

# Chapter 2

## PROPERTIES OF FLUIDS

### Density and Specific Gravity

**2-1C** Intensive properties do not depend on the size (extent) of the system but extensive properties do.

**2-2C** The **specific gravity**, or **relative density**, and is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C, for which  $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$ ). That is,  $SG = \rho / \rho_{\text{H}_2\text{O}}$ . When specific gravity is known, density is determined from  $\rho = SG \times \rho_{\text{H}_2\text{O}}$ .

**2-3C** A gas can be treated as an ideal gas when it is at a high temperature or low pressure relative to its critical temperature and pressure.

**2-4C**  $R_u$  is the universal gas constant that is the same for all gases whereas  $R$  is the specific gas constant that is different for different gases. These two are related to each other by  $R = R_u / M$ , where  $M$  is the molar mass of the gas.

**2-5** A balloon is filled with helium gas. The mole number and the mass of helium in the balloon are to be determined.

**Assumptions** At specified conditions, helium behaves as an ideal gas.

**Properties** The universal gas constant is  $R_u = 8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K}$ . The molar mass of helium is  $4.0 \text{ kg/kmol}$ .

**Analysis** The volume of the sphere is

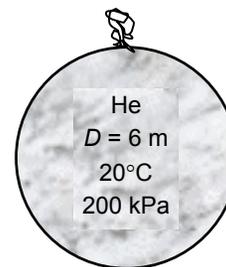
$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (3 \text{ m})^3 = 113.1 \text{ m}^3$$

Assuming ideal gas behavior, the mole numbers of He is determined from

$$N = \frac{PV}{R_u T} = \frac{(200 \text{ kPa})(113.1 \text{ m}^3)}{(8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(293 \text{ K})} = \mathbf{9.286 \text{ kmol}}$$

Then the mass of He can be determined from

$$m = NM = (9.286 \text{ kmol})(4.0 \text{ kg/kmol}) = \mathbf{37.1 \text{ kg}}$$



2-6 A balloon is filled with helium gas. The mole number and the mass of helium in the balloon are to be determined. The effect of the balloon diameter on the mass of helium is to be investigated, and the results are to be tabulated and plotted.

"Given Data"

{D=6"[m]"}

{P=200"[kPa]"}

T=20"[C]"

P=100"[kPa]"

R\_u=8.314"[kJ/kmol\*K]"

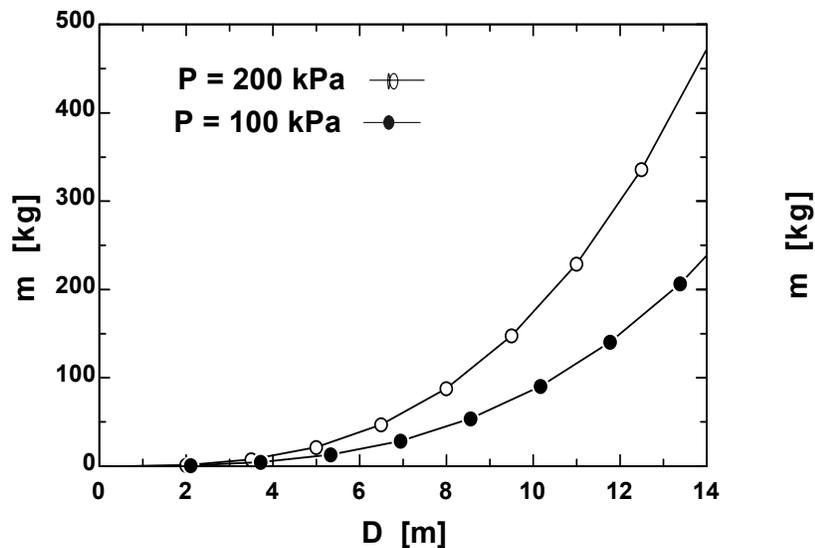
"Solution"

$P*V=N*R_u*(T+273)$

$V=4*pi*(D/2)^3/3"[m^3]"$

$m=N*MOLARMASS(Helium)"[kg]"$

D [m]	m [kg]
0.5	0.01075
2.111	0.8095
3.722	4.437
5.333	13.05
6.944	28.81
8.556	53.88
10.17	90.41
11.78	140.6
13.39	206.5
15	290.4



Mass of Helium in Balloon as function of Diameter

## Chapter 2 Properties of Fluids

**2-7** An automobile tire is inflated with air. The pressure rise of air in the tire when the tire is heated and the amount of air that must be bled off to reduce the temperature to the original value are to be determined.

**Assumptions** 1 At specified conditions, air behaves as an ideal gas. 2 The volume of the tire remains constant.

**Properties** The gas constant of air is  $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ .

**Analysis** Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{atm} = 210 + 100 = 310 \text{ kPa}$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire can be determined from

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{323\text{K}}{298\text{K}} (310\text{kPa}) = 336\text{kPa}$$

Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26 \text{ kPa}}$$

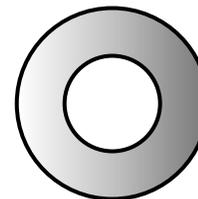
The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(310\text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{K})} = 0.0906\text{kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(310\text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\text{K})} = 0.0836\text{kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070 \text{ kg}}$$

Tire  
25°C  
210 kPa



**2-8E** An automobile tire is under inflated with air. The amount of air that needs to be added to the tire to raise its pressure to the recommended value is to be determined.

**Assumptions** 1 At specified conditions, air behaves as an ideal gas. 2 The volume of the tire remains constant.

**Properties** The gas constant of air is  $R = 0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R}$ .

**Analysis** The initial and final absolute pressures in the tire are

$$P_1 = P_{g1} + P_{atm} = 20 + 14.6 = 34.6 \text{ psia}$$

$$P_2 = P_{g2} + P_{atm} = 30 + 14.6 = 44.6 \text{ psia}$$

Treating air as an ideal gas, the initial mass in the tire is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(34.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550 \text{ R})} = 0.0900 \text{ lbm}$$

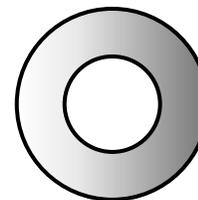
Noting that the temperature and the volume of the tire remain constant, the final mass in the tire becomes

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(44.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550 \text{ R})} = 0.1160 \text{ lbm}$$

Thus the amount of air that needs to be added is

$$\Delta m = m_2 - m_1 = 0.1160 - 0.0900 = \mathbf{0.0260 \text{ lbm}}$$

Tire  
0.53 ft<sup>3</sup>  
90°F  
20 psia



## Chapter 2 *Properties of Fluids*

**2-9E** A rigid tank contains slightly pressurized air. The amount of air that needs to be added to the tank to raise its pressure and temperature to the recommended values is to be determined. ✓

**Assumptions** **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tank remains constant.

**Properties** The gas constant of air is  $R = 0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R}$ .

**Analysis** Treating air as an ideal gas, the initial volume and the final mass in the tank are determined to be

$$V = \frac{m_1 R T_1}{P_1} = \frac{(20 \text{ lbm})(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530 \text{ R})}{20 \text{ psia}} = 196.3 \text{ ft}^3$$

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(35 \text{ psia})(196.3 \text{ ft}^3)}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550 \text{ R})} = 33.73 \text{ lbm}$$

Thus the amount of air added is

$$\Delta m = m_2 - m_1 = 33.73 - 20.0 = \mathbf{13.7 \text{ lbm}}$$

Air, 20 lbm 20 psia 70°F
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## Chapter 2 *Properties of Fluids*

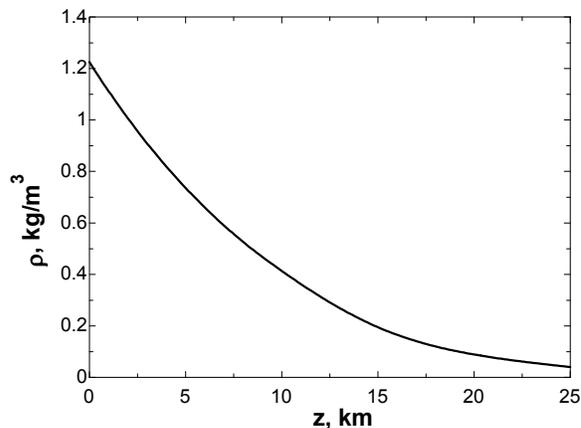
**2-10** The variation of density of atmospheric air with elevation is given in tabular form. A relation for the variation of density with elevation is to be obtained, the density at 7 km elevation is to be calculated, and the mass of the atmosphere using the correlation is to be estimated.  $\checkmark$

**Assumptions** **1** Atmospheric air behaves as an ideal gas. **2** The earth is perfectly sphere with a radius of 6377 km, and the thickness of the atmosphere is 25 km.

**Properties** The density data are given in tabular form as

$r$ , km	$z$ , km	$\rho$ , kg/m <sup>3</sup>
6377	0	1.225
6378	1	1.112
6379	2	1.007
6380	3	0.9093
6381	4	0.8194
6382	5	0.7364
6383	6	0.6601
6385	8	0.5258
6387	10	0.4135
6392	15	0.1948
6397	20	0.08891
6402	25	0.04008

**Analysis** Using EES, (1) Define a trivial function  $\rho = a + z$  in equation window, (2) select new parametric table from Tables, and type the data in a two-column table, (3) select Plot and plot the data, and (4) select plot and click on “curve fit” to get curve fit window. Then specify 2<sup>nd</sup> order polynomial and enter/edit equation. The results are:



$$\rho(z) = a + bz + cz^2 = 1.20252 - 0.101674z + 0.0022375z^2 \quad \text{for the unit of kg/m}^3,$$

$$\text{(or, } \rho(z) = (1.20252 - 0.101674z + 0.0022375z^2) \times 10^9 \quad \text{for the unit of kg/km}^3)$$

where  $z$  is the vertical distance from the earth surface at sea level. At  $z = 7$  km, the equation would give  $\rho = 0.60 \text{ kg/m}^3$ .

## Chapter 2 Properties of Fluids

(b) The mass of atmosphere can be evaluated by integration to be

$$m = \int_V \rho dV = \int_{z=0}^h (a + bz + cz^2) 4\pi(r_0 + z)^2 dz = 4\pi \int_{z=0}^h (a + bz + cz^2)(r_0^2 + 2r_0z + z^2) dz$$
$$= 4\pi \left[ ar_0^2 h + r_0(2a + br_0)h^2 / 2 + (a + 2br_0 + cr_0^2)h^3 / 3 + (b + 2cr_0)h^4 / 4 + ch^5 / 5 \right]$$

where  $r_0 = 6377$  km is the radius of the earth,  $h = 25$  km is the thickness of the atmosphere, and  $a = 1.20252$ ,  $b = -0.101674$ , and  $c = 0.0022375$  are the constants in the density function. Substituting and multiplying by the factor  $10^9$  for the density unity  $\text{kg}/\text{km}^3$ , the mass of the atmosphere is determined to be

$$m = 5.092 \times 10^{18} \text{ kg}$$

**Discussion** Performing the analysis with excel would yield exactly the same results.

EES Solution for final result:

$$a=1.2025166$$

$$b=-0.10167$$

$$c=0.0022375$$

$$r=6377$$

$$h=25$$

$$m=4*\pi*(a*r^2*h+r*(2*a+b*r)*h^2/2+(a+2*b*r+c*r^2)*h^3/3+(b+2*c*r)*h^4/4+c*h^5/5)*1E+9$$

**Vapor Pressure and Cavitation**

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**2-11C** The pressure of a vapor, whether it exists alone or in a mixture with other gases, is called the **vapor pressure**  $P_v$ . During phase change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure.

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**2-12C** Yes. The saturation temperature of a pure substance depends on pressure. The higher the pressure, the higher the saturation or boiling temperature.

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**2-13C** If the pressure of a substance is increased during a boiling process, the temperature will also increase since the boiling (or saturation) temperature of a pure substance depends on pressure and increases with it.

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**2-14C** During liquid flow, vaporization may occur at locations where the pressure drops below the vapor pressure. The vapor bubbles collapse as they are swept away from the low pressure regions, generating highly destructive, extremely high pressure waves. This phenomenon which is a common cause for drop in performance and even the erosion of impeller blades is called **cavitation**.

