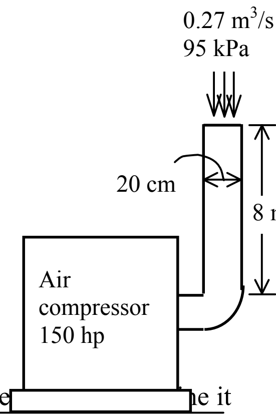
Review Problems

8-112 A compressor takes in air at a specified rate at the outdoor conditions. The useful power used by the compressor to overcome the frictional losses in the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors, and thus minor losses are negligible. **5** The flow section involves no work devices such as fans or turbines.

Properties The properties of air at 1 atm = 101.3 kPa and 15°C are ρ_0 = 1.225 kg/m^3 and $\mu = 1.802\times10^{-5}$ kg/m·s. The roughness of galvanized iron surfaces is ε = 0.00015 m. The dynamic viscosity is independent of pressure, but density of an ideal gas is proportional to pressure. The density of air at 95 kPa is $\rho = (P/P_0)\rho_0 = (95/101.3)(1.225\ \text{kg/m}^3) = 1.149\ \text{kg/m}^3$.

Analysis The average velocity and the Reynolds number are



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.27\ \text{m}^3/\text{s}}{\pi(0.20\ \text{m})^2/4} = 8.594\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.149\ \text{kg/m}^3)(8.594\ \text{m/s})(0.20\ \text{m})}{1.802\times10^{-5}\ \text{kg/m}\cdot\text{s}} = 1.096\times10^5$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-4}\ \text{m}}{0.20\ \text{m}} = 7.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading e[illegible]e it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{7.5\times10^{-4}}{3.7} + \frac{2.51}{1.096\times10^5\sqrt{f}}\right)$$

It gives f = 0.02109. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02109\frac{8\ \text{m}}{0.20\ \text{m}}\frac{(1.149\ \text{kg/m}^3)(8.594\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{Pa}}{1\ \text{N/m}^2}\right) = 35.8\ \text{Pa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.27\ \text{m}^3/\text{s})(35.8\ \text{Pa})\left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{9.66\ W}$$

Discussion Note hat the pressure drop in the duct and the power needed to overcome it is very small (relative to 150 hp), and can be disregarded.

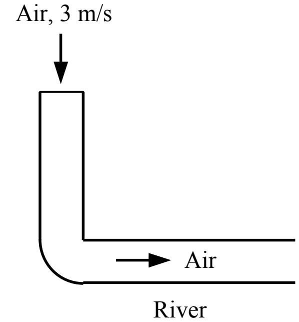
The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02086, which is very close to the Colebrook value. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency (but probably no more than 20 W).

14-113 Air enters the underwater section of a circular duct. The fan power needed to overcome the flow resistance in this section of the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines. **6** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and 15°C are ρ_0 = 1.225 kg/m^3 and μ = 1.802×10^{-5} kg/m·s. The roughness of stainless steel pipes is ε = 0.000005 m.

Analysis The volume flow rate and the Reynolds number are



$$\dot{V} = VA_c = V(\pi D^2/4) = (3\text{ m/s})[\pi(0.20\text{ m})^2/4] = 0.0942\text{ m}^3\text{/s}$$

$$\text{Re} = \frac{\rho VD_h}{\mu} = \frac{(1.225\text{ kg/m}^3)(3\text{ m/s})(0.20\text{ m})}{1.802\times10^{-5}\text{ kg/m}\cdot\text{s}} = 4.079\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{5\times10^{-6}\text{ m}}{0.20\text{ m}} = 2.5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.5\times10^{-5}}{3.7}+\frac{2.51}{4.079\times10^4\sqrt{f}}\right)$$

It gives f = 0.02195. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02195\frac{15\text{ m}}{0.2\text{ m}}\frac{(1.225\text{ kg/m}^3)(3\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 9.07\text{ Pa}$$

$$\dot{W}_{\text{electric}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(0.0942\text{ m}^3\text{/s})(9.07\text{ Pa})}{0.62} = \left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{1.4\ W}$$

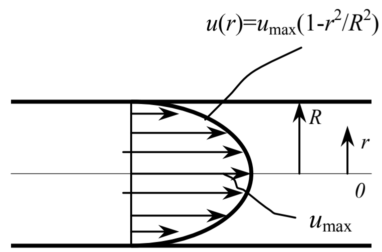
Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02175, which is sufficiently close to 0.02195. Assuming the pipe to be smooth would give 0.02187 for the friction factor, which is almost identical to the f value obtained from the Colebrook relation. Therefore, the duct can be treated as being smooth with negligible error.

8-114 The velocity profile in fully developed laminar flow in a circular pipe is given. The radius of the pipe, the average velocity, and the maximum velocity are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1-\frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 6(1-0.01r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the average velocity to be

$$R^2 = \frac{1}{100} \quad \rightarrow \quad R = 0.10\ \text{m}$$

$$u_{\max} = \textbf{6 m/s}$$

$$V_{avg} = \frac{u_{\max}}{2} = \frac{6\ \text{m/s}}{2} = \textbf{3 m/s}$$

8-115E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions **1** The flow is steady, laminar, and fully developed. **2** The pipe is horizontal.

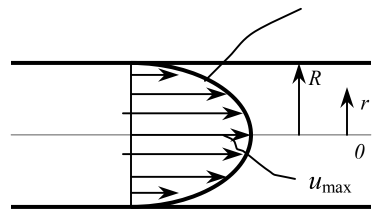
Properties The density and dynamic viscosity of water at 40°F are ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft·h = 1.039×10^{-3} lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1-\frac{r^2}{R^2}\right)$$

The velocity profile in this case is given by

$$u(r) = 0.8(1-625r^2)$$



Comparing the two relations above gives the pipe radius, the maximum velocity, and the average velocity to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04\text{ ft}$$

$$u_{\max} = 0.8\text{ ft/s}$$

$$V = V_{avg} = \frac{u_{\max}}{2} = \frac{0.8\text{ ft/s}}{2} = 0.4\text{ ft/s}$$

Then the volume flow rate and the pressure drop become

$$\dot{V} = VA_c = V(\pi R^2) = (0.4\text{ ft/s})[\pi(0.04\text{ ft})^2] = \mathbf{0.00201\ ft^3/s}$$

$$\dot{V}_{\text{horiz}} = \frac{\Delta P\pi D^4}{128\mu L} \quad \rightarrow \quad 0.00201\text{ ft}^3\text{/s} = \frac{(\Delta P)\pi(0.08\text{ ft})^4}{128(1.039\times10^{-3}\text{ lbm/ft}\cdot\text{s})(80\text{ ft})}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)$$

It gives

$$\Delta P = 5.16\text{ lbf/ft}^2 = \mathbf{0.0358\ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (0.00201\text{ ft}^3\text{/s})(5.16\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.014\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(62.42\text{ lbm/ft}^3)(0.4\text{ ft/s})(0.08\text{ ft})}{1.039\times10^{-3}\text{ lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

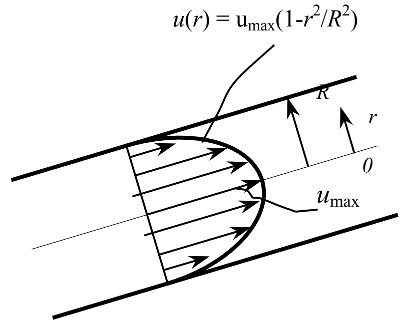
8-116E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Properties The density and dynamic viscosity of water at 40°F are ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft·h = 1.039×10^{-3} lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 0.8(1 - 625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, the average velocity, and the volume flow rate to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04\ \text{ft}$$

$$u_{\max} = 0.8\ \text{ft/s}$$

$$V = V_{avg} = \frac{u_{\max}}{2} = \frac{0.8\ \text{ft/s}}{2} = 0.4\ \text{ft/s}$$

$$\dot{V} = VA_c = V(\pi R^2) = (0.4\ \text{ft/s})[\pi(0.04\ \text{ft})^2] = \mathbf{0.00201\ ft^3/s}$$

For uphill flow with an inclination of 12°, we have θ = +12°, and

$$\rho g L \sin\theta = (62.42\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(80\ \text{ft})\sin 12°\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 1038\ \text{lbf/ft}^2$$

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \quad \rightarrow \quad 0.00201\ \text{ft}^3/\text{s} = \frac{(\Delta P - 1038)\pi(0.08\ \text{ft})^4}{128(1.039\times10^{-3}\ \text{lbm/ft}\cdot\text{s})(80\ \text{ft})}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)$$

It gives

$$\Delta P = 1043\ \text{lbf/ft}^2 = 7.24\ \text{psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (0.00201\ \text{ft}^3/\text{s})(1043\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{2.84\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.42\ \text{lbm/ft}^3)(0.4\ \text{ft/s})(0.08\ \text{ft})}{1.039\times10^{-3}\ \text{lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

