac{dV}{dt}$ gives

$$mg\left(1 - \frac{V^2}{V_F^2}\right) = m\frac{dV}{dt}$$

Canceling m and separating variables, and integrating from $t = 0$ when $V = V_T$ to $t = t$ when $V = V$ gives

$$\frac{dV}{1 - V^2/V_F^2} = gdt \quad \to \quad \int_{V_T}^{V} \frac{dV}{V_F^2 - V^2} = \frac{g}{V_F^2}\int_0^t dt$$

Using $\displaystyle\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x}$ from integral tables and applying the integration limits,

$$\frac{1}{2V_F}\left(\ln\frac{V_F + V}{V_F - V} - \ln\frac{V_F + V_T}{V_F - V_T}\right) = \frac{gt}{V_F^2}$$

Rearranging, the velocity can be expressed explicitly as a function of time as

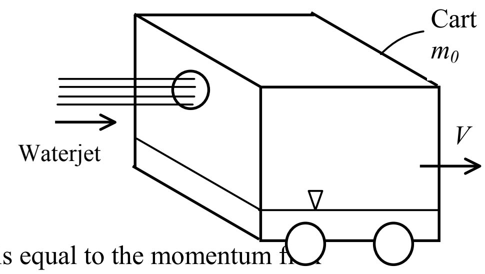
$$V = V_F\,\frac{V_T + V_F + (V_T - V_F)e^{-2gt/V_F}}{V_T + V_F - (V_T - V_F)e^{-2gt/V_F}}$$

Discussion Note that as $t \to \infty$, the velocity approaches the landing velocity of V_F, as expected.

6-69 An empty cart is to be driven by a horizontal water jet that enters from a hole at the rear of the cart. A relation is to be developed for cart velocity versus time.

Assumptions **1** The flow of water is steady, one-dimensional, incompressible, and horizontal. **2** All the water which enters the cart is retained. **3** The path of the cart is level and frictionless. **4** The cart is initially empty and stationary, and thus $V = 0$ at time $t = 0$. **5** Friction between water jet and air is negligible, and the entire momentum of water jet is used to drive the cart with no losses. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis We note that the water jet velocity V_J is constant, but the car velocity V is variable. Noting that $\dot{m} = \rho A(V_J - V)$ where A is the cross-sectional area of the water jet and V_J - V is the velocity of the water jet relative to the cart, the mass of water in the cart at any time t is



$$m_w = \int_0^t \dot{m}dt = \int_0^t \rho A(V_J - V)dt = \rho A V_J t - \rho A \int_0^t V dt \quad (1)$$

Also,

$$\frac{dm_w}{dt} = \dot{m} = \rho A(V_J - V)$$

We take the cart as the system. The net force acting on the cart in this case is equal to the momentum flux of the water jet. Newton's 2nd law $F = ma = d(mV)/dt$ in this case can be expressed as

$$F = \frac{d(m_{\text{total}}V)}{dt} \quad \text{where} \quad F = \dot{m}(V_J - V) = \rho A(V_J - V)^2$$

and

$$\frac{d(m_{\text{total}}V)}{dt} = \frac{d[(m_c + m_w)V]}{dt} = m_c \frac{dV}{dt} + \frac{d(m_w V)}{dt} = m_c \frac{dV}{dt} + m_w \frac{dV}{dt} + V\frac{dm_w}{dt}$$

$$= (m_c + m_w)\frac{dV}{dt} + \rho A(V_J - V)V$$

Substituting,

$$\rho A(V_J - V)^2 = (m_c + m_w)\frac{dV}{dt} + \rho A(V_J - V)V \quad \rightarrow \quad \rho A(V_J - V)(V_J - 2V) = (m_c + m_w)\frac{dV}{dt}$$

Noting that m_w is a function of t (as given by Eq. 1) and separating variables,

$$\frac{dV}{\rho A(V_J - V)(V_J - 2V)} = \frac{dt}{m_c + m_w} \quad \rightarrow \quad \frac{dV}{\rho A(V_J - V)(V_J - 2V)} = \frac{dt}{m_c + \rho A V_J t - \rho A \int_0^t V dt}$$

Integrating from $t = 0$ when $V = 0$ to $t = t$ when $V = V$ gives the desired integral,

$$\int_0^V \frac{dV}{\rho A(V_J - V)(V_J - 2V)} = \int_o^t \frac{dt}{m_c + \rho A V_J t - \rho A \int_0^t V dt}$$

Discussion Note that the time integral involves the integral of velocity, which complicates the solution.