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## Chapter 2 *Properties of Fluids*

**2-15** The minimum pressure in a piping system to avoid cavitation is to be determined.

**Properties** The vapor pressure of water at 40°C is 7.38 kPa.

**Analysis** To avoid cavitation, the pressure anywhere in flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat}@40^{\circ}\text{C}} = \mathbf{7.38 \text{ kPa}}$$

Therefore, the pressure should be maintained above 7.38 kPa everywhere in flow.

**Discussion** Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

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**2-16** The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

**Properties** The vapor pressure of water at 20°C is 2.339 kPa.

**Analysis** To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat}@20^{\circ}\text{C}} = 2.339 \text{ kPa}$$

The minimum pressure in the pump is 2 kPa, which is less than the vapor pressure. Therefore, a **there is** danger of cavitation in the pump.

**Discussion** Note that the vapor pressure increases with increasing temperature, and thus there is a greater danger of cavitation at higher fluid temperatures.

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**2-17E** The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

**Properties** The vapor pressure of water at 70°F is 0.3632 psia.

**Analysis** To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat}@70^{\circ}\text{F}} = 0.3632 \text{ psia}$$

The minimum pressure in the pump is 0.1 psia, which is less than the vapor pressure. Therefore, **there is danger** of cavitation in the pump.

**Discussion** Note that the vapor pressure increases with increasing temperature, and the danger of cavitation increases at higher fluid temperatures.

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**2-18** The minimum pressure in a pump to avoid cavitation is to be determined.

**Properties** The vapor pressure of water at 25°C is 3.17 kPa.

**Analysis** To avoid cavitation, the pressure anywhere in the system should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat}@25^{\circ}\text{C}} = \mathbf{3.17 \text{ kPa}}$$

Therefore, the lowest pressure that can exist in the pump is 3.17 kPa.

**Discussion** Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

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**Energy and Specific Heats**

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**2-19C** The *macroscopic* forms of energy are those a system possesses as a whole with respect to some outside reference frame. The *microscopic* forms of energy, on the other hand, are those related to the molecular structure of a system and the degree of the molecular activity, and are independent of outside reference frames.

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**2-20C** The sum of all forms of the energy a system possesses is called *total energy*. In the absence of magnetic, electrical and surface tension effects, the total energy of a system consists of the kinetic, potential, and internal energies.

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**2-21C** The internal energy of a system is made up of sensible, latent, chemical and nuclear energies. The sensible internal energy is due to translational, rotational, and vibrational effects.

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**2-22C** Thermal energy is the sensible and latent forms of internal energy, and it is referred to as heat in daily life.

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**2-23C** Flow energy or flow work is the energy needed to push a fluid into or out of a control volume. Fluids at rest do not possess any flow energy.

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**2-24C** Flowing fluids possess flow energy in addition to the forms of energy a fluid at rest possesses. The total energy of a fluid at rest consists of internal, kinetic, and potential energies. The total energy of a flowing fluid consists of internal, kinetic, potential, and flow energies.

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**2-25C** Using specific heat values at the average temperature, the changes in internal energy of ideal gases can be determined from  $\Delta u = c_{v,avg} \Delta T$ . For incompressible substances,  $c_p \cong c_v \cong c$  and  $\Delta u = c_{avg} \Delta T$ .

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**2-26C** Using specific heat values at the average temperature, the changes in enthalpy of ideal gases can be determined from  $\Delta h = c_{p,avg} \Delta T$ . For incompressible substances,  $c_p \cong c_v \cong c$  and

$$\Delta h = \Delta u + v\Delta P \cong c_{avg} \Delta T + v\Delta P.$$

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**Coefficient of Compressibility**

**2-27C** The *coefficient of compressibility* represents the variation of pressure of a fluid with volume or density at constant temperature. Isothermal compressibility is the inverse of the coefficient of compressibility, and it represents the fractional change in volume or density corresponding to a change in pressure.

**2-28C** The *coefficient of volume expansion* represents the variation of the *density* of a fluid with temperature at constant pressure. It differs from the coefficient of compressibility in that latter represents the variation of *pressure* of a fluid with density at constant temperature.

**2-29C** The coefficient of compressibility of a fluid cannot be negative, but the coefficient of volume expansion can (e.g., liquid water below 4°C).

**2-30** The percent increase in the density of an ideal gas is given for a moderate pressure. The percent increase in density of the gas when compressed at a higher pressure is to be determined.

**Assumptions** The gas behaves an ideal gas.

**Analysis** For an ideal gas,  $P = \rho RT$  and  $(\partial P / \partial \rho)_T = RT = P / \rho$ , and thus  $\kappa_{\text{ideal gas}} = P$ . Therefore, the coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure.

$$\text{Substituting } \kappa = P \text{ into the definition of the coefficient of compressibility } \kappa \cong -\frac{\Delta P}{\Delta \nu / \nu} \cong \frac{\Delta P}{\Delta \rho / \rho}$$

and rearranging gives

$$\frac{\Delta \rho}{\rho} = \frac{\Delta P}{P}$$

Therefore, the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

$$\text{At 10 atm: } \frac{\Delta \rho}{\rho} = \frac{\Delta P}{P} = \frac{11-10}{10} = 10\%$$

$$\text{At 100 atm: } \frac{\Delta \rho}{\rho} = \frac{\Delta P}{P} = \frac{101-100}{100} = 1\%$$

Therefore, a pressure change of 1 atm causes a density change of 10% at 10 atm and a density change of 1% at 100 atm.

## Chapter 2 Properties of Fluids

**2-31** Using the definition of the coefficient of volume expansion and the expression  $\beta_{\text{ideal gas}} = 1/T$ , it is to be shown that the percent increase in the specific volume of an ideal gas during isobaric expansion is equal to the percent increase in absolute temperature.

**Assumptions** The gas behaves as an ideal gas.

**Analysis** The coefficient of volume expansion  $\beta$  can be expressed as

$$\beta = \frac{1}{\nu} \left( \frac{\partial \nu}{\partial T} \right)_p \approx \frac{\Delta \nu / \nu}{\Delta T}$$

Noting that  $\beta_{\text{ideal gas}} = 1/T$  for an ideal gas and rearranging give

$$\frac{\Delta \nu}{\nu} = \frac{\Delta T}{T}$$

Therefore, the percent increase in the specific volume of an ideal gas during isobaric expansion is equal to the percent increase in absolute temperature.

**2-32** Water at a given temperature and pressure is compressed to a high pressure isothermally. The increase in the density of water is to be determined.

**Assumptions** 1 The isothermal compressibility is constant in the given pressure range. 2 An approximate analysis is performed by replacing differential changes by finite changes.

**Properties** The density of water at 20°C and 1 atm pressure is  $\rho_1 = 998 \text{ kg/m}^3$ . The isothermal compressibility of water is given to be  $\alpha = 4.80 \times 10^{-5} \text{ atm}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta \rho = \alpha \rho \Delta P - \beta \rho \Delta T$$

The change in density due to a change of pressure from 1 atm to 800 atm at constant temperature is

$$\Delta \rho = \alpha \rho \Delta P = (4.80 \times 10^{-5} \text{ atm}^{-1})(998 \text{ kg/m}^3)(800 - 1) \text{ atm} = \mathbf{38.3 \text{ kg/m}^3}$$

**Discussion** Note that the density of water increases from 998 to 1036.3 kg/m<sup>3</sup> while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

## Chapter 2 Properties of Fluids

**2-33** Water at a given temperature and pressure is heated to a higher temperature at constant pressure. The change in the density of water is to be determined.

**Assumptions 1** The coefficient of volume expansion is constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

**Properties** The density of water at 15°C and 1 atm pressure is  $\rho_1 = 999.1 \text{ kg/m}^3$ . The coefficient of volume expansion at the average temperature of  $(15+95)/2 = 55^\circ\text{C}$  is  $\beta = 0.484 \times 10^{-3} \text{ K}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to the change of temperature from 15°C to 95°C at constant pressure is

$$\Delta\rho = -\beta\rho\Delta T = -(0.484 \times 10^{-3} \text{ K}^{-1})(999.1 \text{ kg/m}^3)(95 - 15)\text{K} = \mathbf{-38.7 \text{ kg/m}^3}$$

**Discussion** Noting that  $\Delta\rho = \rho_2 - \rho_1$ , the density of water at 95°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 999.1 + (-38.7) = 960.4 \text{ kg/m}^3$$

which is almost identical to the listed value of 961.5 kg/m<sup>3</sup> at 95°C in water table in the Appendix. This is mostly due to  $\beta$  varying with temperature almost linearly. Note that the density of water decreases while being heated, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

**2-34** Saturated refrigerant-134a at a given temperature is cooled at constant pressure. The change in the density of the refrigerant is to be determined.

**Assumptions 1** The coefficient of volume expansion is constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

**Properties** The density of saturated liquid R-134a at 10°C is  $\rho_1 = 1261 \text{ kg/m}^3$ . The coefficient of volume expansion at the average temperature of  $(10+0)/2 = 5^\circ\text{C}$  is  $\beta = 0.00269 \text{ K}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to the change of temperature from 10°C to 0°C at constant pressure is

$$\Delta\rho = -\beta\rho\Delta T = -(0.00269 \text{ K}^{-1})(1261 \text{ kg/m}^3)(0 - 10)\text{K} = \mathbf{33.9 \text{ kg/m}^3}$$

**Discussion** Noting that  $\Delta\rho = \rho_2 - \rho_1$ , the density of R-134a at 0°C is

$$\rho_2 = \rho_1 + \Delta\rho = 1261 + 33.9 = 1294.9 \text{ kg/m}^3$$

which is almost identical to the listed value of 1295 kg/m<sup>3</sup> at 0°C in R-134a table in the Appendix. This is mostly due to  $\beta$  varying with temperature almost linearly. Note that the density increases during cooling, as expected.

## Chapter 2 Properties of Fluids

**2-35** A water tank completely filled with water can withstand tension caused by a volume expansion of 2%. The maximum temperature rise allowed in the tank without jeopardizing safety is to be determined.

**Assumptions 1** The coefficient of volume expansion is constant. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes. **3** The effect of pressure is disregarded.

**Properties** The average volume expansion coefficient is given to be  $\beta = 0.377 \times 10^{-3} \text{ K}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

A volume increase of 2% corresponds to a density decrease of 2%, which can be expressed as  $\Delta\rho = -0.02\rho$ . Then the decrease in density due to a temperature rise of  $\Delta T$  at constant pressure is

$$-0.02\rho = -\beta\rho\Delta T$$

Solving for  $\Delta T$  and substituting, the maximum temperature rise is determined to be

$$\Delta T = \frac{0.02}{\beta} = \frac{0.02}{0.377 \times 10^{-3} \text{ K}^{-1}} = \mathbf{53.0 \text{ K} = 53.0^\circ\text{C}}$$

**Discussion** This result is conservative since in reality the increasing pressure will tend to compress the water and increase its density.

**2-36** A water tank completely filled with water can withstand tension caused by a volume expansion of 1%. The maximum temperature rise allowed in the tank without jeopardizing safety is to be determined.

**Assumptions 1** The coefficient of volume expansion is constant. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes. **3** The effect of pressure is disregarded.

**Properties** The average volume expansion coefficient is given to be  $\beta = 0.377 \times 10^{-3} \text{ K}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

A volume increase of 1% corresponds to a density decrease of 1%, which can be expressed as  $\Delta\rho = -0.01\rho$ . Then the decrease in density due to a temperature rise of  $\Delta T$  at constant pressure is

$$-0.01\rho = -\beta\rho\Delta T$$

Solving for  $\Delta T$  and substituting, the maximum temperature rise is determined to be

$$\Delta T = \frac{0.01}{\beta} = \frac{0.01}{0.377 \times 10^{-3} \text{ K}^{-1}} = \mathbf{26.5 \text{ K} = 26.5^\circ\text{C}}$$

**Discussion** This result is conservative since in reality the increasing pressure will tend to compress the water and increase its density.

## Chapter 2 Properties of Fluids

**2-37** The density of seawater at the free surface and the bulk modulus of elasticity are given. The density and pressure at a depth of 2500 m are to be determined.

**Assumptions 1** The temperature and the bulk modulus of elasticity of seawater is constant. **2** The gravitational acceleration remains constant.

**Properties** The density of seawater at free surface where the pressure is given to be  $1030 \text{ kg/m}^3$ , and the bulk modulus of elasticity of seawater is given to be  $2.34 \times 10^9 \text{ N/m}^2$ .