8-117 Water is discharged from a water reservoir through a circular pipe of diameter D at the side wall at a vertical distance H from the free surface with a reentrant section. A relation for the "equivalent diameter" of the reentrant pipe for use in relations for frictionless flow through a hole is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The water level in the reservoir remains constant. **4** The pipe is horizontal. **5** The entrance effects are negligible, and thus the flow is fully developed and the friction factor f is constant. **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.8$ for the reentrant section, and $K_L = 0$ for the "frictionless" flow.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the exit of the pipe, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the reservoir is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

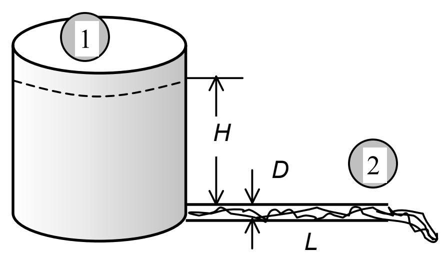
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad H=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the pipe is constant. Substituting and solving for V_2 gives

$$H=\frac{V_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gH}{1+fL/D+K_L}}$$



Then the volume flow rate becomes

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+fL/D+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ and $f = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $V_{2,\text{frictionless}}=\sqrt{2gz_1}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

$$\dot{V}=A_{c,\text{equiv}}V_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+fL/D+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+fL/D+K_L)^{1/4}}=\frac{D}{(1+0.018\times 10/0.04+0.8)^{1/4}}=\mathbf{0.63}\boldsymbol{D}=\mathbf{0.025\ m}$$

Discussion Note that the effect of frictional losses of a pipe with a reentrant section is to reduce the diameter by about 40% in this case. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V}\propto D_{\text{equiv}}^2=(0.63D)^2=0.40D^2$. Therefore, the flow rate through a sharp-edged entrance is about two-thirds less compared to the frictionless flow case.

8-118 A water tank open to the atmosphere is initially filled with water. The tank is drained to the atmosphere through a 90° horizontal bend of negligible length. The flow rate is to be determined for the cases of the bend being a flanged smooth bend and a miter bend without vanes.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **3** The water level in the tank remains constant. **4** The length of the bend and thus the frictional loss associated with its length is negligible. **5** The entrance is well-rounded, and the entrance loss is negligible.

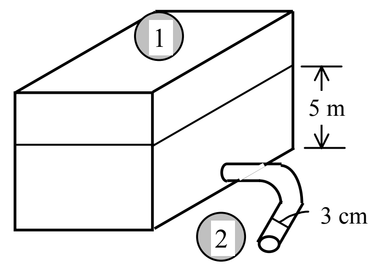
Properties The loss coefficient is $K_L = 0.3$ for a flanged smooth bend and $K_L = 1.1$ for a miter bend without vanes.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the bend, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad z_1 = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

$$z_1 = \alpha_2 \frac{V_2^2}{2g} + K_L \frac{V_2^2}{2g} \quad \rightarrow \quad 2gz_1 = V_2^2(\alpha_2 + K_L) \quad \rightarrow \quad V_2 = \sqrt{\frac{2gz_1}{\alpha_2 + K_L}}$$



Then the flow rate becomes

$$\dot{V} = A_{\text{pipe}} V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2 + K_L}}$$

(***a***) **Case 1** Flanged smooth bend ($K_L = 0.3$):

$$\dot{V} = A_c V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2 + K_L}} = \frac{\pi (0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1.05 + 0.3}} = \mathbf{0.00603\ m^3/s = 6.03\ L/s}$$

(***b***) **Case 2** Miter bend without vanes ($K_L = 1.1$):

$$\dot{V} = A_c V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2 + K_L}} = \frac{\pi (0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1.05 + 1.1}} = \mathbf{0.00478\ m^3/s = 4.78\ L/s}$$

Discussion Note that the type of bend used has a significant effect on the flow rate, and a conscious effort should be made when selecting components in a piping system.

If the effect of the kinetic energy correction factor is neglected, $\alpha_2 = 1$ and the flow rates become

(*a*) Case 1 ($K_L = 0.3$): $$\dot{V} = A_c V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1 + K_L}} = \frac{\pi (0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1 + 0.3}} = 0.00614\ \text{m}^3/\text{s}$$

(*b*) Case 2 ($K_L = 1.1$): $$\dot{V} = A_c V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1 + K_L}} = \frac{\pi (0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1 + 1.1}} = 0.00483\ \text{m}^3/\text{s}$$

Therefore, the effect of the kinetic energy correction factor is (6.14-6.03)/6.03= 1.8% and (4.83-4.78)/4.83= 1.0%, which is negligible.

8-119 The piping system of a geothermal district heating system is being designed. The pipe diameter that will optimize the initial system cost and the energy cost is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses, the only significant energy loss arises from pipe friction. **4** The piping system is horizontal. **5** The properties of geothermal water are the same as fresh water. **6** The friction factor is constant at the given value. **7** The interest rate, the inflation rate, and the salvage value of the system are all zero. **8** The flow rate through the system remains constant.

Properties We take the density of water to be $\rho = 1000$ kg/m^3. The friction factor is given to be $f = 0.015$.

Analysis The system operates in a loop, and thus we can take any point in the system as points 1 and 2 (the same point), and thus $z_1 = z_2$, $V_1 = V_2$, and $P_1 = P_2$. Then the energy equation for this piping system simplifies to

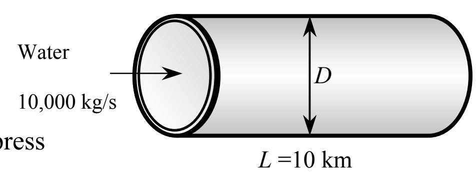
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_1\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2}$$

The flow rate of geothermal water is

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{10{,}000\text{ kg/s}}{1000\text{ kg/m}^3}=10\text{ m}^3\text{/s}$$



To expose the dependence of pressure drop on diameter, we express it in terms of the flow rate as

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2}=f\frac{L}{D}\frac{\rho}{2}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2=f\frac{16L}{D}\frac{\rho\dot{V}^2}{2\pi^2 D^4}=f\frac{8L}{D^5}\frac{\rho\dot{V}^2}{\pi^2}$$

Then the required pumping power can be expressed as

$$\dot{W}_{\text{pump}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{\dot{V}}{\eta_{\text{pump-motor}}}f\frac{8L}{D^5}\frac{\rho\dot{V}^2}{\pi^2}=f\frac{8L}{D^5}\frac{\rho\dot{V}^3}{\eta_{\text{pump-motor}}\pi^2}$$

Note that the pumping power requirement is proportional to f and L, consistent with our intuitive expectation. Perhaps not so obvious is that power is proportional to the *cube* of flow rate. The fact that the *power is inversely proportional to pipe diameter D to the fifth power* averages that a slight increase in pipe diameter will manifest as a tremendous reduction in power dissipation due to friction in a long pipeline. Substituting the given values and expressing the diameter D in meters,

$$\dot{W}_{\text{pump}}=(0.015)\frac{8(10{,}000\text{ m})}{D^5\text{ m}^5}\frac{(1000\text{ kg/m}^3)(10\text{ m}^3\text{/s})^3}{\pi^2(0.80)}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\frac{1.52\times10^5}{D^5}\text{ kW}$$

The number of hours in one year are 24×365 = 8760 h. Then the total amount of electric power used and its cost per year are

$$E_{\text{pump}}=\dot{W}_{\text{pump}}\Delta t=\frac{1.52\times10^5}{D^5}(8760\text{ h})=\frac{1.332\times10^9}{D^5}\text{ kWh/yr}$$

$$\text{Energy cost}=E_{\text{pump}}\times\text{Unit cost}=\left(\frac{1.332\times10^9}{D^5}\text{ kWh/y}\right)(\$0.06/\text{kWh})=\frac{7.99\times10^7}{D^5}\$/\text{yr}$$

The installation cost of the system with a 30-year lifetime is given to be Cost = $\$10^6\,D^2$ where D is in meters. The annual cost of the system is then 1/30$^{\text{th}}$ of it, which is

$$\text{System cost}=\frac{\text{Total cost}}{\text{Life time}}=\frac{\$10^6 D^2}{30\text{ yr}}=\$3.33\times10^4 D^2\text{ (per year)}$$

Then the total annual cost of the system (installation + operating) becomes

$$\text{Total cost} = \text{Energy cost} + \text{System cost} = \frac{7.99\times 10^{7}}{D^{5}} + 3.33\times 10^{4} D^{2} \quad \$/\text{yr}$$

The optimum pipe diameter is the value that minimizes this total, and it is determined by taking the derivative of the total cost with respect to D and setting it equal to zero,

$$\frac{\partial(\text{Total cost})}{\partial D} = -5\times\frac{7.99\times 10^{7}}{D^{6}} + 2\times 3.33\times 10^{4} D = 0$$

Simplifying gives $D^7 = 5998$ whose solution is

D = **3.5 m**

This is the **optimum** pipe diameter that minimizes the total cost of the system under stated assumptions. A larger diameter pipe will increase the system cost more than it decreases the energy cost, and a smaller diameter pipe will increase the system cost more than it decreases the energy cost.

