Piping Systems and Pump Selection

8-62C For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *series*, (*a*) the flow rate through both pipes is the same and (*b*) the pressure drop across smaller diameter pipe is larger.

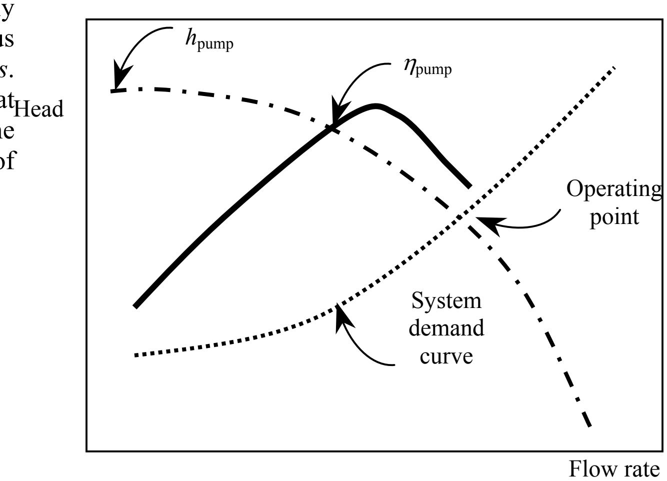
8-63C For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *parallel*, (*a*) the flow rate through the larger diameter pipe is larger and (*b*) the pressure drop through both pipes is the same.

8-64C The pressure drop through both pipes is the *same* since the pressure at a point has a single value, and the inlet and exits of these the pipes connected in parallel coincide.

8-65C Yes, when the head loss is negligible, the required pump head is equal to the elevation difference between the free surfaces of the two reservoirs.

8-66C The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the *operating point*.

8-67C The plot of the head loss versus the flow rate is called the *system curve*. The experimentally determined pump head and pump efficiency versus the flow rate curves are called *characteristic curves*. The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the *operating point*.



8-68 The pumping power input to a piping system with two parallel pipes between two reservoirs is given. The flow rates are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** The flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. Plastic pipes are smooth, and their roughness is zero, ε = 0.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, nowadays the equation solvers such as EES are widely available, and thus below we will simply set up the equations to be solved by an equation solver. The head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect,in}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 7000\text{ W} = \frac{(998\text{ kg/m}^3)\dot{V}(9.81\text{ m/s}^2)h_{\text{pump, u}}}{0.68} \qquad (1)$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are zero ($V_A = V_B = 0$), the energy equation for a control volume between these two points simplifies to

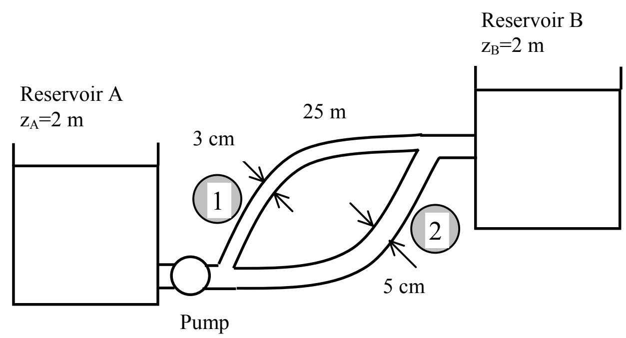
$$\frac{P_A}{\rho g} + \alpha_A \frac{V_A^2}{2g} + z_A + h_{\text{pump, u}} = \frac{P_B}{\rho g} + \alpha_B \frac{V_B^2}{2g} + z_B + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump, u}} = (9-2) + h_L \quad (2)$$

where

$$h_L = h_{L,1} = h_{L,2} \qquad (3)\,(4)$$



We designate the 3-cm diameter pipe by 1 and the 5-cm diameter pipe by 2. The average velocity, Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi (0.03\text{m})^2/4} \qquad (5)$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi (0.05\text{m})^2/4} \qquad (6)$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\text{ kg/m}^3)V_1(0.03\text{ m})}{1.002\times 10^{-3}\text{ kg/m}\cdot s} \qquad (7)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\text{ kg/m}^3)V_2(0.05\text{ m})}{1.002\times 10^{-3}\text{ kg/m}\cdot s} \qquad (8)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (9)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (10$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{25\text{ m}}{0.03\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (11)$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{25\text{ m}}{0.05\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (12)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad (13)$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.0183\ m^3/s}, \quad \dot{V}_1 = \mathbf{0.0037\ m^3/s}, \quad \dot{V}_2 = \mathbf{0.0146\ m^3/s},$$

V_1 = 5.30 m/s, V_2 = 7.42 m/s, $h_L = h_{L,1} = h_{L,2} = 19.5\text{ m}$, $h_{\text{pump,u}}$ = 26.5 m

Re_1 = 158,300, Re_2 = 369,700, f_1 = 0.0164, f_2 = 0.0139

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using an iterative approach, but it will be very time consuming. Equation solvers such as EES are invaluable for this kind of problems.

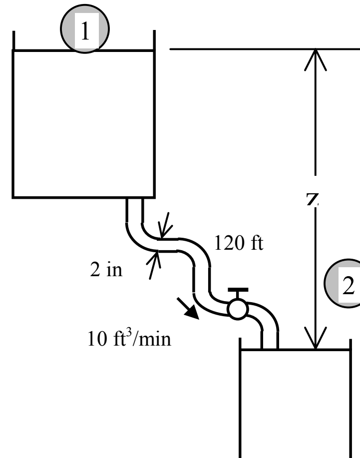
8-69E The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The roughness of cast iron pipe is ε = 0.00085 ft.

Analysis The piping system involves 120 ft of 2-in diameter piping, a well-rounded entrance (K_L = 0.03), 4 standard flanged elbows (K_L = 0.3 each), a fully open gate valve (K_L = 0.2), and a sharp-edged exit (K_L = 1.0). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid velocities at both points are zero ($V_1 = V_2$ =0), the free surface of the lower reservoir is the reference level (z_2 = 0), and that there is no pump or turbine ($h_{pump,u} = h_{turbine} = 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=h_L$$



where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{10/60\text{ ft}^3/\text{s}}{\pi(2/12\text{ ft})^2/4}=7.64\text{ ft/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(62.3\text{ lbm/ft}^3)(7.64\text{ ft/s})(2/12\text{ ft})}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}}=60{,}700$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00085\text{ ft}}{2/12\text{ ft}}=0.0051$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\rightarrow\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{60{,}700\sqrt{f}}\right)$$

It gives f = 0.0320. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+4K_{L,\text{elbow}}+K_{L,\text{valve}}+K_{L,\text{exit}}=0.03+4\times0.3+0.2+1.0=2.43$$

Then the total head loss and the elevation of the source become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left((0.0320)\frac{120\text{ ft}}{2/12\text{ ft}}+2.43\right)\frac{(7.64\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}=23.1\text{ ft}$$

$$z_1=h_L=\mathbf{23.1\ ft}$$

Therefore, the free surface of the first reservoir must be 23.1 ft above the free surface of the lower reservoir to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that fL/D = 23.0 in this case, which is almost 10 folds of the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in an error of about 10%.