**Analysis** The coefficient of compressibility or the bulk modulus of elasticity of fluids is expressed as

$$\kappa = \rho \left( \frac{\partial P}{\partial \rho} \right)_T \quad \text{or} \quad \kappa = \rho \frac{dP}{d\rho} \quad (\text{at constant } T)$$

The differential pressure change across a differential fluid height of  $dz$  is given as

$$dP = \rho g dz$$

Combining the two relations above and rearranging,

$$\kappa = \rho \frac{\rho g dz}{d\rho} = g \rho^2 \frac{dz}{d\rho} \quad \rightarrow \quad \frac{d\rho}{\rho^2} = \frac{g dz}{\kappa}$$

Integrating from  $z = 0$  where  $\rho = \rho_0 = 1030 \text{ kg/m}^3$  to  $z = z$  where  $\rho = \rho$  gives

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \frac{g}{\kappa} \int_0^z dz \quad \rightarrow \quad \frac{1}{\rho_0} - \frac{1}{\rho} = \frac{gz}{\kappa}$$

Solving for  $\rho$  gives the variation of density with depth as

$$\rho = \frac{1}{1/\rho_0 - gz/\kappa}$$

Substituting into the pressure change relation  $dP = \rho g dz$  and integrating from  $z = 0$  where  $P = P_0 = 98 \text{ kPa}$  to  $z = z$  where  $P = P$  gives

$$\int_{P_0}^P dP = \int_0^z \frac{g dz}{1/\rho_0 - gz/\kappa} \quad \rightarrow \quad P = P_0 + \kappa \ln \left( \frac{1}{1 - \rho_0 gz / \kappa} \right)$$

which is the desired relation for the variation of pressure in seawater with depth.

At  $z = 2500 \text{ m}$ , the values of density and pressure are determined by substitution to be

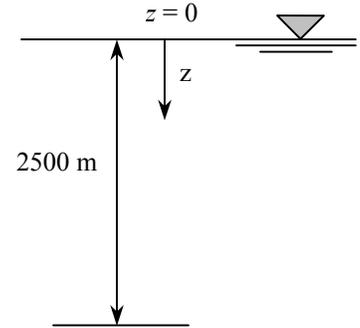
$$\rho = \frac{1}{1/(1030 \text{ kg/m}^3) - (9.81 \text{ m/s}^2)(2500 \text{ m})/(2.34 \times 10^9 \text{ N/m}^2)} = \mathbf{1041 \text{ kg/m}^3}$$

$$\begin{aligned} P &= (98,000 \text{ Pa}) + (2.34 \times 10^9 \text{ N/m}^2) \ln \left( \frac{1}{1 - (1030 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2500 \text{ m})/(2.34 \times 10^9 \text{ N/m}^2)} \right) \\ &= 2.550 \times 10^7 \text{ Pa} \\ &= \mathbf{25.50 \text{ MPa}} \end{aligned}$$

since  $1 \text{ Pa} = 1 \text{ N/m}^2 = 1 \text{ kg/m} \cdot \text{s}^2$  and  $1 \text{ kPa} = 1000 \text{ Pa}$ .

**Discussion** Note that if we assumed  $\rho = \rho_0 = \text{constant}$  at  $1030 \text{ kg/m}^3$ , the pressure at 2500 m would be  $P = P_0 + \rho g z = 0.098 + 25.26 = 25.36 \text{ MPa}$ . Then the density at 2500 m can be estimated to be

$$\Delta \rho = \rho \alpha \Delta P = (1030)(2340 \text{ MPa})^{-1}(25.26 \text{ MPa}) = 11.1 \text{ kg/m}^3 \quad \text{and thus } \rho = 1041 \text{ kg/m}^3$$



**Viscosity**

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**2-38C** Viscosity is a measure of the “stickiness” or “resistance to deformation” of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

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**2-39C** The fluids whose shear stress is proportional to the velocity gradient are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

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**2-40C** When two identical small glass balls dropped into two identical containers, one filled with water and the other with oil, the ball **dropped in water** will reach the bottom of the container first because of the much lower viscosity of water relative to oil.

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**2-41C** (a) The dynamic viscosity of liquids decreases with temperature. (b) The dynamic viscosity of gases increases with temperature.

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**2-42C** For *liquids*, the kinematic viscosity is practically independent of pressure. For *gases*, the kinematic viscosity is inversely proportional to density and thus pressure since the density of a gas is proportional to its pressure.

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## Chapter 2 Properties of Fluids

**2-43** A block is moved at a constant velocity on an inclined surface. The force that needs to be applied in the horizontal direction when the block is dry, and the percent reduction in the required force when an oil film is applied on the surface are to be determined.

**Assumptions** 1 The inclined surface is plane. 2 The friction coefficient and the oil film thickness are uniform. 3 The weight of the oil layer is negligible.

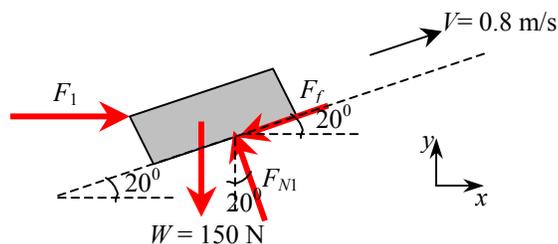
**Properties** The absolute viscosity of oil is given to be  $\mu = 0.012 \text{ Pa}\cdot\text{s} = 0.012 \text{ N}\cdot\text{s}/\text{m}^2$ .

**Analysis** (a) The velocity of the block is constant, and thus its acceleration and the net force acting on it are zero. Free body diagram of the block is given. Then the force balance gives

$$\sum F_x = 0: \quad F_1 - F_f \cos 20^\circ - F_{N1} \sin 20^\circ = 0 \quad (1)$$

$$\sum F_y = 0: \quad F_{N1} \cos 20^\circ - F_f \sin 20^\circ - W = 0 \quad (2)$$

$$\text{Friction force: } F_f = fF_{N1} \quad (3)$$



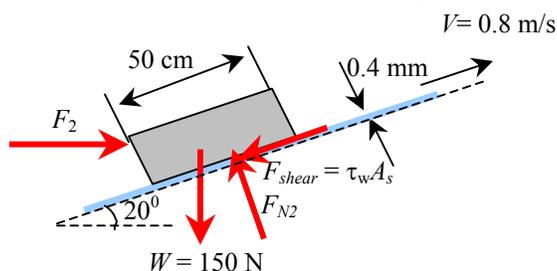
Substituting Eq. (3) into Eq. (2) and solving for  $F_{N1}$  gives

$$F_{N1} = \frac{W}{\cos 20^\circ - f \sin 20^\circ} = \frac{150 \text{ N}}{\cos 20^\circ - 0.27 \sin 20^\circ} = 177.0 \text{ N}$$

Then from Eq. (1):

$$F_1 = F_f \cos 20^\circ + F_{N1} \sin 20^\circ = (0.27 \times 177 \text{ N}) \cos 20^\circ + (177 \text{ N}) \sin 20^\circ = \mathbf{105.5 \text{ N}}$$

(b) In this case, the friction force will be replaced the shear force applied on the bottom surface of the block by oil. Because of the no-slip condition, the oil film will stick to the inclined surface at the bottom and the lower surface of the block at the top. Then the shear force can be expressed as



$$F_{shear} = \tau_w A_s = \mu A_s \frac{V}{h} = (0.012 \text{ N}\cdot\text{s}/\text{m}^2)(0.5 \times 0.2 \text{ m}^2) \frac{0.8 \text{ m/s}}{4 \times 10^{-4} \text{ m}} = 2.4 \text{ N}$$

Replacing the friction force by the shear force in part (a),

$$\sum F_x = 0: \quad F_2 - F_{shear} \cos 20^\circ - F_{N2} \sin 20^\circ = 0 \quad (4)$$

$$\sum F_y = 0: \quad F_{N2} \cos 20^\circ - F_{shear} \sin 20^\circ - W = 0 \quad (5)$$

Eq. (5) gives  $F_{N2} = (F_{shear} \sin 20^\circ + W) / \cos 20^\circ = [(2.4 \text{ N}) \sin 20^\circ + (150 \text{ N})] / \cos 20^\circ = 160.5 \text{ N}$

Substituting into Eq. (4), the required horizontal force is determined to be

$$F_2 = F_{shear} \cos 20^\circ + F_{N2} \sin 20^\circ = (2.4 \text{ N}) \cos 20^\circ + (160.5 \text{ N}) \sin 20^\circ = 57.2 \text{ N}$$

Then,

$$\text{Percentage reduction in required force} = \frac{F_1 - F_2}{F_1} 100 = \frac{105.5 - 57.2}{105.5} 100 = \mathbf{45.8\%}$$

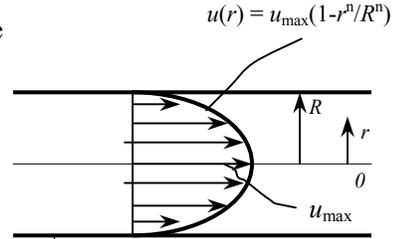
**Discussion** Note that the force required to push the block on the inclined surface reduces significantly by oiling the surface.

## Chapter 2 *Properties of Fluids*

**2-44** The velocity profile of a fluid flowing through a circular pipe is given. The friction drag force exerted on the pipe by the fluid in the flow direction per unit length of the pipe is to be determined.

**Assumptions** The viscosity of the fluid is constant.

**Analysis** The wall shear stress is determined from its definition to be



$$\tau_w = -\mu \left. \frac{du}{dr} \right|_{r=R} = -\mu u_{\max} \left. \frac{d}{dr} \left( 1 - \frac{r^n}{R^n} \right) \right|_{r=R} = -\mu u_{\max} \left. \frac{-nr^{n-1}}{R^n} \right|_{r=R} = \frac{n\mu u_{\max}}{R}$$

Note that the quantity  $du/dr$  is negative in pipe flow, and the negative sign is added to the  $\tau_w$  relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or,  $du/dr = -du/dy$  since  $y = R - r$ ). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F = \tau_w A_w = \frac{n\mu u_{\max}}{R} (2\pi R)L = 2n\pi\mu u_{\max} L$$

Therefore, the drag force per unit length of the pipe is

$$F/L = 2n\pi\mu u_{\max} .$$

**Discussion** Note that the drag force acting on the pipe in this case is independent of the pipe diameter.

## Chapter 2 Properties of Fluids

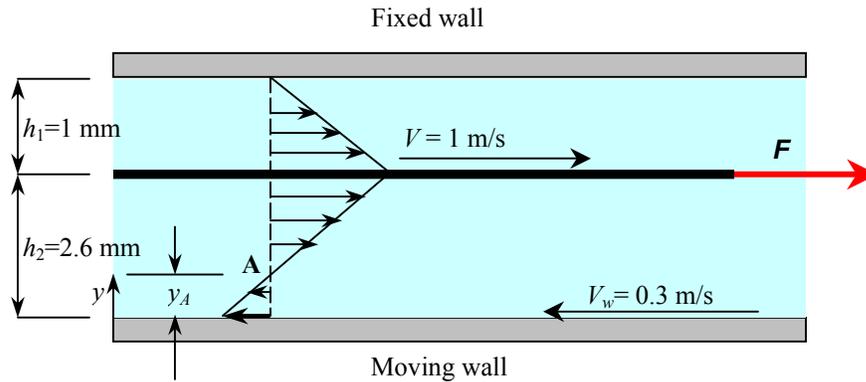
**2-45** A thin flat plate is pulled horizontally through an oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity. The location in oil where the velocity is zero and the force that needs to be applied on the plate are to be determined.

**Assumptions 1** The thickness of the plate is negligible. **2** The velocity profile in each oil layer is linear.

**Properties** The absolute viscosity of oil is given to be  $\mu = 0.027 \text{ Pa}\cdot\text{s} = 0.027 \text{ N}\cdot\text{s}/\text{m}^2$ .