Discussion The assumptions of zero interest and zero inflation are not realistic, and an actual economic analysis must consider these factors as they have a major effect on the pipe diameter. This is done by considering the time value of money, and expressing all the costs at the same time. Pipe purchase is a present cost, and energy expenditures are future annual costs spread out over the project lifetime. Thus, to provide consistent dollar comparisons between initial and future costs, all future energy costs must be expressed as a single present lump sum to reflect the time-value of money. Then we can compare pipe and energy costs on a consistent basis. Economists call the necessary factor the "Annuity Present Value Factor", F. If interest rate is 10% per year with n = 30 years, then F = 9.427. Thus, if power costs $1,000,000/year for the next 30 years, then the present value of those future payments is $9,427,000 (and not $30,000,000!) if money is worth 10%. Alternatively, if you must pay $1,000,000 every year for 30 years, and you can today invest $9,437,000 at 10%, then you can meet 30 years of payments at the end of each year. The energy cost in this case can be determined by dividing the energy cost expression developed above by 9.427. This will result in a pipe diameter of D = 2.5 m. In an actual design, we also need to calculate the average flow velocity and the pressure head to make sure that they are reasonable. For a pipe diameter of 2.5 m, for example, the average flow velocity is 1.47 m/s and the pump pressure head is 5.6 m.

8-120 Water is drained from a large reservoir through two pipes connected in series at a specified rate using a pump. The required pumping head and the minimum pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involve no components such as bends, valves, and other connectors that cause additional minor losses. **6** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **7** The water level in the reservoir remains constant. **8** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s. The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The roughness of cast iron pipes is ε = 0.00026 m.

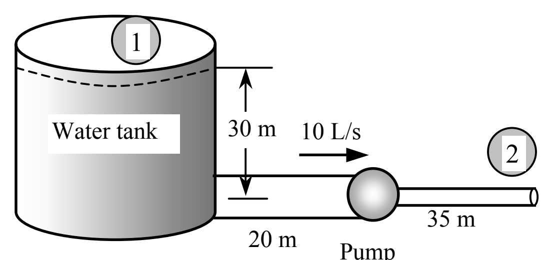
Analysis We take point 1 at the free surface of the tank, and point 2 and the reference level at the centerline of the pipe ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\sum\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$$

and the summation is over two pipes. Noting that the two pipes are connected in series and thus the flow rate through each of them is the same, the head loss for each pipe is determined as follows (we designate the first pipe by 1 and the second one by 2):



Pipe 1: $$V_1=\frac{\dot{V}}{A_{c1}}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.018\text{ m}^3/\text{s}}{\pi(0.06\text{ m})^2/4}=6.366\text{ m/s}$$

$$\text{Re}_1=\frac{\rho V_1 D_1}{\mu}=\frac{(999.1\text{ kg/m}^3)(6.366\text{ m/s})(0.06\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot\text{s}}=335{,}300$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\text{ m}}{0.06\text{ m}}=0.00433$$

The friction factor corresponding to this relative roughness and the Reynolds number is, from the Colebrook equation,

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{\varepsilon/D_1}{3.7}+\frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)\rightarrow\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{0.00433}{3.7}+\frac{2.51}{335{,}300\sqrt{f_1}}\right)$$

It gives $f_1 = 0.02941$. The only minor loss is the entrance loss, which is $K_L = 0.5$. Then the total head loss of the first pipe becomes

$$h_{L1}=\left(f_1\frac{L_1}{D_1}+\sum K_L\right)\frac{V_1^2}{2g}=\left((0.02941)\frac{20\text{ m}}{0.06\text{ m}}+0.5\right)\frac{(6.366\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=21.3\text{ m}$$

Pipe 2: $$V_2=\frac{\dot{V}}{A_{c2}}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.018\text{m}^3/\text{s}}{\pi(0.04\text{ m})^2/4}=14.32\text{ m/s}$$

$$\text{Re}_2=\frac{\rho V_2 D_2}{\mu}=\frac{(999.1\text{ kg/m}^3)(14.32\text{ m/s})(0.04\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot\text{s}}=502{,}900$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_2=\frac{0.00026\text{ m}}{0.04\text{ m}}=0.0065$$

The friction factor corresponding to this relative roughness and the Reynolds number is, from the

Colebrook equation,

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon / D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.0065}{3.7} + \frac{2.51}{502{,}900\sqrt{f_2}}\right)$$

It gives f_2 = 0.03309. The second pipe involves no minor losses. Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the head loss for the second pipe becomes

$$h_{L2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} = (0.03309)\frac{35\text{ m}}{0.04\text{ m}}\frac{(14.32\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 302.6\text{ m}$$

The total head loss for two pipes connected in series is the sum of the head losses of the two pipes,

$$h_L = h_{L,\text{total}} = h_{L1} + h_{L2} = 21.3 + 302.6 = 323.9\text{ m}$$

Then the pumping head and the minimum pumping power required (the pumping power in the absence of any inefficiencies of the pump and the motor, which is equivalent to the useful pumping power) become

$$h_{\text{pump, u}} = \alpha_2\frac{V_2^2}{2g} + h_L - z_1 = (1)\frac{(14.32\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 323.9 - 30 = 304.4\text{ m}$$

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = \rho\dot{V}gh_{\text{pump, u}}$$

$$= (999.1\text{ kg/m}^3)(0.018\text{ m}^3\text{/s})(9.81\text{ m/s}^2)(304.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{53.7\ kW}$$

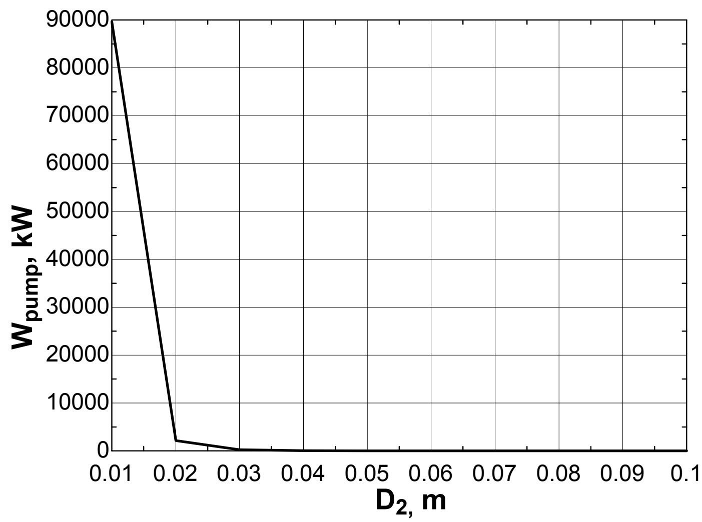
Therefore, the pump must supply a minimum of 53.7 kW of useful mechanical energy to water.

Discussion Note that the shaft power of the pump must be greater than this to account for the pump inefficiency, and the electrical power supplied must be even greater to account for the motor inefficiency.

8-121 In Prob. 8-120, the effect of second pipe diameter on required pumping head for the same flow rate is to be investigated by varying the pipe diameter from 1 cm to 10 cm in increments of 1 cm.

```
rho=999.1
mu=0.001138
nu=mu/rho
Vdot=0.018 "m3/s"
g=9.81 "m/s2"
z1=30 "m"
L1=20 "m"
D1=0.06 "m"
Ac1=pi*D1^2/4
Re1=V1*D1/nu
V1=Vdot/Ac1
eps1=0.00026
rf1=eps1/D1
1/sqrt(f1)=-2*log10(rf1/3.7+2.51/(Re1*sqrt(f1)))
KL1=0.5
HL1=(f1*L1/D1+KL1)*V1^2/(2*g)
L2=35
Re2=V2*D2/nu
V2=Vdot/(pi*D2^2/4)
eps2=0.00026
rf2=eps2/D2
1/sqrt(f2)=-2*log10(rf2/3.7+2.51/(Re2*sqrt(f2)))
HL2=f2*(L2/D2)*V2^2/(2*g)
HL=HL1+HL2
hpump=V2^2/(2*g)+HL-z1
Wpump=rho*Vdot*g*hpump/1000 "kW"
```

D_2, m	W_{pump}, kW	h_{L2}, m	Re
0.01	89632.5	505391.6	2.012E+06
0.02	2174.7	12168.0	1.006E+06
0.03	250.8	1397.1	6.707E+05
0.04	53.7	302.8	5.030E+05
0.05	15.6	92.8	4.024E+05
0.06	5.1	35.4	3.353E+05
0.07	1.4	15.7	2.874E+05
0.08	-0.0	7.8	2.515E+05
0.09	-0.7	4.2	2.236E+05
0.10	-1.1	2.4	2.012E+05



