PROPRIETARY AND CONFIDENTIAL

Chapter 7 Dimensional Analysis

Dimensions and Units, Primary Dimensions

7-1C

Solution We are to explain the difference between a dimension and a unit, and give examples.

Analysis **A dimension is a measure of a physical quantity (without numerical values), while a unit is a way to assign a number to that dimension**. Examples are numerous – length and meter, temperature and ^{o}C, weight and lbf, mass and kg, time and second, power and watt,…

Discussion When performing dimensional analysis, it is important to recognize the difference between dimensions and units.

7-2C

Solution We are to append Table P7-2 with other parameters and their primary dimensions.

Analysis Students' tables will differ, but they should add entries such as angular velocity, kinematic viscosity, work, energy, power, specific heat, thermal conductivity, torque or moment, stress, etc.

Discussion This problem should be assigned as an ongoing homework problem throughout the semester, and then collected towards the end. Individual instructors can determine how many entries to be required in the table.

7-3C

Solution We are to list the seven primary dimensions and explain their significance.

Analysis The seven primary dimensions are **mass, length, time, temperature, electrical current, amount of light, and amount of matter**. Their significance is that **all other dimensions can be formed by combinations of these seven primary dimensions**.

Discussion One of the first steps in a dimensional analysis is to write down the primary dimensions of every variable or parameter that is important in the problem.

7-4

Solution We are to write the primary dimensions of the universal ideal gas constant.

Analysis From the given equation,

Primary dimensions of the universal ideal gas constant:

$$\{R_u\} = \left\{\frac{\text{pressure}\times\text{volume}}{\text{mol}\times\text{temperature}}\right\} = \left\{\frac{\dfrac{\text{m}}{\text{t}^2\text{L}}\times\text{L}^3}{\text{N}\times\text{T}}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t^2TN}}\right\}$$

Or, in exponent form, $\{R_u\}$ = $\{\mathbf{m^1\,L^2\,t^{-2}\,T^{-1}\,N^{-1}}\}$.

Discussion The standard value of R_u is 8314.3 J/kmol·K. You can verify that these units agree with the dimensions of the result.

7-5

Solution We are to write the primary dimensions of atomic weight.

Analysis By definition, atomic weight is mass per mol,

Primary dimensions of atomic weight:
$$\{M\} = \left\{\frac{\text{mass}}{\text{mol}}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{N}}\right\} \tag{1}$$

Or, in exponent form, $\{M\}$ = $\{\mathbf{m^1\,N^{-1}}\}$.

Discussion In terms of primary dimensions, atomic mass is *not* dimensionless, although many authors treat it as such. Note that mass and amount of matter are defined as two separate primary dimensions.

7-6

Solution We are to write the primary dimensions of the universal ideal gas constant in the alternate system where force replaces mass as a primary dimension.

Analysis From Newton's second law, force equals mass times acceleration. Thus, mass is written in terms of force as

Primary dimensions of mass in the alternate system:

$$\{\text{mass}\} = \left\{\frac{\text{force}}{\text{acceleration}}\right\} = \left\{\frac{\text{F}}{\text{L/t}^2}\right\} = \left\{\frac{\text{Ft}^2}{\text{L}}\right\} \qquad (1)$$

We substitute Eq. 1 into the results of Problem 7-4,

Primary dimensions of the universal ideal gas constant:

$$\{R_u\} = \left\{\frac{\text{mL}^2}{\text{Nt}^2\text{T}}\right\} = \left\{\frac{\dfrac{\text{Ft}^2}{\text{L}}\text{L}^2}{\text{Nt}^2\text{T}}\right\} = \left\{\frac{\mathbf{FL}}{\mathbf{TN}}\right\} \qquad (2)$$

Or, in exponent form, $\{R_u\} = \{\mathbf{F^1\,L^1\,T^{-1}\,N^{-1}}\}$.

Discussion The standard value of R_u is 8314.3 J/kmol·K. You can verify that these units agree with the dimensions of Eq. 2.

7-7

Solution We are to write the primary dimensions of the specific ideal gas constant, and verify are result by comparing to the standard SI units of R_{air}.

Analysis We can approach this problem two ways. If we have already worked through Problem 7-4, we can use our results. Namely,

Primary dimensions of specific ideal gas constant:

$$\{R_{gas}\} = \left\{\frac{R_u}{M}\right\} = \left\{\frac{\frac{mL^2}{Nt^2T}}{\frac{m}{N}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \qquad (1)$$

Or, in exponent form, $\{R_{gas}\}$ = $\{\mathbf{L^2\,t^{-2}\,T^{-1}}\}$. Alternatively, we can use either form of the ideal gas law,

Primary dimensions of specific ideal gas constant:

$$\{R_{gas}\} = \left\{\frac{\text{pressure} \times \text{volume}}{\text{mass} \times \text{temperature}}\right\} = \left\{\frac{\frac{m}{t^2L} \times L^3}{m \times T}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \qquad (2)$$

For air, R_{air} = 287.0 J/kg·K. We transform these units into primary dimensions,

Primary dimensions of the specific ideal gas constant for air:

$$\{R_{air}\} = \left\{287.0\frac{J}{kg \times K}\right\} = \left\{\frac{\frac{mL^2}{t^2}}{m \times T}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \qquad (3)$$

Equation 3 agrees with Eq. 1 and Eq. 2, increasing our confidence that we have performed the algebra correctly.

Discussion Notice that numbers, like the value 287.0 in Eq. 3 have no influence on the dimensions.

7-8

Solution We are to write the primary dimensions of torque and list its units.

Analysis Torque is the product of a length and a force,

Primary dimensions of torque:

$$\{\vec{M}\} = \left\{\text{length} \times \text{mass}\frac{\text{length}}{\text{time}^2}\right\} = \left\{\mathbf{m}\frac{\mathbf{L}^2}{\mathbf{t}^2}\right\} \qquad (1)$$

Or, in exponent form, $\{\vec{M}\} = \{\mathbf{m}^1\,\mathbf{L}^2\,\mathbf{t}^{-2}\}$. The common units of torque are newton-meter (SI) and inch-pound (English). In *primary units*, however, we write the primary SI units according to Eq. 1,

Primary SI units: Units of torque = $\mathbf{kg \cdot m^2/s^2}$

and in primary English units,

Primary English units: Units of torque = $\mathbf{lbm \cdot ft^2/s^2}$

Discussion Since torque is the product of a force and a length, it has the same dimensions as energy. Primary *units* are not required for dimensional analysis, but are often useful for unit conversions and for verification of proper units when solving a problem.

7-9

Solution We are to determine the primary dimensions of each variable.

Analysis
(*a*) Energy is force times length (the same dimensions as work),

Primary dimensions of energy:

$$\{E\} = \{\text{force} \times \text{length}\} = \left\{\frac{\text{mass} \times \text{length}}{\text{time}^2} \times \text{length}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t^2}}\right\} \quad (1)$$

Or, in exponent form, $\{E\}$ = $\{\mathbf{m^1\,L^2\,t^{-2}}\}$.

(*b*) Specific energy is energy per unit mass,

Primary dimensions of specific energy:

$$\{e\} = \left\{\frac{\text{energy}}{\text{mass}}\right\} = \left\{\frac{\text{mass} \times \text{length}^2}{\text{time}^2} \times \frac{1}{\text{mass}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2}}\right\} \quad (2)$$

Or, in exponent form, $\{e\}$ = $\{\mathbf{L^2\,t^{-2}}\}$.

(*c*) Power is the rate of change of energy, i.e. energy per unit time,

Primary dimensions of power:

$$\{\dot{W}\} = \left\{\frac{\text{energy}}{\text{time}}\right\} = \left\{\frac{\text{mass} \times \text{length}^2}{\text{time}^2} \times \frac{1}{\text{time}}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t^3}}\right\} \quad (3)$$

Or, in exponent form, $\{\dot{W}\}$ = $\{\mathbf{m^1\,L^2\,t^{-3}}\}$.

Discussion In dimensional analysis it is important to distinguish between energy, specific energy, and power.

7-10

Solution We are to determine the primary dimensions of electrical voltage.

Analysis From the hint,

Primary dimensions of voltage:

$$\{\text{voltage}\} = \left\{\frac{\text{power}}{\text{current}}\right\} = \left\{\frac{\dfrac{\text{mass} \times \text{length}^2}{\text{time}^3}}{\text{current}}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t^3 I}}\right\} \quad (1)$$

Or, in exponent form, $\{E\}$ = $\{\mathbf{m^1\,L^2\,t^{-3}\,I^{-1}}\}$.

Discussion We see that all dimensions, even those of electrical properties, can be expressed in terms of primary dimensions.

7-11

Solution We are to write the primary dimensions of electrical resistance.

Analysis From Ohm's law, we see that resistance has the dimensions of voltage difference divided by electrical current,

Primary dimensions of resistance:

$$\{R\} = \left\{\frac{\Delta E}{I}\right\} = \left\{\frac{\dfrac{\text{mass}\times\text{length}^2}{\text{time}^3\times\text{current}}}{\text{current}}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t^3I^2}}\right\}$$

Or, in exponent form, $\{R\}$ = $\{\mathbf{m^1\ L^2\ t^{-3}\ I^{-2}}\}$, where we have also used the result of Problem 7-10.

Discussion *All* dimensions can be written in terms of primary dimensions.

7-12

Solution We are to determine the primary dimensions of each variable.

Analysis

(*a*) Acceleration is the rate of change of velocity,

Primary dimensions of acceleration:

$$\{a\} = \left\{\frac{\text{velocity}}{\text{time}}\right\} = \left\{\frac{\text{length}}{\text{time}}\times\frac{1}{\text{time}}\right\} = \left\{\frac{\mathbf{L}}{\mathbf{t^2}}\right\} \tag{1}$$

Or, in exponent form, $\{a\}$ = $\{\mathbf{L^1\ t^{-2}}\}$.

(*b*) Angular velocity is the rate of change of angle,

Primary dimensions of angular velocity:

$$\{\omega\} = \left\{\frac{\text{angle}}{\text{time}}\right\} = \left\{\frac{1}{\text{time}}\right\} = \left\{\frac{\mathbf{1}}{\mathbf{t}}\right\} \tag{2}$$

Or, in exponent form, $\{\omega\}$ = $\{\mathbf{t^{-1}}\}$.

(*c*) Angular acceleration is the rate of change of angular velocity,

Primary dimensions of angular acceleration:

$$\{\alpha = \dot{\omega}\} = \left\{\frac{\text{angular velocity}}{\text{time}}\right\} = \left\{\frac{1}{\text{time}}\times\frac{1}{\text{time}}\right\} = \left\{\frac{\mathbf{1}}{\mathbf{t^2}}\right\} \tag{3}$$

Or, in exponent form, $\{\alpha\}$ = $\{\mathbf{t^{-2}}\}$.

Discussion In Part (*b*) we note that the unit of angle is radian, which is a dimensionless unit. Therefore the dimensions of angle are unity.

7-13

Solution We are to write the primary dimensions of angular momentum and list its units.

Analysis Angular momentum is the product of length, mass, and velocity, *Primary dimensions of angular momentum*:

$$\left\{\vec{H}\right\} = \left\{\text{length} \times \text{mass} \times \frac{\text{length}}{\text{time}}\right\} = \left\{\frac{\mathbf{mL^2}}{\mathbf{t}}\right\} \qquad (1)$$

Or, in exponent form, { $\vec{H}$ } = $\{\mathbf{m^1\,L^2\,t^{-1}}\}$. We write the primary SI units according to Eq. 1,

Primary SI units: $$\text{Units of angular momentum} = \frac{\mathbf{kg \cdot m^2}}{\mathbf{s}}$$

and in primary English units,

Primary English units: $$\text{Units of angular momentum} = \frac{\mathbf{lbm \cdot ft^2}}{\mathbf{s}}$$

Discussion Primary units are not required for dimensional analysis, but are often useful for unit conversions and for verification of proper units when solving a problem.

7-14

Solution We are to determine the primary dimensions of each variable.

Analysis

(*a*) Specific heat is energy per unit mass per unit temperature,

Primary dimensions of specific heat at constant pressure:

$$\{c_p\} = \left\{\frac{\text{energy}}{\text{mass}\times\text{temperature}}\right\} = \left\{\frac{\dfrac{\text{mL}^2}{\text{t}^2}}{\text{m}\times\text{T}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \tag{1}$$

Or, in exponent form, $\{c_p\}$ = $\{\mathbf{L^2\,t^{-2}\,T^{-1}}\}$.

(*b*) Specific weight is density times gravitational acceleration,

Primary dimensions of specific weight: $\{\rho g\} = \left\{\frac{\text{mass}}{\text{volume}}\frac{\text{length}}{\text{time}^2}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{L^2t^2}}\right\}$ (2)

Or, in exponent form, $\{\rho g\}$ = $\{\mathbf{m^1\,L^{-2}\,t^{-2}}\}$.

(*c*) Specific enthalpy has dimensions of energy per unit mass,

Primary dimensions of specific enthalpy: $\{h\} = \left\{\frac{\text{energy}}{\text{mass}}\right\} = \left\{\frac{\dfrac{\text{mL}^2}{\text{t}^2}}{\text{m}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2}}\right\}$ (3)

Or, in exponent form, $\{h\}$ = $\{\mathbf{L^2\,t^{-2}}\}$.

Discussion As a check, from our study of thermodynamics we know that $dh = c_p dT$ for an ideal gas. Thus, the dimensions of dh must equal the dimensions of c_p times the dimensions of dT. Comparing Eqs. 1 and 3 above, we see that this is indeed the case.

7-15

Solution We are to determine the primary dimensions of thermal conductivity.

Analysis The primary dimensions of $\dot{Q}_{\text{conduction}}$ are energy/time, and the primary dimensions of *dT*/*dx* are temperature/length. From the given equation,

Primary dimensions of thermal conductivity:

$$\{k\} = \left\{ \frac{\dfrac{\text{energy}}{\text{time}}}{\text{length}^2 \times \dfrac{\text{temperature}}{\text{length}}} \right\} = \left\{ \frac{\dfrac{\text{mL}^2}{\text{t}^3}}{\text{L} \times \text{T}} \right\} = \left\{ \frac{\mathbf{mL}}{\mathbf{t^3 T}} \right\} \tag{1}$$

Or, in exponent form, $\{k\}$ = $\{\mathbf{m^1\ L^1\ t^{-3}\ T^{-1}}\}$. We obtain a value of *k* from a reference book. E.g. k_{copper} = 401 W/m·K. These units have dimensions of power/length·temperature. Since power is energy/time, we see immediately that Eq. 1 is correct. Alternatively, we can transform the units of *k* into primary units,

Primary SI units of thermal conductivity:

$$k_{\text{copper}} = 401 \frac{\text{W}}{\text{m K}} \left(\frac{\text{N m}}{\text{s W}} \right) \left(\frac{\text{kg m}}{\text{N s}^2} \right) = 401 \frac{\mathbf{kg \cdot m}}{\mathbf{s^3 \cdot K}} \tag{2}$$

Discussion We have used the principle of dimensional homogeneity to determine the primary dimensions of *k*. Namely, we utilized the fact that the dimensions of both terms of the given equation must be identical.

7-16

Solution We are to determine the primary dimensions of each variable.

Analysis

(*a*) Heat generation rate is energy per unit volume per unit time,

Primary dimensions of heat generation rate:

$$\{\dot{g}\} = \left\{\frac{\text{energy}}{\text{volume} \times \text{time}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{L}^3\text{t}}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{Lt^3}}\right\} \tag{1}$$

Or, in exponent form, $\{\dot{g}\} = \mathbf{\{m^1\,L^{-1}\,t^{-3}\}}$.

(*b*) Heat flux is energy per unit area per unit time,

Primary dimensions of heat flux:

$$\{\dot{q}\} = \left\{\frac{\text{energy}}{\text{area} \times \text{time}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{L}^2\text{t}}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{t^3}}\right\} \tag{2}$$

Or, in exponent form, $\{\dot{q}\} = \mathbf{\{m^1\,t^{-3}\}}$.

(*c*) Heat flux is energy per unit area per unit time per unit temperature,

Primary dimensions of heat transfer coefficient:

$$\{h\} = \left\{\frac{\text{energy}}{\text{area} \times \text{time} \times \text{temperature}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{L}^2 \times \text{t} \times \text{T}}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{t^3T}}\right\} \tag{3}$$

Or, in exponent form, $\{h\} = \mathbf{\{m^1\,t^{-3}\,T^{-1}\}}$.

Discussion In the field of heat transfer it is critical that one be careful with the dimensions (and units) of heat transfer variables.

7-17

Solution We are to choose three properties or constants and write out their names, their SI units, and their primary dimensions.

Analysis There are many options. For example,

Students may choose c_v (specific heat at constant volume). The units are kJ/kg·K, which is energy per mass per temperature. Thus,

Primary dimensions of specific heat at constant volume:

$$\{c_v\} = \left\{\frac{\text{energy}}{\text{mass}\times\text{temperature}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m}\times\text{T}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \tag{1}$$

Or, in exponent form, $\{c_v\}$ = $\{\mathbf{L^2\, t^{-2}\, T^{-1}}\}$.

Students may choose v (specific volume). The units are m^3/kg, which is volume per mass. Thus,

Primary dimensions of specific volume: $$\{v\} = \left\{\frac{\text{volume}}{\text{mass}}\right\} = \left\{\frac{\mathbf{L^3}}{\mathbf{m}}\right\} \tag{2}$$

Or, in exponent form, $\{v\}$ = $\{\mathbf{m^{-1}\, L^3}\}$.

Students may choose h_{fg} (latent heat of vaporization). The units are kJ/kg, which is energy per mass. Thus,

Primary dimensions of latent heat of vaporization:

$$\{h_{fg}\} = \left\{\frac{\text{energy}}{\text{mass}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2}}\right\} \tag{3}$$

Or, in exponent form, $\{h_{fg}\}$ = $\{\mathbf{L^2\, t^{-2}}\}$. (The same dimensions hold for h_f and h_g.)

Students may choose s_f (specific entropy of a saturated liquid). The units are kJ/kg·K, which is energy per mass per temperature. Thus,

Primary dimensions of specific entropy of a saturated liquid:

$$\{s_f\} = \left\{\frac{\text{energy}}{\text{mass}\times\text{temperature}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m}\times\text{T}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2T}}\right\} \tag{4}$$

Or, in exponent form, $\{s_f\}$ = $\{\mathbf{L^2\, t^{-2}\, T^{-1}}\}$. (The same dimensions hold for s_{fg} and s_g.)

Discussion Students' answers will vary. There are some other choices.

7-18E

Solution We are to choose three properties or constants and write out their names, their SI units, and their primary dimensions.

Analysis There are many options. For example,

Students may choose c_v (specific heat at constant volume). The units are Btu/lbm·R, which is energy per mass per temperature. Thus,

Primary dimensions of specific heat at constant volume:

$$\{c_v\} = \left\{\frac{\text{energy}}{\text{mass} \times \text{temperature}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m} \times \text{T}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2 T}}\right\} \tag{1}$$

Or, in exponent form, $\{c_v\}$ = $\{\mathbf{L^2\,t^{-2}\,T^{-1}}\}$.

Students may choose v (specific volume). The units are ft^3/lbm, which is volume per mass. Thus,

Primary dimensions of specific volume: $$\{v\} = \left\{\frac{\text{volume}}{\text{mass}}\right\} = \left\{\frac{\mathbf{L^3}}{\mathbf{m}}\right\} \tag{2}$$

Or, in exponent form, $\{v\}$ = $\{\mathbf{m^{-1}\,L^3}\}$.

Students may choose h_{fg} (latent heat of vaporization). The units are Btu/lbm, which is energy per mass. Thus,

Primary dimensions of latent heat of vaporization:

$$\{h_{fg}\} = \left\{\frac{\text{energy}}{\text{mass}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2}}\right\} \tag{3}$$

Or, in exponent form, $\{h_{fg}\}$ = $\{\mathbf{L^2\,t^{-2}}\}$. (The same dimensions hold for h_f and h_g.)

Students may choose s_f (specific entropy of a saturated liquid). The units are Btu/lbm·R, which is energy per mass per temperature. Thus,

Primary dimensions of specific entropy of a saturated liquid:

$$\{s_f\} = \left\{\frac{\text{energy}}{\text{mass} \times \text{temperature}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m} \times \text{T}}\right\} = \left\{\frac{\mathbf{L^2}}{\mathbf{t^2 T}}\right\} \tag{4}$$

Or, in exponent form, $\{s_f\}$ = $\{\mathbf{L^2\,t^{-2}\,T^{-1}}\}$. (The same dimensions hold for s_{fg} and s_g.)

Discussion Students' answers will vary. There are some other choices.

Dimensional Homogeneity

7-19C

Solution We are to explain the law of dimensional homogeneity.

Analysis The law of dimensional homogeneity states that **every additive term in an equation must have the same dimensions**. As a simple counter example, an equation with one term of dimensions length and another term of dimensions temperature would clearly violate the law of dimensional homogeneity – you cannot add length and temperature. All terms in the equation must have the *same* dimensions.

Discussion If in the solution of an equation you realize that the dimensions of two terms are not equivalent, this is a sure sign that you have made a mistake somewhere!

7-20

Solution We are to determine the primary dimensions of the gradient operator, and then verify that primary dimensions of each additive term in the equation are the same.

Analysis

(*a*) By definition, the gradient operator is a three-dimensional derivative operator. For example, in Cartesian coordinates,

Gradient operator in Cartesian coordinates:

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

Therefore its dimensions must be 1/length. Thus,

Primary dimensions of the gradient operator:

$$\left\{ \vec{\nabla} \right\} = \left\{ \frac{\mathbf{1}}{\mathbf{L}} \right\}$$

Or, in exponent form, $\{ \vec{\nabla} \} = \mathbf{\{ L^{-1} \}}$.

(*b*) Similarly, the primary dimensions of a time derivative ($\partial/\partial t$) are 1/time. Also, the primary dimensions of velocity are length/time, and the primary dimensions of acceleration are length/time^2. Thus each term in the given equation can be written in terms of primary dimensions,

$$\{ \vec{a} \} = \left\{ \frac{\text{length}}{\text{time}^2} \right\} \qquad \{ \vec{a} \} = \left\{ \frac{\text{L}}{\text{t}^2} \right\}$$

$$\left\{ \frac{\partial \vec{V}}{\partial t} \right\} = \left\{ \frac{\frac{\text{length}}{\text{time}}}{\text{time}} \right\} = \left\{ \frac{\text{length}}{\text{time}^2} \right\} \qquad \left\{ \frac{\partial \vec{V}}{\partial t} \right\} = \left\{ \frac{\text{L}}{\text{t}^2} \right\}$$

$$\left\{ \left(\vec{V} \cdot \vec{\nabla} \right) \vec{V} \right\} = \left\{ \frac{\text{length}}{\text{time}} \times \frac{1}{\text{length}} \times \frac{\text{length}}{\text{time}} \right\} = \left\{ \frac{\text{length}}{\text{time}^2} \right\} \qquad \left\{ \left(\vec{V} \cdot \vec{\nabla} \right) \vec{V} \right\} = \left\{ \frac{\text{L}}{\text{t}^2} \right\}$$

Indeed, **all three additive terms have the same dimensions, namely $\mathbf{\{L^1\,t^{-2}\}}$**.

Discussion If the dimensions of any of the terms were different from the others, it would be a sure sign that an error was made somewhere in deriving or copying the equation.

7-21

Solution We are to determine the primary dimensions of each additive term in the equation, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of the time derivative (∂/∂*t*) are 1/time. The primary dimensions of the gradient vector are 1/length, and the primary dimensions of velocity are length/time. Thus each term in the equation can be written in terms of primary dimensions,

$$\left\{\frac{\vec{F}}{m}\right\} = \left\{\frac{\text{force}}{\text{mass}}\right\} = \left\{\frac{\dfrac{\text{mass}\times\text{length}}{\text{time}^2}}{\text{mass}}\right\} \qquad \left\{\frac{\vec{F}}{m}\right\} = \left\{\frac{\text{L}}{\text{t}^2}\right\}$$

$$\left\{\frac{\partial\vec{V}}{\partial t}\right\} = \left\{\frac{\dfrac{\text{length}}{\text{time}}}{\text{time}}\right\} \qquad \left\{\frac{\partial\vec{V}}{\partial t}\right\} = \left\{\frac{\text{L}}{\text{t}^2}\right\}$$

$$\left\{\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}\right\} = \left\{\frac{\text{length}}{\text{time}}\times\frac{1}{\text{length}}\times\frac{\text{length}}{\text{time}}\right\} \qquad \left\{\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}\right\} = \left\{\frac{\text{L}}{\text{t}^2}\right\}$$

Indeed, **all three additive terms have the same dimensions, namely $\{\mathbf{L}^1\,\mathbf{t}^{-2}\}$**.

Discussion The dimensions are, in fact, those of acceleration.