**Analysis (a)** The velocity profile in each oil layer relative to the fixed wall is as shown in the figure below. The point of zero velocity is indicated by point *A*, and its distance from the lower plate is determined from geometric considerations (the similarity of the two triangles in the lower oil layer) to be

$$\frac{2.6 - y_A}{y_A} = \frac{1}{0.3} \quad \rightarrow \quad y_A = \mathbf{0.60 \text{ mm}}$$



(b) The magnitudes of shear forces acting on the upper and lower surfaces of the plate are

$$F_{\text{shear, upper}} = \tau_{w, \text{upper}} A_s = \mu A_s \left| \frac{du}{dy} \right| = \mu A_s \frac{V - 0}{h_1} = (0.027 \text{ N}\cdot\text{s}/\text{m}^2)(0.2 \times 0.2 \text{ m}^2) \frac{1 \text{ m/s}}{1.0 \times 10^{-3} \text{ m}} = 1.08 \text{ N}$$

$$F_{\text{shear, lower}} = \tau_{w, \text{lower}} A_s = \mu A_s \left| \frac{du}{dy} \right| = \mu A_s \frac{V - V_w}{h_2} = (0.027 \text{ N}\cdot\text{s}/\text{m}^2)(0.2 \times 0.2 \text{ m}^2) \frac{[1 - (-0.3)] \text{ m/s}}{2.6 \times 10^{-3} \text{ m}} = 0.54 \text{ N}$$

Noting that both shear forces are in the opposite direction of motion of the plate, the force  $F$  is determined from a force balance on the plate to be

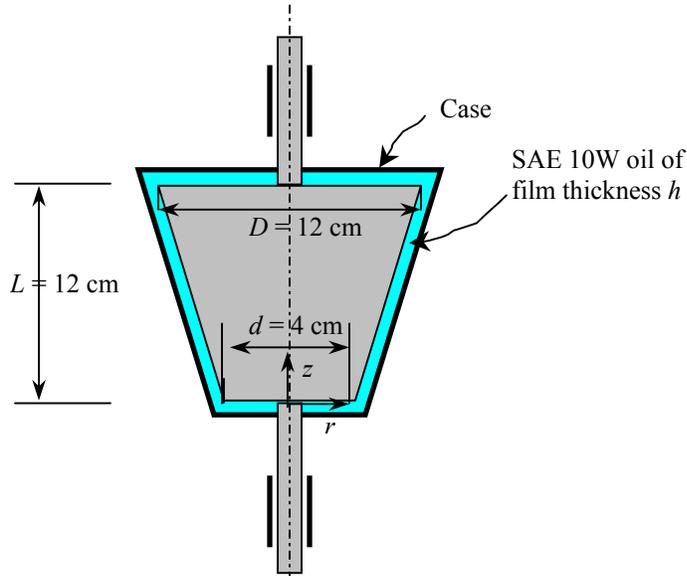
$$F = F_{\text{shear, upper}} + F_{\text{shear, lower}} = 1.08 + 0.54 = \mathbf{1.62 \text{ N}}$$

**Discussion** Note that wall shear is a friction force between a solid and a liquid, and it acts in the opposite direction of motion.

**2-46** A frustum shaped body is rotating at a constant angular speed in an oil container. The power required to maintain this motion, and the reduction in the required power input when the oil temperature rises are to be determined.

**Assumptions** The thickness of the oil layer remains constant.

**Properties** The absolute viscosity of oil is given to be  $\mu = 0.1 \text{ Pa}\cdot\text{s} = 0.1 \text{ N}\cdot\text{s}/\text{m}^2$  at  $20^\circ\text{C}$  and  $0.0078 \text{ Pa}\cdot\text{s}$  at  $80^\circ\text{C}$ .



**Analysis** The velocity gradient anywhere in the oil of film thickness  $h$  is  $V/h$  where  $V = \omega r$  is the tangential velocity. Then the wall shear stress anywhere on the surface of the frustum at a distance  $r$  from the axis of rotation can be expressed as

$$\tau_w = \mu \frac{du}{dr} = \mu \frac{V}{h} = \mu \frac{\omega r}{h}$$

Then the shear force acting on a differential area  $dA$  on the surface, the torque it generates, and the shaft power associated with it can be expressed as

$$dF = \tau_w dA = \mu \frac{\omega r}{h} dA$$

$$dT = r dF = \mu \frac{\omega r^2}{h} dA$$

$$T = \frac{\mu \omega}{h} \int_A r^2 dA$$

$$\dot{W}_{\text{sh}} = \omega T = \frac{\mu \omega^2}{h} \int_A r^2 dA$$

**Top surface:** For the top surface,  $dA = 2\pi r dr$ . Substituting and integrating,

$$\dot{W}_{\text{sh, top}} = \frac{\mu \omega^2}{h} \int_{r=0}^{D/2} r^2 (2\pi r) dr = \frac{2\pi \mu \omega^2}{h} \int_{r=0}^{D/2} r^3 dr = \frac{2\pi \mu \omega^2}{h} \frac{r^4}{4} \Big|_{r=0}^{D/2} = \frac{\pi \mu \omega^2 D^4}{32h}$$

**Bottom surface:** A relation for the bottom surface is obtained by replacing  $D$  by  $d$ ,

$$\dot{W}_{\text{sh, bottom}} = \frac{\pi \mu \omega^2 d^4}{32h}$$

**Side surface:** The differential area for the side surface can be expressed as  $dA = 2\pi r dz$ . From geometric considerations, the variation of radius with axial distance can be expressed as

$$r = \frac{d}{2} + \frac{D-d}{2L} z .$$

Differentiating gives  $dr = \frac{D-d}{2L} dz$  or  $dz = \frac{2L}{D-d} dr$ . Therefore,  $dA = 2\pi dz = \frac{4\pi L}{D-d} r dr$ .

Substituting and integrating,

$$\dot{W}_{sh, top} = \frac{\mu\omega^2}{h} \int_{r=0}^{D/2} r^2 \frac{4\pi L}{D-d} r dr = \frac{4\pi\mu\omega^2 L}{h(D-d)} \int_{r=d/2}^{D/2} r^3 dr = \frac{4\pi\mu\omega^2 L}{h(D-d)} \frac{r^4}{4} \Big|_{r=d/2}^{D/2} = \frac{\pi\mu\omega^2 L(D^2 - d^2)}{16h(D-d)}$$

Then the total power required becomes

$$\dot{W}_{sh, total} = \dot{W}_{sh, top} + \dot{W}_{sh, bottom} + \dot{W}_{sh, side} = \frac{\pi\mu\omega^2 D^4}{32h} \left[ 1 + (d/D)^4 + \frac{2L[1 - (d/D)^4]}{D-d} \right]$$

where  $d/D = 4/12 = 1/3$ . Substituting,

$$\dot{W}_{sh, total} = \frac{\pi(0.1 \text{ N}\cdot\text{s/m}^2)(200/\text{s})^2(0.12 \text{ m})^4}{32(0.0012 \text{ m})} \left[ 1 + (1/3)^4 + \frac{2(0.12 \text{ m})[1 - (1/3)^4]}{(0.12 - 0.04) \text{ m}} \right] \left( \frac{1 \text{ W}}{1 \text{ Nm/s}} \right) = \mathbf{270 \text{ W}}$$

Noting that power is proportional to viscosity, the power required at 80°C is

$$\dot{W}_{sh, total, 80^\circ\text{C}} = \frac{\mu_{80^\circ\text{C}}}{\mu_{20^\circ\text{C}}} \dot{W}_{sh, total, 20^\circ\text{C}} = \frac{0.0078 \text{ N}\cdot\text{s/m}^2}{0.1 \text{ N}\cdot\text{s/m}^2} (270 \text{ W}) = 21.1 \text{ W}$$

Therefore, the reduction in the requires power input at 80°C is

$$\text{Reduction} = \dot{W}_{sh, total, 20^\circ\text{C}} - \dot{W}_{sh, total, 80^\circ\text{C}} = 270 - 21.1 = \mathbf{249 \text{ W}} \quad (92\%)$$

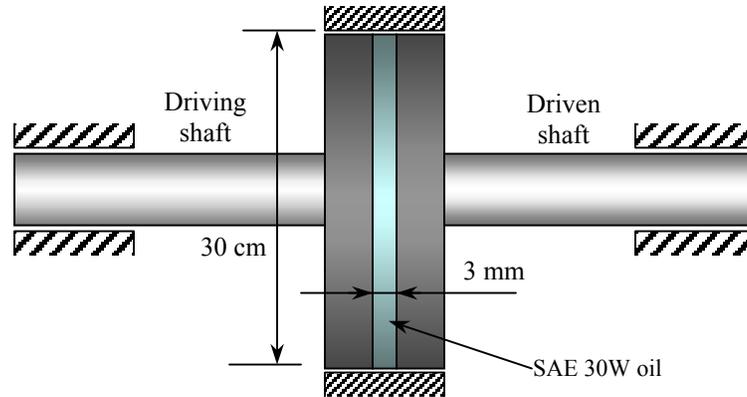
**Discussion** Note that the power required to overcome shear forces in a viscous fluid greatly depends on temperature.

## Chapter 2 Properties of Fluids

**2-47** A clutch system is used to transmit torque through an oil film between two identical disks. For specified rotational speeds, the torque transmitted is to be determined.

**Assumptions 1** The thickness of the oil film is uniform. **2** The rotational speeds of disks remains constant.

**Properties** The absolute viscosity of oil is given to be  $\mu = 0.38 \text{ N}\cdot\text{s}/\text{m}^2$ .



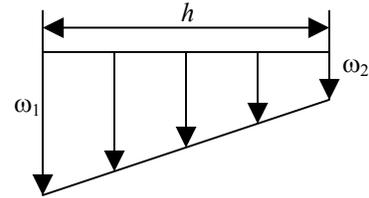
**Analysis** The disks are rotating in the same direction at different angular speeds of  $\omega_1$  and of  $\omega_2$ . Therefore, we can assume one of the disks to be stationary and the other to be rotating at an angular speed of  $\omega_1 - \omega_2$ . The velocity gradient anywhere in the oil of film thickness  $h$  is  $V/h$  where  $V = (\omega_1 - \omega_2)r$  is the tangential velocity. Then the wall shear stress anywhere on the surface of the faster disk at a distance  $r$  from the axis of rotation can be expressed as

$$\tau_w = \mu \frac{du}{dr} = \mu \frac{V}{h} = \mu \frac{(\omega_1 - \omega_2)r}{h}$$

Then the shear force acting on a differential area  $dA$  on the surface and the torque generation associated with it can be expressed as

$$dF = \tau_w dA = \mu \frac{(\omega_1 - \omega_2)r}{h} (2\pi r) dr$$

$$d\Gamma = r dF = \mu \frac{(\omega_1 - \omega_2)r^2}{h} (2\pi r) dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h} r^3 dr$$



**Integrating,**

$$\Gamma = \frac{2\pi\mu(\omega_1 - \omega_2)}{h} \int_{r=0}^{D/2} r^3 dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h} \frac{r^4}{4} \Big|_{r=0}^{D/2} = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$$

Noting that  $\omega = 2\pi\dot{n}$ , the relative angular speed is

$$\omega_1 - \omega_2 = 2\pi(\dot{n}_1 - \dot{n}_2) = (2\pi \text{ rad/rev})[(1450 - 1398) \text{ rev/min}] \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 5.445 \text{ rad/s},$$

Substituting, the torque transmitted is determined to be

$$\Gamma = \frac{\pi(0.38 \text{ N}\cdot\text{s}/\text{m}^2)(5.445 \text{ /s})(0.30 \text{ m})^4}{32(0.003 \text{ m})} = \mathbf{0.55 \text{ N}\cdot\text{m}}$$

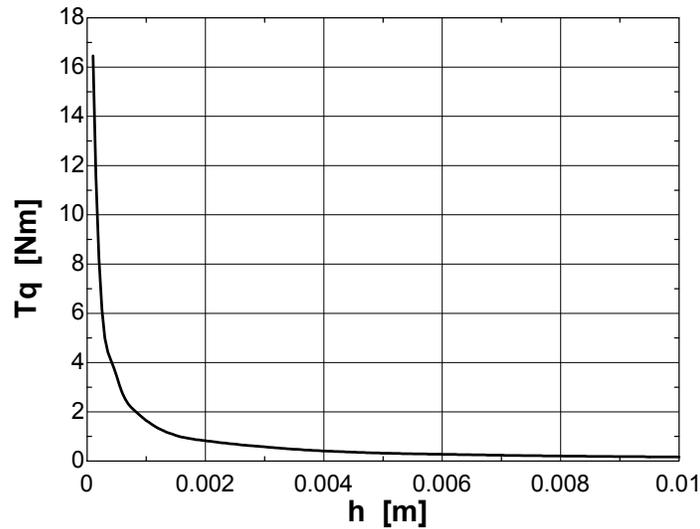
**Discussion** Note that the torque transmitted is proportional to the fourth power of disk diameter, and is inversely proportional to the thickness of the oil film.