8-70 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **3** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. We also take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

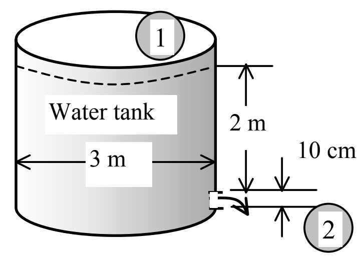
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gz_1=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+K_L}}$$

where $\alpha_2 = 1$. Noting that initially $z_1 = 2$ m, the initial velocity is determined to be

$$V_2=\sqrt{\frac{2gz_1}{1+K_L}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.5}}=\mathbf{5.11\ m/s}$$



The average discharge velocity through the orifice at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the orifice by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the orifice area,

$$\dot{V}=A_{\text{orifice}}V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$d\mathbf{V}=\dot{\mathbf{V}}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}dt=-\frac{\pi D_0^2}{4}dz \quad\rightarrow\quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2gz}}dz \quad\rightarrow\quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}z^{-1/2}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \quad \rightarrow \quad t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}\left|\frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}z_1^{1/2}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+K_L)}{g}} = \frac{(3\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})(1+0.5)}{9.81\,\text{m/s}^2}} = \mathbf{704\ s} = \mathbf{11.7\ min}$$

Discussion The effect of the loss coefficient K_L on the draining time can be assessed by setting it equal to zero in the draining time relation. It gives

$$t_{f,\text{zero loss}} = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1}{g}} = \frac{(3\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})}{9.81\,\text{m/s}^2}} = 575\,\text{s} = 9.6\,\text{min}$$

Note that the loss coefficient causes the draining time of the tank to increase by (11.7- 9.6)/11.7 = 0.18 or 18%, which is quite **significant**. Therefore, the loss coefficient should always be considered in draining processes.

8-71 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe. The initial velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes). **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The friction factor of the pipe is given to be 0.015.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad z_1 = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where

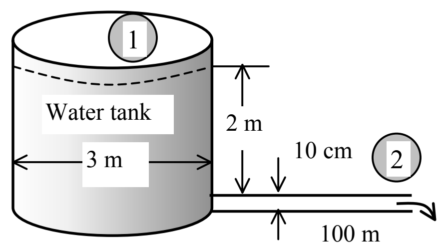
$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L \right) \frac{V^2}{2g} = \left(f\frac{L}{D} + K_L \right) \frac{V^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for V_2 gives

$$z_1 = \alpha_2 \frac{V_2^2}{2g} + \left(f\frac{L}{D} + K_L \right) \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{\frac{2gz_1}{\alpha_2 + fL/D + K_L}}$$

where $\alpha_2 = 1$. Noting that initially $z_1 = 2$ m, the initial velocity is determined to be

$$V_{2,i} = \sqrt{\frac{2gz_1}{1 + fL/D + K_L}} = \sqrt{\frac{2(9.81 \text{ m/s}^2)(2 \text{ m})}{1 + 0.015(100 \text{ m})/(0.1 \text{ m}) + 0.5}} = \mathbf{1.54 \text{ m/s}}$$



The average discharge velocity at any given time, in general, can be expressed as

$$V_2 = \sqrt{\frac{2gz}{1 + fL/D + K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the pipe by D, and the diameter of the tank by D_o. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V} = A_{pipe} V_2 = \frac{\pi D^2}{4} \sqrt{\frac{2gz}{1 + fL/D + K_L}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV = \dot{V} dt = \frac{\pi D^2}{4} \sqrt{\frac{2gz}{1 + fL/D + K_L}} dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\tan k}(-dz) = -\frac{\pi D_0^2}{4} dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt=-\frac{\pi D_0^2}{4}dz \rightarrow dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0}z^{-1/2}dz \rightarrow t_f=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f=\frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}}=\frac{(3\text{ m})^2}{(0.1\text{ m})^2}\sqrt{\frac{2(2\text{ m})[1+(0.015)(100\text{ m})/(0.1\text{ m})+0.5]}{9.81\text{ m/s}^2}}=2334\text{ s}=\mathbf{38.9\ min}$$

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only 11.7 min. Therefore, the pipe in this case increases the draining time by more than 3 folds.

8-72 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump. For a specified initial velocity, the required useful pumping power and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant. **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The friction factor of the pipe is given to be 0.015. The density of water at 30°C is $\rho = 996$ kg/m^3.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. Substituting and noting that the initial discharge velocity is 4 m/s, the required useful pumping head and power are determined to be

$$\dot m=\rho A_c V_2=\rho(\pi D^2/4)V_2=(996\text{ kg/m}^3)[\pi(0.1\text{ m})^2/4](4\text{ m/s})=31.3\text{ kg/m}^3$$

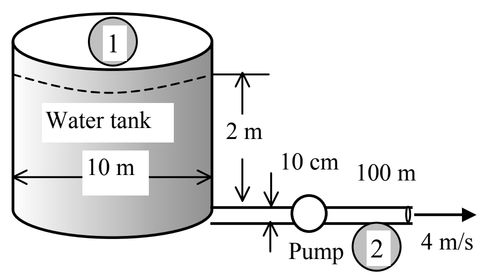
$$h_{\text{pump, u}}=\left(1+f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g}-z_1=\left(1+(0.015)\frac{100\text{ m}}{0.1\text{ m}}+0.5\right)\frac{(4\text{ m/s})^2}{2(9.81\text{ m/s}^2)}-(2\text{ m})=11.46\text{ m}$$

$$\dot W_{\text{pump, u}}=\dot V\Delta P=\dot m g h_{\text{pump, u}}=(31.3\text{ kg/s})(9.81\text{ m/s}^2)(11.46\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\mathbf{3.52\ kW}$$

Therefore, the pump must supply 3.52 kW of mechanical energy to water. Note that the shaft power of the pump must be greater than this to account for the pump inefficiency.

(*b*) When the discharge velocity remains constant, the flow rate of water becomes

$$\dot V=A_c V_2=(\pi D^2/4)V_2=[\pi(0.1\text{ m})^2/4](4\text{ m/s})=0.03142\text{ m}^3\text{/s}$$



The volume of water in the tank is

$$V=A_{\text{tank}}z_1=(\pi D_0^2/4)z_1=[\pi(3\text{ m})^2/4](2\text{ m})=14.14\text{ m}^3$$

Then the discharge time becomes

$$\Delta t=\frac{V}{\dot V}=\frac{14.14\text{ m}^3}{0.03142\text{ m}^3\text{/s}}=450\text{ s}=\mathbf{7.5\ min}$$

Discussion **1** Note that the pump reduces the discharging time from 38.9 min to 7.5 min. The assumption of constant discharge velocity can be justified on the basis of the pump head being much larger than the elevation head (therefore, the pump will dominate the discharging process). The answer obtained assumes that the elevation head remains constant at 2 m (rather than decreasing to zero eventually), and thus it under predicts the actual discharge time. By an exact analysis, it can be shown that when the effect of the decrease in elevation is considered, the discharge time becomes 468 s = 7.8 min. This is demonstrated below.

2 The required pump head (of water) is 11.46 m, which is more than 10.3 m of water column which corresponds to the atmospheric pressure at sea level. If the pump exit is at 1 atm, then the absolute pressure at pump inlet must be negative (= -1.16 m or – 11.4 kPa), which is impossible. Therefore, the system cannot work if the pump is installed near the pipe exit, and cavitation will occur long before the pipe exit where the pressure drops to 4.2 kPa and thus the pump must be installed close to the pipe entrance. A detailed analysis is given below.