7-22

Solution We are to determine the primary dimensions of each additive term in Eq. 1, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of the material derivative (*D*/*Dt*) are 1/time. The primary dimensions of volume are length^3, and the primary dimensions of velocity are length/time. Thus each term in the equation can be written in terms of primary dimensions,

$$\left\{\frac{1}{\mathcal{V}}\frac{D\mathcal{V}}{Dt}\right\} = \left\{\frac{1}{\text{length}^3}\times\frac{\text{length}^3}{\text{time}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{1}{\mathcal{V}}\frac{D\mathcal{V}}{Dt}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{\partial u}{\partial x}\right\} = \left\{\frac{\frac{\text{length}}{\text{time}}}{\text{length}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{\partial u}{\partial x}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{\partial v}{\partial y}\right\} = \left\{\frac{\frac{\text{length}}{\text{time}}}{\text{length}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{\partial v}{\partial y}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{\partial w}{\partial z}\right\} = \left\{\frac{\frac{\text{length}}{\text{time}}}{\text{length}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{\partial w}{\partial z}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

Indeed, **all four additive terms have the same dimensions, namely {t^{-1}}**.

Discussion If the dimensions of any of the terms were different from the others, it would be a sure sign that an error was made somewhere in deriving or copying the equation.

7-23

Solution We are to determine the primary dimensions of each additive term, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of the velocity components are length/time. The primary dimensions of coordinates r and z are length, and the primary dimensions of coordinate θ are unity (it is a dimensionless angle). Thus each term in the equation can be written in terms of primary dimensions,

$$\left\{\frac{1}{r}\frac{\partial\left(ru_r\right)}{\partial r}\right\} = \left\{\frac{1}{\text{length}}\times\frac{\text{length}\,\dfrac{\text{length}}{\text{time}}}{\text{length}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{1}{r}\frac{\partial\left(ru_r\right)}{\partial r}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right\} == \left\{\frac{1}{\text{length}}\times\frac{\dfrac{\text{length}}{\text{time}}}{1}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{\partial u_z}{\partial z}\right\} = \left\{\frac{\dfrac{\text{length}}{\text{time}}}{\text{length}}\right\} = \left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{\partial u_z}{\partial z}\right\} = \left\{\frac{1}{\text{t}}\right\}$$

Indeed, **all three additive terms have the same dimensions, namely $\{t^{-1}\}$**.

Discussion If the dimensions of any of the terms were different from the others, it would be a sure sign that an error was made somewhere in deriving or copying the equation.

7-24

Solution We are to determine the primary dimensions of each additive term in the equation, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of heat transfer rate are energy/time. The primary dimensions of mass flow rate are mass/time, and those of specific heat are energy/mass·temperature, as found in Problem 7-14. Thus each term in the equation can be written in terms of primary dimensions,

$$\{\dot{Q}\} = \left\{\frac{\text{energy}}{\text{time}}\right\} = \left\{\frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{t}}\right\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\} \qquad \{\dot{Q}\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\}$$

$$\{\dot{m}C_pT_{\text{out}}\} = \left\{\frac{\text{mass}}{\text{time}} \times \frac{\text{energy}}{\text{mass} \times \text{temperature}} \times \text{temperature}\right\}$$
$$= \left\{\frac{\text{m}}{\text{t}} \times \frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m} \times \text{T}} \times \text{T}\right\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\} \qquad \{\dot{m}C_pT_{\text{out}}\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\}$$

$$\{\dot{m}C_pT_{\text{in}}\} = \left\{\frac{\text{mass}}{\text{time}} \times \frac{\text{energy}}{\text{mass} \times \text{temperature}} \times \text{temperature}\right\}$$
$$= \left\{\frac{\text{m}}{\text{t}} \times \frac{\frac{\text{mL}^2}{\text{t}^2}}{\text{m} \times \text{T}} \times \text{T}\right\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\} \qquad \{\dot{m}C_pT_{\text{in}}\} = \left\{\frac{\text{mL}^2}{\text{t}^3}\right\}$$

Indeed, **all three additive terms have the same dimensions, namely $\{\mathbf{m^1\,L^2\,t^{-3}}\}$**.

Discussion We could also have left the temperature difference in parentheses as a temperature difference (same dimensions as the individual temperatures), and treated the equation as having only two terms.

7-25

Solution We are to determine the primary dimensions of each additive term, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of the time derivative (d/dt) are 1/time. The primary dimensions of density are mass/length^3, those of volume are length^3, those of area are length^2, and those of velocity are length/time. The primary dimensions of unit vector $\vec{n}$ are unity, i.e. $\{1\}$ (in other words $\vec{n}$ has no dimensions). Finally, the primary dimensions of b, which is defined as B per unit mass, are $\{B/\text{m}\}$. Thus each term in the equation can be written in terms of primary dimensions,

$$\left\{\frac{dB_{\text{sys}}}{dt}\right\} = \left\{\frac{B}{\text{time}}\right\} \qquad \left\{\frac{dB_{\text{sys}}}{dt}\right\} = \left\{\frac{B}{\text{t}}\right\}$$

$$\left\{\frac{d}{dt}\int_{\text{CV}} \rho b d\mathcal{V}\right\} = \left\{\frac{1}{\text{time}} \times \frac{\text{mass}}{\text{length}^3} \times \frac{B}{\text{mass}} \times \text{length}^3\right\} \qquad \left\{\frac{d}{dt}\int_{\text{CV}} \rho b d\mathcal{V}\right\} = \left\{\frac{B}{\text{t}}\right\}$$

$$\left\{\int_{\text{CS}} \rho b \vec{V}_{\text{r}} \cdot \vec{n} dA\right\} = \left\{\frac{\text{mass}}{\text{length}^3} \times \frac{B}{\text{mass}} \times \frac{\text{length}}{\text{time}} \times 1 \times \text{length}^2\right\} \qquad \left\{\int_{\text{CS}} \rho b \vec{V}_{\text{r}} \cdot \vec{n} dA\right\} = \left\{\frac{B}{\text{t}}\right\}$$

Indeed, **all three additive terms have the same dimensions, namely $\{B\ \text{t}^{-1}\}$**.

Discussion The RTT for property B has dimensions of rate of change of B.

7-26

Solution We are to determine the primary dimensions of the first three additive term, and we are to verify that those terms are dimensionally homogeneous. Then we are to evaluate the dimensions of the adsorption coefficient.

Analysis The primary dimensions of the time derivative (d/dt) are 1/time. Those of A_s are length^2, those of $\mathcal{V}$ are length^3, those of c are mass/length^3, and those of $\dot{\mathcal{V}}$ are length^3/time. Thus the primary dimensions of the first three terms are

$$\left\{\mathcal{V}\frac{dc}{dt}\right\} = \left\{\text{length}^3 \frac{\dfrac{\text{mass}}{\text{length}^3}}{\text{time}}\right\} = \left\{\frac{\text{mass}}{\text{time}}\right\} \qquad \left\{\mathcal{V}\frac{dc}{dt}\right\} = \left\{\frac{\text{m}}{\text{t}}\right\}$$

$$\{S\} = \left\{\frac{\text{mass}}{\text{time}}\right\} \qquad \{S\} = \left\{\frac{\text{m}}{\text{t}}\right\}$$

$$\{\dot{\mathcal{V}}c\} = \left\{\frac{\text{length}^3}{\text{time}} \times \frac{\text{mass}}{\text{length}^3}\right\} = \left\{\frac{\text{mass}}{\text{time}}\right\} \qquad \{\dot{\mathcal{V}}c\} = \left\{\frac{\text{m}}{\text{t}}\right\}$$

Indeed, the first three additive terms have the same dimensions, namely $\{\mathbf{m}^1\ \mathbf{t}^{-1}\}$. Since the equation must be dimensionally homogeneous, the last term must have the same dimensions as well. We use this fact to find the dimensions of k_w,

$$\{cA_s k_w\} = \left\{\frac{\text{mass}}{\text{time}}\right\} \qquad \{k_w\} = \left\{\frac{\dfrac{\text{mass}}{\text{time}}}{cA_s}\right\} = \left\{\frac{\dfrac{\text{mass}}{\text{time}}}{\dfrac{\text{mass}}{\text{length}^3}\times\text{length}^2}\right\} \qquad \{k_w\} = \left\{\frac{\mathbf{L}}{\mathbf{t}}\right\}$$

Or, in exponent form, $\{k_w\}$ = $\{\mathbf{L}^1\ \mathbf{t}^{-1}\}$. The dimensions of wall adsorption coefficient are those of velocity.

Discussion In fact, some authors call k_w a “deposition velocity”.

Nondimensionalization of Equations

7-27C

Solution We are to give the primary reason for nondimensionalizing an equation.

Analysis The primary reason for nondimensionalizing an equation is **to reduce the number of parameters in the problem**.

Discussion As shown in the examples in the text, nondimensionalization of an equation reduces the number of independent parameters in the problem, simplifying the analysis.

7-28

Solution We are to nondimensionalize all the variables, and then re-write the equation in nondimensionalized form.

Assumptions **1** The air in the room is well mixed so that c is only a function of time.

Analysis
(a) We nondimensionalize the variables by inspection according to their dimensions,

Nondimensionalized variables:

$$\mathcal{V}^* = \frac{\mathcal{V}}{L^3},\; c^* = \frac{c}{c_{\text{limit}}},\; t^* = t\frac{\dot{\mathcal{V}}}{L^3},\; A_s^* = \frac{A_s}{L^2},\; k_w^* = k_w\frac{L^2}{\dot{\mathcal{V}}},\; \text{and}\; S^* = \frac{S}{c_{\text{limit}}\dot{\mathcal{V}}}$$

(b) We substitute these into the equation to generate the nondimensionalized equation,

$$\mathcal{V}^*L^3\frac{d\left(c^*c_{\text{limit}}\right)}{d\left(t^*\dfrac{L^3}{\dot{\mathcal{V}}}\right)} = S^*c_{\text{limit}}\dot{\mathcal{V}} - \dot{\mathcal{V}}c^*c_{\text{limit}} - \left(c^*c_{\text{limit}}\right)\left(A_s^*L^2\right)\left(k_w^*\frac{\dot{\mathcal{V}}}{L^2}\right) \tag{1}$$

We notice that every term in Eq. 1 contains the quantity $\dot{\mathcal{V}}c_{\text{limit}}$. We divide every term by this quantity to get a nondimensionalized form of the equation,

Nondimensionalized equation:
$$\boxed{\mathcal{V}^*\frac{dc^*}{dt^*} = S^* - c^* - c^*A_s^*k_w^*}$$

No dimensionless groups have arisen in this nondimensionalization.

Discussion Since all the characteristic scales disappear, no dimensionless groups have arisen. Since there are no dimensionless parameters, one solution in nondimensionalized variables is valid for all combinations of L, $\dot{\mathcal{V}}$, and c_{limit}.

7-29

Solution We are to nondimensionalize the equation, and identify the dimensionless parameters that appear in the nondimensionalized equation.

Assumptions **1** The flow is steady. **2** The flow is incompressible.

Analysis We plug the nondimensionalized variables into the equation. For example, $u = u^*U$ and $x = x^*L$ in the first term. The result is

$$\frac{U}{L}\frac{\partial u^*}{\partial x^*}+\frac{U}{L}\frac{\partial v^*}{\partial y^*}+\frac{U}{L}\frac{\partial w^*}{\partial z^*}=0$$

or, after simplifying,

Nondimensionalized incompressible flow relationship:

$$\boxed{\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0} \qquad (1)$$

There are no nondimensional parameters in the nondimensionalized equation. The original equation comes from pure kinematics – there are no fluid properties involved in the equation, and therefore it is not surprising that no nondimensional parameters appear in the nondimensionalized form of the equation, Eq. 1.

Discussion We show in Chap. 9 that the equation given in this problem is the differential equation for conservation of mass for an incompressible flow field – the *incompressible continuity equation*.

7-30

Solution We are to nondimensionalize the equation of motion and identify the dimensionless parameters that appear in the nondimensionalized equation.

Analysis We plug in the nondimensionalized variables. For example, $u = u^*V$ and $x = x^*L$ in the first term. The result is

$$\frac{L^3 f}{L^3}\frac{1}{\mathcal{V}^*}\frac{D\mathcal{V}^*}{Dt^*} = \frac{V}{L}\frac{\partial u^*}{\partial x^*} + \frac{V}{L}\frac{\partial v^*}{\partial y^*} + \frac{V}{L}\frac{\partial w^*}{\partial z^*}$$

or, after simplifying,

$$\left(\frac{fL}{V}\right)\frac{1}{\mathcal{V}^*}\frac{D\mathcal{V}^*}{Dt^*} = \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} \qquad (1)$$

We recognize the nondimensional parameter in parentheses in Eq. 1 as St, the **Strouhal number**. We can re-write Eq. 1 as

Nondimensionalized oscillating compressible flow relationship:

$$\boxed{\mathrm{St}\frac{1}{\mathcal{V}^*}\frac{D\mathcal{V}^*}{Dt^*} = \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}}$$

Discussion We show in Chap. 9 that the given equation of motion is the differential equation for conservation of mass for an unsteady, compressible flow field – the *general continuity equation*. We may also use angular frequency ω (radians per second) in place of physical frequency f (cycles per second), with the same result.

7-31

Solution We are to determine the primary dimensions of the stream function, nondimensionalize the variables, and then re-write the definition of ψ in nondimensionalized form.

Assumptions **1** The flow is incompressible. **2** The flow is two-dimensional in the *x*-*y* plane.

Analysis (*a*) We use that fact that all equations must be dimensionally homogeneous. We solve for the dimensions of ψ,

Primary dimensions of stream function: $\{\psi\} = \{u\}\times\{y\} = \left\{\frac{\mathrm{L}}{\mathrm{t}}\times\mathrm{L}\right\} = \left\{\frac{\mathbf{L}^2}{\mathbf{t}}\right\}$

Or, in exponent form, $\{\psi\}$ = $\{\mathbf{L^2\,t^{-1}}\}$.

(*b*) We nondimensionalize the variables by inspection according to their dimensions,

Nondimensionalized variables:

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad u^* = u\frac{t}{L} \qquad v^* = v\frac{t}{L} \qquad \psi^* = \psi\frac{t}{L^2}$$

(*c*) We generate the nondimensionalized equations,

$$u^*\left(\frac{L}{t}\right) = \frac{\partial\psi^*\left(\frac{L^2}{t}\right)}{\partial y^*(L)} \qquad v^*\left(\frac{L}{t}\right) = -\frac{\partial\psi^*\left(\frac{L^2}{t}\right)}{\partial x^*(L)}$$

We notice that every term in both parts of the above equation contains the ratio L/t. We divide every term by L/t to get the final nondimensionalized form of the equations,

Nondimensionalized stream function equations: $u^* = \frac{\partial\psi^*}{\partial y^*} \qquad v^* = -\frac{\partial\psi^*}{\partial x^*}$

No dimensionless groups have arisen in this nondimensionalization.

Discussion Since all the nondimensionalized variables scale with L and t, no dimensionless groups have arisen.

7-32

Solution We are to nondimensionalize the equation of motion and identify the dimensionless parameters that appear in the nondimensionalized equation.

Analysis We plug the nondimensionalized variables into the equation. For example, $t = t^*/\omega$ and $\vec{V} = V_\infty \vec{V}^*$ in the first term on the right hand side. The result is

$$\omega^2 L\left(\vec{F}/m\right)^* = \omega U \frac{\partial \vec{V}^*}{\partial t^*} + \frac{U^2}{L}\left(\vec{V}^* \cdot \vec{\nabla}^*\right)\vec{V}^*$$

or, after simplifying by multiplying each term by L/V_∞^2,

$$\left(\frac{\omega L}{V_\infty}\right)^2 \left(\vec{F}/m\right)^* = \left(\frac{\omega L}{V_\infty}\right)\frac{\partial \vec{V}^*}{\partial t^*} + \left(\vec{V}^* \cdot \vec{\nabla}^*\right)\vec{V}^* \qquad (1)$$

We recognize the nondimensional parameter in parentheses in Eq. 1 as St, the **Strouhal number**. We re-write Eq. 1 as

Nondimensionalized Newton's second law for incompressible oscillatory flow:

$$(\mathrm{St})^2 \left(\vec{F}/m\right)^* = (\mathrm{St})\frac{\partial \vec{V}^*}{\partial t^*} + \left(\vec{V}^* \cdot \vec{\nabla}^*\right)\vec{V}^*$$

Discussion We used angular frequency ω in this problem. The same result would be obtained if we used physical frequency. Equation 1 is the basis for forming the differential equation for conservation of linear momentum for an unsteady, incompressible flow field.

7-33

Solution We are to nondimensionalize the Bernoulli equation and generate an expression for the pressure coefficient.

Assumptions **1** The flow is incompressible. **2** Gravitational terms in the Bernoulli equation are negligible compared to the other terms.

Analysis We nondimensionalize the equation by dividing each term by the **dynamic pressure**, $\frac{1}{2}\rho V_\infty^2$,

Nondimensionalization:

$$\frac{P}{\frac{1}{2}\rho V_\infty^2} + \frac{V^2}{V_\infty^2} = \frac{P_\infty}{\frac{1}{2}\rho V_\infty^2} + 1$$

Rearranging,

Pressure coefficient:

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \frac{V^2}{V_\infty^2}$$

Discussion Pressure coefficient is a useful dimensionless parameter that is inversely related to local air speed – as local air speed V increases, C_p decreases.

Dimensional Analysis and Similarity

7-34C

Solution We are to list the three primary purposes of dimensional analysis.

Analysis The three primary purposes of *dimensional analysis* are:

1. **To generate nondimensional parameters that help in the design of experiments and in the reporting of experimental results**.
2. **To obtain scaling laws so that prototype performance can be predicted from model performance**.
3. **To (sometimes) predict trends in the relationship between parameters**.

Discussion Dimensional analysis is most useful for difficult problems that cannot be solved analytically.

7-35C

Solution We are to list and describe the three necessary conditions for complete similarity between a model and a prototype.

Analysis The three necessary conditions for complete similarity between a model and a prototype are:

1. **Geometric similarity** – the model must be the same shape as the prototype, but scaled by some constant scale factor.
2. **Kinematic similarity** – the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.
3. **Dynamic similarity** – all *forces* in the model flow scale by a constant factor to corresponding forces in the prototype flow.

Discussion Complete similarity is achievable only when all three of the above similarity conditions are met.

7-36

Solution For a scale model of a submarine being tested in air, we are to calculate the wind tunnel speed required to achieve similarity with the prototype submarine that moves through water at a given speed.

Assumptions **1** Compressibility of the air is assumed to be negligible. **2** The wind tunnel walls are far enough away so as to not interfere with the aerodynamic drag on the model sub. **3** The model is geometrically similar to the prototype.

Properties For water at $T = 15^\circ\text{C}$ and atmospheric pressure, $\rho = 999.1\ \text{kg/m}^3$ and $\mu = 1.138 \times 10^{-3}$ kg/m·s. For air at $T = 25^\circ\text{C}$ and atmospheric pressure, $\rho = 1.184\ \text{kg/m}^3$ and $\mu = 1.849 \times 10^{-5}$ kg/m·s.

Analysis Similarity is achieved when the Reynolds number of the model is equal to that of the prototype,

Similarity:

$$\text{Re}_\text{m} = \frac{\rho_\text{m} V_\text{m} L_\text{m}}{\mu_\text{m}} = \text{Re}_\text{p} = \frac{\rho_\text{p} V_\text{p} L_\text{p}}{\mu_\text{p}} \qquad (1)$$

We solve Eq. 1 for the unknown wind tunnel speed,

$$V_\text{m} = V_\text{p}\left(\frac{\mu_\text{m}}{\mu_\text{p}}\right)\left(\frac{\rho_\text{p}}{\rho_\text{m}}\right)\left(\frac{L_\text{p}}{L_\text{m}}\right)$$

$$= (0.560\ \text{m/s})\left(\frac{1.849\times10^{-5}\ \text{kg/m}\cdot\text{s}}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}}\right)\left(\frac{999.1\ \text{kg/m}^3}{1.184\ \text{kg/m}^3}\right)(8) = \mathbf{61.4\ m/s}$$

Discussion At this air temperature, the speed of sound is around 346 m/s. Thus the Mach number in the wind tunnel is equal to 61.4/346 = 0.177. This is sufficiently low that the incompressible flow approximation is reasonable.

7-37

Solution For a scale model of a submarine being tested in air, we are to calculate the wind tunnel speed required to achieve similarity with the prototype submarine that moves through water at a given speed.

Assumptions **1** Compressibility of the air is assumed to be negligible. **2** The wind tunnel walls are far enough away so as to not interfere with the aerodynamic drag on the model sub. **3** The model is geometrically similar to the prototype.

Properties For water at $T = 15^{o}$C and atmospheric pressure, $\rho = 999.1$ kg/m^3 and $\mu = 1.138 \times 10^{-3}$ kg/m·s. For air at $T = 25^{o}$C and atmospheric pressure, $\rho = 1.184$ kg/m^3 and $\mu = 1.849 \times 10^{-5}$ kg/m·s.

Analysis Similarity is achieved when the Reynolds number of the model is equal to that of the prototype,

Similarity:
$$\text{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \text{Re}_p = \frac{\rho_p V_p L_p}{\mu_p} \qquad (1)$$

We solve Eq. 1 for the unknown wind tunnel speed,

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$
$$= 0.560 \text{ m/s}\left(\frac{1.849\times10^{-5} \text{ kg/m}\cdot\text{s}}{1.138\times10^{-3} \text{ kg/m}\cdot\text{s}}\right)\left(\frac{999.1 \text{ kg/m}^3}{1.184 \text{ kg/m}^3}\right)(24) = \mathbf{184 \text{ m/s}}$$

At this air temperature, the speed of sound is around 346 m/s. Thus the Mach number in the wind tunnel is equal to 184/346 = 0.532. **The Mach number is sufficiently high that the incompressible flow approximation is not reasonable**. The wind tunnel should be run at a flow speed at which the Mach number is less than one-third of the speed of sound. At this lower speed, the Reynolds number of the model will be too small, but the results may still be usable, either by extrapolation to higher Re, or if we are fortunate enough to have Reynolds number independence, as discussed in Section 7-5.

Discussion It is also unlikely that a small instructional wind tunnel can achieve such a high speed.

7-38E

Solution For a prototype parachute and its model we are to calculate drag coefficient, and determine the wind tunnel speed that ensures dynamic similarity. Then we are to estimate the aerodynamic drag on the model.

Assumptions **1** The model is geometrically similar to the prototype.

Properties For air at 60°F and standard atmospheric pressure, $\rho = 0.07633$ lbm/ft^3 and $\mu = 1.213 \times 10^{-5}$ lbm/ft·s.

Analysis (*a*) The aerodynamic drag on the prototype parachute is equal to the total weight. We can then easily calculate the drag coefficient C_D,

Drag coefficient:

$$C_D = \frac{F_{D,\mathrm{p}}}{\frac{1}{2}\rho_\mathrm{p} V_\mathrm{p}^{\,2} A_\mathrm{p}} = \frac{230\ \text{lbf}}{\frac{1}{2}\left(0.07633\ \text{lbm/ft}^3\right)\left(20\ \text{ft/s}\right)^2 \pi \frac{\left(24\ \text{ft}\right)^2}{4}}\left(\frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}\right) = \mathbf{1.07}$$

(*b*) We must match model and prototype Reynolds numbers in order to achieve dynamic similarity,

Similarity:

$$\mathrm{Re}_\mathrm{m} = \frac{\rho_\mathrm{m} V_\mathrm{m} L_\mathrm{m}}{\mu_\mathrm{m}} = \mathrm{Re}_\mathrm{p} = \frac{\rho_\mathrm{p} V_\mathrm{p} L_\mathrm{p}}{\mu_\mathrm{p}} \qquad (1)$$

We solve Eq. 1 for the unknown wind tunnel speed,

Wind tunnel speed:

$$V_\mathrm{m} = V_\mathrm{p}\left(\frac{\mu_\mathrm{m}}{\mu_\mathrm{p}}\right)\left(\frac{\rho_\mathrm{p}}{\rho_\mathrm{m}}\right)\left(\frac{L_\mathrm{p}}{L_\mathrm{m}}\right) = \left(20\ \text{ft/s}\right)\left(1\right)\left(1\right)\left(12\right) = \mathbf{240\ ft/s} \qquad (2)$$

(*c*) As discussed in the text, if the fluid is the same and dynamic similarity between the model and the prototype is achieved, the aerodynamic drag force on the model is the same as that on the prototype. Thus,

Aerodynamic drag on model:

$$F_{D,\mathrm{m}} = F_{D,\mathrm{p}} = \mathbf{230\ lbf} \qquad (3)$$

Discussion We should check that the wind tunnel speed of Eq. 2 is not too high that the incompressibility approximation becomes invalid. The Mach number at this speed is about 240/1120 = 0.214. Since this is less than 0.3, compressibility is not an issue in this model test. The drag force on the model is quite large, and a fairly hefty drag balance must be available to measure such a large force.

7-39

Solution We are to discuss why one would pressurize a wind tunnel.