6-70 A plate is maintained in a horizontal position by frictionless vertical guide rails. The underside of the plate is subjected to a water jet. The minimum mass flow rate $\dot{m}_{\text{min}}$ to just levitate the plate is to be determined, and a relation is to be obtained for the steady state upward velocity. Also, the integral that relates velocity to time when the water is first turned on is to be obtained.

Assumptions **1** The flow of water is steady and one-dimensional. **2** The water jet splatters in the plane of he plate. **3** The vertical guide rails are frictionless. **4** Times are short, so the velocity of the rising jet can be considered to remain constant with height. **5** At time t = 0, the plate is at rest. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis (*a*) We take the plate as the system. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. Noting that $\dot{m} = \rho A V_J$ where A is the cross-sectional area of the water jet and $W = m_p g$, the minimum mass flow rate of water needed to raise the plate is determined by setting the net force acting on the plate equal to zero,

$$-W = 0 - \dot{m}_{\text{min}} V_{\text{J}} \quad \rightarrow \quad W = \dot{m}_{\text{min}} V_{\text{J}} \rightarrow \quad m_p g = \dot{m}_{\text{min}} (\dot{m}_{\text{min}} / A V_{\text{J}}) \rightarrow \quad \dot{m}_{\text{min}} = \sqrt{\rho A m_p g}$$

For $\dot{m} > \dot{m}_{\text{min}}$, a relation for the steady state upward velocity V is obtained setting the upward impulse applied by water jet to the weight of the plate (during steady motion, the plate velocity V is constant, and the velocity of water jet relative to plate is $V_{\text{J}} - V$),

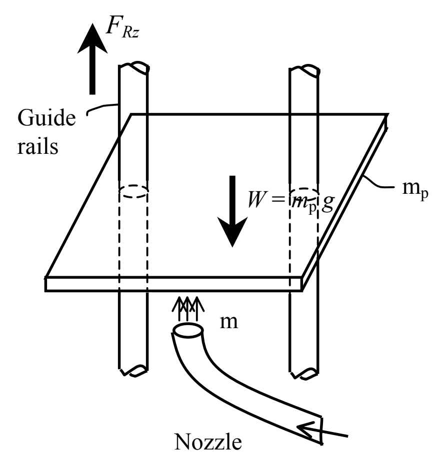
$$W = \dot{m}(V_J - V) \quad \rightarrow \quad m_p g = \rho A (V_J - V)^2 \quad \rightarrow \quad V_J - V = \sqrt{\frac{m_p g}{\rho A}} \quad \rightarrow \quad V = \frac{\dot{m}}{\rho A} - \sqrt{\frac{m_p g}{\rho A}}$$

(*b*) At time $t = 0$ the plate is at rest ($V = 0$), and it is subjected to water jet with $\dot{m} > \dot{m}_{\text{min}}$ and thus the net force acting on it is greater than the weight of the plate, and the difference between the jet impulse and the weight will accelerate the plate upwards. Therefore, Newton's 2[nd] law $F = ma = mdV/dt$ in this case can be expressed as

$$\dot{m}(V_J - V) - W = m_p a \quad \rightarrow \quad \rho A (V_J - V)^2 - m_p g = m_p \frac{dV}{dt}$$

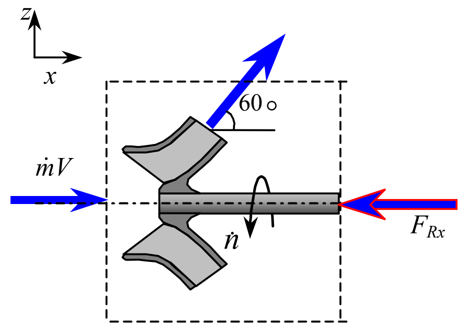
Separating the variables and integrating from $t = 0$ when $V = 0$ to $t = t$ when $V = V$ gives the desired integral,

$$\int_0^{\text{V}} \frac{m_p dV}{\rho A (V_J - V)^2 - m_p g} = \int_{t=0}^{t} dt \quad \rightarrow \quad t = \int_0^{\text{V}} \frac{m_p dV}{\rho A (V_J - V)^2 - m_p g}$$



Discussion This integral can be performed with the help of integral tables. But the relation obtained will be implicit in *V*.