Demonstration 1 for Prob. 8-72 (extra) (the effect of drop in water level on discharge time)
Noting that the water height z in the tank is variable, the average discharge velocity through the pipe at any given time, in general, can be expressed as

$$h_{\text{pump, u}} = \left(1 + f\frac{L}{D} + K_L\right)\frac{V_2^2}{2g} - z \quad \rightarrow \quad V_2 = \sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}$$

where z is the water height relative to the center of the orifice at that time. We denote the diameter of the pipe by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the cross-sectional area of the pipe,

$$\dot{V} = A_{pipe}V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\tan k}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}(z + h_{\text{pump,u}})^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\int_{z=z_1}^{0}(z + h_{\text{pump, u}})^{-1/2}dz$$

Performing the integration gives

$$t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\left.\frac{(z + h_{\text{pump}})^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{D_0^2}{D^2}\left(\sqrt{\frac{2(z_1 + h_{\text{pump}})(1 + fL/D + K_L)}{g}} - \sqrt{\frac{2h_{\text{pump}}(1 + fL/D + K_L)}{g}}\right)$$

Substituting the values given, the draining time is determined to be

$$t_f = \frac{(3\text{ m})^2}{(0.1\text{ m})^2}\left(\sqrt{\frac{2(2 + 11.46\text{ m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\text{ m/s}^2}} - \sqrt{\frac{2(11.46\text{ m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\text{ m/s}^2}}\right)$$
$$= 468\text{ s} = \mathbf{7.8\ min}$$

Demonstration 2 for Prob. 8-72 (on cavitation)

We take the pump as the control volume, with point 1 at the inlet and point 2 at the exit. We assume the pump inlet and outlet diameters to be the same and the elevation difference between the pump inlet and the exit to be negligible. Then we have $z_1 \cong z_2$ and $V_1 \cong V_2$. The pump is located near the pipe exit, and thus the pump exit pressure is equal to the pressure at the pipe exit, which is the atmospheric pressure, $P_2 = P_{atm}$. Also, the can take $h_L = 0$ since the frictional effects and loses in the pump are accounted for by the pump efficiency. Then the energy equation for the pump (in terms of heads) reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{abs}}}{\rho g}+h_{\text{pump, u}}=\frac{P_{\text{atm}}}{\rho g}$$

Solving for P_1 and substituting,

$$P_{1,\text{abs}}=P_{\text{atm}}-\rho g h_{\text{pump, u}}$$

$$=(101.3\text{ kPa})-(996\text{ kg/m}^3)(9.81\text{ m/s}^2)(11.46\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=\mathbf{-10.7\text{ kPa}}$$

which is impossible (absolute pressure cannot be negative). The technical answer to the question is that cavitation **will occur** since the pressure drops below the vapor pressure of 4.246 kPa. The practical answer is that the question is invalid (void) since the system will not work anyway. Therefore, we conclude that the pump must be located **near the beginning**, not the end of the pipe. Note that when doing a cavitation analysis, we must work with the absolute pressures. (If the system were installed as indicated, a water velocity of V = 4 m/s could not be established regardless of how much pump power were applied. This is because the atmospheric air and water elevation heads alone are not sufficient to drive such flow, with the pump restoring pressure after the flow.)

To determine the furthest distance from the tank the pump can be located without allowing cavitation, we assume the pump is located at a distance L^* from the exit, and choose the pump and the discharge portion of the pipe (from the pump to the exit) as the system, and write the energy equation. The energy equation this time will be as above, except that h_L (the pipe losses) must be considered and the pressure at 1 (pipe inlet) is the cavitation pressure, P_1 = 4.246 kPa:

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{abs}}}{\rho g}+h_{\text{pump, u}}=\frac{P_{\text{atm}}}{\rho g}+f\frac{L^*}{D}\frac{V^2}{2g}$$

or
$$f\frac{L^*}{D}\frac{V^2}{2g}=\frac{P_{1,\text{abs}}-P_{\text{atm}}}{\rho g}+h_{\text{pump, u}}$$

Substituting the given values and solving for L^* gives

$$(0.015)\frac{L^*}{0.1\text{ m}}\frac{(4\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=\frac{(4.246-101.3)\text{ kN/m}^2}{(996\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)+(11.46\text{ m}) \quad\rightarrow\quad L^*=12.5\text{ m}$$

Therefore, the pump must be at least 12.5 m from the pipe exit to avoid cavitation at the pump inlet (this is where the lowest pressure occurs in the piping system, and where the cavitation is most likely to occur).

Cavitation onset places an upper limit to the length of the pipe on the suction side. A pipe slightly longer would become vapor bound, and the pump could not pull the suction necessary to sustain the flow. Even if the pipe on the suction side were slightly shorter than 100 – 12.5 = 87.5 m, cavitation can still occur in the pump since the liquid in the pump is usually accelerated at the expense of pressure, and cavitation in the pump could erode and destroy the pump.

Also, over time, scale and other buildup inside the pipe can and will increase the pipe roughness, increasing the friction factor f, and therefore the losses. Buildup also decreases the pipe diameter, which increases pressure drop. Therefore, flow conditions and system performance may change (generally decrease) as the system ages. A new system that marginally misses cavitation may degrade to where cavitation becomes a problem. Proper design avoids these problems, or where cavitation cannot be avoided for some reason, it can at least be anticipated.

8-73 Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

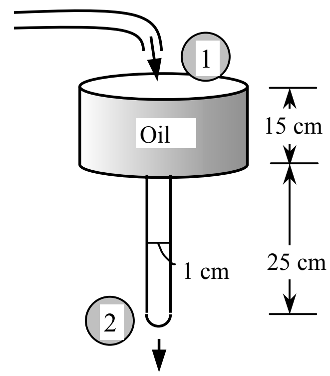
Properties The density and viscosity of oil at 20°C are ρ= 888.1 kg/m^3 and μ = 0.8374 kg/m·s.

Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe which is also taken as the reference level (z_2 = 0). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($V_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad \rightarrow \qquad V_2=V_{2,\max}=\sqrt{2gz_1}$$

Substituting,

$$V_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\text{ m/s}^2)(0.40\text{ m})}=2.801\text{ m/s}$$



This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max}=V_{2,\max}A_2=V_{2,\max}(\pi D_2^2/4)$$
$$=(2.801\text{ m/s})[\pi(0.01\text{ m})^2/4]=2.20\times10^{-4}\text{ m}^3\text{/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(888.1\text{ kg/m}^3)(2.801\text{ m/s})(0.01\text{ m})}{0.8374\text{ kg/m}\cdot\text{s}}=29.71$$

which is less than 2300. Therefore, the flow is laminar, as postulated. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length in this case is

$$L_h=0.05\,\text{Re}\,D=0.05\times29.71\times(0.01\text{ m})=0.015\text{ m}$$

which is much less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected as postulated.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P=P_{\text{pipe inlet}}-P_{\text{pipe exit}}=(P_{atm}+\rho gh_{\text{cylinder}})-P_{atm}=\rho gh_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{pipe}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{\text{cylinder}}+h_{\text{pipe}})\pi D^4}{128\mu L}=\frac{(888.1\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15+0.25\text{ m})\pi(0.01\text{ m})^4}{128(0.8374\text{ kg/m}\cdot\text{s})(0.25\text{ m})}=\mathbf{4.09\times10^{-6}\ m^3/s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{4.09\times10^{-6}\text{ m}^3\text{/s}}{2.20\times10^{-4}\text{ m}^3\text{/s}}=0.0186 \quad \text{or} \quad \mathbf{1.86\%}$$

Discussion Note that the flow is driven by gravity alone, and the actual flow rate is a small fraction of the flow rate that would have occurred if the flow were frictionless.