8-122 Two pipes of identical diameter and material are connected in parallel. The length of one of the pipes is twice the length of the other. The ratio of the flow rates in the two pipes is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully turbulent in both pipes and thus the friction factor is independent of the Reynolds number (it is the same for both pipes since they have the same material and diameter). **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length *L* and diameter *D* can be expressed as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

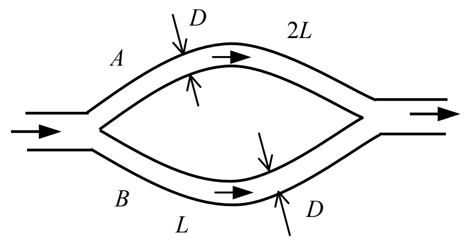
Solving for the flow rate gives

$$\dot{V} = \sqrt{\frac{\pi^2 h_L g D^5}{8fL}} = \frac{k}{\sqrt{L}} \qquad (k \text{ is a constant})$$

When the pipe diameter, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes inversely proportional to the square root of length *L*. Therefore, when the length is doubled, the flow rate will decrease by a factor of $2^{0.5} = 1.41$ since

If $$\dot{V}_A = \frac{k}{\sqrt{L_A}}$$

Then $$\dot{V}_B = \frac{k}{\sqrt{L_B}} = \frac{k}{\sqrt{2L_A}} = \frac{k}{\sqrt{2}\sqrt{L_A}} = \frac{\dot{V}_A}{\sqrt{2}} = 0.707\dot{V}_A$$



Therefore, the ratio of the flow rates in the two pipes is **0.707**.

8-123 A pipeline that transports oil at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Entrance effects are negligible, and thus the flow is fully developed. **3** Minor losses are disregarded. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of oil at 40°C are ρ = 876 kg/m^3 and μ = 0.2177 kg/m·s. The roughness of commercial steel pipes is ε = 0.000045 m.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

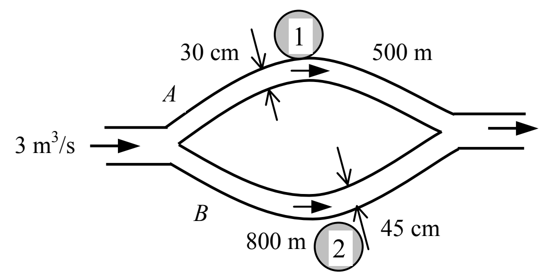
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \;\rightarrow\; \dot{V}_1 + \dot{V}_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \;\rightarrow\; V_1 = \frac{\dot{V}_1}{\pi(0.30\text{m})^2/4} \qquad (3)$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \;\rightarrow\; V_2 = \frac{\dot{V}_2}{\pi(0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(876\text{ kg/m}^3)\mathbf{V}_1(0.30\text{ m})}{0.2177\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(876\text{ kg/m}^3)\mathbf{V}_2(0.45\text{ m})}{0.2177\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and solving them simultaneously by an equation solver gives

$$\dot{V}_1 = \mathbf{0.91\text{ m}^3/s},\quad \dot{V}_2 = \mathbf{2.09\text{ m}^3/s},$$

V_1 = 12.9 m/s, V_2 =13.1 m/s, $h_{L,1} = h_{L,2} = 392$ m

Re_1 = 15,540, Re_2 = 23,800, f_1 = 0.02785, f_2 = 0.02505

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using an iterative approach, but it will be very time consuming. Equations solvers such as EES are invaluable for theses kinds of problems.

8-124 The piping of a district heating system that transports hot water at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are given to be negligible. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 100°C are $\rho = 957.9$ kg/m^3 and $\mu = 0.282\times10^{-3}$ kg/m·s. The roughness of commercial steel pipes is $\varepsilon = 0.000045$ m.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

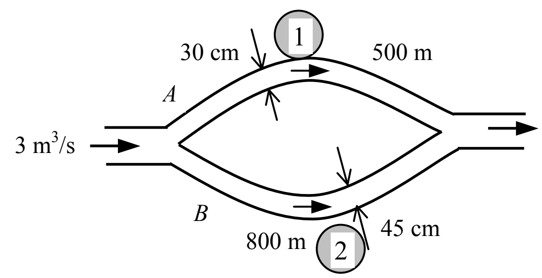
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \;\rightarrow\; \dot{V}_1 + \dot{V}_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \;\rightarrow\; V_1 = \frac{\dot{V}_1}{\pi(0.30\text{m})^2/4} \qquad (3)$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \;\rightarrow\; V_2 = \frac{\dot{V}_2}{\pi(0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(957.9\text{ kg/m}^3)V_1(0.30\text{ m})}{0.282\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(957.9\text{ kg/m}^3)V_2(0.45\text{ m})}{0.282\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V}_1 = \mathbf{0.919\ m^3/s},\quad \dot{V}_2 = \mathbf{2.08\ m^3/s},$$

$V_1 = 13.0$ m/s, $V_2 = 13.1$ m/s, $h_{L,1} = h_{L,2} = 187$ m

$\text{Re}_1 = 1.324\times10^7$, $\text{Re}_2 = 2.00\times10^7$, $f_1 = 0.0131$, $f_2 = 0.0121$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using a trial-and-error approach, but it will be very time consuming. Equations solvers such as EES are invaluable for these kinds of problems.

8-125E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia. **5** Elevation difference between the pipe and the fountain is negligible ($z_2 = z_1$) . **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s. The roughness of cast iron pipe is $\varepsilon = 0.00085$ ft. The minor loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 1.1$ for a 90° miter bend without vanes, $K_L = 0.2$ for a fully open gate valve, and $K_L = 5$ for an angle valve.

Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe to be low. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points is

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$

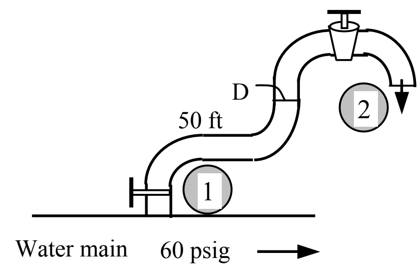
since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60\text{ psi}}{(62.3\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \quad (1)$$

The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4} \quad\rightarrow\quad V_2=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \quad (2)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2 D}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} \quad (3)$$



The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.00085/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{gate valve}}+K_{L,\text{angle valve}}=0.5+3\times1.1+0.2+5=9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \quad\rightarrow\quad h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of V_2, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

$V_2 = 14.3$ ft/s, $h_L = 135.5$ ft, $D = 0.0630$ ft $=$ **0.76 in**, $\text{Re} = 85{,}540$, and $f = 0.04263$

Therefore, the diameter of the pipe must be at least 0.76 in (or roughly 3/4 in).

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give $D = 0.73$ in, which is within 5% of the result obtained above.

8-126E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia. **5** Elevation difference between the pipe and the fountain is negligible ($z_2 = z_1$) . **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The minor loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 1.1$ for a 90° miter bend without vanes, $K_L = 0.2$ for a fully open gate valve, and $K_L = 5$ for an angle valve.

Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe to be low. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points is

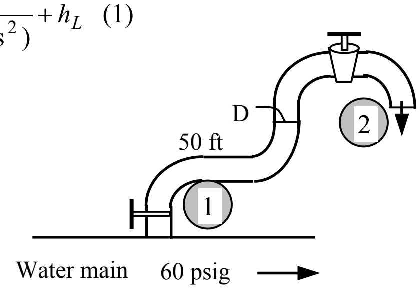
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$

since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60\text{ psi}}{(62.3\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \quad (1)$$

The average velocity in the pipe and the Reynolds number are



$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4} \quad\rightarrow\quad V_2=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \quad (2)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2 D}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} \quad (3)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{gate valve}}+K_{L,\text{angle valve}}=0.5+3\times1.1+0.2+5=9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \quad\rightarrow\quad h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of V_2, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

$V_2 = 18.4$ ft/s, $h_L = 133.4$ ft, $D = 0.05549$ ft = **0.67 in**, Re = 97,170, and $f = 0.0181$

Therefore, the diameter of the pipe must be at least 0.67 in.

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give $D = 0.62$ in, which is within 7% of the result obtained above.

8-127 In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The electric power output of the plant is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The minor losses are given to be negligible. **4** The water level in the reservoir remains constant.

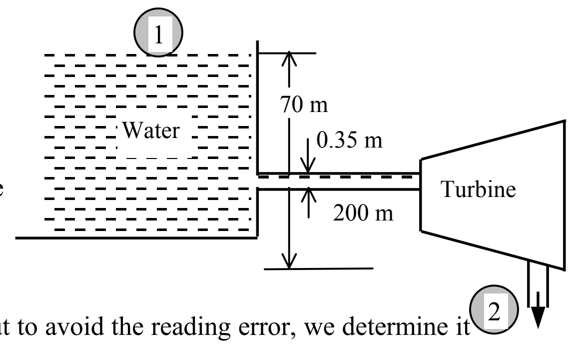
Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. The roughness of cast iron pipes is ε = 0.00026 m.

Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site (z_2 = 0). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($V_1 \cong V_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=z_1-h_L$$

The average velocity, Reynolds number, friction factor, and head loss in the pipe are

$$V=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(0.35\ \text{m})^2/4}=8.315\ \text{m/s}$$



$$\text{Re}=\frac{\rho VD_h}{\mu}=\frac{(998\ \text{kg/m}^3)(8.513\ \text{m/s})(0.35\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=2.899\times10^6$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_h=\frac{0.00026\ \text{m}}{0.35\ \text{m}}=7.43\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\ \rightarrow\ \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt{f}}\right)$$

It gives f = 0.0184. When the minor losses are negligible, the head loss in the pipe and the available turbine head are determined to be

$$h_L=f\frac{L}{D}\frac{V^2}{2g}=0.0184\frac{200\ \text{m}}{0.35\ \text{m}}\frac{(8.315\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=37.05\ \text{m}$$

$$h_{\text{turbine, e}}=z_1-h_L=70-37.05=32.95\ \text{m}$$

Then the extracted power from water and the actual power output of the turbine become

$$\dot{W}_{\text{turbine, e}}=\dot{m}gh_{\text{turbine, e}}=\rho\dot{V}gh_{\text{turbine, e}}$$

$$=(998\ \text{kg/m}^3)(0.8\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(32.95\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=258\ \text{kW}$$

$$\dot{W}_{\text{turbine-gen}}=\eta_{\text{turbine-gen}}\dot{W}_{\text{turbine, e}}=(0.84)(258\ \text{kW})=\mathbf{217\ kW}$$

Discussion Note that a perfect turbine-generator would generate 258 kW of electricity from this resource. The power generated by the actual unit is only 217 kW because of the inefficiencies of the turbine and the generator. Also note that more than half of the elevation head is lost in piping due to pipe friction.

8-128 In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The percent increase in the electric power output of the plant is to be determined when the pipe diameter is tripled.

Assumptions **1** The flow is steady and incompressible. **2** Entrance effects are negligible, and thus the flow is fully developed and friction factor is constant. **3** Minor losses are negligible. **4** Water level is constant.

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. The roughness of cast iron pipes is ε = 0.00026 m.

Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site (z_2 = 0). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($V_1 \cong V_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=z_1-h_L$$

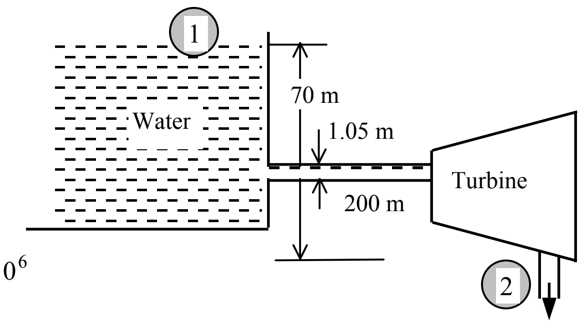
The average velocity, Reynolds number, friction factor, and head loss in the pipe for both cases (pipe diameter being 0.35 m and 1.05 m) are

$$V_1=\frac{\dot V}{A_{c1}}=\frac{\dot V}{\pi D_1^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(0.35\ \text{m})^2/4}=8.315\ \text{m/s}\,,$$

$$V_2=\frac{\dot V}{A_{c2}}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(1.05\ \text{m})^2/4}=0.9239\ \text{m/s}$$

$$\text{Re}_1=\frac{\rho V_1 D_1}{\mu}=\frac{(998\ \text{kg/m}^3)(8.315\ \text{m/s})(0.35\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=2.899\times10^6$$

$$\text{Re}_2=\frac{\rho V_2 D_1}{\mu}=\frac{(998\ \text{kg/m}^3)(0.9239\ \text{m/s})(1.05\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=0.9662\times10^6$$



which are greater than 4000. Therefore, the flow is turbulent for both cases. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\ \text{m}}{0.35\ \text{m}}=7.43\times10^{-4} \qquad \text{and} \qquad \varepsilon/D_2=\frac{0.00026\ \text{m}}{1.05\ \text{m}}=2.476\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt{f_1}}\right) \quad \text{and} \quad \frac{1}{\sqrt{f_2}}=-2.0\log\left(\frac{2.476\times10^{-4}}{3.7}+\frac{2.51}{0.9662\times10^6\sqrt{f_2}}\right)$$

The friction factors are determined to be f_1 = 0.01842 and f_2 = 0.01520. When the minor losses are negligible, the head losses in the pipes and the head extracted by the turbine are determined to be

$$h_{L1}=f_1\frac{L}{D_1}\frac{V_1^2}{2g}=0.01842\frac{200\ \text{m}}{0.35\ \text{m}}\frac{(8.315\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=37.09\ \text{m}\,,\ h_{\text{turbine},1}=z_1-h_{L1}=70-37.09=32.91\ \text{m}$$

$$h_{L2}=f_2\frac{L}{D_2}\frac{V_2^2}{2g}=0.0152\frac{200\ \text{m}}{1.05\ \text{m}}\frac{(0.9239\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=0.13\ \text{m}\,,\ h_{\text{turbine},2}=z_2-h_{L2}=70-0.13=69.87\ \text{m}$$

The available or actual power output is proportional to the turbine head. Therefore, the increase in the power output when the diameter is doubled becomes

$$\text{Increase in power output}=\frac{h_{\text{turbine},2}-h_{\text{turbine},1}}{h_{\text{turbine},1}}=\frac{69.87-32.91}{32.91}=\mathbf{1.12\ \ or\ \ 112\%}$$

Discussion Note that the power generation of the turbine more than doubles when the pipe diameter is tripled at the same flow rate and elevation.

8-129E The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened and when it is empty are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing. **4** The water level in the bottle remains nearly constant during filling. **5** The flow is turbulent (to be verified). **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The total minor loss coefficient is given to be 2.8.

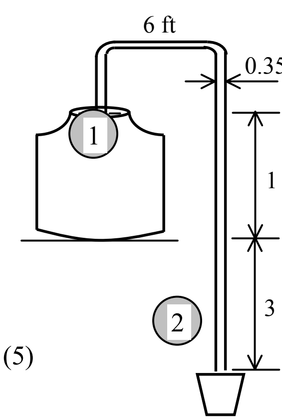
Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose., which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad z_1 = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where $\alpha_2 = 1$ and $h_L = \left(f\dfrac{L}{D} + \sum K_L\right)\dfrac{V_2^2}{2g} \;\rightarrow\; h_L = \left(f\dfrac{6 \text{ ft}}{0.35/12 \text{ ft}} + 2.8\right)\dfrac{V_2^2}{2(32.2 \text{ ft/s}^2)}$ (1)

since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1 = (1)\frac{V_2^2}{2(32.2 \text{ ft/s}^2)} + h_L \qquad (2)$$



The average velocity in the pipe and the Reynolds number are

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \rightarrow V_2 = \frac{\dot{V} \text{ ft}^3\text{/s}}{\pi(0.35/12 \text{ ft})^2/4} \qquad (3)$$

$$\text{Re} = \frac{\rho V_2 D}{\mu} \rightarrow \text{Re} = \frac{(62.3 \text{ lbm/ft}^3)V_2(0.35/12 \text{ ft})}{6.556\times10^{-4} \text{ lbm/ft}\cdot\text{s}} \qquad (4)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad (5)$$

Finally, the filling time of the glass is

$$\Delta t = \frac{V_{\text{glass}}}{\dot{V}} = \frac{0.00835 \text{ ft}^3}{\dot{V} \text{ ft}^3\text{/s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of V_2, $\dot{V}$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus $z_1 = 3+1 = 4$ ft:

$V_2 = 5.185$ ft/s, $h_L = 3.58$ ft, $\dot{V} = 0.00346$ ft^3/s, Re = 14,370, $f = 0.02811$, and Δt = **2.4 s**

Case (*b*): The bottle is almost empty and thus $z_1 = 3$ ft:

$V_2 = 4.436$ ft/s, $h_L = 2.69$ ft, $\dot{V} = 0.00296$ ft^3/s, Re = 12,290, $f = 0.02926$, and Δt = **2.8 s**

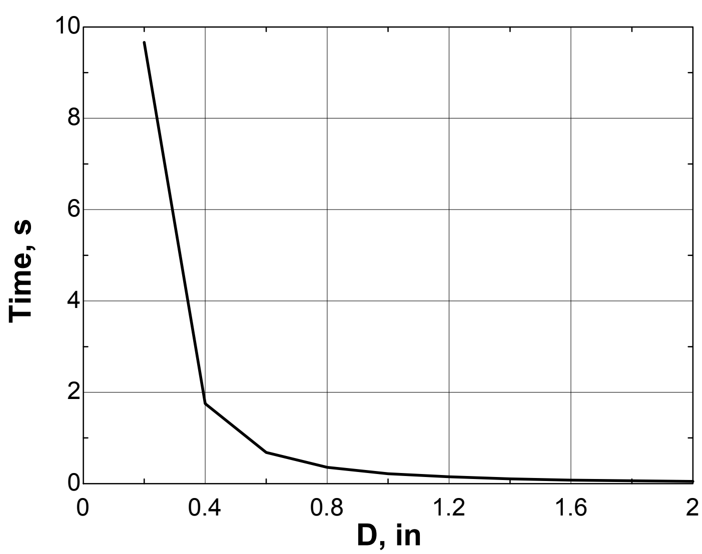
Note that the flow is turbulent for both cases since Re > 4000.