2-48 Prob. 2-47 is reconsidered. Using EES software, the effect of oil film thickness on the torque transmitted is investigated. Film thickness varied from 0.1 mm to 10 mm, and the results are tabulated and plotted. The relation used is  $T = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$ .

mu=0.38  
 n1=1450 "rpm"  
 w1=2\*pi\*n1/60 "rad/s"  
 n2=1398 "rpm"  
 w2=2\*pi\*n2/60 "rad/s"  
 D=0.3 "m"  
 Tq=pi\*mu\*(w1-w2)\*(D^4)/(32\*h)

Film thickness <i>h</i> , mm	Torque transmitted <i>T</i> , Nm
0.1	16.46
0.2	8.23
0.4	4.11
0.6	2.74
0.8	2.06
1	1.65
2	0.82
4	0.41
6	0.27
8	0.21
10	0.16



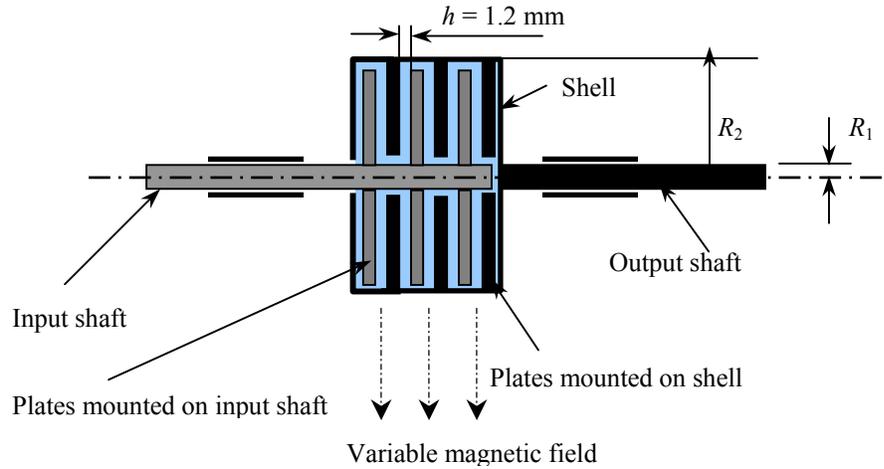
**Conclusion** Torque transmitted is inversely proportional to oil film thickness, and the film thickness should be as small as possible to maximize the transmitted torque.

## Chapter 2 Properties of Fluids

**2-49** A multi-disk Electro-rheological “ER” clutch with a fluid in which shear stress is expressed as  $\tau = \tau_y + \mu(du/dy)$  is considered. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

**Assumptions 1** The thickness of the oil layer between the disks is constant. **2** The Bingham plastic model for shear stress expressed as  $\tau = \tau_y + \mu(du/dy)$  is valid.

**Properties** The constants in shear stress relation are given to be  $\mu = 0.1 \text{ Pa}\cdot\text{s}$  and  $\tau_y = 2.5 \text{ kPa}$ .



**Analysis (a)** The velocity gradient anywhere in the oil of film thickness  $h$  is  $V/h$  where  $V = \omega r$  is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance  $r$  from the axis of rotation can be expressed as

$$\tau_w = \tau_y + \mu \frac{du}{dr} = \tau_y + \mu \frac{V}{h} = \tau_y + \mu \frac{\omega r}{h}$$

Then the shear force acting on a differential area  $dA$  on the surface of a disk and the torque generation associated with it can be expressed as

$$dF = \tau_w dA = \left( \tau_y + \mu \frac{\omega r}{h} \right) (2\pi r) dr$$

$$dT = r dF = r \left( \tau_y + \mu \frac{\omega r}{h} \right) (2\pi r) dr = 2\pi \left( \tau_y r^2 + \mu \frac{\omega r^3}{h} \right) dr$$

**Integrating,**

$$T = 2\pi \int_{r=R_1}^{R_2} \left( \tau_y r^2 + \mu \frac{\omega r^3}{h} \right) dr = 2\pi \left[ \tau_y \frac{r^3}{3} + \frac{\mu \omega r^4}{4h} \right]_{r=R_1}^{R_2} = 2\pi \left[ \frac{\tau_y}{3} (R_2^3 - R_1^3) + \frac{\mu \omega}{4h} (R_2^4 - R_1^4) \right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of  $N$  plates attached to input shaft in the clutch becomes

$$T = 4\pi N \left[ \frac{\tau_y}{3} (R_2^3 - R_1^3) + \frac{\mu \omega}{4h} (R_2^4 - R_1^4) \right]$$

(b) Noting that  $\omega = 2\pi \dot{m} = 2\pi(2400 \text{ rev/min}) = 15,080 \text{ rad/min} = 251.3 \text{ rad/s}$  and substituting,

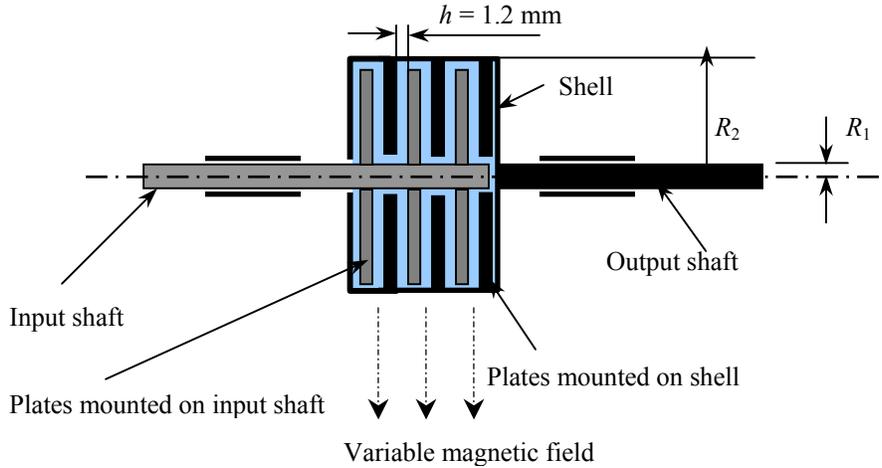
$$T = (4\pi)(11) \left[ \frac{2500 \text{ N/m}^2}{3} [(0.20 \text{ m})^3 - (0.05 \text{ m})^3] + \frac{(0.1 \text{ N}\cdot\text{s/m}^2)(251.3 \text{ /s})}{4(0.0012 \text{ m})} [(0.20 \text{ m})^4 - (0.05 \text{ m})^4] \right] = \mathbf{2060 \text{ N}\cdot\text{m}}$$

## Chapter 2 Properties of Fluids

**2-50** A multi-disk called magnetorheological “MR” clutch with a fluid in which the shear stress is expressed as  $\tau = \tau_y + K(du/dy)^m$  is considered. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

**Assumptions 1** The thickness of the oil layer between the disks is constant. **2** The Herschel-Bulkley model for shear stress expressed as  $\tau = \tau_y + K(du/dy)^m$  is valid.

**Properties** The constants in shear stress relation are given to be  $\tau_y = 900$  Pa,  $K = 58$  Pa·s<sup>m</sup>, and  $m = 0.82$ .



**Analysis (a)** The velocity gradient anywhere in the oil of film thickness  $h$  is  $V/h$  where  $V = \omega r$  is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance  $r$  from the axis of rotation can be expressed as

$$\tau_w = \tau_y + K\left(\frac{du}{dr}\right)^m = \tau_y + K\left(\frac{V}{h}\right)^m = \tau_y + K\left(\frac{\omega r}{h}\right)^m$$

Then the shear force acting on a differential area  $dA$  on the surface of a disk and the torque generation associated with it can be expressed as

$$dF = \tau_w dA = \left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr$$

$$dT = rdF = r\left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr = 2\pi\left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr$$

**Integrating,**

$$T = 2\pi \int_{R_1}^{R_2} \left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr = 2\pi \left[\tau_y \frac{r^3}{3} + \frac{K\omega^m r^{m+3}}{(m+3)h^m}\right]_{R_1}^{R_2} = 2\pi \left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of  $N$  plates attached to input shaft in the clutch becomes

$$T = 4\pi N \left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

(b) Noting that  $\omega = 2\pi\dot{n} = 2\pi(2400 \text{ rev/min}) = 15,080 \text{ rad/min} = 251.3 \text{ rad/s}$  and substituting,

$$T = (4\pi)(11) \left[\frac{900 \text{ N/m}^2}{3} [(0.20 \text{ m})^3 - (0.05 \text{ m})^3] + \frac{(58 \text{ N} \cdot \text{s}^{0.82}/\text{m}^2)(251.3 \text{ /s})^{0.82}}{(0.82 + 3)(0.0012 \text{ m})^{0.82}} [(0.20 \text{ m})^{3.82} - (0.05 \text{ m})^{3.82}]\right]$$

$$= \mathbf{103.4 \text{ kN} \cdot \text{m}}$$

## Chapter 2 *Properties of Fluids*

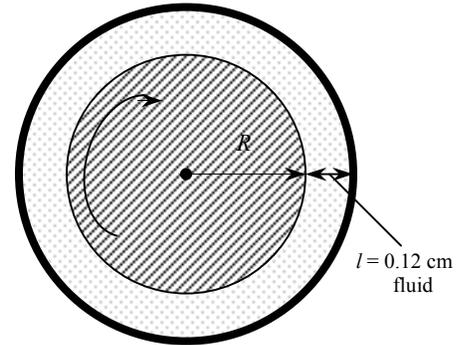
**2-51** The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

**Assumptions** **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

**Analysis** Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T} \ell}{4\pi^2 R^3 \dot{n} L} = \frac{(0.8 \text{ N} \cdot \text{m})(0.0012 \text{ m})}{4\pi^2 (0.075 \text{ m})^3 (200 / 60 \text{ s}^{-1})(0.75 \text{ m})} = \mathbf{0.0231 \text{ N} \cdot \text{s/m}^2}$$

**Discussion** This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



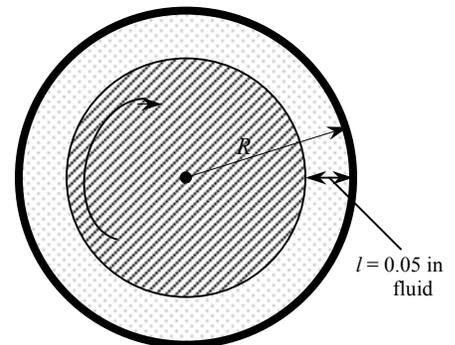
**2-52E** The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

**Assumptions** **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

**Analysis** Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T} \ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.2 \text{ lbf} \cdot \text{ft})(0.05 / 12 \text{ ft})}{4\pi^2 (5.6 / 12 \text{ ft})^3 (250 / 60 \text{ s}^{-1})(3 \text{ ft})} = \mathbf{9.97 \times 10^{-5} \text{ lbf} \cdot \text{s/ft}^2}$$

**Discussion** This is the viscosity value at temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



## Chapter 2 Properties of Fluids

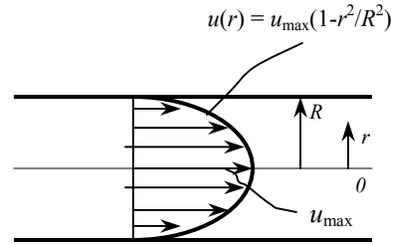
**2-53** The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

**Assumptions 1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

**Properties** The viscosity of water at 20°C is given to be 0.0010 kg/m·s.

**Analysis** The velocity profile is given by  $u(r) = u_{\max} \left( 1 - \frac{r^2}{R^2} \right)$

where  $R$  is the radius of the pipe,  $r$  is the radial distance from the center of the pipe, and  $u_{\max}$  is the maximum flow velocity, which occurs at the center,  $r = 0$ . The shear stress at the pipe surface can be expressed as



$$\tau_w = -\mu \left. \frac{du}{dr} \right|_{r=R} = -\mu u_{\max} \left. \frac{d}{dr} \left( 1 - \frac{r^2}{R^2} \right) \right|_{r=R} = -\mu u_{\max} \left. \frac{-2r}{R^2} \right|_{r=R} = \frac{2\mu u_{\max}}{R}$$