8-74 Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

Properties The density and viscosity of oil at 20°C are ρ= 888.1 kg/m^3 and μ = 0.8374 kg/m·s.

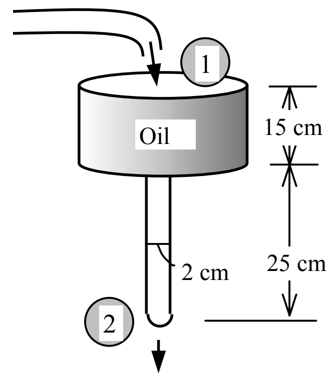
Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe, which is also taken as the reference level ($z_2 = 0$). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($V_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad \rightarrow \qquad V_2=V_{2,\max}=\sqrt{2gz_1}$$

(*a*) **Case 1**: *Pipe length remains constant at 25 cm, but the pipe diameter is doubled* to D_2 = 2 cm:

Substitution gives

$$V_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\text{ m/s}^2)(0.40\text{ m})}=2.801\text{ m/s}$$



This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max}=V_{2,\max}A_2=V_{2,\max}(\pi D_2^2/4)=(2.801\text{ m/s})[\pi(0.02\text{ m})^2/4]$$
$$=8.80\times10^{-4}\text{ m}^3\text{/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(888.1\text{ kg/m}^3)(2.801\text{ m/s})(0.02\text{ m})}{0.8374\text{ kg/m}\cdot\text{s}}=59.41$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h=0.05\,\text{Re}\,D=0.05\times59.41\times(0.02\text{ m})=0.059\text{ m}$$

which is considerably less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected (with reservation).

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P=P_{\text{pipe inlet}}-P_{\text{pipe exit}}=(P_{atm}+\rho gh_{\text{cylinder}})-P_{atm}=\rho gh_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{\text{cylinder}}+h_{\text{pipe}})\pi D^4}{128\mu L}=\frac{(888.1\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15+0.25\text{ m})\pi(0.02\text{ m})^4}{128(0.8374\text{ kg/m}\cdot\text{s})(0.25\text{ m})}=\mathbf{6.54\times10^{-5}\ m^3/s}$$

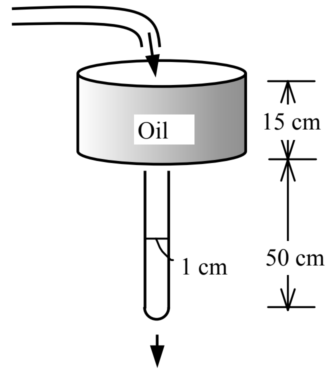
Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{0.654\times10^{-4}\text{ m}^3\text{/s}}{8.80\times10^{-4}\text{ m}^3\text{/s}}=0.074 \quad \text{or} \quad \mathbf{7.4\%}$$

(*b*) **Case 2**: *Pipe diameter remains constant at 1 cm, but the pipe length is doubled to L =* 50 *cm*:

Substitution gives

$$V_{2,\max} = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(0.65\ \text{m})} = 3.571\ \text{m/s}$$



This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max} = V_{2,\max} A_2 = V_{2,\max}(\pi D_2^2/4) = (3.571\ \text{m/s})[\pi(0.01\ \text{m})^2/4]$$
$$= 2.805\times10^{-4}\ \text{m}^3/s$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(888.1\ \text{kg/m}^3)(3.571\ \text{m/s})(0.01\ \text{m})}{0.8374\ \text{kg/m}\cdot\text{s}} = 37.87$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h = 0.05\,\text{Re}\,D = 0.05\times37.87\times(0.01\ \text{m}) = 0.019\ \text{m}$$

which is much less than the 0.50 m pipe length. Therefore, the entrance effects can be neglected.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V} = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P = P_{\text{pipe inlet}} - P_{\text{pipe exit}} = (P_{\text{atm}} + \rho g h_{\text{cylinder}}) - P_{\text{atm}} = \rho g h_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V} = \frac{\rho g(h_{\text{cylinder}} + h_{\text{pipe}})\pi D^4}{128\mu L} = \frac{(888.1\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.15+0.50\ \text{m})\pi(0.01\ \text{m})^4}{128(0.8374\ \text{kg/m}\cdot\text{s})(0.50\ \text{m})} = \mathbf{3.32\times10^{-6}\ m^3/s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff} = \frac{\dot{V}}{\dot{V}_{\max}} = \frac{3.32\times10^{-6}\ \text{m}^3/\text{s}}{2.805\times10^{-4}\ \text{m}^3/\text{s}} = 0.0118 \quad \text{or} \quad \mathbf{1.18\%}$$

Discussion Note that the funnel effectiveness increases as the pipe diameter is increased, and decreases as the pipe length is increased. This is because the frictional losses are proportional to the length but inversely proportional to the diameter of the flow sections.

8-75 Water is drained from a large reservoir through two pipes connected in series. The discharge rate of water from the reservoir is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involve no components such as bends, valves, and other connectors. **6** The piping section involves no work devices such as pumps and turbines. **7** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **8** The water level in the reservoir remains constant. **9** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

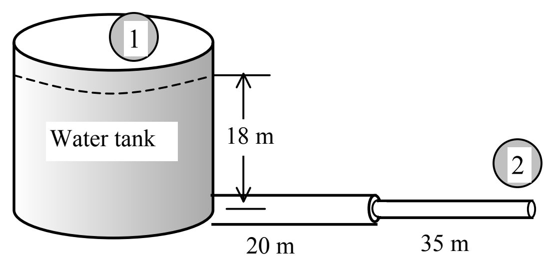
Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s, respectively. The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, and it is 0.46 for the sudden contraction, corresponding to $d^2/D^2 = 4^2/10^2 = 0.16$. The pipes are made of plastic and thus they are smooth, $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the reservoir, and point 2 at the exit of the pipe, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid level in the reservoir is constant ($V_1 = 0$), and that there are no work devices such as pumps and turbines, the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$. Substituting,

$$18\text{ m}=\frac{V_2^2}{2(9.81\text{m/s}^2)}+h_L \qquad (1)$$



where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\sum\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$$

Note that the diameters of the two pipes, and thus the flow velocities through them are different. Denoting the first pipe by 1 and the second pipe by 2, and using conservation of mass, the velocity in the first pipe can be expressed in terms of V_2 as

$$\dot m_1=\dot m_2 \;\rightarrow\; \rho V_1A_1=\rho V_2A_2 \;\rightarrow\; V_1=\frac{A_2}{A_1}V_2=\frac{D_2^2}{D_1^2}V_2=\frac{(4\text{ cm})^2}{(10\text{ cm})^2}V_2 \;\rightarrow\; V_1=0.16V_2 \qquad (2)$$

Then the head loss can be expressed as

$$h_L=\left(f_1\frac{L_1}{D_1}+K_{L,\text{entrance}}\right)\frac{V_1^2}{2g}+\left(f_2\frac{L_2}{D_2}+K_{L,\text{contraction}}\right)\frac{V_2^2}{2g}$$

or

$$h_L=\left(f_1\frac{20\text{ m}}{0.10\text{ m}}+0.5\right)\frac{V_1^2}{2(9.81\text{m/s}^2)}+\left(f_2\frac{35\text{ m}}{0.04\text{ m}}+0.46\right)\frac{V_2^2}{2(9.81\text{m/s}^2)} \qquad (3)$$