Discussion The filling time of the glass increases as the water level in the bottle drops, as expected.

8-130E In Prob. 8-129E, the effect of the hose diameter on the time required to fill a glass when the bottle is full is to be investigated by varying the pipe diameter from 0.2 to 2 in. in increments of 0.2 in.

```
rho=62.3
mu=2.36/3600
nu=mu/rho
g=32.2
z1=4
Volume=0.00835
D=Din/12
Ac=pi*D^2/4
L=6
KL=2.8
eps=0
rf=eps/D
V=Vdot/Ac
"Reynolds number"
Re=V*D/nu
1/sqrt(f)=-2*log10(rf/3.7+2.51/(Re*sqrt(f)))
HL=(f*L/D+KL)*(V^2/(2*g))
z1=V^2/(2*g)+HL
Time=Volume/Vdot
```

D, in	*Time,* s	h_L, ft	Re
0.2	9.66	3.76	6273
0.4	1.75	3.54	17309
0.6	0.68	3.40	29627
0.8	0.36	3.30	42401
1.0	0.22	3.24	55366
1.2	0.15	3.20	68418
1.4	0.11	3.16	81513
1.6	0.08	3.13	94628
1.8	0.06	3.11	107752
2.0	0.05	3.10	120880



8-131E The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing. **4** The water level in the bottle remains constant during filling. **5** The flow is turbulent (to be verified). **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The total minor loss coefficient is given to be 2.8 during filling.

Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L=\left(f\dfrac{L}{D}+\sum K_L\right)\dfrac{V_2^2}{2g} \;\rightarrow\; h_L=\left(f\dfrac{12\text{ ft}}{0.35/12\text{ ft}}+2.8\right)\dfrac{V_2^2}{2(32.2\text{ ft/s}^2)}$ (1)

since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1=(1)\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \qquad (2)$$

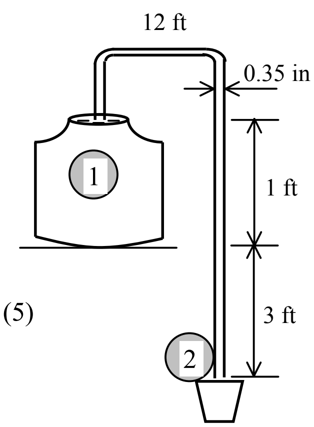
The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4} \quad\rightarrow\quad V_2=\frac{\dot{V}\text{ ft}^3\text{/s}}{\pi(0.35/12\text{ ft})^2/4} \qquad (3)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2(0.35/12\text{ ft})}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \qquad (4)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad (5)$$



Finally, the filling time of the glass is

$$\Delta t=\frac{V_{\text{glass}}}{\dot{V}}=\frac{0.00835\text{ ft}^3}{\dot{V}\text{ ft}^3\text{/s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of V_2, $\dot{V}$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus $z_1 = 3+1 = 4$ ft:

$V_2 = 3.99$ ft/s, $h_L = 3.75$ ft, $\dot{V} = 0.002667$ ft^3/s, Re = 11,060, $f = 0.03007$, and $\Delta t =$ **3.1 s**

Case (*b*): The bottle is almost empty and thus $z_1 = 3$ ft:

$V_2 = 3.40$ ft/s, $h_L = 2.82$ ft, $\dot{V} = 0.002272$ ft^3/s, Re = 9426, $f = 0.03137$, and $\Delta t =$ **3.7 s**

Note that the flow is turbulent for both cases since Re > 4000.

Discussion The filling times in Prob. 8-129E were 2.4 s and 2.8 s, respectively. Therefore, doubling the tube length increases the filling time by 0.7 s when the bottle is full, and by 0.9 s when it is empty.

8-132 A water pipe has an abrupt expansion from diameter D_1 to D_2. It is to be shown that the loss coefficient is $K_L = (1 - D_1^2 / D_2^2)^2$, and K_L and P_2 are to be calculated.

Assumptions **1** The flow is steady and incompressible. **2** The pressure is uniform at the cross-section where expansion occurs, and is equal to the upstream pressure P_1. **3** The flow section is horizontal (or the elevation difference across the expansion section is negligible). **4** The flow is turbulent, and the effects of kinetic energy and momentum-flux correction factors are negligible, $\beta \approx 1$ and $\alpha \approx 1$.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis We designate the cross-section where expansion occurs by x. We choose cross-section 1 in the smaller diameter pipe shortly before x, and section 2 in the larger diameter pipe shortly after x. We take the region occupied by the fluid between cross-sections 1 and 2 as the control volume, with an inlet at 1 and exit at 2. The velocity, pressure, and cross-sectional area are V_1, P_1, and A_1 at cross-section 1, and V_2, P_2, and A_2 at cross-section 2. We assume the pressure along the cross-section x to be P_1 so that $P_x = P_1$. Then the continuity, momentum, and energy equations applied to the control volume become

(1) *Continuity*: $\dot m_1 = \dot m_2 \ \rightarrow\ \rho V_1 A_1 = \rho V_2 A_2 \ \rightarrow\ V_2 = \dfrac{A_1}{A_2} V_1$ (1)

(2) *Momentum*: $\sum \vec F = \sum_{\text{out}} \beta \dot m V - \sum_{\text{in}} \beta \dot m V \ \rightarrow\ P_1 A_1 + P_1 (A_x - A_1) - P_2 A_2 = \dot m (V_2 - V_1)$

But $P_1 A_1 + P_1 (A_x - A_1) = P_1 A_x = P_1 A_2$

$$\dot m (V_2 - V_1) = \rho A_2 V_2 (V_2 - V_1) = \rho A_2 \frac{A_1}{A_2} V_1 \left(\frac{A_1}{A_2} V_1 - V_1 \right) = \rho A_2 \frac{A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) V_1^2$$

Therefore, $P_1 A_2 - P_2 A_2 = \rho A_2 \dfrac{A_1}{A_2} \left(\dfrac{A_1}{A_2} - 1 \right) V_1^2 \ \rightarrow\ \dfrac{P_1 - P_2}{\rho} = \dfrac{A_1}{A_2} \left(\dfrac{A_1}{A_2} - 1 \right) V_1^2$ (2)

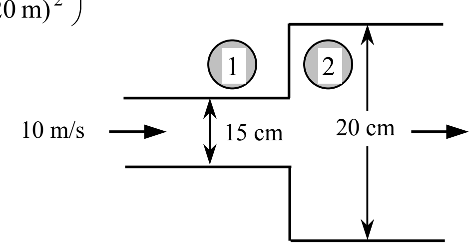
(3) *Energy*: $\dfrac{P_1}{\rho g} + \alpha_1 \dfrac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \dfrac{P_2}{\rho g} + \alpha_2 \dfrac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \ \rightarrow\ h_L = \dfrac{P_1 - P_2}{\rho g} + \dfrac{V_1^2 - V_2^2}{2g}$ (3)

Substituting Eqs. (1) and (2) and $h_L = K_L \dfrac{V_1^2}{2g}$ into Eq. (3) gives

$$K_L \frac{V_1^2}{2g} = \frac{A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) \frac{V_1^2}{g} + \frac{V_1^2 - (A_1^2 / A_2^2) V_1^2}{2g} \ \rightarrow\ K_L = \frac{2A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) + \left(1 - \frac{A_1^2}{A_2^2} \right)$$

Simplifying and substituting $A = \pi D^2/4$ gives the desired relation and its value,

$$K_L = \left(1 - \frac{A_1}{A_1} \right)^2 = \left(1 - \frac{\pi D_1^2 / 4}{\pi D_2^2 / 4} \right)^2 = \left(1 - \frac{D_1^2}{D_2^2} \right)^2 = \left(1 - \frac{(0.15\ \text{m})^2}{(0.20\ \text{m})^2} \right)^2 = 0.1914$$



Also, $h_L = K_L \dfrac{V_1^2}{2g} = (0.1914) \dfrac{(10\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 0.9756\ \text{m}$

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1 = \frac{(0.15\ \text{m})^2}{(0.20\ \text{m})^2} (10\ \text{m/s}) = 5.625\ \text{m/s}$$

Solving for P_2 from Eq. (3) and substituting,

$$P_2 = P_1 + \rho \{ (V_1{}^2 - V_2{}^2)/2 - g h_L \}$$

$$= (120\ \text{kPa}) + (1000\ \text{kg/m}^3) \left\{ \frac{(10\ \text{m/s})^2 - (5.625\ \text{m/s})^2}{2} - (9.81\ \text{m/s}^2)(0.9756\ \text{m}) \right\} \left(\frac{1\ \text{kPa} \cdot \text{m}^2}{1000\ \text{kg} \cdot \text{m/s}^2} \right) = \mathbf{145\ kPa}$$

Note that the pressure increases by 25 kPa after the expansion due to the conversion of dynamic pressure to static pressure when the velocity is decreased. Also, $K_L = 1$ when $D_2 >> D_1$ (discharging into a reservoir).