Note that the quantity  $du/dr$  is negative in pipe flow, and the negative sign is added to the  $\tau_w$  relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or,  $du/dr = -du/dy$  since  $y = R - r$ ). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{\max}}{R} (2\pi RL) = 4\pi\mu L u_{\max}$$

Substituting,

$$F_D = 4\pi\mu L u_{\max} = 4\pi(0.0010 \text{ kg/m}\cdot\text{s})(15 \text{ m})(3 \text{ m/s}) \left( \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right) = \mathbf{0.565 \text{ N}}$$

**Discussion** In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be greater.

## Chapter 2 Properties of Fluids

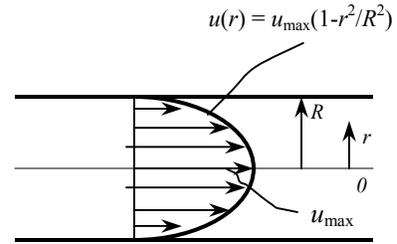
**2-54** The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

**Assumptions 1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

**Properties** The viscosity of water at 20°C is given to be 0.0010 kg/m·s.

**Analysis** The velocity profile is given by  $u(r) = u_{\max} \left( 1 - \frac{r^2}{R^2} \right)$

where  $R$  is the radius of the pipe,  $r$  is the radial distance from the center of the pipe, and  $u_{\max}$  is the maximum flow velocity, which occurs at the center,  $r = 0$ . The shear stress at the pipe surface can be expressed as



$$\tau_w = -\mu \left. \frac{du}{dr} \right|_{r=R} = -\mu u_{\max} \left. \frac{d}{dr} \left( 1 - \frac{r^2}{R^2} \right) \right|_{r=R} = -\mu u_{\max} \left. \frac{-2r}{R^2} \right|_{r=R} = \frac{2\mu u_{\max}}{R}$$

Note that the quantity  $du/dr$  is negative in pipe flow, and the negative sign is added to the  $\tau_w$  relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or,  $du/dr = -du/dy$  since  $y = R - r$ ). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{\max}}{R} (2\pi RL) = 4\pi\mu L u_{\max}$$

Substituting,

$$F_D = 4\pi\mu L u_{\max} = 4\pi(0.0010 \text{ kg/m}\cdot\text{s})(15 \text{ m})(5 \text{ m/s}) \left( \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right) = \mathbf{0.942 \text{ N}}$$

**Discussion** In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be larger.

**Surface Tension and Capillary Effect**

**2-55C** The magnitude of the pulling force at the surface of a liquid per unit length is called *surface tension*  $\sigma_s$ . It is caused by the attractive forces between the molecules. The surface tension is also surface energy since it represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

**2-56C** The pressure inside a soap bubble is greater than the pressure outside, as evidenced by the stretch of the soap film.

**2-57C** The *capillary effect* is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. It is caused by the net effect of the *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between dislike molecules, like water and glass). The capillary effect is proportional to the cosine of the *contact angle*, which is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

**2-58C** The liquid level in the tube will drop since the contact angle is greater than  $90^\circ$ , and  $\cos 110^\circ < 0$ .

**2-59C** The capillary rise is inversely proportional to the diameter of the tube, and thus it is greater in the smaller-diameter tube.

**2-60E** A slender glass tube is inserted into kerosene. The capillary rise of kerosene in the tube is to be determined.

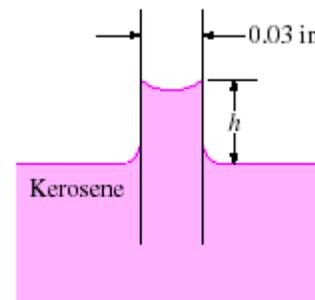
**Assumptions 1** There are no impurities in the kerosene, and no contamination on the surfaces of the glass tube. **2** The kerosene is open to the atmospheric air.

**Properties** The surface tension of kerosene-glass at  $68^\circ\text{F}$  ( $20^\circ\text{C}$ ) is  $\sigma_s = 0.028 \times 0.06852 = 0.00192$  lbf/ft. The density of kerosene at  $68^\circ\text{F}$  is  $\rho = 51.2$  lbm/ft<sup>3</sup>. The contact angle of kerosene with the glass surface is given to be  $26^\circ$ .

**Analysis** Substituting the numerical values, the capillary rise is determined to be

$$\begin{aligned}
 h &= \frac{2\sigma_s \cos \phi}{\rho g R} = \frac{2(0.00192 \text{ lbf/ft})(\cos 26^\circ)}{(51.2 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(0.015/12 \text{ ft})} \left( \frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) \\
 &= 0.0539 \text{ ft} \\
 &= \mathbf{0.65 \text{ in}}
 \end{aligned}$$

**Discussion** The capillary rise in this case more than half of an inch, and thus it is clearly noticeable.



## Chapter 2 *Properties of Fluids*

**2-61** A glass tube is inserted into a liquid, and the capillary rise is measured. The surface tension of the liquid is to be determined.

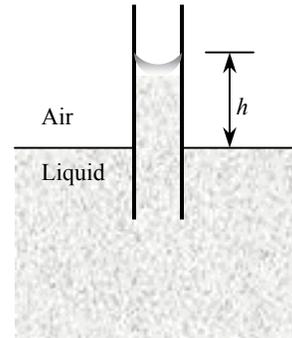
**Assumptions 1** There are no impurities in the liquid, and no contamination on the surfaces of the glass tube.  
**2** The liquid is open to the atmospheric air.

**Properties** The density of the liquid is given to be  $960 \text{ kg/m}^3$ . The contact angle is given to be  $15^\circ$ .

**Analysis** Substituting the numerical values, the surface tension is determined from the capillary rise relation to be

$$\sigma_s = \frac{\rho g R h}{2 \cos \phi} = \frac{(960 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0019 / 2 \text{ m})(0.005 \text{ m})}{2(\cos 15^\circ)} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{0.0232 \text{ N/m}}$$

**Discussion** The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.



**2-62** The diameter of a soap bubble is given. The gage pressure inside the bubble is to be determined.

**Assumptions** The soap bubble is in atmospheric air.

**Properties** The surface tension of soap water at  $20^\circ\text{C}$  is  $\sigma_s = 0.025 \text{ N/m}$ .

**Analysis** The pressure difference between the inside and the outside of a bubble is given by

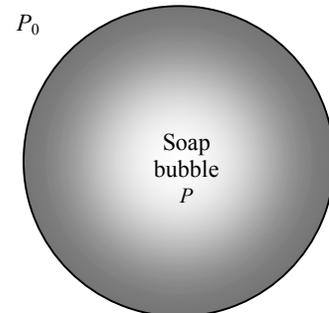
$$\Delta P_{\text{bubble}} = P_i - P_0 = \frac{4\sigma_s}{R}$$

In the open atmosphere  $P_0 = P_{\text{atm}}$ , and thus  $\Delta P_{\text{bubble}}$  is equivalent to the gage pressure. Substituting,

$$P_{i,\text{gage}} = \Delta P_{\text{bubble}} = \frac{4(0.025 \text{ N/m})}{0.002/2 \text{ m}} = 100 \text{ N/m}^2 = 100 \text{ Pa}$$

$$P_{i,\text{gage}} = \Delta P_{\text{bubble}} = \frac{4(0.025 \text{ N/m})}{0.05/2 \text{ m}} = 4 \text{ N/m}^2 = 4 \text{ Pa}$$

**Discussion** Note that the gage pressure in a soap bubble is inversely proportional to the radius. Therefore, the excess pressure is larger in smaller bubbles.



## Chapter 2 *Properties of Fluids*

**2-63** Nutrients dissolved in water are carried to upper parts of plants. The height that the water solution will rise in a tree as a result of the capillary effect is to be determined.

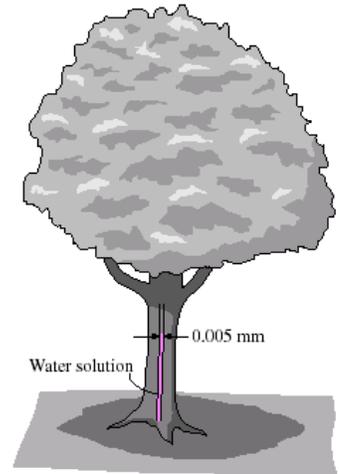
**Assumptions** **1** The solution can be treated as water with a contact angle of  $15^\circ$ . **2** The diameter of the tube is constant. **3** The temperature of the water solution is  $20^\circ\text{C}$ .

**Properties** The surface tension of water at  $20^\circ\text{C}$  is  $\sigma_s = 0.073 \text{ N/m}$ . The density of water solution can be taken to be  $1000 \text{ kg/m}^3$ . The contact angle is given to be  $15^\circ$ .

**Analysis** Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos \phi}{\rho g R} = \frac{2(0.073 \text{ N/m})(\cos 15^\circ)}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.5 \times 10^{-6} \text{ m})} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = \mathbf{5.75 \text{ m}}$$

**Discussion** Other effects such as the chemical potential difference also cause the fluid to rise in trees.



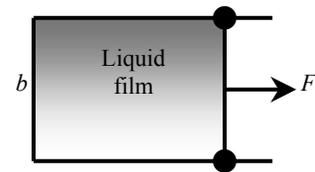
**2-64** The force acting on the movable wire of a liquid film suspended on a U-shaped wire frame is measured. The surface tension of the liquid in the air is to be determined.

**Assumptions** **1** There are no impurities in the liquid, and no contamination on the surfaces of the wire frame. **2** The liquid is open to the atmospheric air.

**Analysis** Substituting the numerical values, the surface tension is determined from the surface tension force relation to be

$$\sigma_s = \frac{F}{2b} = \frac{0.012 \text{ N}}{2(0.08 \text{ m})} = \mathbf{0.075 \text{ N/m}}$$

**Discussion** The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.

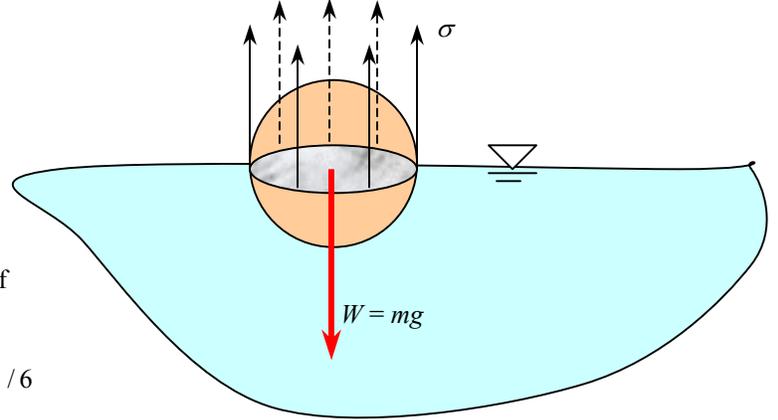


## Chapter 2 *Properties of Fluids*

**2-65** A steel ball floats on water due to the surface tension effect. The maximum diameter of the ball is to be determined, and the calculations are to be repeated for aluminum.

**Assumptions** **1** The water is pure, and its temperature is constant. **2** The ball is dropped on water slowly so that the inertial effects are negligible. **3** The contact angle is taken to be  $0^\circ$  for maximum diameter.

**Properties** The surface tension of water at  $20^\circ\text{C}$  is  $\sigma_s = 0.073 \text{ N/m}$ . The contact angle is taken to be  $0^\circ$ . The densities of steel and aluminum are given to be  $\rho_{\text{steel}} = 7800 \text{ kg/m}^3$  and  $\rho_{\text{Al}} = 2700 \text{ kg/m}^3$ .



**Analysis** The surface tension force and the weight of the ball can be expressed as

$$F_s = \pi D \sigma_s \quad \text{and} \quad W = mg = \rho g \mathcal{V} = \rho g \pi D^3 / 6$$

When the ball floats, the net force acting on the ball in the vertical direction is zero. Therefore, setting  $F_s = W$  and solving for diameter  $D$  gives

$$D = \sqrt{\frac{6\sigma_s}{\rho g}}$$

Substituting the known quantities, the maximum diameters for the steel and aluminum balls become

$$D_{\text{steel}} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073 \text{ N/m})}{(7800 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)} = 2.4 \times 10^{-3} \text{ m} = \mathbf{2.4 \text{ mm}}$$

$$D_{\text{Al}} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073 \text{ N/m})}{(2700 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)} = 4.1 \times 10^{-3} \text{ m} = \mathbf{4.1 \text{ mm}}$$

**Discussion** Note that the ball diameter is inversely proportional to the square root of density, and thus for a given material, the smaller balls are more likely to float.