Analysis As we see in some of the example problems and homework problems in this chapter, it is often difficult to achieve a high-enough wind tunnel speed to match the Reynolds number between a small model and a large prototype. Even if we were able to match the speed, the Mach number would often be too high. A pressurized wind tunnel has higher density air. At the same Reynolds number, the larger density leads to a lower air speed requirement. In other words, **a pressurized wind tunnel can achieve higher Reynolds numbers for the same scale model**.

If the pressure were to be increased by a factor of 1.5, the air density would also go up by a factor of 1.5 (ideal gas law), assuming that the air temperature remains constant. Then the Reynolds number, Re = $\rho VL/\mu$, would go up by approximately **1.5**. Note that we are also assuming that the viscosity does not change significantly with pressure, which is a reasonable assumption.

Discussion The speed of sound is not a strong function of pressure, so Mach number is not affected significantly by pressurizing the wind tunnel. However, the *power* requirement for the wind tunnel blower increases significantly as air density is increased, so this must be taken into account when designing the wind tunnel.

7-40

Solution We are to estimate the drag on a prototype submarine in water, based on aerodynamic drag measurements performed in a wind tunnel.

Assumptions **1** The model is geometrically similar. **2** The wind tunnel is run at conditions which ensure similarity between model and prototype.

Properties For water at T = 15°C and atmospheric pressure, ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m·s. For air at T = 25°C and atmospheric pressure, ρ = 1.184 kg/m^3 and μ = 1.849 × 10^{-5} kg/m·s.

Analysis Since the Reynolds numbers have been matched, the nondimensionalized drag coefficient of the model equals that of the prototype,

$$\frac{F_{D,\mathrm{m}}}{\rho_\mathrm{m} V_\mathrm{m}^2 L_\mathrm{m}^2} = \frac{F_{D,\mathrm{p}}}{\rho_\mathrm{p} V_\mathrm{p}^2 L_\mathrm{p}^2} \tag{1}$$

We solve Eq. 1 for the unknown aerodynamic drag force on the prototype, $F_{D,\mathrm{p}}$,

$$F_{D,\mathrm{p}} = F_{D,\mathrm{m}}\left(\frac{\rho_\mathrm{p}}{\rho_\mathrm{m}}\right)\left(\frac{V_\mathrm{p}}{V_\mathrm{m}}\right)^2\left(\frac{L_\mathrm{p}}{L_\mathrm{m}}\right)^2 = (2.3\ \mathrm{N})\left(\frac{999.1\ \mathrm{kg/m\cdot s}}{1.184\ \mathrm{kg/m\cdot s}}\right)\left(\frac{0.560\ \mathrm{m/s}}{61.4\ \mathrm{m/s}}\right)^2(8)^2 = \mathbf{10.3\ N}$$

where we have used the wind tunnel speed calculated in Problem 7-36.

Discussion Although the prototype moves at a much slower speed than the model, the density of water is much higher than that of air, and the prototype is eight times larger than the model. When all of these factors are combined, the drag force on the prototype is much larger than that on the model.

7-41E

Solution The concept of similarity will be utilized to determine the speed of the wind tunnel.

Assumptions **1** Compressibility of the air is ignored (the validity of this assumption will be discussed later). **2** The wind tunnel walls are far enough away so as to not interfere with the aerodynamic drag on the model car. **3** The model is geometrically similar to the prototype. **4** Both the air in the wind tunnel and the air flowing over the prototype car are at standard atmospheric pressure.

Properties For air at T = 25°C and atmospheric pressure, ρ = 1.184 kg/m^3 and μ = 1.849 × 10^{-5} kg/m·s.

Analysis Since there is only one independent Π in this problem, similarity is achieved if $\Pi_{2,m} = \Pi_{2,p}$, where Π_2 is the Reynolds number. Thus, we can write

$$\Pi_{2,m} = \mathrm{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \Pi_{2,p} = \mathrm{Re}_p = \frac{\rho_p V_p L_p}{\mu_p}$$

which can be solved for the unknown wind tunnel speed for the model tests, V_m,

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right) = (60.0 \text{ mph})(1)(1)(4) = \mathbf{240\ mph}$$

Thus, to ensure similarity, the wind tunnel should be run at 240 miles per hour (to three significant digits).

Discussion This speed is quite high, and the wind tunnel may not be able to run at that speed. We were never given the actual length of either car, but the ratio of L_p to L_m is known because the prototype is four times larger than the scale model. The problem statement contains a mixture of SI and English units, but it does not matter since we use ratios in the algebra.

7-42E

Solution We are to estimate the drag on a prototype car, based on aerodynamic drag measurements performed in a wind tunnel.

Assumptions **1** The model is geometrically similar. **2** The wind tunnel is run at conditions which ensure similarity between model and prototype.

Properties For air at T = 25°C and atmospheric pressure, ρ = 1.184 kg/m^3 and μ = 1.849 × 10^{-5} kg/m·s.

Analysis Following the example in the text, since the Reynolds numbers have been matched, the nondimensionalized drag coefficient of the model equals that of the prototype,

$$\frac{F_{D,\mathrm{m}}}{\rho_\mathrm{m} V_\mathrm{m}^2 L_\mathrm{m}^2} = \frac{F_{D,\mathrm{p}}}{\rho_\mathrm{p} V_\mathrm{p}^2 L_\mathrm{p}^2} \qquad (1)$$

We solve Eq. 1 for the unknown aerodynamic drag force on the prototype, $F_{D,\mathrm{p}}$,

$$F_{D,\mathrm{p}} = F_{D,\mathrm{m}} \left(\frac{\rho_\mathrm{p}}{\rho_\mathrm{m}}\right)\left(\frac{V_\mathrm{p}}{V_\mathrm{m}}\right)^2 \left(\frac{L_\mathrm{p}}{L_\mathrm{m}}\right)^2 = \left(36.5 \text{ lbf}\right)\left(1\right)\left(\frac{60.0 \text{ mph}}{240 \text{ mph}}\right)^2 \left(4\right)^2 = \mathbf{36.5\ lbf}$$

where we have used the wind tunnel speed calculated in Problem 7-41E.

Discussion Since the air properties of the wind tunnel are identical to those of the air flowing around the prototype car, it turns out that the aerodynamic drag force on the prototype is the same as that on the model. This would not be the case if the wind tunnel air were at a different temperature or pressure compared to that of the prototype.

7-43

Solution We are to discuss whether cold or hot air in a wind tunnel is better, and we are to support our answer by comparing air at two given temperatures.

Properties For air at atmospheric pressure and at $T = 10^{\circ}$C, $\rho = 1.246$ kg/m^3 and $\mu = 1.778 \times 10^{-5}$ kg/m·s. At $T = 50^{\circ}$C, $\rho = 1.092$ kg/m^3 and $\mu = 1.963 \times 10^{-5}$ kg/m·s.

Analysis As we see in some of the example problems and homework problems in this chapter, it is often difficult to achieve a high-enough wind tunnel speed to match the Reynolds number between a small model and a large prototype. Even if we were able to match the speed, the Mach number would often be too high. Cold air has higher density than warm air. In addition, the viscosity of cold air is lower than that of hot air. Thus, at the same Reynolds number, the colder air leads to a lower air speed requirement. In other words, **a cold wind tunnel can achieve higher Reynolds numbers than can a hot wind tunnel for the same scale model, all else being equal**. We support our conclusion by comparing air at two temperatures,

Comparison of Reynolds numbers:

$$\frac{\mathrm{Re}_{\mathrm{cold}}}{\mathrm{Re}_{\mathrm{hot}}} = \frac{\dfrac{\rho_{\mathrm{cold}} V L}{\mu_{\mathrm{cold}}}}{\dfrac{\rho_{\mathrm{hot}} V L}{\mu_{\mathrm{hot}}}} = \frac{\rho_{\mathrm{cold}}}{\rho_{\mathrm{hot}}} \frac{\mu_{\mathrm{hot}}}{\mu_{\mathrm{cold}}} = \frac{1.246 \text{ kg/m}^3}{1.092 \text{ kg/m}^3} \frac{1.963 \times 10^{-5} \text{ kg/m} \cdot \text{s}}{1.778 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = \mathbf{1.26}$$

Thus we see that **the colder wind tunnel can achieve approximately 26% higher Reynolds number, all else being equal**.

Discussion There are other issues however. First of all, the denser air of the cold wind tunnel is harder to pump – the cold wind tunnel may not be able to achieve the same wind speed as the hot wind tunnel. Furthermore, the speed of sound is proportional to the square root of temperature. Thus, at colder temperatures, the Mach number is higher than at warmer temperatures for the same value of V, and compressibility effects are therefore more significant at lower temperatures.

7-44

Solution We are to calculate the speed and angular velocity (rpm) of a spinning baseball in a water channel such that flow conditions are dynamically similar to that of the actual baseball moving and spinning in air.

Properties For air at T = 20°C and atmospheric pressure, ρ = 1.204 kg/m^3 and μ = 1.825 × 10^{-5} kg/m·s. For water at T = 20°C and atmospheric pressure, ρ = 998.0 kg/m^3 and μ = 1.002 × 10^{-3} kg/m·s.

Analysis The model (in the water) and the prototype (in the air) are actually the same baseball, so their characteristic lengths are equal, $L_m = L_p$. We match Reynolds number,

$$\mathrm{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \mathrm{Re}_p = \frac{\rho_p V_p L_p}{\mu_p} \quad (1)$$

and solve for the required water tunnel speed for the model tests, V_m,

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right) = (80.0\text{ mph})\left(\frac{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}}{1.825\times10^{-5}\text{ kg/m}\cdot\text{s}}\right)\left(\frac{1.204\text{ kg/m}^3}{998.0\text{ kg/m}^3}\right)(1) = \mathbf{5.30\ mph} \quad (2)$$

We also match Strouhal numbers, recognizing that $\dot{n}$ is proportional to f,

$$\mathrm{St}_m = \frac{f_m L_m}{V_m} = \mathrm{St}_p = \frac{f_p L_p}{V_p} \quad \rightarrow \quad \frac{\dot{n}_m L_m}{V_m} = \frac{\dot{n}_p L_p}{V_p} \quad (3)$$

from which we solve for the required spin rate in the water tunnel,

$$\dot{n}_m = \dot{n}_p\left(\frac{L_p}{L_m}\right)\left(\frac{V_m}{V_p}\right) = (300\text{ rpm})(1)\left(\frac{5.30\text{ mph}}{80.0\text{ mph}}\right) = \mathbf{19.9\ rpm} \quad (4)$$

Discussion Because of the difference in fluid properties between air and water, the required water tunnel speed is much lower than that in air. In addition, the spin rate is much lower, making flow visualization easier.

Dimensionless Parameters and the Method of Repeating Variables

7-45

Solution We are to verify that the Archimedes number is dimensionless.

Analysis Archimedes number is defined as

Archimedes number:
$$\text{Ar} = \frac{\rho_s g L^3}{\mu^2}\left(\rho_s - \rho\right) \tag{1}$$

We know the primary dimensions of density, gravitational acceleration, length, and viscosity. Thus,

Primary dimensions of Archimedes number:
$$\left\{\text{Ar}\right\} = \left\{ \frac{\frac{\text{m}}{\text{L}^3}\frac{\text{L}}{\text{t}^2}\text{L}^3}{\frac{\text{m}^2}{\text{L}^2\text{t}^2}} \frac{\text{m}}{\text{L}^3} \right\} = \left\{\mathbf{1}\right\} \tag{2}$$

Discussion If the primary dimensions were *not* unity, we would assume that we made an error in the dimensions of one or more of the parameters.

7-46

Solution We are to verify that the Grashof number is dimensionless.

Analysis Grashof number is defined as

Grashof number:
$$\text{Gr} = \frac{g\beta\left|\Delta T\right| L^3 \rho^2}{\mu^2} \tag{1}$$

We know the primary dimensions of density, gravitational acceleration, length, temperature, and viscosity. The dimensions of coefficient of thermal expansion β are 1/temperature. Thus,

Primary dimensions of Grashof number:
$$\left\{\text{Gr}\right\} = \left\{ \frac{\frac{\text{L}}{\text{t}^2}\frac{1}{\text{T}}\text{TL}^3\frac{\text{m}^2}{\text{L}^6}}{\frac{\text{m}^2}{\text{L}^2\text{t}^2}} \right\} = \left\{\mathbf{1}\right\} \tag{2}$$

Discussion If the primary dimensions were *not* unity, we would assume that we made an error in the dimensions of one or more of the parameters.

7-47

Solution We are to verify that the Rayleigh number is dimensionless, and determine what other established nondimensional parameter is formed by the ratio of Ra and Gr.

Analysis Rayleigh number is defined as

Rayleigh number:
$$\text{Ra} = \frac{g\beta\left|\Delta T\right|L^3\rho^2 c_p}{k\mu} \tag{1}$$

We know the primary dimensions of density, gravitational acceleration, length, temperature, and viscosity. The dimensions of coefficient of thermal expansion β are 1/temperature, those of specific heat c_p are length^2/time^2·temperature (Problem 7-14), and those of thermal conductivity k are mass·length/time^3·temperature. Thus,

Primary dimensions of Rayleigh number:
$$\left\{\text{Ra}\right\} = \left\{\frac{\frac{\text{L}}{\text{t}^2}\frac{1}{\text{T}}\text{TL}^3\frac{\text{m}^2}{\text{L}^6}\frac{\text{L}^2}{\text{t}^2\text{T}}}{\frac{\text{mL}}{\text{t}^3\text{T}}\frac{\text{m}}{\text{Lt}}}\right\} = \left\{\mathbf{1}\right\} \tag{2}$$

We take the ratio of Ra and Gr:

Ratio of Rayleigh number and Grashof number:
$$\frac{\text{Ra}}{\text{Gr}} = \frac{\dfrac{g\beta\left|\Delta T\right|L^3\rho^2 c_p}{k\mu}}{\dfrac{g\beta\left|\Delta T\right|L^3\rho^2}{\mu^2}} = \frac{c_p\mu}{k} \tag{3}$$

We recognize Eq. 3 as the **Prandtl number**,

Prandtl number:
$$\boxed{\text{Pr} = \frac{\text{Ra}}{\text{Gr}} = \frac{c_p\mu}{k} = \frac{\rho c_p\mu}{\rho k} = \frac{\nu}{\alpha}} \tag{4}$$

Discussion Many of the established nondimensional parameters are formed by the ratio or product of two (or more) other established nondimensional parameters.

7-48

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $h = f\left(\omega, \rho, g, R\right) \qquad n = 5 \qquad (1)$

Step 2 The primary dimensions of each parameter are listed,

h	ω	ρ	g	R
$\{L^1\}$	$\{t^{-1}\}$	$\{m^1L^{-3}\}$	$\{L^1t^{-2}\}$	$\{L^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since j = 3. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. We choose

Repeating parameters: ω, ρ, and R

Step 5 The dependent Π is generated:

$$\Pi_1 = h\omega^{a_1}\rho^{b_1}R^{c_1} \qquad \{\Pi_1\} = \left\{\left(L^1\right)\left(t^{-1}\right)^{a_1}\left(m^1L^{-3}\right)^{b_1}\left(L^1\right)^{c_1}\right\}$$

mass: $\{m^0\} = \{m^{b_1}\}$ $\qquad 0 = b_1$ $\qquad b_1 = 0$

time: $\{t^0\} = \{t^{-a_1}\}$ $\qquad 0 = -a_1$ $\qquad a_1 = 0$

length: $\{L^0\} = \{L^1L^{-3b_1}L^{c_1}\}$ $\qquad 0 = 1 - 3b_1 + c_1$ $\qquad c_1 = -1$

The dependent Π is thus

Π_1: $\Pi_1 = \dfrac{h}{R}$

The second Pi (the only independent Π in this problem) is generated:

$$\Pi_2 = g\omega^{a_2}\rho^{b_2}R^{c_2} \qquad \{\Pi_2\} = \left\{\left(\mathrm{L}^1\mathrm{t}^{-2}\right)\left(\mathrm{t}^{-1}\right)^{a_2}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_2}\left(\mathrm{L}^1\right)^{c_2}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^{b_2}\}$ $\qquad 0 = b_2 \qquad b_2 = 0$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-2}\mathrm{t}^{-a_2}\}$ $\qquad 0 = -2 - a_2 \qquad a_2 = -2$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{-3b_2}\mathrm{L}^{c_2}\}$ $\qquad 0 = 1 - 3b_2 + c_2 \qquad c_2 = -1$

$$0 = 1 + c_2$$

which yields

Π_2: $$\Pi_2 = \frac{g}{\omega^2 R}$$

If we take Π_2 to the power –1/2 and recognize that ωR is the speed of the rim, we see that Π_2 can be modified into a **Froude number**,

Modified Π_2: $$\Pi_2 = \mathrm{Fr} = \frac{\omega R}{\sqrt{gR}}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\frac{h}{R} = f\left(\mathrm{Fr}\right)} \qquad (2)$$

Discussion In the generation of the first Π, h and R have the same dimensions. Thus, we could have immediately written down the result, $\Pi_1 = h/R$. Notice that density ρ does not appear in the result. Thus, density is not a relevant parameter after all.

7-49

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are seven parameters in this problem; $n = 7$,

List of relevant parameters: $$h = f(\omega, \rho, g, R, t, \mu) \qquad n = 7 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

h	ω	ρ	g	R	t	μ
$\{L^1\}$	$\{t^{-1}\}$	$\{m^1L^{-3}\}$	$\{L^1t^{-2}\}$	$\{L^1\}$	$\{t^1\}$	$\{m^1L^{-1}t^{-1}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $$k = n - j = 7 - 3 = 4$$

Step 4 We need to choose three repeating parameters since $j = 3$. For convenience we choose the same repeating parameters that we used in Problem 7-48,

Repeating parameters: ω, ρ, and R

Step 5 The first two Πs are identical to those of Problem 7-48:

Π_1: $$\Pi_1 = \frac{h}{R}$$

and

Π_2: $$\Pi_2 = \frac{\omega R}{\sqrt{gR}}$$

where Π_2 is identified as a form of the **Froude number**. The third Π is formed with time t. Since repeating parameter ω has dimensions of 1/time, it is the only one that remains in the Π. Thus, without the formal algebra,

Π_3: $$\Pi_3 = \omega t$$

Finally, Π_4 is generated with liquid viscosity,

$$\Pi_4 = \mu\omega^{a_4}\rho^{b_4}R^{c_4} \qquad \{\Pi_4\} = \left\{\left(m^1L^{-1}t^{-1}\right)\left(t^{-1}\right)^{a_4}\left(m^1L^{-3}\right)^{b_4}\left(L^1\right)^{c_4}\right\}$$

mass:	$\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^1\mathrm{m}^{b_4}\right\}$	$0 = 1 + b_4$	$b_4 = -1$
time:	$\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{-a_4}\right\}$	$0 = -1 - a_4$	$a_4 = -1$
length:	$\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^{-1}\mathrm{L}^{-3b_1}\mathrm{L}^{c_1}\right\}$	$0 = -1 - 3b_4 + c_4$ $c_4 = 1 + 3b_4$	$c_4 = -2$

The final Π is thus

Π_4:
$$\Pi_4 = \frac{\mu}{\rho\Omega R^2} \tag{2}$$

If we invert Π_4 and recognize that ωR is the speed of the rim, it becomes clear that Π_4 of Eq. 2 can be modified into a **Reynolds number**,

Modified Π_4:
$$\Pi_4 = \frac{\rho\omega R^2}{\mu} = \mathrm{Re} \tag{3}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*:
$$\boxed{\frac{h}{R} = f\left(\mathrm{Fr}, \omega t, \mathrm{Re}\right)} \tag{4}$$

Discussion Notice that this time density ρ *does* appear in the result. There are other acceptable answers, but this one has the most established dimensionless groups.

7-50

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $f_k = f\left(V, \rho, \mu, D\right) \qquad n = 5$

Step 2 The primary dimensions of each parameter are listed,

f_k	V	ρ	μ	D
$\left\{\mathrm{t}^{-1}\right\}$	$\left\{\mathrm{L}^1\mathrm{t}^{-1}\right\}$	$\left\{\mathrm{m}^1\mathrm{L}^{-3}\right\}$	$\left\{\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right\}$	$\left\{\mathrm{L}^1\right\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. We choose

Repeating parameters: V, ρ, and D

Step 5 The dependent Π is generated:

$$\Pi_1 = f_k V^{a_1} \rho^{b_1} D^{c_1}$$

mass: $\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^{b_1}\right\}$ $\qquad 0 = b_1$ $\qquad b_1 = 0$

time: $\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{-a_1}\right\}$ $\qquad 0 = -1 - a_1$ $\qquad a_1 = -1$

length: $\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^{a_1}\mathrm{L}^{-3b_1}\mathrm{L}^{c_1}\right\}$ $\qquad 0 = a_1 - 3b_1 + c_1$ $\qquad c_1 = 1$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{f_k D}{V} = \mathrm{St}$$

where we have identified this Pi as the **Strouhal number**.

The second Pi (the only independent Π in this problem) is generated:

$$\Pi_2 = \mu V^{a_2} \rho^{b_2} D^{c_2} \qquad \{\Pi_2\} = \left\{ \left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right) \left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{a_2} \left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_2} \left(\mathrm{L}^1\right)^{c_2} \right\}$$

mass: $\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^1\mathrm{m}^{b_2}\right\}$ $\qquad 0 = 1 + b_2 \qquad b_2 = -1$

time: $\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{-a_2}\right\}$ $\qquad 0 = -1 - a_2 \qquad a_2 = -1$

length: $\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^{-1}\mathrm{L}^{a_2}\mathrm{L}^{-3b_2}\mathrm{L}^{c_2}\right\}$ $\qquad \begin{array}{l} 0 = -1 + a_2 - 3b_2 + c_2 \\ 0 = -1 - 1 + 3 + c_2 \end{array} \qquad c_2 = -1$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho V D}$$

We recognize this Π as the inverse of the **Reynolds number**. So, after inverting,

Modified Π_2: $$\Pi_2 = \frac{\rho V D}{\mu} = \text{Reynolds number} = \mathrm{Re}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\mathrm{St} = f\left(\mathrm{Re}\right)$$

Discussion We cannot tell from dimensional analysis the exact form of the functional relationship. However, experiments verify that the Strouhal number is indeed a function of Reynolds number.

7-51

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are six parameters in this problem; $n = 6$,

List of relevant parameters: $f_k = f\left(V, \rho, \mu, D, c\right) \qquad n = 6 \qquad (1)$

Step 2 The primary dimensions of each parameter are listed,

f_k	V	ρ	μ	D	c
$\{t^{-1}\}$	$\{L^1t^{-1}\}$	$\{m^1L^{-3}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1\}$	$\{L^1t^{-1}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 6 - 3 = 3$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. We choose

Repeating parameters: V, ρ, and D

Step 5 The dependent Π is generated:

$$\Pi_1 = f_k V^{a_1} \rho^{b_1} D^{c_1} \qquad \{\Pi_1\} = \left\{\left(t^{-1}\right)\left(L^1t^{-1}\right)^{a_1}\left(m^1L^{-3}\right)^{b_1}\left(L^1\right)^{c_1}\right\}$$

mass: $\{m^0\} = \{m^{b_1}\} \qquad 0 = b_1 \qquad b_1 = 0$

time: $\{t^0\} = \{t^{-1}t^{-a_1}\} \qquad 0 = -1 - a_1 \qquad a_1 = -1$

length: $\{L^0\} = \{L^{a_1}L^{-3b_1}L^{c_1}\} \qquad 0 = a_1 - 3b_1 + c_1 \qquad c_1 = 1$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{f_k D}{V} = \mathrm{St}$$

where we have identified this Pi as the **Strouhal number**.

The second Pi (the first independent Π in this problem) is generated:

$$\Pi_2 = \mu V^{a_2} \rho^{b_2} D^{c_2} \qquad \{\Pi_2\} = \left\{ \left(\mathrm{m^1L^{-1}t^{-1}}\right)\left(\mathrm{L^1t^{-1}}\right)^{a_2}\left(\mathrm{m^1L^{-3}}\right)^{b_2}\left(\mathrm{L^1}\right)^{c_2} \right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{b_2}\}$ $\quad 0 = 1 + b_2 \quad$ $b_2 = -1$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{-a_2}\}$ $\quad 0 = -1 - a_2 \quad$ $a_2 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{-1}\mathrm{L}^{a_2}\mathrm{L}^{-3b_2}\mathrm{L}^{c_2}\}$ $\quad 0 = -1 + a_2 - 3b_2 + c_2$, $\; 0 = -1 - 1 + 3 + c_2 \quad$ $c_2 = -1$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho V D}$$

We recognize this Π as the inverse of the **Reynolds number**. So, after inverting,

Modified Π_2: $$\Pi_2 = \frac{\rho V D}{\mu} = \text{Reynolds number} = \text{Re}$$

The third Pi (the second independent Π in this problem) is generated:

$$\Pi_3 = c V^{a_3} \rho^{b_3} D^{c_3} \qquad \{\Pi_3\} = \left\{ \left(\mathrm{L^1t^{-1}}\right)\left(\mathrm{L^1t^{-1}}\right)^{a_3}\left(\mathrm{m^1L^{-3}}\right)^{b_3}\left(\mathrm{L^1}\right)^{c_3} \right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^{b_3}\}$ $\quad 0 = b_3 \quad$ $b_3 = 0$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{-a_3}\}$ $\quad 0 = -1 - a_3 \quad$ $a_3 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{1}\mathrm{L}^{a_3}\mathrm{L}^{-3b_3}\mathrm{L}^{c_3}\}$ $\quad 0 = 1 + a_3 - 3b_3 + c_3$, $\; 0 = 1 - 1 + c_3 \quad$ $c_3 = 0$

which yields

Π_3: $$\Pi_3 = \frac{c}{V}$$

We recognize this Π as the inverse of the **Mach number**. So, after inverting,

Modified Π_3: $$\Pi_3 = \frac{V}{c} = \text{Mach number} = \text{Ma}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\text{St} = f\left(\text{Re}, \text{Ma}\right)$$

Discussion We have shown all the details. After you become comfortable with the method of repeating variables, you can do some of the algebra in your head.