6-71 Water enters a centrifugal pump axially at a specified rate and velocity, and leaves at an angle from the axial direction. The force acting on the shaft in the axial direction is to be determined.



Assumptions **1** The flow is steady and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces. **4** Water flow is nearly uniform at the outlet and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis From conservation of mass we have $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and thus $\dot{V}_1 = \dot{V}_2$ and $A_{c1}V_1 = A_{c2}V_2$. Noting that the discharge area is half the inlet area, the discharge velocity is twice the inlet velocity. That is,

$$A_{c1}V_2 = \frac{A_{c1}}{A_{c2}}V_1 = 2V_1 = 2(5 \text{ m/s}) = 10 \text{ m/s}$$

We take the pump as the control volume, and the inlet direction of flow as the positive direction of x axis. The linear momentum equation in this case in the x direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V} \quad \rightarrow \quad -F_{Rx} = \dot{m}V_2\cos\theta - \dot{m}V_1 \quad \rightarrow \quad F_{Rx} = \dot{m}(V_1 - V_2\cos\theta)$$

where the mass flow rate it

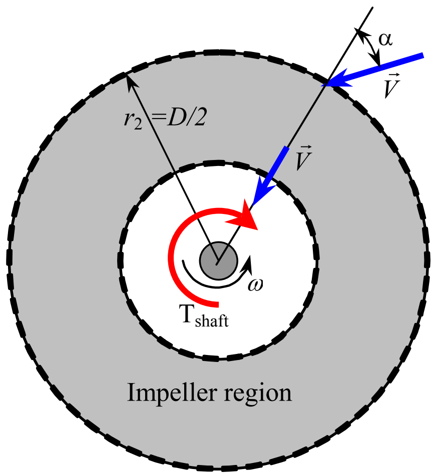
$$\dot{m} = \rho\dot{V} = (1000 \text{ kg/m}^3)(0.20 \text{ m}^3\text{/s}) = 200 \text{ kg/s}$$

Substituting the known quantities, the reaction force is determined to be (note that cos60° = 0.5)

$$F_{\text{Rx}} = (200 \text{ kg/s})[(5 \text{ m/s}) - (10 \text{ m/s})\cos 60]\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0}$$

Discussion Note that at this angle of discharge, the bearing is not subjected to any horizontal loading. Therefore, the loading in the system can be controlled by adjusting the discharge angle.

6-72 Water enters the impeller of a turbine through its outer edge of diameter D with velocity $\mathscr{V}$ making an angle α with the radial direction at a mass flow rate of $\dot{m}$, and leaves the impeller in the radial direction. The maximum power that can be generated is to be shown to be $\dot{W}_{\text{shaft}} = \pi\dot{n}\dot{m}D\mathscr{V}\sin\alpha$.



Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Analysis We take the impeller region as the control volume. The tangential velocity components at the inlet and the outlet are $V_{1,t} = 0$ and $V_{2,t} = V\sin\alpha$.

Normal velocity components as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$\mathrm{T}_{\text{shaft}} = \dot{m}(r_2V_{2,t} - r_1V_{1,t}) = \dot{m}r_2V_{2,t} - 0 = \dot{m}D(V\sin\alpha)/2$$

The angular velocity of the propeller is $\omega = 2\pi\dot{n}$. Then the shaft power becomes