The flow rate, the Reynolds number, and the friction factor are expressed as

$$\dot V=V_2A_2=V_2(\pi D_2^2/4) \quad\rightarrow\quad \dot V=V_2[\pi(0.04\text{ m})^2/4] \qquad (4)$$

$$\text{Re}_1=\frac{\rho V_1D_1}{\mu} \;\rightarrow\; \text{Re}_1=\frac{(999.1\text{ kg/m}^3)\mathbf{V}_1(0.10\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2=\frac{\rho V_2D_2}{\mu} \;\rightarrow\; \text{Re}_2=\frac{(999.1\text{ kg/m}^3)\mathbf{V}_2(0.04\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{\varepsilon/D_1}{3.7}+\frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}}=-2.0\log\left(0+\frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon / D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

This is a system of 8 equations in 8 unknowns, and their simultaneous solution by an equation solver gives

$\dot{V}$ = **0.00595 m³/s**, V_1 = 0.757 m/s, V_2 = 4.73 m/s, $h_L = h_{L1} + h_{L2}$ = 0.13 + 16.73 = 16.86 m,

Re_1 = 66,500, Re_2 =166,200, f_1 = 0.0196, f_2 = 0.0162

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is valid.

Discussion This problem can also be solved by using an iterative approach by assuming an exit velocity, but it will be very time consuming. Equation solvers such as EES are invaluable for this kind of problems.

8-76E The flow rate through a piping system between a river and a storage tank is given. The power input to the pump is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surfaces of the tank and the river remains constant. **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

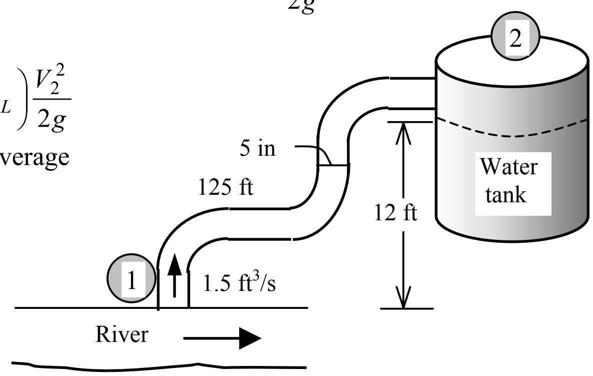
Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s. The roughness of galvanized iron pipe is $\varepsilon = 0.0005$ ft.

Analysis The piping system involves 125 ft of 5-in diameter piping, an entrance with negligible loses, 3 standard flanged 90° smooth elbows ($K_L = 0.3$ each), and a sharp-edged exit ($K_L = 1.0$). We choose points 1 and 2 at the free surfaces of the river and the tank, respectively. We note that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), and the fluid velocity is 6 ft/s at point 1 and zero at point 2 (V_1 = 6 ft/s and V_2 =0). We take the free surface of the river as the reference level ($z_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \alpha_1\frac{V_1^2}{2g}+h_{\text{pump, u}}=z_2+h_L$$

where $\alpha_1 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4}=11.0\text{ ft/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(62.3\text{ lbm/ft}^3)(11.0\text{ ft/s})(5/12\text{ ft})}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}}=435{,}500$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.0005\text{ ft}}{5/12\text{ ft}}=0.0012$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0012}{3.7}+\frac{2.51}{435{,}500\sqrt f}\right)$$

It gives $f = 0.0211$. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{exit}}=0+3\times0.3+1.0=1.9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left((0.0211)\frac{125\text{ ft}}{5/12\text{ ft}}+1.90\right)\frac{(11.0\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}=15.5\text{ ft}$$

The useful pump head input and the required power input to the pump are

$$h_{\text{pump, u}}=z_2+h_L-\frac{V_1^2}{2g}=12+15.5-\frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}=26.9\text{ ft}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump}}} = \frac{\dot{V}\rho g h_{\text{pump, u}}}{\eta_{\text{pump}}}$$

$$= \frac{(1.5\ \text{ft}^3/\text{s})(62.30\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(26.9\ \text{ft})}{0.70}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{kW}}{737\ \text{lbf}\cdot\text{ft/s}}\right)$$

$$= \mathbf{4.87\ kW}$$

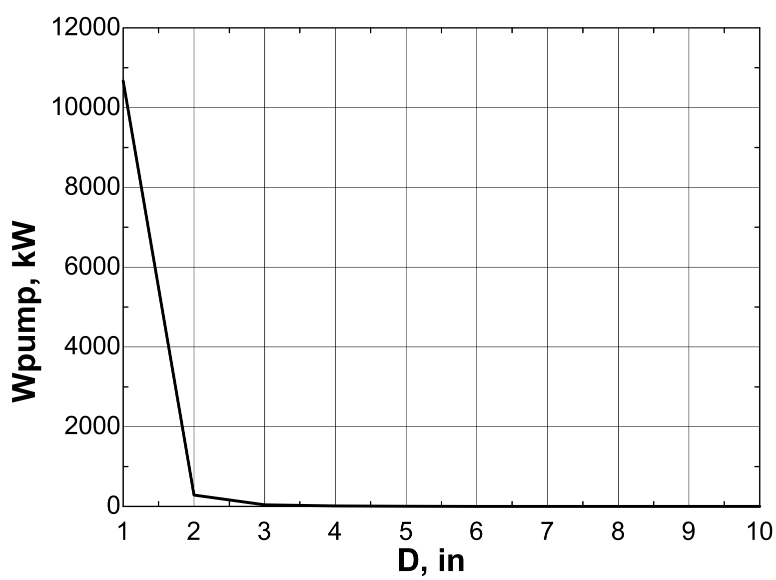
Therefore, 4.87 kW of electric power must be supplied to the pump.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0211$, which is identical to the calculated value. The friction coefficient would drop to 0.0135 if smooth pipes were used. Note that $fL/D = 6.3$ in this case, which is about 3 times the total minor loss coefficient of 1.9. Therefore, the frictional losses in the pipe dominate the minor losses, but the minor losses are still significant.

8-47 In Prob. 8-76E, the effect of the pipe diameter on pumping power for the same constant flow rate is to be investigated by varying the pipe diameter from 1 in to 10 in in increments of 1 in.

```
g=32.2
L=125
D=Dinch/12
z2=12
rho=62.30
nu=mu/rho
mu=0.0006556
eff=0.70
Re=V2*D/nu
A=pi*(D^2)/4
V2=Vdot/A
Vdot=1.5
V1=6
eps1=0.0005
rf1=eps1/D
1/sqrt(f1)=-2*log10(rf1/3.7+2.51/(Re*sqrt(f1)))
KL=1.9
HL=(f1*(L/D)+KL)*(V2^2/(2*g))
hpump=z2+HL-V1^2/(2*32.2)
Wpump=(Vdot*rho*hpump)/eff/737
```

D, in	W_{pump}, kW	*V*, ft/s	Re
1	2.178E+06	275.02	10667.48
2	1.089E+06	68.75	289.54
3	7.260E+05	30.56	38.15
4	5.445E+05	17.19	10.55
5	4.356E+05	11.00	4.88
6	3.630E+05	7.64	3.22
7	3.111E+05	5.61	2.62
8	2.722E+05	4.30	2.36
9	2.420E+05	3.40	2.24
10	2.178E+05	2.75	2.17



8-78 A solar heated water tank is to be used for showers using gravity driven flow. For a specified flow rate, the elevation of the water level in the tank relative to showerhead is to be determined.