8-133 A swimming pool is initially filled with water. A pipe with a well-rounded entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow). **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

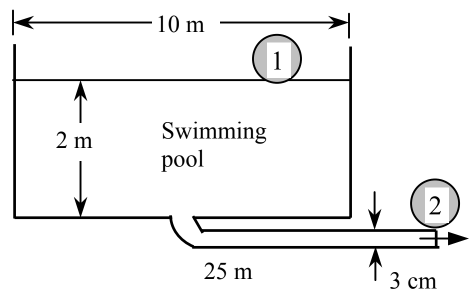
Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s. The friction factor of the pipe is given to be 0.022. Plastic pipes are considered to be smooth, and their surface roughness is $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where

$$h_L=f\frac{L}{D}\frac{V^2}{2g}$$



since the minor losses are negligible. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+\left(f\frac{L}{D}\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+fL/D}}$$

Noting that $\alpha_2 = 1$ and initially $z_1 = 2$ m, the initial velocity and flow rate are determined to be

$$V_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.022(25\text{ m})/(0.03\text{ m})}}=1.425\text{ m/s}$$

$$\dot{V}_{initial}=V_{2,i}A_c=V_{2,i}(\pi D^2/4)=(1.425\text{ m/s})[\pi(0.03\text{ m})^2/4]=1.01\times10^{-3}\text{ m}^3\text{/s}=1.01\text{ L/s}$$

The average discharge velocity at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+fL/D}}$$

where z is the water height relative to the center of the orifice at that time.

We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$dV=A_{c,\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt = -\frac{\pi D_0^2}{4}dz \;\rightarrow\; dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2gz}}dz = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \;\rightarrow\; t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D)}{g}} = \frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})[1+(0.022)(25\text{ m})/(0.03\text{ m})]}{9.81\text{ m/s}^2}} = 312{,}000\text{ s} = \mathbf{86.7\ h}$$

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re} = \frac{\rho V_2 D}{\mu} = \frac{(998\text{ kg/m}^3)(1.425\text{ m/s})(0.03\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = 42{,}580$$

which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}} = -2.0\log\left(0 + \frac{2.51}{42{,}570\sqrt{f}}\right)$$

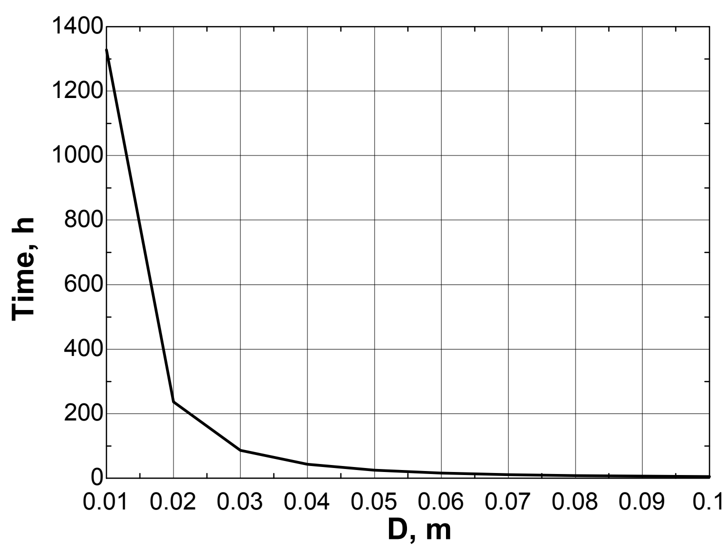
It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 18 h. Therefore, the pipe in this case increases the draining time by about 5 folds.

8-134 In Prob. 8-133, the effect of the discharge pipe diameter on the time required to empty the pool completely is to be investigated by varying the pipe diameter from 1 cm to 10 cm in increments of 1 cm.

```
rho=998
mu=0.001002
g=9.81
Dtank=10
Ac=pi*D^2/4
L=25
f=0.022
z1=2
V=(2*g*z1/(1+f*L/D))^0.5
Vdot=V*Ac
Time=(Dtank/D)^2*(2*z1*(1+f*L/D)/g)^0.5/3600
```

D, m	*Time,* h	$V_{initial}$, m/s	Re
0.01	1327.4	0.84	8337
0.02	236.7	1.17	23374
0.03	86.7	1.42	42569
0.04	42.6	1.63	64982
0.05	24.6	1.81	90055
0.06	15.7	1.96	117406
0.07	10.8	2.10	146750
0.08	7.8	2.23	177866
0.09	5.8	2.35	210572
0.10	4.5	2.46	244721



8-135 A swimming pool is initially filled with water. A pipe with a sharp-edged entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow). **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s. The loss coefficient for the sharp-edged entrance is $K_L = 0.5$. Plastic pipes are considered to be smooth, and their surface roughness is $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for V_2 gives

$$z_1=\frac{V_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}$$

Noting that initially $z_1 = 2$ m, the initial velocity and flow rate are determined to be

$$V_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.022(25\text{ m})/(0.03\text{ m})+0.5}}=1.407\text{ m/s}$$

$$\dot{V}_{initial}=V_{2,i}A_c=V_{2,i}(\pi D^2/4)=(1.407\text{ m/s})[\pi(0.03\text{ m})^2/4]=9.94\times10^{-4}\text{ m}^3/\text{s}=0.994\text{ L/s}$$

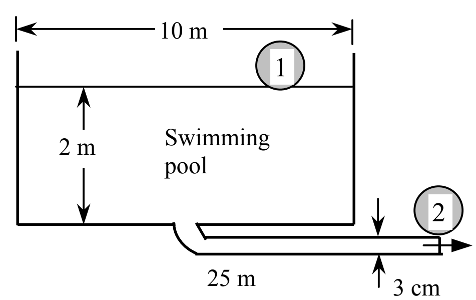
The average discharge velocity at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{\boldsymbol{V}}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}$$



Then the amount of water that flows through the pipe during a differential time interval dt is

$$d\boldsymbol{V}=\dot{\boldsymbol{V}}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$d\boldsymbol{V}=A_{c,\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt=-\frac{\pi D_0^2}{4}dz \;\rightarrow\; dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0}z^{-1/2}dz \;\rightarrow\; t_f=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f=\frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}}=\frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})[1+(0.022)(25\text{ m})/(0.03\text{ m})+0.5]}{9.81\text{ m/s}^2}}=316{,}000\text{ s}=\mathbf{87.8\text{ h}}$$

This is a change of (87.8-86.7)/86.7 = 0.013 or 1.3%. Therefore, the minor loss in this case is **truly minor**.

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(998\text{ kg/m}^3)(1.407\text{ m/s})(0.03\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}}=42{,}030$$

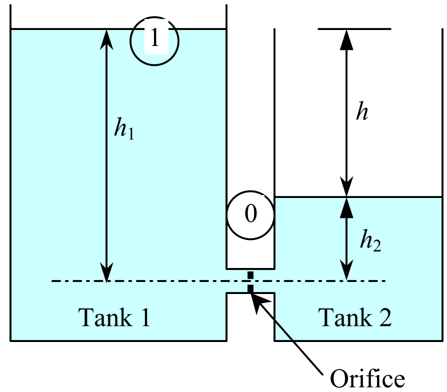
which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{42{,}030\sqrt{f}}\right)$$

It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 24 h. Therefore, the pipe in this case increases the draining time more than 3 folds.

8-136 A system that concsists of two interconnected cylindrical tanks is used to determine the discharge coefficient of a short 5-mm diameter orifice. For given initial fluid heights and discharge time, the discharge coefficient of the orifice is to be determined.