7-52

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $$\dot{W} = f\left(\omega, \rho, \mu, D\right) \qquad n = 5 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

$\dot{W}$	ω	ρ	μ	D
$\left\{m^1L^2t^{-3}\right\}$	$\left\{t^{-1}\right\}$	$\left\{m^1L^{-3}\right\}$	$\left\{m^1L^{-1}t^{-1}\right\}$	$\left\{L^1\right\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $$k = n - j = 5 - 3 = 2$$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. We choose

Repeating parameters: ω, ρ, and D

Step 5 The dependent Π is generated:

$$\Pi_1 = \dot{W}\omega^{a_1}\rho^{b_1}D^{c_1} \qquad \left\{\Pi_1\right\} = \left\{\left(m^1L^2t^{-3}\right)\left(t^{-1}\right)^{a_1}\left(m^1L^{-3}\right)^{b_1}\left(L^1\right)^{c_1}\right\}$$

mass: $$\left\{m^0\right\} = \left\{m^1m^{b_1}\right\} \qquad 0 = 1 + b_1 \qquad b_1 = -1$$

time: $$\left\{t^0\right\} = \left\{t^{-3}t^{-a_1}\right\} \qquad 0 = -3 - a_1 \qquad a_1 = -3$$

length: $$\left\{L^0\right\} = \left\{L^2L^{-3b_1}L^{c_1}\right\} \qquad 0 = 2 - 3b_1 + c_1 \qquad c_1 = -5$$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{\dot{W}}{\rho D^5\omega^3} = N_P$$

where we have defined this Pi as the **power number** (Table 7-5).

The second Pi (the only independent Π in this problem) is generated:

$$\Pi_2 = \mu\omega^{a_2}\rho^{b_2}D^{c_2} \qquad \{\Pi_2\} = \left\{\left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right)\left(\mathrm{t}^{-1}\right)^{a_2}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_2}\left(\mathrm{L}^1\right)^{c_2}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{b_2}\}$ $\qquad 0 = 1 + b_2 \qquad b_2 = -1$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{-a_2}\}$ $\qquad 0 = -1 - a_2 \qquad a_2 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{-1}\mathrm{L}^{-3b_2}\mathrm{L}^{c_2}\}$ $\qquad 0 = -1 - 3b_2 + c_2$, $\; 0 = -1 + 3 + c_2 \qquad c_2 = -2$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho D^2\omega}$$

Since Dω is the speed of the tip of the rotating stirrer blade, we recognize this Π as the inverse of a **Reynolds number**. So, after inverting,

Modified Π_2: $$\Pi_2 = \frac{\rho D^2\omega}{\mu} = \frac{\rho(D\omega)D}{\mu} = \text{Reynolds number} = \text{Re}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$N_P = f(\text{Re}) \qquad (2)$$

Discussion After some practice you should be able to do some of the algebra with the exponents in your head. Also, we usually expect a type of Reynolds number when we combine viscosity with a density, a length, and some kind of speed, be it angular speed or linear speed.

7-53

Solution We are to determine the dimensionless relationship between the given parameters

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The dimensional analysis is identical to the Problem 7-52 except that we add two additional independent parameters, both of which have dimensions of length. The two Πs of Problem 7-52 remain. We get two additional Πs since n is now equal to 7 instead of 5. There is no need to go through all the algebra for the two additional Πs – since their dimensions match those of one of the repeating variables (D), we know that all the exponents in the Π will be zero except the exponent for D, which will be –1. The two additional Πs are

Π_3 and Π_4:

$$\Pi_3 = \frac{D_{\text{tank}}}{D} \qquad \Pi_4 = \frac{h_{\text{surface}}}{D}$$

The final functional relationship is

Relationship between Πs:

$$\boxed{N_P = f\left(\text{Re}, \frac{D_{\text{tank}}}{D}, \frac{h_{\text{surface}}}{D}\right)} \qquad (1)$$

Discussion We could also manipulate our Πs so that we have other length ratios like $h_{\text{surface}}/D_{\text{tank}}$, etc. Any such combination is acceptable.

7-54

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters:

$$\delta = f(x, V, \rho, \mu) \qquad n = 5 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

δ	x	V	ρ	μ
$\{\text{L}^1\}$	$\{\text{L}^1\}$	$\{\text{L}^1\text{t}^{-1}\}$	$\{\text{m}^1\text{L}^{-3}\}$	$\{\text{m}^1\text{L}^{-1}\text{t}^{-1}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction:

$$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since j = 3. We pick length scale x, density ρ, and freestream velocity V.

Repeating parameters: x, ρ, and V

Step 5 The Πs are generated. Note that for the first Π we can do the algebra in our heads since the relationship is very simple. Namely, the dimensions of δ are identical to those of one of the repeating variables (x). In such a case we know that all the exponents in the Π group are zero except the one for x, which is –1. The dependent Π is thus

Π_1: $\Pi_1 = \dfrac{\delta}{x}$

The second Π is formed with viscosity,

$\Pi_2 = \mu x^a \rho^b V^c$ $\{\Pi_2\} = \left\{\left(\text{m}^1\text{L}^{-1}\text{t}^{-1}\right)\left(\text{L}^1\right)^a\left(\text{m}^1\text{L}^{-3}\right)^b\left(\text{L}^1\text{t}^{-1}\right)^c\right\}$

mass: $\{\text{m}^0\} = \{\text{m}^1\text{m}^b\}$ $0 = 1 + b$ $b = -1$

time: $\{\text{t}^0\} = \{\text{t}^{-1}\text{t}^{-c}\}$ $0 = -1 - c$ $c = -1$

length: $\{\text{L}^0\} = \{\text{L}^{-1}\text{L}^a\text{L}^{-3b}\text{L}^c\}$ $0 = -1 + a - 3b + c$ $a = -1$
$0 = -1 + a + 3 - 1$

which yields

Π_2: $\Pi_2 = \dfrac{\mu}{\rho V x}$

We recognize this Π as the inverse of the **Reynolds number**,

Modified Π_2 *= Reynolds number based on x*: $\Pi_2 = \text{Re}_x \dfrac{\rho V x}{\mu}$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $\boxed{\dfrac{\delta}{x} = f\left(\text{Re}_x\right)}$

Discussion We cannot determine the *form* of the relationship by purely dimensional reasoning since there are two Πs. However, in Chap. 10 we shall see that for a laminar boundary layer, Π_1 is proportional to the square root of Π_2.

7-55

Solution We are to create a scale for volume flow rate and then define an appropriate Richardson number.

Analysis By "back of the envelope" reasoning (or by inspection), we define a volume flow rate scale as L^2V. Then the Richardson number can be defined as

Richardson number: $$\mathrm{Ri} = \frac{L^5 g\Delta\rho}{\rho\dot{V}^2} = \frac{L^5 g\Delta\rho}{\rho\left(L^2V\right)^2} = \frac{Lg\Delta\rho}{\rho V^2} \qquad (1)$$

Discussion It is perhaps more clear from the form of Eq. 1 that Richardson number is a ratio of buoyancy forces to inertial forces.

7-56

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are six parameters in this problem; $n = 6$,

List of relevant parameters: $$u = f\left(\mu, V, h, \rho, y\right) \qquad n = 6 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

u	μ	V	h	ρ	y
$\{L^1t^{-1}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1t^{-1}\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{L^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $$k = n - j = 6 - 3 = 3$$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. It is better to pick a fixed length (h) rather than a variable length (y); otherwise y would appear in each Pi, which would not be desirable. We choose

Repeating parameters: V, ρ, and h

Step 5 The dependent Π is generated:

$$\Pi_1 = uV^{a_1}\rho^{b_1}h^{c_1} \qquad \{\Pi_1\} = \left\{\left(\mathrm{L}^1\mathrm{t}^{-1}\right)\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{a_1}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_1}\left(\mathrm{L}^1\right)^{c_1}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^{b_1}\}$ $\quad 0 = b_1 \quad b_1 = 0$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{-a_1}\}$ $\quad 0 = -1 - a_1 \quad a_1 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{a_1}\mathrm{L}^{-3b_1}\mathrm{L}^{c_1}\}$ $\quad 0 = 1 + a_1 - 3b_1 + c_1 \quad c_1 = 0$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{u}{V}$$

The second Pi (the first independent Π in this problem) is generated:

$$\Pi_2 = \mu V^{a_2}\rho^{b_2}h^{c_2} \qquad \{\Pi_2\} = \left\{\left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right)\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{a_2}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_2}\left(\mathrm{L}^1\right)^{c_2}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{b_2}\}$ $\quad 0 = 1 + b_2 \quad b_2 = -1$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{-a_2}\}$ $\quad 0 = -1 - a_2 \quad a_2 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{-1}\mathrm{L}^{a_2}\mathrm{L}^{-3b_2}\mathrm{L}^{c_2}\}$ $\quad 0 = -1 + a_2 - 3b_2 + c_2 \quad c_2 = -1$

$\quad 0 = -1 - 1 + 3 + c_2$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho V h}$$

We recognize this Π as the inverse of the **Reynolds number**. So, after inverting,

Modified Π_2: $$\Pi_2 = \frac{\rho V h}{\mu} = \text{Reynolds number} = \text{Re}$$

The third Pi (the second independent Π in this problem) is generated:

$$\Pi_3 = yV^{a_3}\rho^{b_3}h^{c_3} \qquad \{\Pi_3\} = \left\{\left(\mathrm{L}^1\right)\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{a_3}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_3}\left(\mathrm{L}^1\right)^{c_3}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^{b_3}\}$ $\quad 0 = b_3 \quad b_3 = 0$

time: $\{t^0\} = \{t^{-a_3}\}$ $0 = -a_3$ $a_3 = 0$

length: $\{L^0\} = \{L^1 L^{a_3} L^{-3b_3} L^{c_3}\}$ $0 = 1 + a_3 - 3b_3 + c_3$ $c_3 = -1$
$0 = 1 + c_3$

which yields

Π_3: $\Pi_3 = \dfrac{y}{h}$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $\boxed{\dfrac{u}{V} = f\left(\mathrm{Re}, \dfrac{y}{h}\right)}$ (2)

Discussion We notice in the first and third Πs that when the parameter on which we are working has the same dimensions as one of the repeating parameters, the Π is simply the ratio of those two parameters (here u/V and y/h).

7-57

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are seven parameters in this problem; $n = 7$,

List of relevant parameters: $u = f(\mu, V, h, \rho, y, t)$ $n = 7$ (1)

Step 2 The primary dimensions of each parameter are listed,

u	μ	V	h	ρ	y	t
$\{L^1 t^{-1}\}$	$\{m^1 L^{-1} t^{-1}\}$	$\{L^1 t^{-1}\}$	$\{L^1\}$	$\{m^1 L^{-3}\}$	$\{L^1\}$	$\{t^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 7 - 3 = 4$

Step 4 We need to choose three repeating parameters since j = 3. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. It is better to pick a fixed length (h) rather than a variable length (y); otherwise y would appear in each Pi, which would not be desirable. It would also not be wise to have time appear in each parameter. We choose

Repeating parameters: V, ρ, and h

Step 5 The Πs are generated. The first three Πs are identical to those of Problem 7-56, so we do not include the details here. The fourth Π is formed by joining the new parameter t to the repeating variables,

$$\Pi_4 = tV^{a_4}\rho^{b_4}h^{c_4} \qquad \{\Pi_4\} = \left\{\left(\mathrm{t}^1\right)\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{a_4}\left(\mathrm{m}^1\mathrm{L}^{-3}\right)^{b_4}\left(\mathrm{L}^1\right)^{c_4}\right\}$$

mass: $\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^{b_4}\right\}$ $\quad 0 = b_4$ $\quad b_4 = 0$

time: $\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^1\mathrm{t}^{-a_4}\right\}$ $\quad 0 = 1 - a_4$ $\quad a_4 = 1$

length: $\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^{a_4}\mathrm{L}^{-3b_4}\mathrm{L}^{c_4}\right\}$ $\quad 0 = a_4 - 3b_4 + c_4$ $\quad c_4 = -1$

This Π is thus

Π_4: $$\Pi_4 = \frac{tV}{h}$$

Step 6 Combining this result with the first three Πs from Problem 7-56,

Relationship between Πs: $$\boxed{\frac{u}{V} = f\left(\mathrm{Re}, \frac{y}{h}, \frac{tV}{h}\right)} \qquad (2)$$

Discussion As $t \to \infty$, Π_4 becomes irrelevant and the result degenerates into that of Problem 7-56.

7-58

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are four parameters in this problem; n = 4,

List of relevant parameters: $$c = f\left(k, T, R_{\mathrm{gas}}\right) \qquad n = 4 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed; the ratio of specific heats k is dimensionless.

$$\begin{array}{cccc} c & k & T & R_{\text{gas}} \\ \left\{\text{L}^1\text{t}^{-1}\right\} & \left\{1\right\} & \left\{\text{T}^1\right\} & \left\{\text{L}^2\text{t}^{-2}\text{T}^{-1}\right\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (T, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 4 - 3 = 1$$

Thus we expect only one Π.

Step 4 We need to choose three repeating parameters since $j = 3$. We only have one choice in this problem, since there are only three independent parameters on the right-hand side of Eq. 1. However, one of these is already dimensionless, so it is a Π all by itself. In this situation we reduce j by one and continue,

Reduction: $$j = 3 - 1 = 2$$

If this revised value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 4 - 2 = 2$$

We now expect two Πs. We choose two repeating parameters since $j = 2$,

Repeating parameters: T and R_{gas}

Step 5 The dependent Π is generated:

$$\Pi_1 = cT^{a_1}R_{\text{gas}}^{\ \ b_1} \qquad \left\{\Pi_1\right\} = \left\{\left(\text{L}^1\text{t}^{-1}\right)\left(\text{T}^1\right)^{a_1}\left(\text{L}^2\text{t}^{-2}\text{T}^{-1}\right)^{b_1}\right\}$$

time: $$\left\{\text{t}^0\right\} = \left\{\text{t}^{-1-2b_1}\right\} \qquad 0 = -1 - 2b_1 \qquad b_1 = -1/2$$

temperature: $$\left\{\text{T}^0\right\} = \left\{\text{T}^{a_1 - b_1}\right\} \qquad a_1 = b_1 \qquad a_1 = -1/2$$

length: $$\left\{\text{L}^0\right\} = \left\{\text{L}^1\text{L}^{2b_1}\right\} \qquad 0 = 1 + 2b_1 \qquad b_1 = -1/2$$

Fortunately the two results for exponent b_1 agree. The dependent Π is thus

Π_1: $$\Pi_1 = \frac{c}{\sqrt{R_{\text{gas}}T}}$$

The independent Π is already known,

Π_2: $\qquad \Pi_2 = k$

Step 6 We write the final functional relationship as

Relationship between Π*s*:

$$\boxed{\frac{c}{\sqrt{R_{\text{gas}}T}} = f\left(k\right)} \qquad (2)$$

Discussion We cannot tell from dimensional analysis the exact form of the functional relationship. However, in this case the result agrees with the known equation for speed of sound in an ideal gas, $c = \sqrt{kR_{\text{gas}}T}$.

7-59

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; n = 5,

List of relevant parameters: $\qquad c = f\left(k, T, R_u, M\right) \qquad n = 5 \qquad (1)$

Step 2 The primary dimensions of each parameter are listed; the ratio of specific heats k is dimensionless.

c	k	T	R_u	M
$\{L^1t^{-1}\}$	$\{1\}$	$\{T^1\}$	$\{m^1L^2t^{-2}T^{-1}N^{-1}\}$	$\{m^1N^{-1}\}$

Step 3 As a first guess, j is set equal to 5, the number of primary dimensions represented in the problem (m, T, L, N, and t).

Reduction: $\qquad j = 5$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $\qquad k = n - j = 5 - 5 = 0$

Obviously we cannot have zero Πs. We check that we have not missed a relevant parameter. Convinced that we have included all the relevant parameters we reduce j by 1:

Reduction: $\qquad j = 5 - 1 = 4$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 5 - 4 = 1$

Step 4 We need to choose four repeating parameters since $j = 4$. We only have one choice in this problem, since there are only four independent parameters on the right-hand side of Eq. 1. However, one of these is already dimensionless, so it is a Π all by itself. In this situation we reduce j by one (*again*) and continue,

Reduction: $j = 4 - 1 = 3$

If this revised value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 5 - 3 = 2$

We now expect two Πs. Since $j = 3$ we choose three repeating parameters,

Repeating parameters: T, M, and R_u

Step 5 The dependent Π is generated:

$$\Pi_1 = cT^{a_1}M^{b_1}R_u^{c_1} \qquad \left\{\Pi_1\right\} = \left\{\left(\mathrm{L}^1\mathrm{t}^{-1}\right)\left(\mathrm{T}^1\right)^{a_1}\left(\mathrm{m}^1\mathrm{n}^{-1}\right)^{b_1}\left(\mathrm{m}^1\mathrm{L}^2\mathrm{t}^{-2}\mathrm{T}^{-1}\mathrm{N}^{-1}\right)^{c_1}\right\}$$

time: $\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{-2c_1}\right\}$ $\quad 0 = -1 - 2c_1$ $\quad c_1 = -1/2$

mass: $\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^{b_1}\mathrm{m}^{c_1}\right\}$ $\quad 0 = b_1 + c_1 \quad b_1 = -c_1$ $\quad b_1 = 1/2$

amount of matter: $\left\{\mathrm{N}^0\right\} = \left\{\mathrm{N}^{-b_1}\mathrm{N}^{-c_1}\right\}$ $\quad 0 = -b_1 - c_1 \quad b_1 = -c_1$ $\quad b_1 = 1/2$

temperature: $\left\{\mathrm{T}^0\right\} = \left\{\mathrm{T}^{a_1}\mathrm{T}^{-c_1}\right\}$ $\quad 0 = a_1 - c_1 \quad a_1 = c_1$ $\quad a_1 = -1/2$

length: $\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^1\mathrm{L}^{2c_1}\right\}$ $\quad 0 = 1 + 2c_1$ $\quad c_1 = -1/2$

Fortunately the two results for exponent b_1 agree, and the two results for exponent c_1 agree. (If they did not agree, we would search for algebra mistakes. Finding none we would suspect that j is not correct or that we are missing a relevant parameter in the problem.) The dependent Π is thus

Π_1: $$\Pi_1 = \frac{c\sqrt{M}}{\sqrt{R_u T}}$$

The independent Π is already known,

Π_2: $\Pi_2 = k$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\Pi_1 = \frac{c\sqrt{M}}{\sqrt{R_u T}} = f(k)} \qquad (2)$$

Discussion Since we know that $R_{\text{gas}} = R_u/M$, we see that the result here is the same as that of Problem 7-58.

7-60

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are three parameters in this problem; $n = 3$,

List of relevant parameters: $$c = f\left(T, R_{\text{gas}}\right) \qquad n = 3 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

$$\begin{array}{ccc} c & T & R_{\text{gas}} \\ \left\{\mathrm{L}^1\mathrm{t}^{-1}\right\} & \left\{\mathrm{T}^1\right\} & \left\{\mathrm{L}^2\mathrm{t}^{-2}\mathrm{T}^{-1}\right\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (T, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 3 - 3 = 0$$

Obviously this is not correct, so we re-examine our initial assumptions. We can add another variable, k (the ratio of specific heats) to our List of relevant parameters. This problem would then be identical to Problem 7-58. Instead, for instructional purposes we reduce j by one and continue,

Reduction: $$j = 3 - 1 = 2$$

If this revised value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 3 - 2 = 1$$

We now expect only one Π.

Step 4 We need to choose two repeating parameters since $j = 2$. We only have one choice in this problem, since there are only two independent parameters on the right-hand side of Eq. 1, *Repeating parameters*: T and R_{gas}

Step 5 The dependent Π is generated:

$$\Pi_1 = cT^{a_1}R_{\text{gas}}^{\ \ b_1} \qquad \{\Pi_1\} = \left\{\left(\mathrm{L}^1\mathrm{t}^{-1}\right)\left(\mathrm{T}^1\right)^{a_1}\left(\mathrm{L}^2\mathrm{t}^{-2}\mathrm{T}^{-1}\right)^{b_1}\right\}$$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1-2b_1}\}$ $\quad 0 = -1 - 2b_1 \quad b_1 = -1/2$

temperature: $\{\mathrm{T}^0\} = \{\mathrm{T}^{a_1-b_1}\}$ $\quad a_1 = b_1 \quad a_1 = -1/2$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{2b_1}\}$ $\quad 0 = 1 + 2b_1 \quad b_1 = -1/2$

Fortunately the two results for exponent b_1 agree. The dependent Π is thus

Π_1: $$\Pi_1 = \frac{c}{\sqrt{R_{\text{gas}}T}}$$

Step 6 Since there is only one Π, it is a function of nothing. This is only possible if we set the Π equal to a constant. We write the final functional relationship as

Relationship between Πs: $$\boxed{\Pi_1 = \frac{c}{\sqrt{R_{\text{gas}}T}} = \text{constant}} \qquad (2)$$

Discussion Our result represents an interesting case of "luck". Although we failed to include the ratio of specific heats k in our analysis, we nevertheless obtain the correct result. In fact, if we set the constant in Eq. 2 as the square root of k, our result agrees with the known equation for speed of sound in an ideal gas, $c = \sqrt{kR_{\text{gas}}T}$.

7-61

Solution We are to use dimensional analysis to find the functional relationship between the given parameters, and compare to the known equation for an ideal gas.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are three parameters in this problem; n = 3,

List of relevant parameters: $$c = f(P, \rho) \qquad n = 3 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

$$\begin{array}{ccc} c & P & \rho \\ \{L^1 t^{-1}\} & \{m^1 L^{-1} t^{-2}\} & \{m^1 L^{-3}\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 3 - 3 = 0$

Obviously this is not correct, so we re-examine our initial assumptions. If we are convinced that c is a function of only P and ρ, we reduce j by one and continue,

Reduction: $j = 3 - 1 = 2$

If this revised value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 3 - 2 = 1$

We now expect only one Π.

Step 4 We need to choose two repeating parameters since j = 2. We only have one choice in this problem, since there are only two independent parameters on the right-hand side of Eq. 1,

Repeating parameters: P and ρ

Step 5 The dependent Π is generated:

$$\Pi_1 = cP^{a_1}\rho^{b_1} \qquad \{\Pi_1\} = \left\{\left(L^1 t^{-1}\right)\left(m^1 L^{-1} t^{-2}\right)^{a_1}\left(m^1 L^{-3}\right)^{b_1}\right\}$$

time: $\{t^0\} = \{t^{-1} t^{-2a_1}\}$ $\quad 0 = -1 - 2a_1 \quad a_1 = -1/2$

mass: $\{m^0\} = \{m^{a_1} m^{b_1}\}$ $\quad 0 = a_1 + b_1 \quad b_1 = 1/2$

length: $\{L^0\} = \{L^1 L^{-2a_1} L^{-3b_1}\}$ $\quad 0 = 1 - a_1 - 3b_1$, $0 = 1 + \frac{1}{2} - \frac{3}{2}$ $\quad 0 = 0$

Fortunately the exponents for length agree with those of mass and time. The dependent Π is thus

Π_1: $\Pi_1 = c\sqrt{\dfrac{\rho}{P}}$

Step 6 Since there is only one Π, it is a function of nothing. This is only possible if we set the Π equal to a constant. We write the final functional relationship as

Relationship between Π*s*: $$\Pi_1 = c\sqrt{\frac{\rho}{P}} = \text{constant, or } c = \text{constant}\sqrt{\frac{P}{\rho}} \qquad (2)$$

The ideal gas equation is $P = \rho R_{\text{gas}}T$, or $P/\rho = R_{\text{gas}}T$. Thus, Eq. 2 can be written as

Alternative result using ideal gas law: $$c = \text{constant}\sqrt{R_{\text{gas}}T} \qquad (3)$$

Equation 3 is indeed consistent with the equation $c = \sqrt{kR_{\text{gas}}T}$.