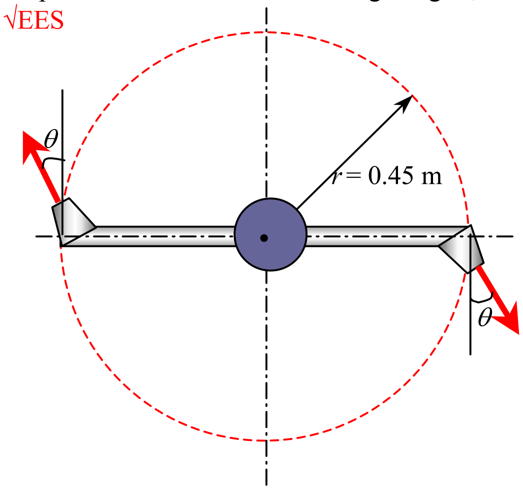
$$\dot{W}_{\text{shaft}} = \omega\mathrm{T}_{\text{shaft}} = 2\pi\dot{n}\dot{m}D(V\sin\alpha)/2$$

Simplifying, the maximum power generated becomes

$$\dot{W}_{\text{shaft}} = \pi\dot{n}\dot{m}DV\sin\alpha$$

which is the desired relation.

6-73 A two-armed sprinkler is used to water a garden. For specified flow rate and discharge angles, the rates of rotation of the sprinkler head are to be determined. √EES



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Frictional effects and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m³ = 1 kg/L.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the two nozzles are identical, we have $\dot{m}_{\text{nozzle}} = \dot{m}/2$ or $\dot{V}_{\text{nozzle}} = \dot{V}_{total}/2$ since the density of water is constant. The average jet outlet velocity relative to the nozzle is

$$V_{\text{jet}} = \frac{\dot{V}_{\text{nozzle}}}{A_{\text{jet}}} = \frac{60\text{ L/s}}{2[\pi(0.02\text{ m})^2/4]}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = 95.49\text{ m/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$. Noting that there are no external moments acting, the angular momentum equation about the axis of rotation becomes

$$0 = -2r\dot{m}_{\text{nozzle}}V_r\cos\theta \quad\rightarrow\quad V_r = 0 \quad\rightarrow\quad V_{\text{jet,t}} - V_{\text{nozzle}} = 0$$

Noting that the tangential component of jet velocity is $V_{\text{jet,t}} = V_{\text{jet}}\cos\theta$, we have

$$V_{\text{nozzle}} = V_{\text{jet}}\cos\theta = (95.49\text{ m/s})\cos\theta$$

Also noting that $V_{\text{nozzle}} = \omega r = 2\pi\dot{n}r$, and angular speed and the rate of rotation of sprinkler head become

1) θ = 0°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 0}{0.45\text{ m}} = \mathbf{212\text{ rad/s}}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{212\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{2026\text{ rpm}}$

2) θ = 30°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 30°}{0.45\text{ m}} = \mathbf{184\text{ rad/s}}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{184\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{1755\text{ rpm}}$

3) θ = 60°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 60°}{0.45\text{ m}} = \mathbf{106\text{ rad/s}}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{106\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{1013\text{ rpm}}$

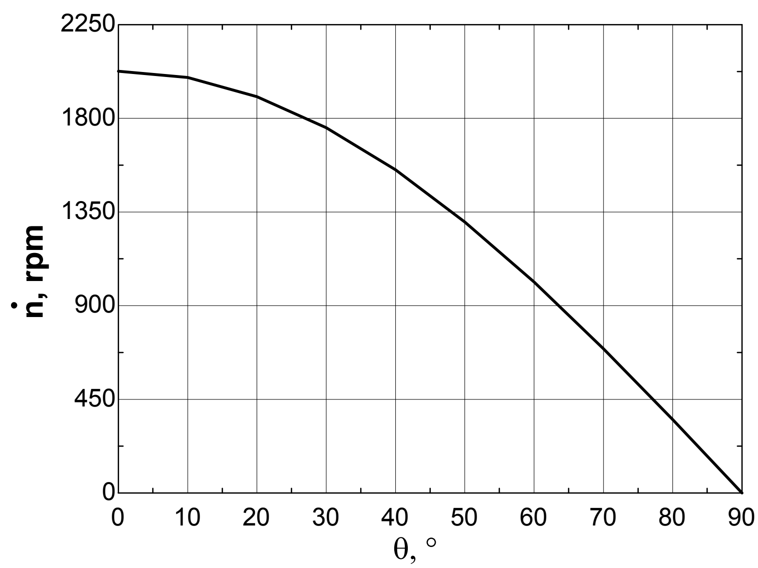
Discussion The rate of rotation in reality will be lower because of frictional effects and air drag.