Assumptions **1** The fluid is incompressible. **2** The entire systems, including the connecting flow section, is horizontal. **3** The discharge coefficient remains constant (in reality, it may change since the flow velocity and thus the Reynolds number changes during flow). **4** Losses other than the ones associated with flow through the orifice are negligible. **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Analysis We take point 1 at the free surface of water in Tank 1, and point 0 at the exit of the orifice. We take the centerline of the orifice as the reference level ($z_1 = h_1$ and $z_0 = 0$). Noting that the fluid at point 1is open to the atmosphere (and thus $P_1 = P_{atm}$ and $P_0 = P_{atm} + \rho g h_2$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the *Bernoulli equation* between these two points gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_0}{\rho g}+\frac{V_0^2}{2g}+z_0 \;\rightarrow\; \frac{P_{atm}}{\rho g}+h_1=\frac{P_{atm}+\rho g h_2}{\rho g}+\frac{V_0^2}{2g} \;\rightarrow\; V_0=\sqrt{2g(h_1-h_2)}=\sqrt{2gh}$$

where $h = h_1 - h_2$ is the vertical distance between the water levels in the two tanks at any time t. Note that h_1, h_2, h, and V_0 are all variable (h_1 decreases while h_2 and h increase during discharge.

Noting that the fluid is a liquid (ρ = constant) and keeping the conservation of mass in mind and the definition of the discharge coefficient C_d, the flow rate through the orifice can be expressed as

$$\dot{V} = C_d V_o A_o = -A_1\frac{dh_1}{dt} = A_2\frac{dh_2}{dt} \;\longrightarrow\; dh_2 = -\frac{A_1}{A_2}dh_1$$

Also, $h = h_1 - h_2 \longrightarrow dh = dh_1 - dh_2 \longrightarrow dh_1 = dh_2 + dh$ (Note that $dh < 0$, $dh_1 < 0$, and $dh_2 > 0$)

Combining the two equations above,

$$dh_1 = \frac{dh}{1 + A_1/A_2}$$

Then, $$\dot{V} = C_d V_o A_o = -A_1\frac{dh_1}{dt} \;\longrightarrow\; C_d A_o\sqrt{2gh} = -A_1\frac{1}{1+A_1/A_2}\frac{dh}{dt}$$

which can be rearranged as $$-dt = \frac{A_1A_2}{A_1+A_2}\frac{1}{C_d A_o\sqrt{2g}}\frac{dh}{\sqrt{h}}$$

Integrating $$-\int_0^t dt = \frac{A_1A_2}{A_1+A_2}\frac{1}{C_d A_o\sqrt{2g}}\int_{h_1}^{h}\frac{dh}{\sqrt{h}}$$

Performing the integration $$t = -\frac{A_1A_2}{A_1+A_2}\frac{2}{C_d A_o\sqrt{2g}}\left[\sqrt{h}-\sqrt{h_1}\right]$$

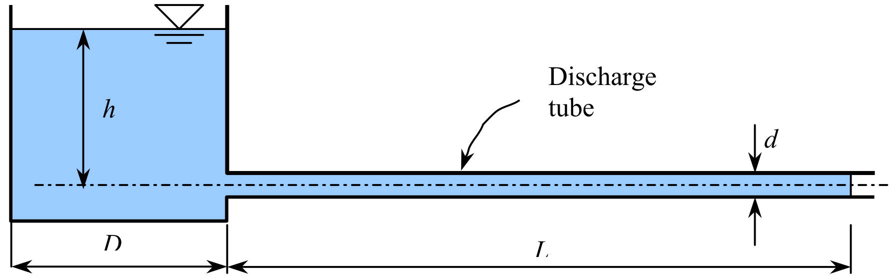
Solving for C_d $$C_d = \frac{2(\sqrt{h_1}-\sqrt{h})}{(A_0/A_2 + A_0/A_1)t\sqrt{2g}}$$

Fluid flow stops when the liquid levels in the two tanks become equal (and thus h = 0). Substituign the given values, the discharge coefficient is determined to be

$$\frac{A_0}{A_2}+\frac{A_0}{A_1}=\left(\frac{D_0}{D_2}\right)^2+\left(\frac{D_0}{D_1}\right)^2=\left(\frac{0.5\text{ cm}}{30\text{ cm}}\right)^2+\left(\frac{0.5\text{ cm}}{12\text{ cm}}\right)^2=0.002014\ ,$$

$$C_d=\frac{2\sqrt{0.5\text{ m}}}{(0.002014)(170\text{ s})\sqrt{2\times 9.81\text{ m/s}^2}}=\mathbf{0.933}$$

8-137 A highly viscous liquid discharges from a large container through a small diameter tube in laminar flow. A relation is to be obtained for the variation of fluid depth in the tank with time.



Assumptions **1** The fluid is incompressible. **2** The discharge tube is horizontal, and the flow is laminar. **3** Entrance effects and the velocity heads are negligible.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the exit of the pipe. We take the centerline of the pipe as the reference level ($z_1 = h$ and $z_2 = 0$). Noting that the fluid at both points 1 and 2 are open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and the velocity heads are negligible, the energy equation for a control volume between these two points gives

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L \quad\to\quad \frac{P_{atm}}{\rho g}+h=\frac{P_{atm}}{\rho g}+h_L \quad\to\quad h_L=h \qquad (1)$$

where h is the liquid height in the tank at any time t. The total head loss through the pipe consists of major losses in the pipe since the minor losses are negligible. Also, the entrance effects are negligible and thus the friction factor for the entire tube is constant at the fully developed value. Noting that $f = \mathrm{Re}/64$ for fully developed laminar flow in a circular pipe of diameter d, the head loss can be expressed as

$$h_L = f\frac{L}{d}\frac{V^2}{2g}=\frac{64}{\mathrm{Re}}\frac{L}{d}\frac{V^2}{2g}=\frac{64}{Vd/\nu}\frac{L}{d}\frac{V^2}{2g}=\frac{64\nu L}{d^2}\frac{V}{2g} \qquad (2)$$

The average velocity can be expressed in terms of the flow rate as $V=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi d^2/4}$. Substituting into (2),

$$h_L=\frac{64\nu L}{d^2}\frac{1}{2g}\left(\frac{\dot{V}}{\pi d^2/4}\right)=\frac{64\nu L}{d^2}\frac{4\dot{V}}{2g\pi d^2}=\frac{128\nu L\dot{V}}{g\pi d^4} \qquad (3)$$

Combining Eqs. (1) and (3):
$$h=\frac{128\nu L\dot{V}}{g\pi d^4} \qquad (4)$$

Noting that the liquid height h in the tank decreases during flow, the flow rate can also be expressed in terms of the rate of change of liquid height in the tank as

$$\dot{V}=-A_{\text{tank}}\frac{dh}{dt}=-\frac{\pi D^2}{4}\frac{dh}{dt} \qquad (5)$$

Substituting Eq. (5) into (4):
$$h=-\frac{128\nu L}{g\pi d^4}\frac{\pi D^2}{4}\frac{dh}{dt}=-\frac{32\nu LD^2}{gd^4}\frac{dh}{dt} \qquad (6)$$

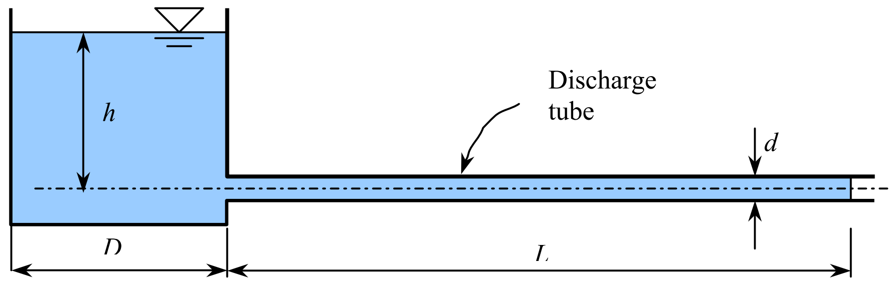
To separate variables, it can be rearranged as
$$dt=-\frac{32\nu LD^2}{gd^4}\frac{dh}{h}$$

Integrating from $t = 0$ (at which $h = H$) to $t = t$ (at which $h = h$) gives

$$t=\frac{32\nu LD^2}{gd^4}\ln(H/h)$$

which is the desired relation for the variation of fluid depth h in the tank with time t.

8-138 Using the setup described in the previous problem, the viscosity of an oil is to be determined for a given set of data.



Assumptions **1** The oil is incompressible. **2** The discharge tube is horizontal, and the flow is laminar. **3** Entrance effects and the inlet and the exit velocity heads are negligible.

Analysis The variation of fluid depth *h* in the tank with time *t* was determined in the previous problem to be

$$t = \frac{32\nu L D^2}{g d^4} \ln(H/h)$$

Solving for ν and substituting the given values, the kinematic viscosity of the oil is determined to be

$$\nu = \frac{g d^4}{32 L D^2 \ln(H/h)} t = \frac{(9.81\ \text{m/s}^2)(0.006\ \text{m})^4}{32(0.65\ \text{m})(0.63\ \text{m})^2 \ln(0.4/0.36)}(2842\ \text{s}) = \mathbf{4.15 \times 10^{-5}\ m^2/s}$$

Discussion Note that the entrance effects are not considered, and the velocity heads are disregarded. Also, the value of the viscosity strongly depends on temperature, and thus the oil temperature should be maintained constant during the test.

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