Discussion There is no way to obtain the value of the constant in Eq. 2 or 3 solely by dimensional analysis, but it turns out that the constant is the square root of k.

7-62

Solution We are to use dimensional analysis to find a functional relationship between F_D and variables V, L, and μ.

Assumptions **1** We assume Re << 1 so that the creeping flow approximation applies. **2** Gravitational effects are irrelevant. **3** No parameters other than those listed in the problem statement are relevant to the problem.

Analysis We follow the step-by-step method of repeating variables.

Step 1 There are four variables and constants in this problem; n = 4. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

List of relevant parameters: $$F_D = f\left(V, L, \mu\right) \qquad n = 4$$

Step 2 The primary dimensions of each parameter are listed.

F_D	V	L	μ
$\{\text{m}^1\text{L}^1\text{t}^{-2}\}$	$\{\text{L}^1\text{t}^{-1}\}$	$\{\text{L}^1\}$	$\{\text{m}^1\text{L}^{-1}\text{t}^{-1}\}$

Step 3 As a first guess, we set j equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the number of Πs expected is

Number of expected Π*s*: $$k = n - j = 4 - 3 = 1$$

Step 4 Now we need to choose three repeating parameters since j = 3. Since we cannot choose the dependent variable, our only choices are V, L, and μ.

Step 5 Now we combine these repeating parameters into a product with the dependent variable F_D to create the dependent Π, *Dependent* Π: $$\Pi_1 = F_D V^{a_1} L^{b_1} \mu^{c_1} \qquad (1)$$

We apply the primary dimensions of Step 2 into Eq. 1 and force the Π to be dimensionless,

Dimensions of Π_1:
$$\{\Pi_1\} = \{m^0L^0t^0\} = \{F_D V^{a_1} L^{b_1} \mu^{c_1}\} = \left\{\left(m^1L^1t^{-2}\right)\left(L^1t^{-1}\right)^{a_1}\left(L^1\right)^{b_1}\left(m^1L^{-1}t^{-1}\right)^{c_1}\right\}$$

Now we equate the exponents of each primary dimension to solve for exponents a_1 through c_1.

mass: $\{m^0\} = \{m^1m^{c_1}\}$ $\quad 0 = 1 + c_1 \quad c_1 = -1$

time: $\{t^0\} = \{t^{-2}t^{-a_1}t^{-c_1}\}$ $\quad 0 = -2 - a_1 - c_1 \quad a_1 = -1$

length: $\{L^0\} = \{L^1L^{a_1}L^{b_1}L^{-c_1}\}$ $\quad 0 = 1 + a_1 + b_1 - c_1 \quad b_1 = -1$

Equation 1 thus becomes

Π_1: $$\Pi_1 = \frac{F_D}{\mu V L} \qquad (2)$$

Step 6 We now write the functional relationship between the nondimensional parameters. In the case at hand, there is only one Π, which is a function of *nothing*. This is possible only if the Π is constant. Putting Eq. 2 into standard functional form,

Relationship between Πs: $$\Pi_1 = \frac{F_D}{\mu V L} = f(\text{nothing}) = \text{constant} \qquad (3)$$

or

Result of dimensional analysis: $$F_D = \text{constant} \cdot \mu V L \qquad (4)$$

Thus we have shown that for creeping flow around an object, the aerodynamic drag force is simply a constant multiplied by μVL, regardless of the shape of the object.

Discussion This result is very significant because all that is left to do is find the constant, which will be a function of the shape of the object (and its orientation with respect to the flow).

7-63

Solution We are to find the functional relationship between the given parameters and name any established dimensionless parameters.

Assumptions **1** The given parameters are the only ones relevant to the flow at hand.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $V = f\left(d_p, \left(\rho_p - \rho\right), \mu, g\right) \qquad n = 5 \qquad (1)$

Step 2 The primary dimensions of each parameter are listed,

V	d_p	$(\rho_p - \rho)$	μ	g
$\left\{L^1t^{-1}\right\}$	$\left\{L^1\right\}$	$\left\{m^1L^{-3}\right\}$	$\left\{m^1L^{-1}t^{-1}\right\}$	$\left\{L^1t^{-2}\right\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale d_p, density difference (ρ_p - ρ), and gravitational constant g.

Repeating parameters: d_p, ρ, and g

Step 5 The Πs are generated. Note that for the first Π we do the algebra in our heads since the relationship is very simple. The dependent Π is

Π_1 = *a Froude number*: $\Pi_1 = \dfrac{V}{\sqrt{gd_p}}$

This Π is a type of **Froude number**. Similarly, the Π formed with viscosity is generated,

$$\Pi_2 = \mu d_p^{\,a}\left(\rho_p - \rho\right)^b g^c \qquad \left\{\Pi_2\right\} = \left\{\left(m^1L^{-1}t^{-1}\right)\left(L^1\right)^a\left(m^1L^{-3}\right)^b\left(L^1t^{-2}\right)^c\right\}$$

mass: $\left\{m^0\right\} = \left\{m^1m^b\right\} \qquad 0 = 1 + b \qquad b = -1$

time: $\left\{t^0\right\} = \left\{t^{-1}t^{-2c}\right\} \qquad 0 = -1 - 2c \qquad c = -\dfrac{1}{2}$

length: $\left\{L^0\right\} = \left\{L^{-1}L^aL^{-3b}L^c\right\} \qquad \begin{array}{l} 0 = -1 + a - 3b + c \\ 0 = -1 + a + 3 - \frac{1}{2} \end{array} \qquad a = -\dfrac{3}{2}$

which yields

Π_2:

$$\Pi_2 = \frac{\mu}{\left(\rho_p - \rho\right) d_p^{\frac{3}{2}} \sqrt{g}}$$

We recognize this Π as the inverse of a kind of **Reynolds number** if we split the d_p terms to separate them into a length scale and (when combined with g) a velocity scale. The final form is

Modified Π_2 = *a Reynolds number*:

$$\Pi_2 = \frac{\left(\rho_p - \rho\right) d_p \sqrt{g d_p}}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*:

$$\boxed{\frac{V}{\sqrt{g d_p}} = f\left(\frac{\left(\rho_p - \rho\right) d_p \sqrt{g d_p}}{\mu}\right)} \qquad (2)$$

Discussion We cannot determine the *form* of the relationship by purely dimensional reasoning since there are two Πs. However, in Chap. 10 we shall see that Π_1 is a constant times Π_2.

7-64

Solution We are to develop an equation for the settling speed of an aerosol particle falling in air under creeping flow conditions.

Assumptions **1** The particle falls at steady speed V. **2** The Reynolds number is small enough that the creeping flow approximation is valid.

Analysis We start by recognizing that as the particle falls at steady settling speed, its net weight W must equal the aerodynamic drag F_D on the particle. We also know that W is proportional to $(\rho_p - \rho)g d_p^3$. Thus,

Equating forces:

$$W = \text{constant}_1 \left(\rho_p - \rho\right) g d_p^{\,3} = F_D = \text{constant}_2 \mu V d_p \qquad (1)$$

where we have converted the notation of Problem 7-62 to that of Problem 7-63, and we have defined two different constants. The two constants in Eq. 1 can be combined into one new constant for simplicity. Solving for V,

Settling speed:

$$\boxed{V = \text{constant} \frac{\left(\rho_p - \rho\right) g d_p^{\,2}}{\mu}} \qquad (2)$$

If we divide both sides of Eq. 2 by $\sqrt{g d_p}$ we see that the functional relationship given by Eq. 2 of Problem 7-63 is consistent.

Discussion This result is valid only if the Reynolds number is much smaller than one, as will be discussed in Chap. 10. If the particle is less dense than the fluid (e.g. bubbles rising in water), our result is still valid, but the particle rises instead of falls.

7-65

Solution We are to determine how the settling speed of an aerosol particle falling in air under creeping flow conditions changes when certain parameters are doubled.

Assumptions **1** The particle falls at steady speed V. **2** The Reynolds number is small enough that the creeping flow approximation is valid.

Analysis From the results of Problem 7-64, we see that **if particle size doubles, the settling speed will increase by a factor of 2^2 = 4**. Similarly, **if density difference doubles, the settling speed will increase by a factor of 2^1 = 2**.

Discussion This result is valid only if the Reynolds number remains much smaller than unity, as will be discussed in Chap. 10. As the particle's settling speed increases by a factor of 2 or 4, the Reynolds number will also increase by that same factor. If the new Reynolds number is not small enough, the creeping flow approximation will be invalid and our results will not be correct, although the error will probably be small.

7-66

Solution We are to generate a nondimensional relationship between the given parameters.

Assumptions **1** The flow is fully developed. **2** The fluid is incompressible. **3** No other parameters are significant in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters.

Step 1 All the relevant parameters in the problem are listed in functional form:

List of relevant parameters: $\Delta P = f\left(V, \varepsilon, \rho, \mu, D, L\right) \qquad n = 7$

Step 2 The primary dimensions of each parameter are listed:

ΔP	V	ε	ρ	μ	D	L
$\left\{\mathrm{m^1L^{-1}t^{-2}}\right\}$	$\left\{\mathrm{L^1t^{-1}}\right\}$	$\left\{\mathrm{L^1}\right\}$	$\left\{\mathrm{m^1L^{-3}}\right\}$	$\left\{\mathrm{m^1L^{-1}t^{-1}}\right\}$	$\left\{\mathrm{L^1}\right\}$	$\left\{\mathrm{L^1}\right\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 7 - 3 = 4$

Step 4 We need to choose three repeating parameters since j = 3. Following the guidelines listed in Table 7-3, we cannot pick the dependent variable, ΔP. We cannot choose any two of parameters ε, L, and D since their dimensions are identical. It is not desirable to have μ or ε appear in all the Πs. The best choice of repeating parameters is thus V, D, and ρ.

Repeating parameters: V, D, and ρ

Step 5 The dependent Π is generated:

$$\Pi_1 = \Delta P V^{a_1} D^{b_1} \rho^{c_1} \qquad \{\Pi_1\} = \left\{ \left(\mathrm{m^1L^{-1}t^{-2}}\right)\left(\mathrm{L^1t^{-1}}\right)^{a_1}\left(\mathrm{L^1}\right)^{b_1}\left(\mathrm{m^1L^{-3}}\right)^{c_1} \right\}$$

mass: $\{\mathrm{m^0}\} = \{\mathrm{m^1m^{c_1}}\}$ $\quad 0 = 1 + c_1 \quad c_1 = -1$

time: $\{\mathrm{t^0}\} = \{\mathrm{t^{-2}t^{-a_1}}\}$ $\quad 0 = -2 - a_1 \quad a_1 = -2$

length: $\{\mathrm{L^0}\} = \{\mathrm{L^{-1}L^{a_1}L^{b_1}L^{-3c_1}}\}$ $\quad 0 = -1 + a_1 + b_1 - 3c_1 \quad b_1 = 0$

$$0 = -1 - 2 + b_1 + 3$$

The dependent Π is thus

Π_1: $\Pi_1 = \dfrac{\Delta P}{\rho V^2}$

From Table 7-5, the established nondimensional parameter most similar to our Π_1 is the **Euler number** Eu. No manipulation is required.

We form the second Π with μ. By now we know that we will generate a **Reynolds number**,

$$\Pi_2 = \mu V^{a_2} D^{b_2} \rho^{c_2} \qquad \Pi_2 = \frac{\rho V D}{\mu} = \text{Reynolds number} = \text{Re}$$

The final two Π groups are formed with ε and then with L. The algebra is trivial for these cases since their dimension (length) is identical to that of one of the repeating variables (D). The results are

$$\Pi_3 = \varepsilon V^{a_3} D^{b_3} \rho^{c_3} \qquad \Pi_3 = \frac{\varepsilon}{D} = \text{Roughness ratio}$$

$$\Pi_4 = L V^{a_4} D^{b_4} \rho^{c_4} \qquad \Pi_4 = \frac{L}{D} = \text{Length-to-diameter ratio or aspect ratio}$$

Step 6 We write the final functional relationship as

Relationship between Πs:

$$\boxed{\text{Eu} = \frac{\Delta P}{\rho V^2} = f\left(\text{Re}, \frac{\varepsilon}{D}, \frac{L}{D}\right)} \qquad (1)$$

Discussion The result applies to both laminar and turbulent fully developed pipe flow; it turns out, however, that the second independent Π (roughness ratio) is not nearly as important in laminar pipe flow as in turbulent pipe flow. Since ΔP drops linearly with distance down the pipe, we know that ΔP is linearly proportional to L/D. It is not possible to determine the functional relationships between the other Πs by dimensional reasoning alone.

7-67

Solution We are to determine by what factor volume flow rate increases in the case of fully developed laminar pipe flow when pipe diameter is doubled.

Assumptions **1** The flow is steady. **2** The flow is fully developed, meaning that dP/dx is constant and the velocity profile does not change downstream.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters.

Step 1 All the relevant parameters in the problem are listed in functional form:

List of relevant parameters: $$\dot{V} = f\left(D, \mu, \frac{dP}{dx}\right) \qquad n = 4$$

Step 2 The primary dimensions of each parameter are listed:

$$\begin{array}{cccc} \dot{V} & D & \mu & dP/dx \\ \{L^3t^{-1}\} & \{L^1\} & \{m^1L^{-1}t^{-1}\} & \{m^1L^{-2}t^{-2}\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 4 - 3 = 1$

Step 4 We need to choose three repeating parameters since $j = 3$. Here we must pick all three independent parameters,

Repeating parameters: D, μ, and dP/dx

Step 5 The Π is generated:

$$\Pi_1 = \dot{V}\left(dP/dx\right)^{a_1} D^{b_1} \mu^{c_1} \qquad \{\Pi_1\} = \left\{\left(L^3t^{-1}\right)\left(m^1L^{-2}t^{-2}\right)^{a_1}\left(L^1\right)^{b_1}\left(m^1L^{-1}t^{-1}\right)^{c_1}\right\}$$

mass: $\left\{m^0\right\} = \left\{m^{a_1} m^{c_1}\right\}$ $\quad 0 = a_1 + c_1 \quad c_1 = -a_1$

time: $\left\{t^0\right\} = \left\{t^{-1} t^{-2a_1} t^{-c_1}\right\}$ $\quad 0 = -1 - 2a_1 - c_1$, $\quad 0 = -1 - 2a_1 + a_1 \quad a_1 = -1 \quad c_1 = 1$

length: $\left\{L^0\right\} = \left\{L^3 L^{-2a_1} L^{b_1} L^{-c_1}\right\}$ $\quad 0 = 3 - 2a_1 + b_1 - c_1$, $\quad b_1 = -3 + 2a_1 + c_1 \quad b_1 = -4$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{\dot{V}\mu}{D^4 \dfrac{dP}{dx}}$$

Step 6 Since there is only one Π, we set it equal to a constant. We write the final functional relationship as

Relationship between Πs: $$\boxed{\Pi_1 = \text{constant}, \qquad \dot{V} = \text{constant}\frac{D^4}{\mu}\frac{dP}{dx}} \tag{1}$$

We see immediately that **if the pipe diameter is doubled with all other parameters fixed, the volume flow rate will increase by a factor of $2^4 = 16$.**

Discussion We will see in Chap. 9 that the constant is π/8. There is no way to obtain the value of the constant from dimensional analysis alone.

7-68

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are four parameters in this problem; $n = 4$,

List of relevant parameters: $$\dot{Q} = f\left(\dot{m}, c_p, \left(T_{out} - T_{in}\right)\right) \qquad n = 4 \tag{1}$$

Step 2 The primary dimensions of each parameter are listed,

$\dot{Q}$	$\dot{m}$	c_p	$T_{out} - T_{in}$
$\left\{m^1 L^2 t^{-3}\right\}$	$\left\{m^1 t^{-1}\right\}$	$\left\{L^2 t^{-2} T^{-1}\right\}$	$\left\{T^1\right\}$

Step 3 As a first guess, j is set equal to 4, the number of primary dimensions represented in the problem (m, T, L, and t).

Reduction: $j = 4$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 4 - 4 = 0$

Obviously this is not correct, so we re-examine our initial assumptions. We are convinced that our list of parameters is sufficient, so we reduce j by one and continue,

Reduction: $j = 4 - 1 = 3$

If this revised value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 4 - 3 = 1$

We now expect only one Π.

Step 4 We need to choose three repeating parameters since $j = 3$. We only have one choice in this problem, since there are only three independent parameters on the right-hand side of Eq. 1,

Repeating parameters: $\dot{m}$, c_p, and $\left(T_{\text{out}} - T_{\text{in}}\right)$

Step 5 The dependent Π is generated:

$$\Pi_1 = \dot{Q}\dot{m}^{a_1} c_p^{\ b_1} \left(T_{\text{out}} - T_{\text{in}}\right)^{c_1} \qquad \left\{\Pi_1\right\} = \left\{\left(\text{m}^1\text{L}^2\text{t}^{-3}\right)\left(\text{m}^1\text{t}^{-1}\right)^{a_1}\left(\text{L}^2\text{t}^{-2}\text{T}^{-1}\right)^{b_1}\left(\text{T}^1\right)^{c_1}\right\}$$

mass:	$\left\{\text{m}^0\right\} = \left\{\text{m}^{1+a_1}\right\}$	$0 = 1 + a_1$	$a_1 = -1$
length:	$\left\{\text{L}^0\right\} = \left\{\text{L}^2\text{L}^{2b_1}\right\}$	$0 = 2 + 2b_1$	$b_1 = -1$
temperature:	$\left\{\text{T}^0\right\} = \left\{\text{T}^{-b_1+c_1}\right\}$	$c_1 = b_1$	$c_1 = -1$
time:	$\left\{\text{t}^0\right\} = \left\{\text{t}^{-3-a_1-2b_1}\right\}$	$0 = -3 - a_1 - 2b_1$	$3 = 1 + 2$

Fortunately the result for the time exponents is consistent with that of the other dimensions. The dependent Π is thus

Π_1: $$\Pi_1 = \frac{\dot{Q}}{\dot{m}c_p\left(T_{\text{out}} - T_{\text{in}}\right)}$$

Step 6 Since there is only one Π, it is a function of nothing. This is only possible if we set the Π equal to a constant. We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\Pi_1 = \frac{\dot{Q}}{\dot{m}c_p\left(T_{\text{out}} - T_{\text{in}}\right)} = \text{constant}} \qquad (2)$$

Discussion When there is only one Π, we know the functional relationship to within some (unknown) constant. In this particular case, comparing to Eq. 1 of Problem 7-24, we see that the constant is unity, $\dot{Q} = \dot{m}c_p\left(T_{\text{out}} - T_{\text{in}}\right)$. There is no way to obtain the constant in Eq. 2 from dimensional analysis; however, *one* experiment would be sufficient to determine the constant.

Experimental Testing and Incomplete Similarity

7-69C

Solution We are to define wind tunnel blockage and discuss its acceptable limit. We are also to discuss the source of measurement errors at high values of blockage.

Analysis Wind tunnel blockage is defined as the **ratio of model frontal area to cross-sectional area of the test-section**. The rule of thumb is that the blockage should be no more than 7.5%. If the blockage were significantly higher than this value, the flow would have to accelerate around the model much more than if the model were in an unbounded situation. Hence, similarity would not be achieved. We might expect the aerodynamic drag on the model to be too high since the *effective* freestream speed is too large due to the blockage.

Discussion There are formulas to correct for wind tunnel blockage, but they become less and less reliable as blockage increases.

7-70C

Solution We are to discuss the rule of thumb concerning Mach number and incompressibility.

Analysis **The rule of thumb is that the Mach number must stay below about 0.3 in order for the flow field to be considered "incompressible"**. What this really means is that compressibility effects, although present at all Mach numbers, are negligibly small compared to other effects driving the flow. If Ma is larger than about 0.3 in a wind tunnel test, the model flow field loses both kinematic and dynamic similarity, and the measured results are questionable. Of course, the error increases as Ma increases.

Discussion Compressible flow is discussed in detail in Chap. 12. There you will see where the value 0.3 comes from.

7-71C

Solution We are to discuss some situations in which a model should be larger than its prototype.

Analysis There are many possible situations, and students' examples should vary. Generally, any flow field that is very small and/or very fast benefits from simulation with a larger model. In most cases these are situations in which we want the model to be larger and slower so that experimental measurements and flow visualization are easier. Here are a few examples:

- **Modeling a hard disk drive.**
- **Modeling insect flight.**
- **Modeling the settling of very small particles in air or water.**
- **Modeling the motion of water droplets in clouds.**
- **Modeling flow through very fine tubing.**
- **Modeling biological systems like blood flow through capillaries, flow in the bronchi of lungs, etc.**

Discussion You can think of several more examples.

7-72C

Solution We are to discuss the purpose of a moving ground belt and suggest an alternative.

Analysis From the frame of reference of a moving car, both the air and the ground approach the car at freestream speed. When we test a model car in a wind tunnel, the air approaches at freestream speed, but the ground (floor of the wind tunnel) is *stationary*. Therefore we are not modeling the same flow. A boundary layer builds up on the wind tunnel floor, and the flow under the car cannot be expected to be the same as that under a real car. A moving ground belt solves this problem. Another way to say the same thing is to say that without the moving ground belt, there would not be kinematic similarity between the underside of the model and the underside of the prototype.

If a moving ground belt is unavailable, we could instead **install a false wall – i.e., a thin flat plate just above the boundary layer on the floor of the wind tunnel**. A sketch is shown in Fig. 1. At least then the boundary layer will be very thin and will not have as much influence on the flow under the model.

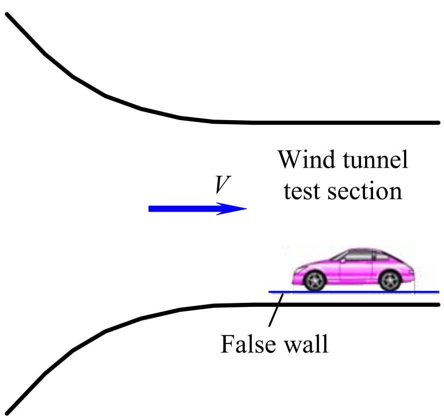

FIGURE 1
A false wall along the floor of a wind tunnel to reduce the size of the ground boundary layer.

Discussion We discuss boundary layer growth in Chap. 10.

7-73

Solution We are to show that Froude number and Reynolds number are the dimensionless parameters that appear in a problem involving shallow water waves.

Assumptions **1** Wave speed c is a function only of depth h, gravitational acceleration g, fluid density ρ, and fluid viscosity μ.

Analysis We perform a dimensional analysis using the method of repeating variables.

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $c = f\left(h, \rho, \mu, g\right) \qquad n = 5$

Step 2 The primary dimensions of each parameter are listed,

c	h	ρ	μ	g
$\left\{L^1 t^{-1}\right\}$	$\left\{L^1\right\}$	$\left\{m^1 L^{-3}\right\}$	$\left\{m^1 L^{-1} t^{-1}\right\}$	$\left\{L^1 t^{-2}\right\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale h, density difference ρ, and gravitational constant g.

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. Note that for the first Π we do the algebra in our heads since the relationship is very simple. The dependent Π is

Π_1 = *Froude number*:
$$\Pi_1 = \mathrm{Fr} = \frac{c}{\sqrt{gh}} \qquad (1)$$

This Π is the *Froude number*. Similarly, the Π formed with viscosity is generated,

$$\Pi_2 = \mu h^a \rho^b g^c \qquad \left\{\Pi_2\right\} = \left\{\left(m^1 L^{-1} t^{-1}\right)\left(L^1\right)^a \left(m^1 L^{-3}\right)^b \left(L^1 t^{-2}\right)^c\right\}$$

mass: $\left\{m^0\right\} = \left\{m^1 m^b\right\} \qquad 0 = 1 + b \qquad b = -1$

time: $\left\{t^0\right\} = \left\{t^{-1} t^{-2c}\right\} \qquad 0 = -1 - 2c \qquad c = -\frac{1}{2}$

length: $\left\{L^0\right\} = \left\{L^{-1} L^a L^{-3b} L^c\right\} \qquad \begin{array}{l} 0 = -1 + a - 3b + c \\ 0 = -1 + a + 3 - \frac{1}{2} \end{array} \qquad a = -\frac{3}{2}$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho h^{\frac{3}{2}}\sqrt{g}}$$

We can manipulate this Π into the Reynolds number if we invert it and then multiply by Fr (Eq. 1) The final form is

Modified Π_2 = *Reynolds number*: $$\Pi_2 = \mathrm{Re} = \frac{\rho c h}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\mathrm{Fr} = \frac{c}{\sqrt{gh}} = f\left(\mathrm{Re}\right) \quad \text{where} \quad \mathrm{Re} = \frac{\rho c h}{\mu}}$$

Discussion As discussed in this chapter, it is often difficult to match both Fr and Re between a model and a prototype.

7-74

Solution We are to nondimensionalize experimental pipe data, plot the data, and determine if Reynolds number independence has been achieved. We are then to extrapolate to a higher speed.