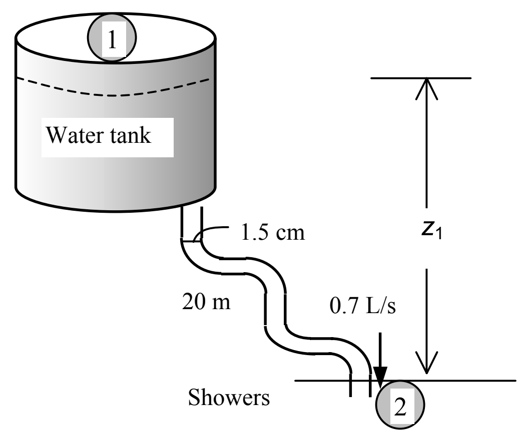
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surface of water in the tank and the shower head remains constant. **5** There are no pumps or turbines in the piping system. **6** The losses at the entrance and at the showerhead are said to be negligible. **7** The water tank is open to the atmosphere. **8** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 40°C are $\rho = 992.1$ kg/m^3 and $\mu = 0.653\times10^{-3}$ kg/m·s, respectively. The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The roughness of galvanized iron pipe is $\varepsilon = 0.00015$ m.

Analysis The piping system involves 20 m of 1.5-cm diameter piping, an entrance with negligible loss, 4 miter bends (90°) without vanes ($K_L = 1.1$ each), and a wide open globe valve ($K_L = 10$). We choose point 1 at the free surface of water in the tank, and point 2 at the shower exit, which is also taken to be the reference level ($z_2 = 0$). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$), and $V_1 = 0$. Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.0007\text{ m}^3/\text{s}}{\pi(0.015\text{ m})^2/4}=3.961\text{ m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(992.1\text{ kg/m}^3)(3.961\text{ m/s})(0.015\text{ m})}{0.653\times10^{-3}\text{ kg/m}\cdot\text{s}}=90{,}270$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00015\text{ m}}{0.015\text{ m}}=0.01$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{90{,}270\sqrt f}\right)$$

It gives $f = 0.03857$. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+4K_{L,\text{elbow}}+K_{L,\text{valve}}+K_{L,\text{exit}}=0+4\times1.1+10+0=14.4$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the total head loss and the elevation of the source become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.03857)\frac{20\text{ m}}{0.015\text{ m}}+14.4\right)\frac{(3.961\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=52.6\text{ m}$$

$$z_1=\alpha_2\frac{V_2^2}{2g}+h_L=(1)\frac{(3.961\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+52.6\text{ m}=\mathbf{53.4\text{ m}}$$

since $\alpha_2 = 1$. Therefore, the free surface of the tank must be 53.4 m above the shower exit to ensure water flow at the specified rate.

8-79 The flow rate through a piping system connecting two water reservoirs with the same water level is given. The absolute pressure in the pressurized reservoir is to be determined.

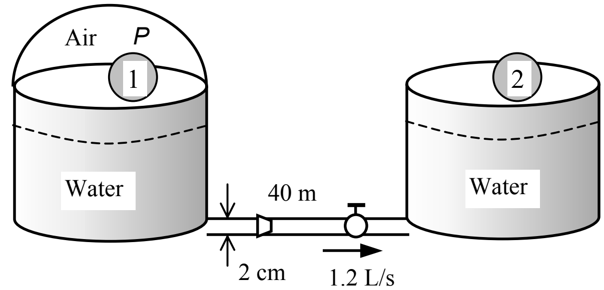
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 10°C are ρ = 999.7 kg/m^3 and μ =1.307×10^{-3} kg/m·s. The loss coefficient is K_L = 0.5 for a sharp-edged entrance, K_L = 2 for swing check valve, K_L = 0.2 for the fully open gate valve, and K_L = 1 for the exit. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis We choose points 1 and 2 at the free surfaces of the two reservoirs. We note that the fluid velocities at both points are zero ($V_1 = V_2 = 0$), the fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{atm}$), both points are at the same level ($z_1 = z_2$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+h_L \quad\rightarrow\quad P_1=P_{\text{atm}}+\rho g h_L$$

where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$



since the diameter of the piping system is constant. The average flow velocity and the Reynolds number are

$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.0012\text{ m}^3/\text{s}}{\pi(0.02\text{ m})^2/4}=3.82\text{ m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(999.7\text{ kg/m}^3)(3.82\text{ m/s})(0.02\text{ m})}{1.307\times10^{-3}\text{ kg/m}\cdot\text{s}}=58{,}400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.02\text{ m}}=0.013$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.013}{3.7}+\frac{2.51}{58{,}400\sqrt{f}}\right)$$

It gives f = 0.0424. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+K_{L,\text{check valve}}+K_{L,\text{ gate valve}}+K_{L,\text{exit}}=0.5+2+0.2+1=3.7$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0424)\frac{40\text{ m}}{0.02\text{ m}}+3.7\right)\frac{(3.82\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=65.8\text{ m}$$

Substituting,

$$P_1=P_{\text{atm}}+\rho g h_L=(88\text{ kPa})+(999.7\text{ kg/m}^3)(9.81\text{ m/s}^2)(65.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=\mathbf{734\ kPa}$$

Discussion The absolute pressure above the first reservoir must be 734 kPa, which is quite high. Note that the minor losses in this case are negligible (about 4% of total losses). Also, the friction factor could be determined easily from the explicit Haaland relation (it gives the same result, 0.0424). The friction coefficient would drop to 0.0202 if smooth pipes were used.

8-80 A tanker is to be filled with fuel oil from an underground reservoir using a plastic hose. The required power input to the pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Fuel oil level remains constant. **5** Reservoir is open to the atmosphere.

Properties The density and dynamic viscosity of fuel oil are given to be ρ = 920 kg/m^3 and μ = 0.045 kg/m·s. The loss coefficient is K_L = 0.12 for a slightly-rounded entrance and K_L = 0.3 for a 90° smooth bend (flanged). The plastic pipe is smooth and thus ε = 0. The kinetic energy correction factor at hose discharge is given to be α = 1.05.