**Review Problems**

**2-66** The pressure in an automobile tire increases during a trip while its volume remains constant. The percent increase in the absolute temperature of the air in the tire is to be determined.

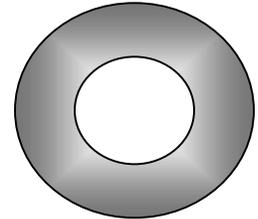
**Assumptions** 1 The volume of the tire remains constant. 2 Air is an ideal gas.

**Analysis** Noting that air is an ideal gas and the volume is constant, the ratio of absolute temperatures after and before the trip are

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{310 \text{ kPa}}{290 \text{ kPa}} = 1.069$$

Therefore, the absolute temperature of air in the tire will increase by **6.9%** during this trip.

TIRE  
200 kPa  
0.022 m<sup>3</sup>



**2-67** A large tank contains nitrogen at a specified temperature and pressure. Now some nitrogen is allowed to escape, and the temperature and pressure of nitrogen drop to new values. The amount of nitrogen that has escaped is to be determined.

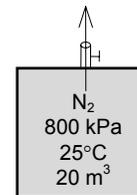
**Analysis** Treating N<sub>2</sub> as an ideal gas, the initial and the final masses in the tank are determined to be

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(800 \text{ kPa})(20 \text{ m}^3)}{(0.2968 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(298 \text{ K})} = 180.9 \text{ kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(600 \text{ kPa})(20 \text{ m}^3)}{(0.2968 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(293 \text{ K})} = 138.0 \text{ kg}$$

Thus the amount of N<sub>2</sub> that escaped is

$$\Delta m = m_1 - m_2 = 180.9 - 138.0 = \mathbf{42.9 \text{ kg}}$$



## Chapter 2 Properties of Fluids

**2-68** Suspended solid particles in water are considered. A relation is to be developed for the specific gravity of the suspension in terms of the mass fraction  $C_{s, \text{mass}}$  and volume fraction  $C_{s, \text{vol}}$  of the particles.  $\sqrt$

**Assumptions 1** The solid particles are distributed uniformly in water so that the solution is homogeneous. 2 The effect of dissimilar molecules on each other is negligible.

**Analysis** Consider solid particles of mass  $m_s$  and volume  $V_s$  dissolved in a fluid of mass  $m_f$  and volume  $V_f$ . The total volume of the suspension (or mixture) can be expressed as

$$V_m = V_s + V_f$$

Dividing by  $V_m$  and using the definition  $C_{s, \text{vol}} = V_s / V_m$  give

$$1 = C_{s, \text{vol}} + \frac{V_f}{V_m} \quad \rightarrow \quad \frac{V_f}{V_m} = 1 - C_{s, \text{vol}} \quad (1)$$

The total mass of the suspension (or mixture) can be expressed as

$$m_m = m_s + m_f$$

Dividing by  $m_m$  and using the definition  $C_{s, \text{mass}} = m_s / m_m$  give

$$1 = C_{s, \text{mass}} + \frac{m_f}{m_m} = C_{s, \text{mass}} + \frac{\rho_f V_f}{\rho_m V_m} \quad \rightarrow \quad \frac{\rho_f}{\rho_m} = (1 - C_{s, \text{mass}}) \frac{V_m}{V_f} \quad (2)$$

Combining equations 1 and 2 gives

$$\frac{\rho_f}{\rho_m} = \frac{1 - C_{s, \text{mass}}}{1 - C_{s, \text{vol}}}$$

When the fluid is water, the ratio  $\rho_f / \rho_m$  is the inverse of the definition of specific gravity. Therefore, the desired relation for the specific gravity of the mixture is

$$SG_m = \frac{\rho_m}{\rho_f} = \frac{1 - C_{s, \text{vol}}}{1 - C_{s, \text{mass}}}$$

which is the desired result.

## Chapter 2 *Properties of Fluids*

**2-69** The specific gravities of solid particles and carrier fluids of a slurry are given. The relation for the specific gravity of the slurry is to be obtained in terms of the mass fraction  $C_{s, \text{mass}}$  and the specific gravity  $SG_s$  of solid particles.  $\surd$

**Assumptions 1** The solid particles are distributed uniformly in water so that the solution is homogeneous. **2** The effect of dissimilar molecules on each other is negligible.

**Analysis** Consider solid particles of mass  $m_s$  and volume  $V_s$  dissolved in a fluid of mass  $m_f$  and volume  $V_f$ . The total volume of the suspension (or mixture) can be expressed as

$$V_m = V_s + V_f$$

Dividing by  $V_m$  gives

$$1 = \frac{V_s}{V_m} + \frac{V_f}{V_m} \rightarrow \frac{V_f}{V_m} = 1 - \frac{V_s}{V_m} = 1 - \frac{m_s / \rho_s}{m_m / \rho_m} = 1 - \frac{m_s}{m_m} \frac{\rho_m}{\rho_s} = 1 - C_{s, \text{mass}} \frac{SG_m}{SG_s} \quad (1)$$

since ratio of densities is equal two the ratio of specific gravities, and  $m_s / m_m = C_{s, \text{mass}}$ .

The total mass of the suspension (or mixture) can be expressed as

$$m_m = m_s + m_f$$

Dividing by  $m_m$  and using the definition  $C_{s, \text{mass}} = m_s / m_m$  give

$$1 = C_{s, \text{mass}} + \frac{m_f}{m_m} = C_{s, \text{mass}} + \frac{\rho_f V_f}{\rho_m V_m} \rightarrow \frac{\rho_m}{\rho_f} = \frac{V_f}{(1 - C_{s, \text{mass}}) V_m} \quad (2)$$

Taking the fluid to be water so that  $\rho_m / \rho_f = SG_m$  and combining equations 1 and 2 give

$$SG_m = \frac{1 - C_{s, \text{mass}} SG_m / SG_s}{1 - C_{s, \text{mass}}}$$

Solving for  $SG_m$  and rearranging gives

$$SG_m = \frac{1}{1 + C_{s, \text{mass}} (1/SG_s - 1)}$$

which is the desired result.

## Chapter 2 *Properties of Fluids*

**2-70E** The minimum pressure on the suction side of a water pump is given. The maximum water temperature to avoid the danger of cavitation is to be determined.

**Properties** The saturation temperature of water at 0.95 psia is 100°F.

**Analysis** To avoid cavitation at a specified pressure, the fluid temperature everywhere in the flow should remain below the saturation temperature at the given pressure, which is

$$T_{\max} = T_{\text{sat}@0.95\text{psia}} = \mathbf{100^{\circ}\text{F}}$$

Therefore, the  $T < 100^{\circ}\text{F}$  to avoid cavitation.

**Discussion** Note that saturation temperature increases with pressure, and thus cavitation may occur at higher pressure locations at higher fluid temperatures.

**2-71** Air in a partially filled closed water tank is evacuated. The absolute pressure in the evacuated space is to be determined.

**Properties** The saturation pressure of water at 60°C is 19.94 kPa.

**Analysis** When air is completely evacuated, the vacated space is filled with water vapor, and the tank contains a saturated water-vapor mixture at the given pressure. Since we have a two-phase mixture of a pure substance at a specified temperature, the vapor pressure must be the saturation pressure at this temperature. That is,

$$P_v = P_{\text{sat}@60^{\circ}\text{C}} = \mathbf{19.94\text{ kPa}}$$

**Discussion** If there is any air left in the container, the vapor pressure will be less. In that case the sum of the component pressures of vapor and air would equal 19.94 kPa.

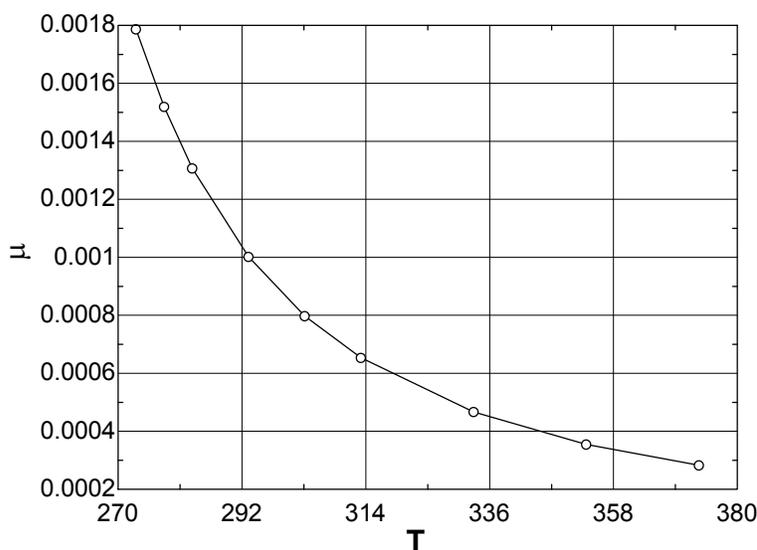
## Chapter 2 *Properties of Fluids*

**2-72** The variation of the dynamic viscosity of water with absolute temperature is given. Using tabular data, a relation is to be obtained for viscosity as a 4<sup>th</sup> degree polynomial. The result is to be compared to Andrade's equation in the form of  $\mu = D \cdot e^{B/T}$ .

**Properties** The viscosity data are given in tabular form as

$T(K)$	$\mu$ (Pa·s)
273.15	$1.787 \times 10^{-3}$
278.15	$1.519 \times 10^{-3}$
283.15	$1.307 \times 10^{-3}$
293.15	$1.002 \times 10^{-3}$
303.15	$7.975 \times 10^{-4}$
313.15	$6.529 \times 10^{-4}$
333.15	$4.665 \times 10^{-4}$
353.15	$3.547 \times 10^{-4}$
373.15	$2.828 \times 10^{-4}$

**Analysis** Using EES, (1) Define a trivial function  $a=\mu+T$  in equation window, (2) select new parametric table from Tables, and type the data in a two-column table, (3) select Plot and plot the data, and (4) select plot and click on "curve fit" to get curve fit window. Then specify polynomial and enter/edit equation. The results are:



$$\mu = 0.489291758 - 0.00568904387T + 0.0000249152104T^2 - 4.86155745 \times 10^{-8}T^3 + 3.56198079 \times 10^{-11}T^4$$

$$\mu = 0.000001475 \cdot \text{EXP}(1926.5/T) \quad [\text{used initial guess of } a_0=1.8 \times 10^{-6} \text{ and } a_1=1800 \text{ in } \mu=a_0 \cdot \exp(a_1/T)]$$

At  $T = 323.15$  K, the polynomial and exponential curve fits give

<i>Polynomial:</i> $\mu(323.15 \text{ K}) = 0.0005529 \text{ Pa}\cdot\text{s}$	(1.1% error, relative to 0.0005468 Pa·s)
<i>Exponential:</i> $\mu(323.15 \text{ K}) = 0.0005726 \text{ Pa}\cdot\text{s}$	(4.7% error, relative to 0.0005468 Pa·s)

**Discussion** This problem can also be solved using an Excel worksheet, with the following results:

Polynomial:  $A = 0.4893$ ,  $B = -0.005689$ ,  $C = 0.00002492$ ,  $D = -0.000000048612$ , and  $E = 0.00000000003562$

Andrade's equation:  $\mu = 1.807952E - 6 * e^{1864.06/T}$

## Chapter 2 *Properties of Fluids*

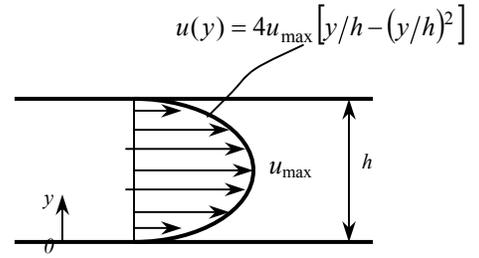
**2-73** The velocity profile for laminar one-dimensional flow between two parallel plates is given. A relation for friction drag force exerted on the plates per unit area of the plates is to be obtained.