Assumptions **1** The flow is fully developed. **2** The flow is steady and incompressible.

Properties For water at $T = 20^\circ$C and atmospheric pressure, $\rho = 998.0$ kg/m^3 and $\mu = 1.002 \times 10^{-3}$ kg/m·s.

Analysis (*a*) We convert each data point in Table P7-74 from V and ΔP to Reynolds number and Euler number. The calculations at the last (highest speed) data point are shown here:

Reynolds number:

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{\left(998.0\text{ kg/m}^3\right)\left(50\text{ m/s}\right)\left(0.104\text{ m}\right)}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = 5.18\times10^6 \tag{1}$$

and

Euler number:

$$\text{Eu} = \frac{\Delta P}{\rho V^2} = \frac{758{,}700\text{ N/m}^2}{\left(998.0\text{ kg/m}^3\right)\left(50\text{ m/s}\right)^2}\left(\frac{\text{kg m}}{\text{s}^2\text{N}}\right) = 0.304 \tag{2}$$

We plot Eu versus Re in Fig. 1. Although there is experimental scatter in the data, it appears that Reynolds number independence has been achieved beyond a Reynolds number of about 2 × 10^6. The average value of Eu based on the last 6 data points is 0.3042.

(*b*) We extrapolate to higher speeds. At V = 80 m/s, we calculate ΔP, assuming that Eu remains constant to higher values of Re,

Extrapolated value:

$$\Delta P = \text{Eu}\times\rho V^2 = 0.3042\left(998.0\text{ kg/m}^3\right)\left(80\text{ m/s}\right)^2\left(\frac{\text{s}^2\text{N}}{\text{kg m}}\right) = \mathbf{1{,}940{,}000\text{ N/m}^2} \tag{3}$$

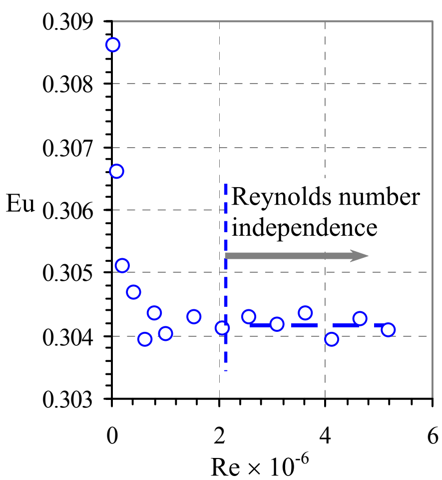

FIGURE 1
Nondimensionalized experimental data from a section of pipe.

Discussion It is shown in Chap. 8 that Reynolds number independence is indeed achieved at high-enough values of Re. The threshold value above which Re independence is achieved is a function of relative roughness height, ε/D.

7-75

Solution We are to calculate the wind tunnel blockage of a model truck in a wind tunnel and determine if it is within acceptable limits.

Assumptions **1** The frontal area is equal to truck width times height. (Note that the actual area of the truck may be somewhat smaller than this due to rounded corners and the air gap under the truck, but a truck looks nearly like a rectangle from the front, so this is not a bad approximation.)

Analysis Wind tunnel blockage is defined as the ratio of model frontal area to cross-sectional area of the test-section,

Blockage:
$$Blockage = \frac{A_{\text{model}}}{A_{\text{wind tunnel}}} = \frac{(0.159\text{ m})(0.257\text{ m})}{(1.2\text{ m})(1.0\text{ m})} = 0.034 = \mathbf{3.4\%} \qquad (1)$$

The rule of thumb is that the blockage should be no more than 7.5%. Since we are well below this value, we need not worry about blockage effects.

Discussion The *length* of the model does not enter our analysis since we are only concerned with the frontal area of the model.

7-76C

Solution We are to discuss whether Reynolds number independence has been achieved, and whether the researchers can be confident about it.

Analysis We remove the last four data points from Table 7-7 and from Fig. 7-42. From the remaining data it appears that the drag coefficient is beginning to level off, but is still decreasing with Re. Thus, **the researchers do not know if they have achieved Reynolds number independence or not**.

Discussion The wind tunnel speed is too low to achieve Reynolds number independence.

7-77E

Solution We are to calculate the size and scale of the model truck to be constructed, and calculate its maximum Reynolds number. Then we are to determine whether this model in this wind tunnel will achieve Reynolds number independence.

Assumptions **1** The model will be constructed carefully so as to achieve approximate geometric similarity. **2** The wind tunnel air is at the same temperature and pressure as that flowing over the prototype truck.

Properties For air at $T = 80^{\circ}$F and atmospheric pressure, $\rho = 0.07350$ lbm/ft^3 and $\mu = 1.248 \times 10^{-5}$ lbm/ft·s.

Analysis (*a*) The rule of thumb about blockage is that we should keep the blockage below 7.5%. Thus, the frontal area of the model truck must be no more than $0.075 \times A_{\text{wind tunnel}}$. The ratio of height to width of the full-scale truck is $H_p/W_p = 12/8.33 = 1.44$. Thus, for the geometrically similar model truck,

Equation for model truck width:

$$W_m = \frac{A_m}{H_m} = \frac{7.5\% A_{\text{wind tunnel}}}{1.44 W_m} \qquad (1)$$

We solve Eq. 1 for W_m,

Model truck width:

$$W_m = \sqrt{\frac{7.5\% A_{\text{wind tunnel}}}{1.44}} = \sqrt{\frac{0.075\left(400 \text{ in}^2\right)}{1.44}} = 4.56 \text{ in} \qquad (2)$$

Scaling the height and length geometrically,

Model truck dimensions:

$$W_m = \mathbf{4.56\ in},\ H_m = \mathbf{6.57\ in},\ L_m = \mathbf{28.5\ in} \qquad (3)$$

These dimensions represent a model that is scaled at approximately 1:22.

(*b*) At the maximum speed, with Re based on truck width,

Maximum Re:

$$\text{Re} = \frac{\rho W_m V_{max}}{\mu} = \frac{\left(0.07350 \text{ lbm/ft}^3\right)\left(4.56 \text{ in}\right)\left(160 \text{ ft/s}\right)}{1.248\times 10^{-5} \text{ lbm/ft}\cdot\text{s}}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \mathbf{3.58\times 10^5} \qquad (4)$$

(*c*) Based on the data of Fig. 7-42, **this Reynolds number is shy of the value needed to achieve Reynolds number independence**.

Discussion The students should run at the highest wind tunnel speed. Their measured values of C_D will probably be higher than those of the prototype, but the *relative* difference in C_D due to their modifications should still be valid.

7-78

Solution We are to calculate and plot C_D as a function of Re for a given set of wind tunnel measurements, and determine if dynamic similarity and/or Reynolds number independence have been achieved. Finally, we are to estimate the aerodynamic drag force acting on the prototype car.

Assumptions **1** The model car is geometrically similar to the prototype car. **2** The aerodynamic drag on the strut holding the model car is negligible.

Properties For air at atmospheric pressure and at $T = 25^{\circ}$C, $\rho = 1.184$ kg/m^3 and $\mu = 1.849 \times 10^{-5}$ kg/(m s).

Analysis We calculate C_D and Re for the last data point listed in Table P7-78 (at the fastest wind tunnel speed),

Model drag coefficient at last data point:

$$C_{D,\mathrm{m}} = \frac{F_{D,\mathrm{m}}}{\frac{1}{2}\rho_\mathrm{m} V_\mathrm{m}{}^2 A_\mathrm{m}} = \frac{4.91\text{ N}}{\frac{1}{2}\left(1.184\text{ kg/m}^3\right)\left(55\text{ m/s}\right)^2 \frac{\left(1.69\text{ m}\right)\left(1.30\text{ m}\right)}{16^2}}\left(\frac{\text{kg m}}{\text{s}^2\text{N}}\right) = 0.319$$

and

Model Reynolds number at last data point:

$$\mathrm{Re}_\mathrm{m} = \frac{\rho_\mathrm{m} V_\mathrm{m} W_\mathrm{m}}{\mu_\mathrm{m}} = \frac{\left(1.184\text{ kg/m}^3\right)\left(55\text{ m/s}\right)\left(\frac{1.69}{16}\text{ m}\right)}{1.849\times 10^{-5}\text{ kg/m}\cdot\text{s}} = 3.72\times 10^5 \qquad (1)$$

We repeat the above calculations for all the data points in Table P7-78, and we plot C_D verses Re in Fig. 1.

Have we achieved dynamic similarity? Well, we have *geometric* similarity between model and prototype, but the Reynolds number of the prototype car is

Reynolds number of prototype car:

$$\mathrm{Re}_\mathrm{p} = \frac{\rho_\mathrm{p} V_\mathrm{p} W_\mathrm{p}}{\mu_\mathrm{p}} = \frac{\left(1.184\text{ kg/m}^3\right)\left(29\text{ m/s}\right)\left(1.69\text{ m}\right)}{1.849\times 10^{-5}\text{ kg/m}\cdot\text{s}} = 3.14\times 10^6 \qquad (2)$$

where the width and speed of the prototype are used in the calculation of Re_p. Comparison of Eqs. 1 and 2 reveals that the prototype Reynolds number is more than eight times larger than that of the model. Since we cannot match the independent Πs in the problem, **dynamic similarity has not been achieved**.

Have we achieved Reynolds number independence? From Fig. 1 we see that **Reynolds number independence has indeed been achieved** – at Re greater than about 3×10^5, C_D has leveled off to a value of about 0.32 (to two significant digits).

Since we have achieved Reynolds number independence, we can extrapolate to the full scale prototype, assuming that C_D remains constant as Re is increased to that of the full scale prototype.

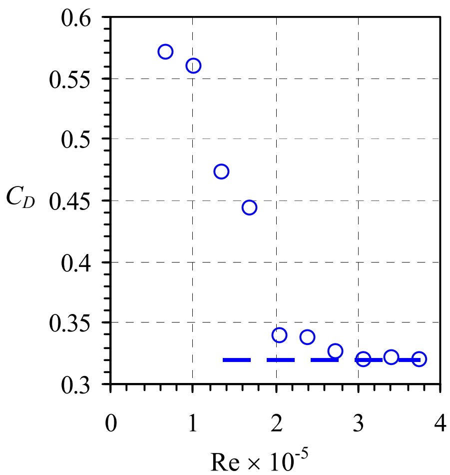

FIGURE 1
Aerodynamic drag coefficient as a function of Reynolds number – results nondimensionalized from wind tunnel test data on a model car.

Aerodynamic drag on the prototype:

$$F_{D,\mathrm{p}} = \frac{1}{2}\rho_\mathrm{p} V_\mathrm{p}^{\,2} A_\mathrm{p} C_{D,\mathrm{p}}$$

$$= \frac{1}{2}\left(1.184\ \mathrm{kg/m^3}\right)\left(29\ \mathrm{m/s}\right)^2\left(1.69\ \mathrm{m}\right)\left(1.30\ \mathrm{m}\right)0.32\left(\frac{\mathrm{s^2N}}{\mathrm{kg\ m}}\right) = \mathbf{350\ N}$$

Discussion We give our final result to two significant digits.

Review Problems

7-79C

Solution

(*a*) **False**: Kinematic similarity is a necessary but not sufficient condition for dynamic similarity.

(*b*) **True**: You cannot have dynamic similarity if the model and prototype are not geometrically similar.

(*c*) **True**: You cannot have kinematic similarity if the model and prototype are not geometrically similar.

(d) **False**: It is possible to have kinematic similarity (scaled velocities at corresponding points), yet not have dynamic similarity (forces do not scale at corresponding points).

7-80C

Solution We are to think of and describe a prototype and model flow in which there is geometric but not kinematic similarity even though $\mathrm{Re_m} = \mathrm{Re_p}$.

Analysis Students' responses will vary. Here are some examples:

- A model car is being tested in a wind tunnel such that there is geometric similarity and the wind tunnel speed is adjusted so that $\mathrm{Re_m} = \mathrm{Re_p}$. However, there is not a moving ground belt, so there is not kinematic similarity between the model and prototype.
- A model airplane is being tested in a wind tunnel such that there is geometric similarity and the wind tunnel speed is adjusted so that $\mathrm{Re_m} = \mathrm{Re_p}$. However, the Mach numbers are quite different, and therefore kinematic similarity is not achieved.
- A model of a river or waterfall or other open surface flow problem in which there is geometric similarity and the speed is adjusted so that $\mathrm{Re_m} = \mathrm{Re_p}$. However, the Froude numbers do not match and therefore the velocity fields are not similar and kinematic similarity is not achieved.

Discussion There are many more acceptable cases that students may imagine.

7-81C

Solution We are to find at least three established nondimensional parameters not listed in Table 7-5, and list these following the format of that table.

Analysis Students' responses will vary. Here are some examples:

Name	Definition	Ratio of significance
Bingham number	$\mathrm{Bm} = \dfrac{\tau L}{\mu V}$	$\dfrac{\text{yield stress}}{\text{viscous stress}}$
Elasticity number	$\mathrm{El} = \dfrac{t_c \mu}{\rho L^2}$	$\dfrac{\text{elastic force}}{\text{inertial force}}$
Galileo number	$\mathrm{Ga} = \dfrac{g D^3 \rho^2}{\mu^2}$	$\dfrac{\text{gravitational force}}{\text{viscous force}}$

In the above, t_c is a characteristic time.

Discussion There are many more established dimensionless parameters in the literature. Some sneaky students may make up their own!

-82

Solution We are to determine the primary dimensions of each variable, and then show that Hooke's law is dimensionally homogeneous.

Analysis
(*a*) Moment of inertia has dimensions of length^4,

Primary dimensions of moment of inertia: $$\{I\} = \{\text{length}^4\} = \{\mathbf{L^4}\} \qquad (1)$$

(*b*) Modulus of elasticity has the same dimensions as pressure,

Primary dimensions of modulus of elasticity:
$$\{E\} = \left\{\frac{\text{force}}{\text{area}}\right\} = \left\{\frac{\text{mass}\times\text{length}}{\text{time}^2}\times\frac{1}{\text{length}^2}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{Lt^2}}\right\} \qquad (2)$$

Or, in exponent form, $\{E\}$ = $\{\mathbf{m^1\,L^{-1}\,t^{-2}}\}$.

(*c*) Strain is defined as change in length per unit length, so it is dimensionless.

Primary dimensions of strain: $$\{\varepsilon\} = \left\{\frac{\text{length}}{\text{length}}\right\} = \{\mathbf{1}\} \qquad (3)$$

(*d*) Stress is force per unit area, again just like pressure.

Primary dimensions of stress: $$\{\sigma\} = \left\{\frac{\text{force}}{\text{area}}\right\} = \left\{\frac{\text{mass}\times\text{length}}{\text{time}^2\times\text{length}^2}\right\} = \left\{\frac{\mathbf{m}}{\mathbf{Lt^2}}\right\} \qquad (4)$$

Or, in exponent form, $\{\sigma\}$ = $\{\mathbf{m^1\,L^{-1}\,t^{-2}}\}$.

(*e*) Hooke's law is $\sigma = E\varepsilon$. We write the primary dimensions of both sides:

Primary dimensions of Hooke's law: $$\{\sigma\} = \left\{\frac{\mathbf{m}}{\mathbf{Lt^2}}\right\} = \{E\varepsilon\} = \left\{\frac{\text{m}}{\text{Lt}^2}\times 1\right\} = \left\{\frac{\mathbf{m}}{\mathbf{Lt^2}}\right\} \qquad (5)$$

Or, in exponent form, the dimensions of both sides of the equation are $\{\mathbf{m^1\,L^{-1}\,t^{-2}}\}$. Thus we see that **Hooke's law is indeed dimensionally homogeneous**.

Discussion If the dimensions of Eq. 5 were not homogeneous, we would surely expect that we made an error somewhere.

7-83

Solution We are to find the functional relationship between the given parameters and name any established dimensionless parameters.

Assumptions **1** The given parameters are the only ones relevant to the flow at hand.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$, *List of relevant parameters*: $z_d = f\left(F, L, E, I\right) \qquad n = 5 \qquad (1)$

Step 2 The primary dimensions of each parameter are listed,

z_d	F	L	E	I
$\{L^1\}$	$\{m^1L^1t^{-2}\}$	$\{L^1\}$	$\{m^1L^{-1}t^{-2}\}$	$\{L^4\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since j = 3. We cannot pick both length L and moment of inertia I since their dimensions differ only by a power. We also notice that we cannot choose F, L, and E since these three parameters can form a Π all by themselves. So, we set j = 3 – 1 = 2, and we choose two repeating parameters, expecting 5 – 2 = 3 Πs,

Repeating parameters: L and E

Step 5 The Πs are generated. Note that for the first Π we do the algebra in our heads since z_d has the same dimensions as L. The dependent Π is

Π_1: $\Pi_1 = \dfrac{z_d}{L}$

This Π is not an established dimensionless group, although it is a ratio of two lengths, **similar to an aspect ratio**.

We form the second Π with force F:

$$\Pi_2 = FL^aE^b \qquad \{\Pi_2\} = \left\{\left(m^1L^1t^{-2}\right)\left(L^1\right)^a\left(m^1L^{-1}t^{-2}\right)^b\right\}$$

mass: $\{m^0\} = \{m^1m^b\} \qquad 0 = 1 + b \qquad b = -1$

time: $\{t^0\} = \{t^{-2}t^{-2b}\} \qquad 0 = -2 - 2b \qquad b = -1$

length: $\{L^0\} = \{L^1L^aL^{-b}\} \qquad 0 = 1 + a - b, \quad a = -1 + b \qquad a = -2$

which yields

Π_2: $\Pi_2 = \dfrac{F}{L^2E}$

We do not recognize Π_2 as a named dimensionless parameter.

The final Π is formed with moment of inertia. Since {I} = {L^4}, there is no need to go through the algebra – we write

Π_3: $$\Pi_3 = \frac{I}{L^4}$$

Again, we do not recognize Π_2 as a named dimensionless parameter.

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\frac{z_d}{L} = f\left(\frac{F}{L^2 E}, \frac{I}{L^4}\right) \tag{2}$$

Discussion We cannot determine the *form* of the relationship by purely dimensional reasoning since there are three Πs.

7-84

Solution We are to generate dimensionless relationships among given parameters, and then we are to discuss how ΔP decreases if the time is doubled.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis (*a*) We perform dimensional analyses using the method of repeating variables. First we analyze ΔP:

Step 1 There are four parameters in this problem; $n = 4$,

List of relevant parameters: $$\Delta P = f(t, c, E) \qquad n = 4 \tag{1}$$

Step 2 The primary dimensions of each parameter are listed,

ΔP	t	c	E
$\{m^1L^{-1}t^{-2}\}$	$\{t^1\}$	$\{L^1t^{-1}\}$	$\{m^1L^2t^{-2}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 4 - 3 = 1$$

Step 4 We need to choose three repeating parameters since j = 3. We pick all the independent parameters – time t, speed of sound c, and energy E,

Repeating parameters: t, c, and E

Step 5 The dependent Π is generated:

$$\Pi_1 = \Delta P \times t^{a_1} c^{b_1} E^{c_1} \qquad \{\Pi_1\} = \left\{\left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-2}\right)\left(\mathrm{t}^1\right)^{a_1}\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^{b_1}\left(\mathrm{m}^1\mathrm{L}^2\mathrm{t}^{-2}\right)^{c_1}\right\}$$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{c_1}\}$ $\qquad 0 = 1 + c_1$ $\qquad c_1 = -1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{-1}\mathrm{L}^{b_1}\mathrm{L}^{2c_1}\}$ $\qquad 0 = -1 + b_1 + 2c_1$, $b_1 = 1 - 2c_1$ $\qquad b_1 = 3$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-2}\mathrm{t}^{a_1}\mathrm{t}^{-b_1}\mathrm{t}^{-2c_1}\}$ $\qquad 0 = -2 + a_1 - b_1 - 2c_1$, $a_1 = 2 + b_1 + 2c_1$ $\qquad a_1 = 3$

The dependent Π is thus

Π_1 for ΔP: $$\Pi_1 = \frac{t^3 c^3 \Delta P}{E}$$

This Π is not an established one, so we leave it as is.

Step 6 We write the final functional relationship as

Relationship between Πs: $$\boxed{\Delta P = \text{constant}\frac{E}{t^3 c^3}} \qquad (2)$$

We perform a similar dimensional analysis using the same repeating variables, but this time for radius r. We do not show the algebra since the Π can be found by inspection. We get

Π_1 for r: $$\Pi_1 = \frac{r}{ct}$$

Since this is the only Π, it must be equal to a constant,

Relationship between Πs: $$\boxed{r = \text{constant} \cdot ct} \qquad (3)$$

(*b*) From Eq. 2 we see that **if *t* is doubled, Δ*P* decreases by a factor of $2^3 = 8$**.

Discussion The pressure rise across the blast wave decays rapidly with time (and with distance from the explosion). The speed of sound depends on temperature. If the explosion is of sufficient strength, T will increase significantly and c will not remain constant.

7-85

Solution We are to find an alternate definition of Archimedes number, and list it following the format of Table 7-5. Then we are to find an established Π group that is similar.

Analysis Students' responses will vary. There seems to be a plethora of definitions of Archimedes number. Here is the one most appropriate for buoyant fluids:

Name	Definition	Ratio of significance
Archimedes number	$\text{Ar} = \dfrac{gL\Delta\rho}{\rho V^2}$	$\dfrac{\text{buoyant force}}{\text{inertial force}}$

In the above, $\Delta\rho$ is a characteristic density difference in the fluid (due to buoyancy) and ρ is a characteristic or average density of the fluid. A glance through Table 7-5 shows that the **Richardson number** is very similar to this alternative definition of Ar. In fact, the alternate form of Ri (Problem 7-55) is *identical* to our new Ar.

Discussion Some students may find other definitions that are also valid. For example, $\Delta\rho/\rho$ may be replaced by $\Delta T/T$.

7-86

Solution We are to generate a dimensionless relationship between the given parameters.

Assumptions **1** The flow is steady. **2** The flow is two-dimensional in the *x*-*y* plane. **3** The flow is fully developed.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters:
$$u = f\left(h, \frac{dP}{dx}, \mu, y\right) \qquad n = 5 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

u	h	dP/dx	μ	y
$\{\text{L}^1\text{t}^{-1}\}$	$\{\text{L}^1\}$	$\{\text{m}^1\text{L}^{-2}\text{t}^{-2}\}$	$\{\text{m}^1\text{L}^{-1}\text{t}^{-1}\}$	$\{\text{L}^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since j = 3. We cannot pick both h and y since they have the same dimensions. We choose

Repeating parameters: h, dP/dx, and μ

Step 5 The dependent Π is generated:

$$\Pi_1 = uh^{a_1}\left(\frac{dP}{dx}\right)^{b_1}\mu^{c_1} \qquad \left\{\Pi_1\right\} = \left\{\left(\mathrm{L}^1\mathrm{t}^{-1}\right)\left(\mathrm{L}^1\right)^{a_1}\left(\mathrm{m}^1\mathrm{L}^{-2}\mathrm{t}^{-2}\right)^{b_1}\left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right)^{c_1}\right\}$$

mass: $\left\{\mathrm{m}^0\right\} = \left\{\mathrm{m}^{b_1}\mathrm{m}^{c_1}\right\}$ $\quad 0 = b_1 + c_1$ $\quad c_1 = -b_1$

time: $\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{-2b_1}\mathrm{t}^{-c_1}\right\}$ $\quad 0 = -1 - 2b_1 - c_1$ $\quad b_1 = -1$

$0 = -1 - b_1$ $\quad c_1 = 1$

length: $\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^1\mathrm{L}^{a_1}\mathrm{L}^{-2b_1}\mathrm{L}^{-c_1}\right\}$ $\quad 0 = 1 + a_1 - 2b_1 - c_1$ $\quad a_1 = -2$

$0 = 1 + a_1 + 1$

The dependent Π is thus

Π_1:
$$\Pi_1 = \frac{\mu u}{h^2 \frac{dP}{dx}}$$

The independent Π is generated with variable y. Since $\{y\} = \{\mathrm{L}\}$, and this is the same as one of the repeating variables (h), Π_2 is simply y/h,

Π_2:
$$\Pi_2 = \frac{y}{h}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*:
$$\boxed{\Pi_1 = \frac{\mu u}{h^2 \frac{dP}{dx}} = f\left(\frac{y}{h}\right)} \qquad (2)$$

Discussion We solve this problem exactly in Chap. 9 where we see that the functional relationship of Eq. 2 is correct.

7-87

Solution We are to generate a dimensionless relationship between the given parameters and then analyze the behavior of u_{max} when an independent variable is doubled.

Assumptions **1** The flow is steady. **2** The flow is two-dimensional in the *x*-*y* plane. **3** The flow is fully developed.