Analysis We choose point 1 at the free surface of oil in the reservoir and point 2 at the exit of the hose in the tanker. We note the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and the fluid velocity at point 1 is zero ($V_1 = 0$). We take the free surface of the reservoir as the reference level ($z_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

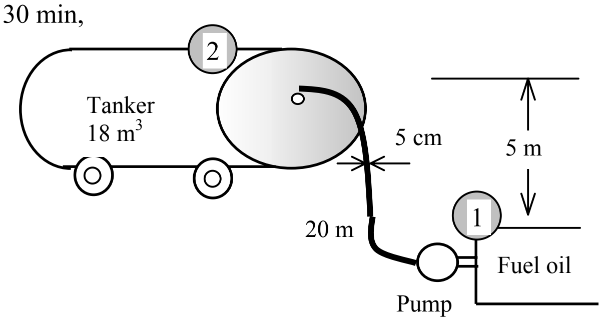
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. The flow rate is determined from the requirement that the tanker must be filled in 30 min,

$$\dot{V}=\frac{V_{\text{tanker}}}{\Delta t}=\frac{18\text{ m}^3}{(30\times 60\text{ s})}=0.01\text{ m}^3/\text{s}$$



Then the average velocity in the pipe and the Reynolds number become

$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.01\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4}=5.093\text{ m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(920\text{ kg/m}^3)(5.093\text{ m/s})(0.05\text{ m})}{0.045\text{ kg/m}\cdot\text{s}}=5206$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{5206\sqrt{f}}\right)$$

It gives f = 0.0370. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+2K_{L,\text{bend}}=0.12+2\times 0.3=0.72$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system (in this case it is not). Then the total head loss, the useful pump head, and the required pumping power become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0370)\frac{20\text{ m}}{0.05\text{ m}}+0.72\right)\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=20.5\text{ m}$$

$$h_{\text{pump, u}}=\frac{V_2^2}{2g}+z_2+h_L=1.05\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+5\text{ m}+20.5\text{ m}=26.9\text{ m}$$

$$\dot{W}_{\text{pump}}=\frac{\dot{V}\rho g h_{\text{pump, u}}}{\eta_{\text{pump}}}=\frac{(0.01\text{ m}^3/\text{s})(920\text{ kg/m}^3)(9.81\text{ m/s}^2)(26.9\text{ m})}{0.82}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\mathbf{2.96\text{ kW}}$$

Discussion Note that the minor losses in this case are negligible (0.72/15.52 = 0.046 or about 5% of total losses). Also, the friction factor could be determined easily from the Haaland relation (it gives 0.0372).

8-81 Two pipes of identical length and material are connected in parallel. The diameter of one of the pipes is twice the diameter of the other. The ratio of the flow rates in the two pipes is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The friction factor is given to be the same for both pipes. **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

Solving for the flow rate gives

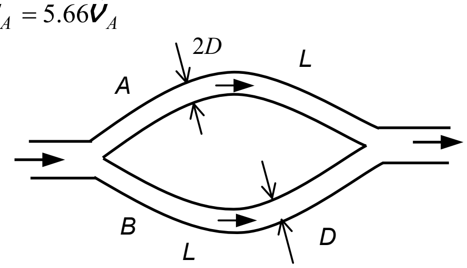
$$\dot{V} = \sqrt{\frac{\pi^2 h_L g}{8fL}}D^{2.5} = kD^{2.5} \qquad (k = \text{constant of proportionality})$$

When the pipe length, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes proportional to the 2.5th power of diameter. Therefore, when the diameter is doubled, the flow rate will increase by a factor of $2^{2.5} = 5.66$ since

If $\qquad \dot{V}_A = kD_A^{2.5}$

Then $\qquad \dot{V}_B = kD_B^{2.5} = k(2D_A)^{2.5} = 2^{2.5}kD_A^{2.5} = 2^{2.5}\dot{V}_A = 5.66\dot{V}_A$

Therefore, the ratio of the flow rates in the two pipes is **5.66**.



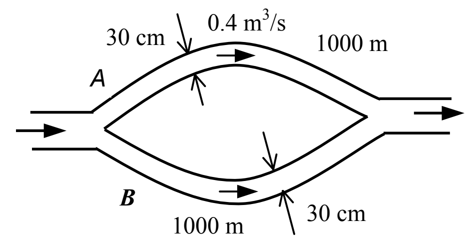
8-82 Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number (to be verified).

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ =1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis The average velocity in pipe A is

$$V_A = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.4 \text{ m}^3/\text{s}}{\pi(0.30 \text{ m})^2/4} = 5.659 \text{ m/s}$$



When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D is

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

Writing this for both pipes and setting them equal to each other, and noting that $D_A = D_B$ (given) and $f_A = f_B$ (to be verified) gives

$$f_A \frac{L_A}{D_A}\frac{V_A^2}{2g} = f_B \frac{L_B}{D_B}\frac{V_B^2}{2g} \quad \rightarrow \quad V_B = V_A\sqrt{\frac{L_A}{L_B}} = (5.659 \text{ m/s})\sqrt{\frac{1000 \text{ m}}{3000 \text{ m}}} = 3.267 \text{ m/s}$$

Then the flow rate in pipe B becomes

$$\dot{V}_B = A_c V_B = [\pi D^2/4]V_B = [\pi(0.30 \text{ m})^2/4](3.267 \text{ m/s}) = \mathbf{0.231 \ m^3/s}$$

Proof *that flow is fully turbulent and thus friction factor is independent of Reynolds number:*

The velocity in pipe B is lower. Therefore, if the flow is fully turbulent in pipe B, then it is also fully turbulent in pipe A. The Reynolds number in pipe B is

$$\text{Re}_B = \frac{\rho V_B D}{\mu} = \frac{(999.1 \text{ kg/m}^3)(3.267 \text{ m/s})(0.30 \text{ m})}{1.138\times10^{-3} \text{ kg/m}\cdot\text{s}} = 0.860\times10^6$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026 \text{ m}}{0.30 \text{ m}} = 0.00087$$

From Moody's chart, we observe that for a relative roughness of 0.00087, the flow is fully turbulent for Reynolds number greater than about 10^6. Therefore, the flow in both pipes is fully turbulent, and thus the assumption that the friction factor is the same for both pipes is valid.

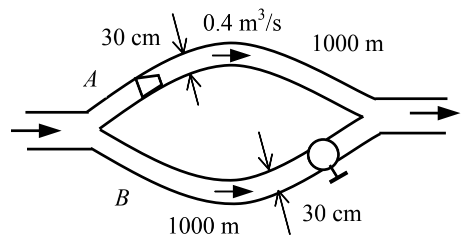
Discussion Note that the flow rate in pipe B is less than the flow rate in pipe A because of the larger losses due to the larger length.

8-83 Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths and different valves. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses other than those for the valves are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ =1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis For pipe *A*, the average velocity and the Reynolds number are



$$V_A = \frac{\dot{V}_A}{A_c} = \frac{\dot{V}_A}{\pi D^2/4} = \frac{0.4 \text{ m}^3/\text{s}}{\pi (0.30 \text{ m})^2/4} = 5.659 \text{ m/s}$$

$$\text{Re}_A = \frac{\rho V_A D}{\mu} = \frac{(999.1 \text{ kg/m}^3)(5.659 \text{ m/s})(0.30 \text{ m})}{1.138\times 10^{-3} \text{ kg/m}\cdot\text{s}} = 1.49\times 10^6$$

The relative roughness of the pipe is

$$\varepsilon / D = \frac{0.00026 \text{ m}}{0.30 \text{ m}} = 8.667\times 10^{-4}$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{8.667\times 10^{-4}}{3.7} + \frac{2.51}{1.49\times 10^6 \sqrt{f}}\right)$$

It gives f = 0.0192. Then the total head loss in pipe *A* becomes

$$h_{L,A} = \left(f\frac{L_A}{D} + K_L\right)\frac{V_A^2}{2g} = \left((0.0192)\frac{1000 \text{ m}}{0.30 \text{ m}} + 2.1\right)\frac{(5.659 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 107.9 \text{ m}$$

When two pipes are parallel in a piping system, the head loss for each pipe must be same. Therefore, the head loss for pipe *B* must also be 107.9 m. Then the average velocity in pipe *B* and the flow rate become

$$h_{L,B} = \left(f\frac{L_B}{D} + K_L\right)\frac{V_B^2}{2g} \rightarrow 107.9 \text{ m} = \left((0.0192)\frac{3000 \text{ m}}{0.30 \text{ m}} + 10\right)\frac{V_B^2}{2(9.81 \text{ m/s}^2)} \rightarrow V_B = 3.24 \text{ m/s}$$

$$\dot{V}_B = A_c V_B = [\pi D^2/4]V_B = [\pi (0.30 \text{ m})^2/4](3.24 \text{ m/s}) = \mathbf{0.229 \ m^3/s}$$

Discussion Note that the flow rate in pipe *B* decreases slightly (from 0.231 to 0.229 m^3/s) due to the larger minor loss in that pipe. Also, minor losses constitute just a few percent of the total loss, and they can be neglected if great accuracy is not required.