**Assumptions 1** The flow between the plates is one-dimensional. **2** The fluid is Newtonian.

**Analysis** The velocity profile is given by  $u(y) = 4u_{\max} \left[ y/h - (y/h)^2 \right]$

where  $h$  is the distance between the two plates,  $y$  is the vertical distance from the bottom plate, and  $u_{\max}$  is the maximum flow velocity that occurs at midplane. The shear stress at the bottom surface can be expressed as

$$\tau_w = \mu \left. \frac{du}{dy} \right|_{y=0} = 4\mu u_{\max} \left. \frac{d}{dy} \left( \frac{y}{h} - \frac{y^2}{h^2} \right) \right|_{y=0} = 4\mu u_{\max} \left. \left( \frac{1}{h} - \frac{2y}{h^2} \right) \right|_{y=0} = \frac{4\mu u_{\max}}{h}$$



Because of symmetry, the wall shear stress is identical at both bottom and top plates. Then the friction drag force exerted by the fluid on the inner surface of the plates becomes

$$F_D = 2\tau_w A_{\text{plate}} = \frac{8\mu u_{\max}}{h} A_{\text{plate}}$$

Therefore, the friction drag per unit plate area is

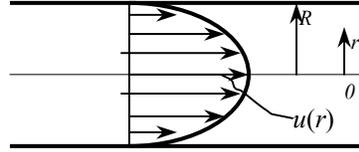
$$F_D / A_{\text{plate}} = \frac{8\mu u_{\max}}{h}$$

**Discussion** Note that the friction drag force acting on the plates is inversely proportional to the distance between plates.

## Chapter 2 Properties of Fluids

**2-74** The laminar flow of a Bingham plastic fluid in a horizontal pipe of radius  $R$  is considered. The shear stress at the pipe wall and the friction drag force acting on a pipe section of length  $L$  are to be determined.

**Assumptions 1** The fluid is a Bingham plastic with  $\tau = \tau_y + \mu(du/dr)$  where  $\tau_y$  is the yield stress. **2** The flow through the pipe is one-dimensional.



**Analysis** The velocity profile is given by  $u(r) = \frac{\Delta P}{4\mu L}(r^2 - R^2) + \frac{\tau_y}{\mu}(r - R)$  where  $\Delta p/L$  is the pressure drop along the pipe per unit length,  $\mu$  is the dynamic viscosity,  $r$  is the radial distance from the centerline. Its gradient at the pipe wall ( $r = R$ ) is

$$\left. \frac{du}{dr} \right|_{r=R} = \frac{d}{dr} \left( \frac{\Delta P}{4\mu L}(r^2 - R^2) + \frac{\tau_y}{\mu}(r - R) \right) \Bigg|_{r=R} = \left( 2r \frac{\Delta P}{4\mu L} + \frac{\tau_y}{\mu} \right) \Bigg|_{r=R} = \frac{1}{\mu} \left( \frac{\Delta P}{2L} R + \tau_y \right)$$

Substituting into  $\tau = \tau_y + \mu(du/dr)$ , the wall shear stress at the pipe surface becomes

$$\tau_w = \tau_y + \mu \left. \frac{du}{dr} \right|_{r=R} = \tau_y + \frac{\Delta P}{2L} R + \tau_y = 2\tau_y + \frac{\Delta P}{2L} R$$

Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

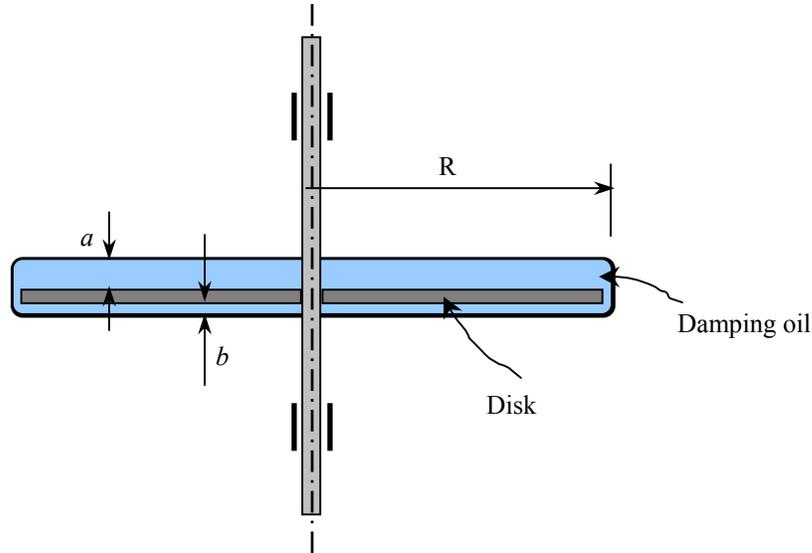
$$F_D = \tau_w A_s = \left( 2\tau_y + \frac{\Delta P}{2L} R \right) (2\pi RL) = 2\pi RL \left( 2\tau_y + \frac{\Delta P}{2L} R \right) = 4\pi RL \tau_y + \pi R^2 \Delta P$$

**Discussion** Note that the total friction drag is proportional to yield shear stress and the pressure drop.

## Chapter 2 Properties of Fluids

**2-75** A circular disk immersed in oil is used as a damper, as shown in the figure. It is to be shown that the damping torque is  $T_{\text{damping}} = C\omega$  where  $C = 0.5\pi\mu(1/a + 1/b)R^4$ .

**Assumptions 1** The thickness of the oil layer on each side remains constant. **2** The velocity profiles are linear on both sides of the disk. **3** The tip effects are negligible. **4** The effect of the shaft is negligible.



**Analysis** The velocity gradient anywhere in the oil of film thickness  $a$  is  $V/a$  where  $V = \omega r$  is the tangential velocity. Then the wall shear stress anywhere on the upper surface of the disk at a distance  $r$  from the axis of rotation can be expressed as

$$\tau_w = \mu \frac{du}{dr} = \mu \frac{V}{a} = \mu \frac{\omega r}{a}$$

Then the shear force acting on a differential area  $dA$  on the surface and the torque it generates can be expressed as

$$dF = \tau_w dA = \mu \frac{\omega r}{a} dA$$

$$d\Gamma = r dF = \mu \frac{\omega r^2}{a} dA$$

Noting that  $dA = 2\pi r dr$  and integrating, the torque on the top surface is determined to be

$$T_{\text{top}} = \frac{\mu\omega}{a} \int_A r^2 dA = \frac{\mu\omega}{a} \int_{r=0}^R r^2 (2\pi r) dr = \frac{2\pi\mu\omega}{a} \int_{r=0}^R r^3 dr = \frac{2\pi\mu\omega}{a} \frac{r^4}{4} \Big|_{r=0}^R = \frac{\pi\mu\omega R^4}{2a}$$

The torque on the bottom surface is obtained by replaying  $a$  by  $b$ ,

$$T_{\text{bottom}} = \frac{\pi\mu\omega R^4}{2b}$$

The total torque acting on the disk is the sum of the torques acting on the top and bottom surfaces,

$$T_{\text{damping, total}} = T_{\text{bottom}} + T_{\text{top}} = \frac{\pi\mu\omega R^4}{2} \left( \frac{1}{a} + \frac{1}{b} \right)$$

or,

$$T_{\text{damping, total}} = C\omega \quad \text{where} \quad C = \frac{\pi\mu R^4}{2} \left( \frac{1}{a} + \frac{1}{b} \right)$$

This completes the proof.

**Discussion** Note that the damping torque (and thus damping power) is inversely proportional to the thickness of oil films on either side, and it is proportional to the 4<sup>th</sup> power of the radius of the damper disk.

## Chapter 2 *Properties of Fluids*

**2-76E** A glass tube is inserted into mercury. The capillary drop of mercury in the tube is to be determined.

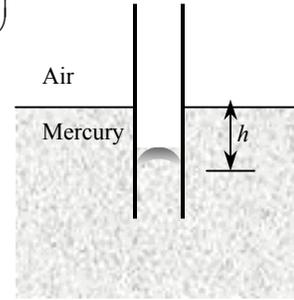
**Assumptions 1** There are no impurities in mercury, and no contamination on the surfaces of the glass tube.  
**2** The mercury is open to the atmospheric air.

**Properties** The surface tension of mercury-glass in atmospheric air at 68°F (20°C) is  $\sigma_s = 0.440 \times 0.06852 = 0.03015$  lbf/ft. The density of mercury is  $\rho = 847$  lbm/ft<sup>3</sup> at 77°F, but we can also use this value at 68°F. The contact angle is given to be 140°.

**Analysis** Substituting the numerical values, the capillary drop is determined to be

$$\begin{aligned} h &= \frac{2\sigma_s \cos \phi}{\rho g R} = \frac{2(0.03015 \text{ lbf/ft})(\cos 140^\circ)}{(847 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(0.45/12 \text{ ft})} \left( \frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) \\ &= -0.00145 \text{ ft} \\ &= -0.0175 \text{ in} \end{aligned}$$

**Discussion** The negative sign indicates capillary drop instead of rise. The drop is very small in this case because of the large diameter of the tube.



**2-77** A relation is to be derived for the capillary rise of a liquid between two large parallel plates a distance  $t$  apart inserted into a liquid vertically. The contact angle is given to be  $\phi$ .

**Assumptions** There are no impurities in the liquid, and no contamination on the surfaces of the plates.

**Analysis** The magnitude of the capillary rise between two large parallel plates can be determined from a force balance on the rectangular liquid column of height  $h$  and width  $w$  between the plates. The bottom of the liquid column is at the same level as the free surface of the liquid reservoir, and thus the pressure there must be atmospheric pressure. This will balance the atmospheric pressure acting from the top surface, and thus these two effects will cancel each other. The weight of the liquid column is

$$W = mg = \rho g V = \rho g (w \times t \times h)$$

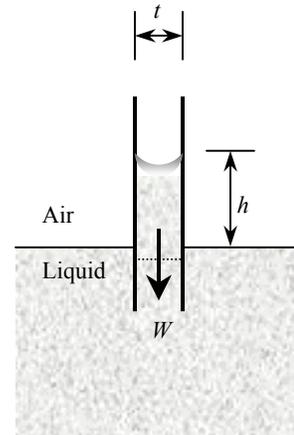
Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \quad \rightarrow \quad \rho g (w \times t \times h) = 2w \sigma_s \cos \phi$$

Canceling  $w$  and solving for  $h$  gives the capillary rise to be

$$\text{Capillary rise:} \quad h = \frac{2\sigma_s \cos \phi}{\rho g t}$$

**Discussion** The relation above is also valid for non-wetting liquids (such as mercury in glass), and gives the capillary drop.



## Chapter 2 Properties of Fluids

**2-78** A journal bearing is lubricated with oil whose viscosity is known. The torques needed to overcome the bearing friction during start-up and steady operation are to be determined.

**Assumptions** **1** The gap is uniform, and is completely filled with oil. **2** The end effects on the sides of the bearing are negligible. **3** The fluid is Newtonian.

**Properties** The viscosity of oil is given to be 0.1 kg/m·s at 20°C, and 0.008 kg/m·s at 80°C.

**Analysis** The radius of the shaft is  $R = 0.04$  m. Substituting the given values, the torque is determined to be

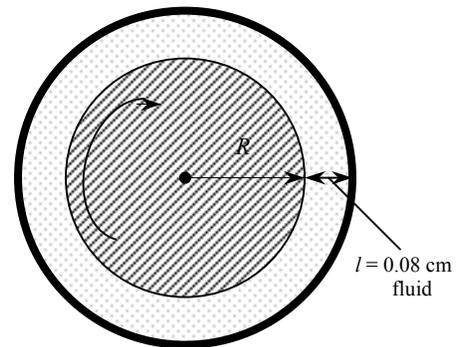
At start up at 20°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n}L}{\ell} = (0.1 \text{ kg/m}\cdot\text{s}) \frac{4\pi^2 (0.04 \text{ m})^3 (500/60 \text{ s}^{-1})(0.30 \text{ m})}{0.0008 \text{ m}} = \mathbf{0.79 \text{ N}\cdot\text{m}}$$

During steady operation at 80°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n}L}{\ell} = (0.008 \text{ kg/m}\cdot\text{s}) \frac{4\pi^2 (0.04 \text{ m})^3 (500/60 \text{ s}^{-1})(0.30 \text{ m})}{0.0008 \text{ m}} = \mathbf{0.063 \text{ N}\cdot\text{m}}$$

**Discussion** Note that the torque needed to overcome friction reduces considerably due to the decrease in the viscosity of oil at higher temperature.



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### 2-79 ... 2-81 Design and Essay Problems

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