Analysis (*a*) A step-by-step dimensional analysis procedure could be performed. However, we notice that u_{max} has the same dimensions as u. Therefore the algebra would be identical to that of Problem 7-86 except that there is only one Π instead of two since y is no longer a parameter. The result is

Relationship between Πs:

$$\Pi_1 = \frac{\mu u_{max}}{h^2 \frac{dP}{dx}} = \text{constant} = C \qquad (1)$$

or

Final relationship for u_{max}:

$$\boxed{u_{max} = C \frac{h^2}{\mu} \frac{dP}{dx}} \qquad (2)$$

Alternatively, we can use the results of Problem 7-86 directly. Namely, since we know that the maximum velocity occurs at the centerline, $y/h = 1/2$ there, and is a constant. Hence, Eq. 2 of Problem 7-86 reduces to Eq. 1 of the present problem.

(*b*) If h doubles, we see from Eq. 2 that u_{max} will increase by a factor of $2^2 = 4$.

(*c*) If dP/dx doubles, we see from Eq. 2 that u_{max} will increase by a factor of $2^1 = 2$.

(*d*) Since there is only one Π in this problem, we would need to conduct only *one experiment* to determine the constant C in Eq. 2.

Discussion The constant turns out to be -1/8, but there is no way to determine this from dimensional analysis alone (See Chap. 9 for an exact solution).

CD-EES 7-88

Solution We are to generate a relationship for Darcy friction factor f in terms of Euler number Eu. We then are to plot f as a function of Re and discuss whether Reynolds number independence has been achieved.

Assumptions **1** The flow is fully developed. **2** The flow is steady and incompressible.

Properties For water at $T = 20^\circ$C and atmospheric pressure, $\rho = 998.0$ kg/m^3 and $\mu = 1.002 \times 10^{-3}$ kg/m·s.

Analysis (*a*) Since the flow is fully developed, the control volume cuts through two cross sections in which the velocity profiles are identical. The flow is also steady, so the control volume momentum equation in the horizontal (x) direction reduces to

Conservation of momentum:
$$\sum F_x = \sum F_{x,\text{ pressure}} + \sum F_{x,\text{ shear stress}} = 0 \qquad (1)$$

We multiply pressure by cross-sectional area to obtain the pressure force, and wall shear stress times inner pipe wall surface area to obtain the shear stress force,

$$\sum F_{x,\text{ pressure}} = \Delta P \frac{\pi D^2}{4} \qquad \sum F_{x,\text{ shear stress}} = -\tau_w \pi D L \qquad (2)$$

Note the negative sign in the shear stress term since τ_w points to the left. We substitute Eq. 2 into Eq. 1. After some algebra,

Result:
$$\Delta P = \frac{4\tau_w L}{D} \qquad (3)$$

Finally, we divide both sides of Eq. 3 by ρV^2 to convert ΔP into an Euler number,

Nondimensional relationship:
$$\text{Eu} = \frac{\Delta P}{\rho V^2} = \frac{4\tau_w L}{\rho V^2 D} = \frac{1}{2}\frac{L}{D}\left(\frac{8\tau_w}{\rho V^2}\right) \qquad (4)$$

We recognize the term in parentheses on the right as the Darcy friction factor. Thus,

Final nondimensional relationship:
$$\boxed{\text{Eu} = \frac{1}{2}\frac{L}{D} f} \quad \text{or} \quad \boxed{f = 2\frac{D}{L}\text{Eu}} \qquad (5)$$

(*b*) We use Eq. 5 to calculate f at each data point of Table P7-74. We plot f as a function of Re in Fig. 1. We see that the behavior of f mimics that of Eu (as it must because of Eq. 5 where we see that f is just a constant times Eu). Since Eu shows Reynolds number independence for Re greater than about 2×10^6, so does f. **We see Reynolds number independence for Re greater than about 2×10^6.** From the plot, the extrapolated value of f at large Re is about 0.04867, which agrees with Eq. 5 when we plug in the Re-independent value of Eu,

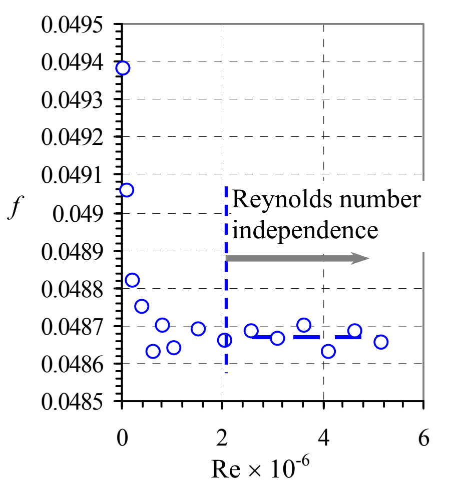

FIGURE 1
Nondimensionalized experimental data from a section of pipe.

Extrapolated value of f:
$$f = 2\frac{D}{L}\text{Eu} = 2\frac{0.104\text{ m}}{1.3\text{ m}}(0.3042) = \mathbf{0.0487} \qquad (6)$$

Discussion We show in Chap. 9 (the Moody chart) that f does indeed flatten out at high enough values of Re, depending on the relative roughness height, ε/D.

7-89

Solution We are to create characteristic scales so that we can define a desired established dimensionless parameter.

Analysis (*a*) For Froude number we need a velocity scale, a length scale, and gravity. We already have a length scale and gravity. We create a velocity scale as $\dot{V} / L$. We then define a Froude number as

Froude number:
$$\mathrm{Fr} = \frac{V}{\sqrt{gL}} = \frac{\dot{V}}{L\sqrt{gL}} = \frac{\dot{V}}{\sqrt{gL^3}}$$

(*b*) For Reynolds number we need a velocity scale, a length scale, and kinematic viscosity. Of these we only have the kinematic viscosity, so we need to create a velocity scale and a length scale. After a "back of the envelope" analysis, we create a velocity scale as $\dot{V} / L$ where L is some undefined characteristic length scale. Thus,

Reynolds number:
$$\mathrm{Re} = \frac{LV}{\nu} = \frac{L\dot{V}}{L\nu} = \frac{\dot{V}}{\nu}$$

Note that in this case, the length scales drop out, so it doesn't matter that we could not define a length scale from the given parameters.

(*c*) For Richardson number we need a length scale, the gravitational constant, a volume flow rate, a density, and a density difference. Of these we have all but the volume flow rate, so we create a volume flow rate scale as $\dot{V} L$. Thus,

Richardson number:
$$\mathrm{Ri} = \frac{L^5 g\Delta\rho}{\rho\dot{V}^2} = \frac{L^5 g\Delta\rho}{\rho\left(\dot{V}L\right)^2} = \frac{L^3 g\Delta\rho}{\rho\left(\dot{V}\right)^2}$$

Discussion You can verify that each of the parameters above is dimensionless.

7-90

Solution We are to find the functional relationship between the given parameters and name any established dimensionless parameters.

Assumptions **1** The given parameters are the only ones relevant to the flow at hand.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are seven parameters in this problem; $n = 7$,

List of relevant parameters: $$V = f\left(d, D, \rho, \mu, h, g\right) \qquad n = 7 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

V	d	D	ρ	μ	h	g

$$\{L^1t^{-1}\} \quad \{L^1\} \quad \{L^1\} \quad \{m^1L^{-3}\} \quad \{m^1L^{-1}t^{-1}\} \quad \{L^1\} \quad \{L^1t^{-2}\}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 7 - 3 = 4$

Step 4 We need to choose three repeating parameters since j = 3. We pick length scale h, fluid density ρ, and gravitational constant g.

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. Note that in this case we do the algebra in our heads since these relationships are very simple. The dependent Π is

Π_1 = *a Froude number*: $\Pi_1 = \dfrac{V}{\sqrt{gh}}$

This Π is a type of **Froude number**. Similarly, the two length-scale Πs are obtained easily,

Π_2: $\Pi_2 = \dfrac{d}{h}$

and

Π_3: $\Pi_3 = \dfrac{D}{h}$

Finally, the Π formed with viscosity is generated,

$$\Pi_4 = \mu h^{a_4}\rho^{b_4}g^{c_4} \qquad \{\Pi_4\} = \left\{\left(m^1L^{-1}t^{-1}\right)\left(L^1\right)^{a_4}\left(m^1L^{-3}\right)^{b_4}\left(L^1t^{-2}\right)^{c_4}\right\}$$

mass: $\{m^0\} = \{m^1m^{b_4}\}$ $\qquad 0 = 1 + b_4$ $\qquad b_4 = -1$

time: $\{t^0\} = \{t^{-1}t^{-2c_4}\}$ $\qquad 0 = -1 - 2c_4$ $\qquad c_4 = -\dfrac{1}{2}$

length: $\{L^0\} = \{L^{-1}L^{a_4}L^{-3b_4}L^{c_4}\}$ $\qquad \begin{array}{l} 0 = -1 + a_4 - 3b_4 + c_4 \\ 0 = -1 + a_4 + 3 - \frac{1}{2} \end{array}$ $\qquad a_4 = -\dfrac{3}{2}$

which yields

Π_4: $\Pi_4 = \dfrac{\mu}{\rho h^{\frac{3}{2}}\sqrt{g}}$

We recognize this Π as the inverse of a kind of **Reynolds number**. We also split the *h* terms to separate them into a length scale and (when combined with *g*) a velocity scale. The final form is

Modified Π_4 = *a Reynolds number*: $$\Pi_4 = \frac{\rho h \sqrt{gh}}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\frac{V}{\sqrt{gh}} = f\left(\frac{d}{h}, \frac{D}{h}, \frac{\rho h\sqrt{gh}}{\mu}\right)} \quad (2)$$

Discussion You may choose different repeating variables, and may generate different nondimensional groups. If you do the algebra correctly, your answer is not "wrong" – you just may not get the same dimensionless groups.

7-91

Solution We are to find a dimensionless relationship among the given parameters.

Assumptions **1** The given parameters are the only ones relevant to the flow at hand.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are seven parameters in this problem; $n = 7$,

List of relevant parameters: $$t_{\text{empty}} = f\left(d, D, \rho, \mu, h, g\right) \qquad n = 7 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

t_{empty}	d	D	ρ	μ	h	g
$\{t^1\}$	$\{L^1\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1\}$	$\{L^1t^{-2}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 7 - 3 = 4$$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale h, fluid density ρ, and gravitational constant g. (Note: these are the same repeating parameters as in Problem 7-90.)

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. We leave out the details since the algebra is trivial and can be done by inspection in most cases. The dependent Π is

Π_1: $$\Pi_1 = t_{\text{empty}}\sqrt{\frac{g}{h}}$$

The rest of the Πs are identical to those of Problem 7-90.

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{t_{\text{empty}}\sqrt{\frac{g}{h}} = f\left(\frac{d}{h}, \frac{D}{h}, \frac{\rho h\sqrt{gh}}{\mu}\right)} \qquad (2)$$

Discussion You may choose different repeating variables, and may generate different nondimensional groups. If you do the algebra correctly, your answer is not “wrong” – you just may not get the same dimensionless groups.

7-92

Solution We are to calculate the temperature of water in a model test to ensure similarity with the prototype, and we are to predict the time required to empty the prototype tank.

Assumptions **1** The parameters specified in Problem 7-91 are the only parameters relevant to the problem. **2** The model and prototype are geometrically similar.

Properties For ethylene glycol at 60°C, $\nu = \mu/\rho = 4.75\times10^{-6}$ m²/s (given).

Analysis

(*a*) We use the functional relationship obtained in Problem 7-91,

Dimensionless relationship:
$$t_{\text{empty}}\sqrt{\frac{g}{h}} = f\left(\frac{d}{h},\frac{D}{h},\frac{\rho h\sqrt{gh}}{\mu}\right) \qquad (1)$$

Since the model and prototype are geometrically similar, $(d/h)_{\text{model}} = (d/h)_{\text{prototype}}$ and $(D/h)_{\text{model}} = (D/h)_{\text{prototype}}$. Thus, we are left with only one Π to match to ensure similarity. Namely, the Reynolds number parameter in Eq. 1 must be matched between model and prototype. Since g remains the same in either case, and using "m" for model and "p" for prototype,

Similarity:
$$\left(\frac{\rho h\sqrt{gh}}{\mu}\right)_{\text{m}} = \left(\frac{\rho h\sqrt{gh}}{\mu}\right)_{\text{p}} \quad \text{or} \quad \frac{\rho_{\text{m}}}{\mu_{\text{m}}} = \frac{\rho_{\text{p}}}{\mu_{\text{p}}}\left(\frac{h_{\text{p}}}{h_{\text{m}}}\right)^{\frac{3}{2}} \qquad (2)$$

We recognize that $\nu = \mu/\rho$, and we know that $h_{\text{p}}/h_{\text{m}} = 4$. Thus, Eq. 2 reduces to

Similarity:
$$\nu_{\text{m}} = \nu_{\text{p}}\left(\frac{h_{\text{p}}}{h_{\text{m}}}\right)^{\frac{-3}{2}} = 4.75\times10^{-6}\ \text{m}^2/\text{s}\,(4)^{\frac{-3}{2}} = 5.94\times10^{-7}\ \text{m}^2/\text{s} \qquad (3)$$

For similarity we need to find the temperature of water where the kinematic viscosity is 5.94×10^{-7} m²/s. By interpolation from the property tables, **the designers should run the model tests at a water temperature of 45.8°C**.

(*b*) At dynamically similar conditions, Eq. 1 yields

At dynamically similar conditions:

$$\left(t_{\text{empty}}\sqrt{\frac{g}{h}}\right)_{\text{p}} = \left(t_{\text{empty}}\sqrt{\frac{g}{h}}\right)_{\text{m}} \quad \rightarrow \quad t_{\text{empty,p}} = t_{\text{empty,m}}\sqrt{\frac{h_{\text{p}}}{h_{\text{m}}}} = 4.53\ \text{min}\sqrt{4} = \mathbf{9.06\ min} \qquad (5)$$

Discussion We set up Eqs. 3 and 5 in terms of ratios of h_{p} to h_{m} so that the actual dimensions are not needed – just the *ratio* is needed, and it is given.

7-93

Solution For the simplified case in which V depends only on h and g, we are to determine how V increases when h is doubled.

Assumptions **1** The given parameters are the only ones relevant to the problem.

Analysis We employ the dimensional analysis results of Problem 7-90. Dropping d, D, ρ, and μ from the list of parameters, we are left with $n = 3$,

List of relevant parameters: $$V = f(h, g) \qquad n = 3 \qquad (1)$$

We perform the analysis in our heads –only one Π remains, and it is therefore set to a constant. The final result of the dimensional analysis is

Relationship between Πs: $$\frac{V}{\sqrt{gh}} = \text{constant} \qquad (2)$$

Thus, when h is doubled, we can easily calculate the factor by which V increases,

Increase in V: $$\boxed{\frac{V_2}{\sqrt{gh_2}} = \frac{V_1}{\sqrt{gh_1}} \quad \text{or} \quad V_2 = V_1\sqrt{\frac{h_2}{h_1}} = V_1\sqrt{2}} \qquad (3)$$

Thus, **when *h* increases by a factor of 2, *V* increases by a factor of** $\sqrt{2}$.

Discussion We don't need to know the constant in Eq. 2 to solve the problem. However, it turns out that the constant is $\sqrt{2}$ (see Chap. 5).

7-94

Solution We are to verify the dimensions of particle relaxation time τ_p, and then identify the established dimensionless parameter formed by nondimensionalization of τ_p.

Analysis First we obtain the primary dimensions of τ_p,

Primary dimensions of τ_p: $$\{\tau_p\} = \left\{\frac{\frac{\text{m}}{\text{L}^3}\times\text{L}^2}{\frac{\text{m}}{\text{Lt}}}\right\} = \{\mathbf{t}\}$$

A characteristic time scale for the air flow is L/V. Thus, we nondimensionalize τ_p,

Nondimensionalized particle relaxation time: $$\tau_p{}^* = \tau_p = \frac{\rho_p d_p{}^2}{18\mu}\frac{V}{L}$$

From Table 7-5 we recognize this as the **Stokes number**, Stk,

Stokes number: $$\boxed{\text{Stk} = \frac{\rho_p d_p{}^2 V}{18\mu L}}$$

Discussion Stokes number is useful when studying the flow of aerosol particles.

7-95

Solution We are to compare the primary dimensions of each property in mass-based and force-based primary dimensions, and discuss.

Analysis From previous problems and examples in this chapter, we can write down the primary dimensions of each property in the mass-based system. We use the fundamental definitions of these quantities to generate the primary dimensions in the force-based system:

(*a*) For pressure *P* the primary dimensions are

Mass-based primary dimensions	**Force-based primary dimensions**
$\{P\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$	$\{P\} = \left\{\frac{\text{force}}{\text{area}}\right\} = \left\{\frac{\text{F}}{\text{L}^2}\right\}$

(*b*) For moment $\vec{M}$ the primary dimensions are

Mass-based primary dimensions	**Force-based primary dimensions**
$\{\vec{M}\} = \left\{\text{m}\frac{\text{L}^2}{\text{t}^2}\right\}$	$\{\vec{M}\} = \{\text{force} \times \text{moment arm}\} = \{\text{FL}\}$

(*c*) For energy *E* the primary dimensions are

Mass-based primary dimensions	**Force-based primary dimensions**
$\{E\} = \left\{\text{m}\frac{\text{L}^2}{\text{t}^2}\right\}$	$\{E\} = \{\text{force} \times \text{distance}\} = \{\text{FL}\}$

We see that (in these three examples anyway), the forced-base cases have only two primary dimensions represented (F and L), whereas the mass-based cases have three primary dimensions represented (m, L, and t). Some authors would prefer the force-based system because of its **reduced complexity** when dealing with forces, pressures, energies, etc.

Discussion Not all variables have a simpler form in the force-based system. Mass itself for example has primary dimensions of {m} in the mass-based system, but has primary dimensions of $\{\text{Ft}^2/\text{L}\}$ in the force-based system. In problems involving mass, mass flow rates, and/or density, the force-based system may not have any advantage.

7-96

Solution The pressure difference between the inside of a soap bubble and the outside air is to be analyzed with dimensional analysis and the method of repeating variables using the force-based system of primary dimensions.

Assumptions **1** The soap bubble is neutrally buoyant in the air, and gravity is not relevant. **2** No other variables or constants are important in this problem.

Analysis The step-by-step method of repeating variables is employed.

Step 1 There are three variables and constants in this problem; $n = 3$,

$$\Delta P = f\left(R, \sigma_s\right) \qquad n = 3 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed. The dimensions of pressure are force per area and those of surface tension are force per length.

ΔP	R	σ_s
$\left\{F^1L^{-2}\right\}$	$\left\{L^1\right\}$	$\left\{F^1L^{-1}\right\}$

Step 3 As a first guess, j is set equal to 2, the number of primary dimensions represented in the problem (F and L).

Reduction: $$j = 2$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 3 - 2 = 1$$

Step 4 We need to choose two repeating parameters since $j = 2$. Our only choices here are R and σ_s since ΔP is the dependent variable.

Step 5 The dependent Π is generated:

$$\Pi_1 = \Delta P R^{a_1} \sigma_s^{\ b_1} \qquad \left\{\Pi_1\right\} = \left\{F^0L^0\right\} = \left\{\left(F^1L^{-2}\right)L^{a_1}\left(F^1L^{-1}\right)^{b_1}\right\}$$

force: $$\left\{F^0\right\} = \left\{F^1F^{b_1}\right\} \qquad 0 = 1 + b_1 \qquad b_1 = -1$$

length: $$\left\{L^0\right\} = \left\{L^{-2}L^{a_1}L^{-b_1}\right\} \qquad \begin{aligned} 0 &= -2 + a_1 - b_1 \\ a_1 &= 2 + b_1 \end{aligned} \qquad a_1 = 1$$

Eq. 1 thus becomes

Π_1: $$\Pi_1 = \frac{\Delta P R}{\sigma_s} \qquad (2)$$

From Table 7-5, the established nondimensional parameter most similar to Eq. 2 is the **Weber number**, defined as a pressure times a length divided by surface tension. There is no need to further manipulate this Π.

Step 6 We now write the functional relationship between the nondimensional parameters. Since there is only one Π, it is a function of *nothing*, which means it must be a constant,

Relationship between Π*s*:

$$\Pi_1 = \frac{\Delta P R}{\sigma_s} = f\left(\text{nothing}\right) = \text{constant} \quad \rightarrow \quad \boxed{\Delta P = \text{constant}\frac{\sigma_s}{R}} \qquad (3)$$

The result using the force-based system of primary dimensions is indeed identical to the previous result using the mass-based system.

Discussion Because only two primary dimensions are represented in the problem when using the force-based system, the algebra is in fact a lot easier.

7-97

Solution We are to a third established nondimensional parameter that is formed by the product or ratio of two given established nondimensional parameters.

Analysis

(*a*) The product of Reynolds number and Prandtl number yields

Reynolds number times Prandtl number:
$$\mathrm{Re}\times\mathrm{Pr} = \frac{\rho L V}{\mu}\times\frac{c_P\mu}{k} = \frac{\rho L V c_P}{k} \qquad (1)$$

We recognize Eq. 1 as the *Peclet number*,

Peclet number:
$$\boxed{\mathrm{Pe} = \mathrm{Re}\times\mathrm{Pr} = \frac{\rho L V c_P}{k} = \frac{LV}{\alpha}} \qquad (2)$$

(*b*) The ratio of Schmidt number and Prandtl number yields

Schmidt number divided by Prandtl number:
$$\frac{\mathrm{Sc}}{\mathrm{Pr}} = \frac{\dfrac{\mu}{\rho D_{AB}}}{\dfrac{c_P\mu}{k}} = \frac{k}{\rho c_P D_{AB}} \qquad (3)$$

We recognize Eq. 3 as the *Lewis number*,

Lewis number:
$$\boxed{\mathrm{Le} = \frac{\mathrm{Sc}}{\mathrm{Pr}} = \frac{k}{\rho c_P D_{AB}} = \frac{\alpha}{D_{AB}}} \qquad (4)$$

(*c*) The product of Reynolds number and Schmidt number yields

Reynolds number times Schmidt number:
$$\mathrm{Re}\times\mathrm{Sc} = \frac{\rho L V}{\mu}\times\frac{\mu}{\rho D_{AB}} = \frac{LV}{D_{AB}} \qquad (5)$$

We recognize Eq. 5 as the *Sherwood number*,

Sherwood number:
$$\boxed{\mathrm{Sh} = \mathrm{Re}\times\mathrm{Sc} = \frac{LV}{D_{AB}}} \qquad (6)$$

Discussion Can you find any other such combinations from Table 7-5?

7-98

Solution We are to determine the relationship between four established nondimensional parameters, and then try to form the Stanton number by some combination of only *two* other established dimensionless parameters.

Analysis We manipulate Re, Nu, and Pr, guided by the known result. After some trial and error,

Stanton number:
$$\mathrm{St} = \frac{\mathrm{Nu}}{\mathrm{Re}\times\mathrm{Pr}} = \frac{\dfrac{Lh}{k}}{\dfrac{\rho VL}{\mu}\times\dfrac{\mu c_P}{k}} = \frac{h}{\rho c_p V} \qquad (1)$$

We recognize from Table 7-5 (or from Problem 7-97) that *Peclet number* is equal to the product of Reynolds number and Prandtl number. Thus,

Stanton number:
$$\mathrm{St} = \frac{\mathrm{Nu}}{\mathrm{Pe}} = \frac{\dfrac{Lh}{k}}{\dfrac{\rho LVc_P}{k}} = \frac{h}{\rho c_p V} \qquad (2)$$

Discussion Not all named, established dimensionless parameters are independent of other named, established dimensionless parameters.

7-99

Solution We are to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis First we do some thinking. If we imagine traveling at the same speed as the bottom plate, the flow would be identical to that of Problem 7-56 except that the top plate speed would be ($V_{\text{top}} - V_{\text{bottom}}$) instead of just V. The step-by-step method of repeating variables is otherwise identical to that of Problem 7-56, and the details are not included here. The final functional relationship is

Relationship between Πs:
$$\frac{u}{V_{\text{top}} - V_{\text{bottom}}} = f\left(\mathrm{Re}, \frac{y}{h}\right) \qquad (1)$$

where

Reynolds number:
$$\mathrm{Re} = \frac{\rho\left(V_{\text{top}} - V_{\text{bottom}}\right)h}{\mu} \qquad (2)$$

Discussion It is always wise to look for shortcuts like this to save us time.

7-100

Solution We are to determine the primary dimensions of electrical charge.

Analysis The fundamental definition of electrical current is charge per unit time. Thus,

Primary dimensions of charge:
$$\{q\} = \{\text{current} \times \text{time}\} = \{\mathbf{I\ t}\} \qquad (1)$$

Or, in exponent form, $\{q\}$ = $\{\mathbf{t^1\, I^1}\}$.

Discussion We see that all dimensions, even those of electrical properties, can be expressed in terms of primary dimensions.

7-101

Solution We are to determine the primary dimensions of electrical capacitance.