6-74 Problem 6-73 is reconsidered. The effect of discharge angle θ on the rate of rotation $\dot{n}$ as θ varies from 0 to 90° in increments of 10° is to be investigated.

```
D=0.02 "m"
r=0.45 "m"
n_nozzle=2 "number of nozzles"
Ac=pi*D^2/4
V_jet=V_dot/Ac/n_nozzle

V_nozzle=V_jet*cos(theta)
V_dot=0.060 "m3/s"
omega=V_nozzle/r
n_dot=omega*60/(2*pi)
```

Angle, θ°	V_{nozzle}, m/s	ω rad/s	$\dot{n}$ rpm
0	95.5	212	2026
10	94.0	209	1996
20	89.7	199	1904
30	82.7	184	1755
40	73.2	163	1552
50	61.4	136	1303
60	47.7	106	1013
70	32.7	73	693
80	16.6	37	352
90	0.0	0	0



6-75 A stationary water tank placed on wheels on a frictionless surface is propelled by a water jet that leaves the tank through a smooth hole. Relations are to be developed for the acceleration, the velocity, and the distance traveled by the tank as a function of time as water discharges.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **3** The surface under the wheeled tank is level and frictionless. **4** The water jet is discharged horizontally and rearward. **5** The mass of the tank and wheel assembly is negligible compared to the mass of water in the tank. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the outlet of the hole, which is also taken to be the reference level ($z_2 = 0$) so that the water height above the hole at any time is z. Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$), it is open to the atmosphere ($P_1 = P_{atm}$), and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

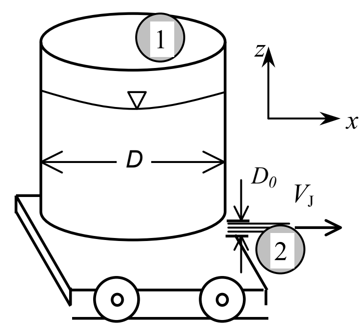
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z=\frac{V_J^2}{2g}+0 \quad\rightarrow\quad V_J=\sqrt{2gz}$$

The discharge rate of water from the tank through the hole is

$$\dot{m}=\rho A V_J=\rho\frac{\pi D_0^2}{4}V_J=\rho\frac{\pi D_0^2}{4}\sqrt{2gz}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{\text{out}}\beta\dot{m}\vec{V}-\sum_{\text{in}}\beta\dot{m}\vec{V}$. Applying it to the water tank, the horizontal force that acts on the tank is determined to be

$$F=\dot{m}V_e-0=\dot{m}V_J=\rho\frac{\pi D_0^2}{4}2gz=\rho g z\frac{\pi D_0^2}{2}$$



The acceleration of the water tank is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of water in the tank, $m=\rho \boldsymbol{V}_{\text{tank}}=\rho(\pi D^2/4)z$,

$$a=\frac{F}{m}=\frac{\rho g z(\pi D_0^2/2)}{\rho z(\pi D^2/4)} \qquad\rightarrow\qquad a=2g\frac{D_0^2}{D^2}$$

Note that the acceleration of the tank is constant.

(*b*) Noting that $a = dV/dt$ and thus $dV = adt$ and acceleration a is constant, the velocity is expressed as

$$V=at \qquad\rightarrow\qquad V=2g\frac{D_0^2}{D^2}t$$

(*c*) Noting that $V = dx/dt$ and thus $dx = Vdt$, the distance traveled by the water tank is determined by integration to be

$$dx=Vdt \qquad\rightarrow\qquad dx=2g\frac{D_0^2}{D^2}tdt \quad\rightarrow\qquad x=g\frac{D_0^2}{D^2}t^2$$

since $x = 0$ at $t = 0$.

Discussion In reality, the flow rate discharge velocity and thus the force acting on the water tank will be less because of the frictional losses at the hole. But these losses can be accounted for by incorporating a discharge coefficient.

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