Analysis Electrical capacitance *C* is measured in units of farads (F). By definition, a one-farad capacitor with an applied electric potential of one volt across it will store one coulomb of electrical charge. Thus,

Primary dimensions of capacitance:
$$\{C\} = \{\text{charge / voltage}\} = \left\{\frac{\mathrm{I\ t}}{\dfrac{\mathrm{mL^2}}{\mathrm{t^3 I}}}\right\} = \left\{\frac{\mathbf{I^2 t^4}}{\mathbf{mL^2}}\right\} \qquad (1)$$

where the primary dimensions of voltage are obtained from Problem 7-10, and those of electric charge are obtained from Problem 7-100. Or, in exponent form, $\{C\}$ = $\{\mathbf{m^{-1}\, L^{-2}\, t^4\, I^2}\}$.

Discussion We see that all dimensions, even those of electrical properties, can be expressed in terms of primary dimensions.

7-102

Solution We are to determine the primary dimensions of electrical time constant *RC*, and discuss the significance of our result.

Analysis The primary dimensions of electrical resistance are obtained from Problem 7-11. Those of electrical capacitance *C* are obtained from Problem 7-101. Thus,

Primary dimensions of electrical time constant RC:

$$\{RC\} = \{\text{resistance} \times \text{capacitance}\} = \left\{\frac{\text{mL}^2}{\text{t}^3\text{I}^2} \times \frac{\text{I}^2\text{t}^4}{\text{mL}^2}\right\} = \{\mathbf{t}\} \tag{1}$$

Thus we see that the primary dimensions of *RC* are those of *time*. This explains why a resistor and capacitor in series is often used in timing circuits.

Discussion The cut-off frequency of the low-pass filter of Fig. P7-102 is proportional to $1/R{\cdot}C$. If the resistor and the capacitor were to swap places we would have a high-pass rather than a low-pass filter.

7-103

Solution We are to determine the primary dimensions of both sides of the equation, and we are to verify that the equation is dimensionally homogeneous.

Analysis The primary dimensions of the time derivative (d/dt) are 1/time. The primary dimensions of capacitance are current^2×time^4 / (mass×length^2), as obtained from Problem 7-101. Thus both sides of the equation can be written in terms of primary dimensions,

$$\{I\} = \{\text{current}\} \qquad \{I\} = \{\text{I}\}$$

$$C\frac{dE}{dt} = \left\{ \frac{\text{current}^2 \times \text{time}^4}{\text{mass} \times \text{length}^2} \frac{\dfrac{\text{mass} \times \text{length}^2}{\text{current} \times \text{time}^3}}{\text{time}} \right\} = \{\text{current}\} \qquad \left\{C\frac{dE}{dt}\right\} = \{\text{I}\}$$

Indeed, **both sides of the equation have the same dimensions, namely {*I*}**.

Discussion Current is one of our seven primary dimensions. These results verify our algebra in Problem 7-101.

7-104

Solution We are to find the functional relationship between the given parameters, and then answer some questions about scaling.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis

(*a*) The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are four parameters in this problem; $n = 4$,

List of relevant parameters: $$\delta P = f\left(\rho, \dot{V}, D\right) \qquad n = 4 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

δP	ρ	$\dot{V}$	D
$\{m^1L^{-1}t^{-2}\}$	$\{m^1L^{-3}\}$	$\{L^3t^{-1}\}$	$\{L^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 4 - 3 = 1$$

Step 4 We need to choose three repeating parameters since $j = 3$. We only have one choice in this problem, since there are only three independent parameters on the right-hand side of Eq. 1,

Repeating parameters: ρ, $\dot{V}$, and D

Step 5 The dependent Π is generated:

$$\Pi_1 = \delta P \rho^{a_1} \dot{V}^{b_1} D^{c_1} \qquad \{\Pi_1\} = \left\{\left(m^1L^{-1}t^{-2}\right)\left(m^1L^{-3}\right)^{a_1}\left(L^3t^{-1}\right)^{b_1}\left(L^1\right)^{c_1}\right\}$$

mass: $$\{m^0\} = \{m^{1+a_1}\} \qquad 0 = 1 + a_1 \qquad a_1 = -1$$

time: $$\{t^0\} = \{t^{-2-b_1}\} \qquad 0 = -2 - b_1 \qquad b_1 = -2$$

length: $$\{L^0\} = \{L^{-1}L^{-3a_1}L^{3b_1}L^{c_1}\} \qquad 0 = -1 - 3a_1 + 3b_1 + c_1 \qquad c_1 = 4$$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{D^4 \delta P}{\rho \dot{V}^2}$$

Step 6 Since there is only one Π, it is a function of nothing. This is only possible if we set the Π equal to a constant. We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\Pi_1 = \frac{D^4 \delta P}{\rho \dot{V}^2} = \text{constant}} \qquad (2)$$

(*b*) We re-write Eq. 2 as

Equation for δP: $$\delta P = \text{constant}\frac{\rho \dot{V}^2}{D^4} \qquad (3)$$

Thus, **if we double the size of the cyclone, the pressure drop will *decrease* by a factor of $2^4 = 16$**.

(c) Also from Eq. 3 we see that **if we double the volume flow rate, the pressure drop will increase by a factor of $2^2 = 4$**.

Discussion The pressure drop would be smallest for the *largest* cyclone operating at the *smallest* volume flow rate. (This agrees with our intuition.)

7-105

Solution We are to find the functional relationship between the given parameters, and then answer some questions about scaling.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis
(*a*) The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $$w = f\left(q_p, E_f, \mu, D_p\right) \qquad n = 5 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

$$\begin{array}{ccccc} w & q_p & E_f & \mu & D_p \\ \left\{\mathrm{L}^1\mathrm{t}^{-1}\right\} & \left\{\mathrm{I}^1\mathrm{t}^1\right\} & \left\{\mathrm{m}^1\mathrm{L}^1\mathrm{t}^{-3}\mathrm{I}^{-1}\right\} & \left\{\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right\} & \left\{\mathrm{L}^1\right\} \end{array}$$

where the primary dimensions of voltage are obtained from Problem 7-10, and those of electric charge are obtained from Problem 7-100.

Step 3 As a first guess, j is set equal to 4, the number of primary dimensions represented in the problem (m, L, t, and I).

Reduction: $$j = 4$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 5 - 4 = 1$$

Step 4 We need to choose four repeating parameters since $j = 4$. We only have one choice in this problem, since there are only four independent parameters on the right-hand side of Eq. 1,

Repeating parameters: q_p, E, D_p, and μ

Step 5 The dependent Π is generated:

$$\Pi_1 = wq_p^{a_1} E_f^{b_1} \mu^{c_1} D_p^{d_1} \qquad \{\Pi_1\} = \left\{ \left(\mathrm{L}^1\mathrm{t}^{-1}\right)\left(\mathrm{I}^1\mathrm{t}^1\right)^{a_1}\left(\mathrm{m}^1\mathrm{L}^1\mathrm{t}^{-3}\mathrm{I}^{-1}\right)^{b_1}\left(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right)^{c_1}\left(\mathrm{L}^1\right)^{d_1} \right\}$$

current: $\{\mathrm{I}^0\} = \{\mathrm{I}^{a_1}\mathrm{I}^{-b_1}\}$ $\qquad 0 = a_1 - b_1$ $\qquad a_1 = b_1$

mass: $\{\mathrm{m}^0\} = \{\mathrm{m}^{b_1}\mathrm{m}^{c_1}\}$ $\qquad 0 = b_1 + c_1$ $\qquad c_1 = -b_1 = -a_1$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-1}\mathrm{t}^{a_1}\mathrm{t}^{-3b_1}\mathrm{t}^{-c_1}\}$ $\qquad 0 = -1 + a_1 - 3b_1 - c_1$ $\qquad a_1 = b_1 = -1$, $c_1 = 1$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{b_1}\mathrm{L}^{-c_1}\mathrm{L}^{d_1}\}$ $\qquad 0 = 1 + b_1 - c_1 + d_1$ $\qquad d_1 = 1$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{w\mu D_p}{q_p E_f}$$

Step 6 Since there is only one Π, it is a function of nothing. This is only possible if we set the Π equal to a constant. We write the final functional relationship as

Relationship between Πs: $$\boxed{\Pi_1 = \frac{w\mu D_p}{q_p E} = \text{constant}} \qquad (2)$$

(*b*) We re-write Eq. 2 as

Equation for w: $$\boxed{w = \text{constant}\,\frac{q_p E_f}{\mu D_p}} \qquad (3)$$

Thus, **if we double the electric field strength, the drift velocity will increase by a factor of 2**.

(c) Also from Eq. 3 we see that **if we double the particle size, the drift velocity will decrease by a factor of 2**.

Discussion These results agree with our intuition. Certainly we would expect the drift velocity to increase if we increase the field strength. Also, larger particles have more aerodynamic drag, so for the same charge, we would expect a larger dust particle to drift more slowly than a smaller dust particle.

7-106

Solution We are to generate a dimensionless functional relationship between the given parameters and then compare our results with a known exact analytical solution.

Assumptions **1** There is no flow (hydrostatics). **2** The parameters listed here are the only relevant parameters in the problem.

Analysis (*a*) We perform a dimensional analysis using the method of repeating variables.

Step 1 There are five parameters in this problem; $n = 6$,

List of relevant parameters: $$h = f\left(\rho, g, \sigma_s, D, \phi\right) \qquad n = 6 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

h	ρ	g	σ_s	D	ϕ
$\{L^1\}$	$\{m^1L^{-3}\}$	$\{L^1t^{-2}\}$	$\{m^1t^{-2}\}$	$\{L^1\}$	$\{1\}$

Note that the dimensions of the contact angle are unity (angles are dimensionless).

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 6 - 3 = 3$$

Step 4 We need to choose three repeating parameters since $j = 3$. We cannot choose ϕ since it is dimensionless. We choose a length (D) and a density (ρ). We'd rather have gravitational constant g than surface tension σ_s in our Πs. So, we choose

Repeating parameters: $$\rho,\ g,\ D$$

Step 5 The dependent Π is generated. Since h has the same dimensions as D, we immediately write

Π_1: $$\Pi_1 = \frac{h}{D}$$

The first independent Π is generated by combining σ_s with the repeating parameters,

$$\Pi_2 = \sigma_s \rho^{a_2} g^{b_2} D^{c_2} \qquad \{\Pi_2\} = \left\{\left(m^1t^{-2}\right)\left(m^1L^{-3}\right)^{a_2}\left(L^1t^{-2}\right)^{b_2}\left(L^1\right)^{c_2}\right\}$$

mass: $$\{m^0\} = \{m^1m^{a_2}\} \qquad 0 = 1 + a_2 \qquad a_2 = -1$$

time: $\left\{t^0\right\} = \left\{t^{-2}t^{-2b_2}\right\}$ $\quad 0 = -2 - 2b_2 \quad b_2 = -1$

length: $\left\{L^0\right\} = \left\{L^{-3a_2}L^{b_2}L^{c_2}\right\}$ $\quad 0 = -3a_2 + b_2 + c_2 \quad c_2 = -2$

$c_2 = 3a_2 - b_2$

The first independent Π is thus

Π_2: $$\Pi_2 = \frac{\sigma_s}{\rho g D^2}$$

Finally, the third Π (second independent Π) is simply angle ϕ itself since it is dimensionless,

Π_3: $$\Pi_3 = \phi$$

Step 6 We write the final functional relationship as

Relationship between Πs: $$\boxed{\frac{h}{D} = f\left(\frac{\sigma_s}{\rho g D^2}, \phi\right)} \quad (2)$$

(*b*) From Chap. 2 we see that the exact analytical solution is

Exact relationship: $$h = \frac{4\sigma_s}{\rho g D}\cos\phi \quad (3)$$

Comparing Eqs. 2 and 3, we see that they are indeed of the same form. In fact,

Functional relationship: $$\boxed{\Pi_1 = \text{constant} \times \Pi_2 \times \cos\Pi_3} \quad (4)$$

Discussion We cannot determine the constant in Eq. 4 by dimensional analysis. However, one experiment is enough to establish the constant. Or, in this case we can find the constant exactly. Viscosity is not relevant in this problem since there is no fluid motion.

7-107

Solution We are to find a functional relationship for the time scale required for the liquid to climb up the capillary tube.

Assumptions **1** t_{rise} is a function of the same parameters listed in Problem 7-106, but there is another relevant parameter.

Analysis Since this is an unsteady problem, the rise time will surely depend also on fluid viscosity μ. The list of parameters now involves seven parameters,

List of relevant parameters: $$t_{\text{rise}} = f\left(\rho, g, \sigma_s, D, \phi, \mu\right) \qquad n = 7 \qquad (1)$$

and we expect four Πs. We choose the same repeating parameters and the algebra is similar to that of the previous problem. It turns out that

Π_1: $$\Pi_1 = t_{\text{rise}}\sqrt{\frac{g}{D}}$$

The second and third Π are the same as those of Problem 7-106. Finally, the fourth Π is formed by combining μ with the repeating parameters. We expect some kind of Reynolds number. We can do the algebra in our head. Specifically, a velocity scale can be formed as $\sqrt{gD}$. Thus,

Π_4: $$\Pi_4 = \text{Re} = \frac{\rho D\sqrt{gD}}{\mu}$$

The final functional relationship is

Relationship between Πs: $$\boxed{t_{\text{rise}}\sqrt{\frac{g}{D}} = f\left(\frac{\sigma_s}{\rho g D^2}, \phi, \text{Re}\right)} \qquad (2)$$

Discussion If we would have defined a time scale as $\sqrt{D/g}$, we could have written Π_1 by inspection as well, saving ourselves some algebra.

7-108

Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are four parameters in this problem; $n = 4$,

List of relevant parameters: $$I = f\left(P, c, \rho\right) \qquad n = 4 \qquad (1)$$

Step 2 The dimensions of I are those of power per area. The primary dimensions of each parameter are listed,

$$\begin{array}{cccc} I & P & c & \rho \\ \{m^1t^{-3}\} & \{m^1L^{-1}t^{-2}\} & \{L^1t^{-1}\} & \{m^1L^{-3}\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 4 - 3 = 1$

Step 4 We need to choose three repeating parameters since j = 3. *The problem is that the three independent parameters form a* Π *all by themselves* ($c^2\rho/P$ is dimensionless). Let's see what happens if we don't notice this, and we pick all three independent parameters as repeating variables,

Repeating parameters: P, ρ, and c

Step 5 The Π is generated:

$$\Pi_1 = I \times P^a \rho^b c^c \qquad \{\Pi_1\} = \left\{\left(m^1t^{-3}\right)\left(m^1L^{-1}t^{-2}\right)^a\left(m^1L^{-3}\right)^b\left(L^1t^{-1}\right)^c\right\}$$

mass: $\{m^0\} = \{m^1m^am^b\}$ $\qquad 0 = 1 + a + b \qquad a = -1 - b$

time: $\{t^0\} = \{t^{-3}t^{-2a}t^{-c}\}$ $\qquad 0 = -3 - 2a - c$, $c = -3 - 2a \qquad c = -1 + 2b$

length: $\{L^0\} = \{L^{-a}L^{-3b}L^c\}$ $\qquad 0 = -a - 3b + c$, $c = a + 3b \qquad c = -1 + 2b$

This is a situation in which two of the equations agree, but we cannot solve for unique exponents. If we knew b, we could get a and c. The problem is that any value of b we choose will make the Π dimensionless. For example, if we choose b = 1, we find that a = –2 and c = 1, yielding

Π_1 for the case with b = 1: $\qquad \Pi_1 = \dfrac{I\rho c}{P^2}$

Since there is only one Π, we write

Functional relationship for the case with b = 1: $\qquad I = \text{constant} \times \dfrac{P^2}{\rho c} \qquad (2)$

However, if we choose a different value of b, say b = –1, then a = 0 and c = –3, yielding

Π_1 for the case with $b = -1$: $\Pi_1 = \dfrac{I}{\rho c^3}$

Since there is only one Π, we write

Functional relationship for the case with $b = -1$: $I = \text{constant} \times \rho c^3$ (3)

Similarly, you can come up with a whole family of possible answers, depending on your choice of b. We double check our algebra and realize that any value of b works. Hence *the problem is indeterminate with three repeating variables*.

We go back now and realize that something is wrong. As stated previously, the problem is that the three independent parameters can form a dimensionless group all by themselves. This is another case where we have to reduce j by 1. Setting $j = 3 - 1 = 2$, we choose two repeating parameters,

Repeating parameters: ρ and c

We jump to Step 5 of the method of repeating variables,

Step 5 The first Π is generated:

$$\Pi_1 = I\rho^a c^b \qquad \{\Pi_1\} = \left\{\left(\text{m}^1\text{t}^{-3}\right)\left(\text{m}^1\text{L}^{-3}\right)^a\left(\text{L}^1\text{t}^{-1}\right)^b\right\}$$

mass: $\{\text{m}^0\} = \{\text{m}^1\text{m}^a\}$ $\quad 0 = 1 + a$ $\quad a = -1$

time: $\{\text{t}^0\} = \{\text{t}^{-3}\text{t}^{-b}\}$ $\quad 0 = -3 - b$ $\quad b = -3$

length: $\{\text{L}^0\} = \{\text{L}^{-3a}\text{L}^b\}$ $\quad 0 = -3a + b$, $b = 3a$ $\quad b = -3$

Fortunately, the results for time and length agree. The dependent Π is thus

Π_1: $\Pi_1 = \dfrac{I}{\rho c^3}$

We form the second Π with sound pressure P,

$$\Pi_2 = P\rho^e c^f \qquad \{\Pi_2\} = \left\{\left(\text{m}^1\text{L}^{-1}\text{t}^{-2}\right)\left(\text{m}^1\text{L}^{-3}\right)^e\left(\text{L}^1\text{t}^{-1}\right)^f\right\}$$

mass: $\{\text{m}^0\} = \{\text{m}^1\text{m}^e\}$ $\quad 0 = 1 + e$ $\quad e = -1$

time: $\{\text{t}^0\} = \{\text{t}^{-2}\text{t}^{-f}\}$ $\quad 0 = -2 - f$ $\quad f = -2$

length: $\{\text{L}^0\} = \{\text{L}^{-1}\text{L}^{-3e}\text{L}^f\}$ $\quad 0 = -1 - 3e + f$, $f = 1 + 3e$ $\quad f = -2$

The second Π is thus

Π_2: $$\Pi_2 = \frac{P}{\rho c^2}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*: $$\boxed{\frac{I}{\rho c^3} = f\left(\frac{P}{\rho c^2}\right)} \qquad (4)$$

(*b*) We try the force-based primary dimension system instead.

Step 1 There are four parameters in this problem; $n = 4$,

List of relevant parameters: $$I = f\left(P, c, \rho\right) \qquad n = 4 \qquad (5)$$

Step 2 The dimensions of I are those of power per area. The primary dimensions of each parameter are listed,

$$\begin{array}{cccc} I & P & c & \rho \\ \left\{\mathrm{F}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right\} & \left\{\mathrm{F}^1\mathrm{L}^{-2}\right\} & \left\{\mathrm{L}^1\mathrm{t}^{-1}\right\} & \left\{\mathrm{F}^1\mathrm{t}^2\mathrm{L}^{-4}\right\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (F, L, and t). Again, however, the three independent parameters form a dimensionless group all by themselves. Thus we lower j by 1.

Reduction: $$j = 2$$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $$k = n - j = 4 - 2 = 2$$

Step 4 We need to choose two repeating parameters since $j = 2$. We pick the same two parameters as in Part (*a*),

Repeating parameters: ρ and c

Step 5 The first Π is generated:

$$\Pi_1 = I \times \rho^b c^c \qquad \left\{\Pi_1\right\} = \left\{\left(\mathrm{F}^1\mathrm{L}^{-1}\mathrm{t}^{-1}\right)\left(\mathrm{F}^1\mathrm{t}^2\mathrm{L}^{-4}\right)^b\left(\mathrm{L}^1\mathrm{t}^{-1}\right)^c\right\}$$

force: $$\left\{\mathrm{F}^0\right\} = \left\{\mathrm{F}^1\mathrm{F}^b\right\} \qquad 0 = 1 + b \qquad b = -1$$

time: $$\left\{\mathrm{t}^0\right\} = \left\{\mathrm{t}^{-1}\mathrm{t}^{2b}\mathrm{t}^{-c}\right\} \qquad \begin{array}{l} 0 = -1 + 2b - c \\ c = -1 + 2b \end{array} \qquad c = -3$$

length: $$\left\{\mathrm{L}^0\right\} = \left\{\mathrm{L}^{-1}\mathrm{L}^{-4b}\mathrm{L}^c\right\} \qquad \begin{array}{l} 0 = -1 - 4b + c \\ c = 1 + 4b \end{array} \qquad c = -3$$

Again the two results for length and time agree. The dependent Π is thus

Π_1: $$\Pi_1 = \frac{I}{\rho c^3}$$

We form the second Π with sound pressure *P*,

$$\Pi_2 = P \times \rho^e c^f \qquad \{\Pi_2\} = \left\{\left(\mathrm{F^1L^{-2}}\right)\left(\mathrm{F^1t^2L^{-4}}\right)^e\left(\mathrm{L^1t^{-1}}\right)^f\right\}$$

force: $\{\mathrm{F}^0\} = \{\mathrm{F}^1\mathrm{F}^e\}$ $\quad 0 = 1 + e \quad$ $e = -1$

time: $\{\mathrm{t}^0\} = \{\mathrm{t}^{-2e}\mathrm{t}^{-f}\}$ $\quad 0 = -2e - f$, $f = -2e \quad$ $f = -2$

length: $\{\mathrm{L}^0\} = \{\mathrm{L}^{-2}\mathrm{L}^{-4e}\mathrm{L}^f\}$ $\quad 0 = -2 - 4e + f$, $f = 2 + 4e \quad$ $f = -2$

The second Π is thus

Π_2: $$\Pi_2 = \frac{P}{\rho c^2}$$

Step 6 We write the final functional relationship as

Relationship between Πs: $$\boxed{\frac{I}{\rho c^3} = f\left(\frac{P}{\rho c^2}\right)} \qquad (6)$$

Discussion Equations 4 and 6 are the same. This exercise shows that you should get the same results using mass-based or force-based primary dimensions.

7-109

Solution We are to find the dimensionless relationship between the given parameters.

Assumptions **1** The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are now five parameters in this problem; $n = 5$,

List of relevant parameters: $$I = f(P, c, \rho, r) \qquad n = 5 \qquad (1)$$

Step 2 The dimensions of *I* are those of power per area. The primary dimensions of each parameter are listed,

I	P	c	ρ	r
$\{m^1t^{-3}\}$	$\{m^1L^{-1}t^{-2}\}$	$\{L^1t^{-1}\}$	$\{m^1L^{-3}\}$	$\{L^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s*: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since j = 3. We pick the three simplest independent parameters (r instead of P),

Repeating parameters: r, ρ, and c

Step 5 The first Π is generated:

$$\Pi_1 = I \times r^a \rho^b c^c \qquad \{\Pi_1\} = \left\{\left(m^1t^{-3}\right)\left(L^1\right)^a\left(m^1L^{-3}\right)^b\left(L^1t^{-1}\right)^c\right\}$$

mass: $\{m^0\} = \{m^1m^b\}$ $\qquad 0 = 1 + b \qquad b = -1$

time: $\{t^0\} = \{t^{-3}t^{-c}\}$ $\qquad 0 = -3 - c \qquad c = -3$

length: $\{L^0\} = \{L^aL^{-3b}L^c\}$ $\qquad 0 = a - 3b + c$, $a = 3b - c \qquad a = 0$

The first Π is thus

Π_1: $$\Pi_1 = \frac{I}{\rho c^3}$$

We form the second Π with sound pressure P,

$$\Pi_2 = P \times r^d \rho^e c^f \qquad \{\Pi_2\} = \left\{\left(m^1L^{-1}t^{-2}\right)\left(L^1\right)^d\left(m^1L^{-3}\right)^e\left(L^1t^{-1}\right)^f\right\}$$

mass: $\{m^0\} = \{m^1m^e\}$ $\qquad 0 = 1 + e \qquad e = -1$

time: $\{t^0\} = \{t^{-2}t^{-f}\}$ $\qquad 0 = -2 - f \qquad f = -2$

length: $\{L^0\} = \{L^{-1}L^dL^{-3e}L^f\}$ $\qquad 0 = -1 + d - 3e + f$, $d = 1 + 3e - f \qquad d = 0$

The second Π is thus

Π_2:
$$\Pi_2 = \frac{P}{\rho c^2}$$

Step 6 We write the final functional relationship as

Relationship between Π*s*:
$$\boxed{\frac{I}{\rho c^3} = f\left(\frac{P}{\rho c^2}\right)} \qquad (2)$$

Discussion This is an interesting case in which we added another independent parameter (r), yet this new parameter does not even appear in the final functional relationship! The list of independent parameters is thus *over specified*. (It turns out that P is a function of r, so r is not needed in the problem.) The result here is identical to the result of Problem 7-108. It turns out that the function in Eq. 2 is a constant times $\Pi_2^{\,2}$, which yields the correct analytical equation for I, namely

Analytical result:
$$I = \text{constant} \times \frac{P^2}{\rho c} \qquad (3)$$