8-84 Geothermal water is supplied to a city through stainless steel pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

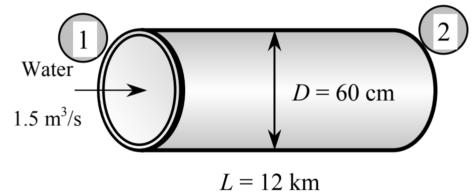
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m·s, and C_p = 4.229 kJ/kg·°C. The roughness of stainless steel pipes is 2×10^{-6} m.

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are



$$V=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{2\times10^{-6}\text{ m}}{0.60\text{ m}}=3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{3.33\times10^{-6}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.00829. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=\Delta P_L=f\frac{L}{D}\frac{\rho V^2}{2}=0.00829\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=2218\text{ kPa}$$

$$h_L=\frac{\Delta P_L}{\rho g}=f\frac{L}{D}\frac{V^2}{2g}=(0.00829)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=238\text{ m}$$

$$\dot{W}_{\text{electric, in}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3/\text{s})(2218\text{ kPa})}{0.74}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{4496\text{ kW}}$$

Therefore, the pumps will consume 4496 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 2218 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount}=\dot{W}_{\text{elect,in}}\Delta t=(4496\text{ kW})(24\text{ h/day})=107{,}900\text{ kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (107{,}900 \text{ kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$6474/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 4496 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho \dot{V} c_p \Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} c_p} = \frac{0.74 \times (4496 \text{ kJ/s})}{(950.6 \text{ kg/m}^3)(1.5 \text{ m}^3/\text{s})(4.229 \text{ kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{0.55^\circ C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-85 Geothermal water is supplied to a city through cast iron pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

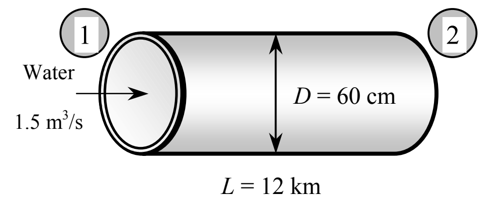
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, $\mu = 0.255\times10^{-3}$ kg/m·s, and c_p = 4.229 kJ/kg·°C. The roughness of cast iron pipes is 0.00026 m.

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are



$$V=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.60\text{ m}}=4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{4.33\times10^{-4}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.0162. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=\Delta P_L=f\frac{L}{D}\frac{\rho V^2}{2}=0.0162\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=4334\text{ kPa}$$

$$h_L=\frac{\Delta P_L}{\rho g}=f\frac{L}{D}\frac{V^2}{2g}=(0.0162)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=465\text{ m}$$

$$\dot{W}_{\text{elect, in}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3/\text{s})(4334\text{ kPa})}{0.74}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{8785\text{ kW}}$$

Therefore, the pumps will consume 8785 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 4334 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\Delta t = (8785\text{ kW})(24\text{ h/day}) = 210{,}800\text{ kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (210{,}800\text{ kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$12{,}650/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 8785 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho\dot{V}c_p\Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p} = \frac{0.74\times(8785\text{ kJ/s})}{(950.6\text{ kg/m}^3)(1.5\text{ m}^3/\text{s})(4.229\text{ kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{1.1^\circ C}$$

Therefore, the temperature of water will rise at least 1.1°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-86E The air discharge rate of a clothes drier with no ducts is given. The flow rate when duct work is attached is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects in the duct are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **4** The losses at the vent and its proximity are negligible. **5** The effect of the kinetic energy correction factor on discharge stream is negligible, $\alpha = 1$.

Properties The density of air at 1 atm and 120°F is $\rho = 0.06843$ lbm/ft^3. The roughness of galvanized iron pipe is $\varepsilon = 0.0005$ ft. The loss coefficient is $K_L \approx 0$ for a well-rounded entrance with negligible loss, $K_L = 0.3$ for a flanged 90° smooth bend, and $K_L = 1.0$ for an exit. The friction factor of the duct is given to be 0.019.

Analysis To determine the useful fan power input, we choose point 1 inside the drier sufficiently far from the vent, and point 2 at the exit on the same horizontal level so that $z_1 = z_2$ and $P_1 = P_2$, and the flow velocity at point 1 is negligible ($V_1 = 0$) since it is far from the inlet of the fan. Also, the frictional piping losses between 1 and 2 are negligible, and the only loss involved is due to fan inefficiency. Then the energy equation for a control volume between 1 and 2 reduces to

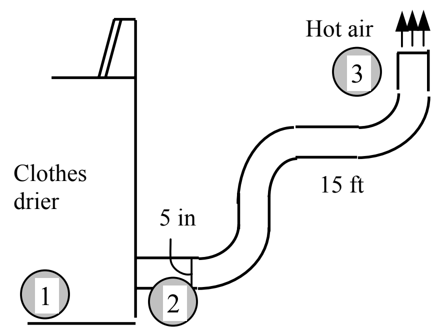
$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{fan}} = \dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad\to\quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2} \qquad (1)$$

since $\alpha = 1$ and $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, fan}} + \dot{E}_{\text{mech loss, piping}}$ and $\dot{W}_{\text{fan, u}} = \dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}}$.

The average velocity is $V_2 = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{V}}{\pi D^2/4} = \dfrac{1.2\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4} = 8.80$ ft/s

Now we attach the ductwork, and take point 3 to be at the duct exit so that the duct is included in the control volume. The energy equation for this control volume simplifies to

$$\dot{W}_{\text{fan, u}} = \dot{m}\frac{V_3^2}{2}+\dot{m}gh_L \qquad (2)$$



Combining (1) and (2),

$$\rho\dot{V}_2\frac{V_2^2}{2} = \rho\dot{V}_3\frac{V_3^2}{2}+\rho\dot{V}_3 gh_L \quad\to\quad \dot{V}_2\frac{V_2^2}{2} = \dot{V}_3\frac{V_3^2}{2}+\dot{V}_3 gh_L \qquad (3)$$

where

$$V_3 = \frac{\dot{V}_3}{A_c} = \frac{\dot{V}_3}{\pi D^2/4} = \frac{\dot{V}_3\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4} = 7.33\dot{V}_3\text{ ft/s}$$

$$h_L = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_3^2}{2g} = \left(0.019\frac{15\text{ ft}}{5/12\text{ ft}}+3\times 0.3+1\right)\frac{V_3^2}{2g} = 2.58\frac{V_3^2}{2g}$$

Substituting into Eq. (3),

$$\dot{V}_2\frac{V_2^2}{2} = \dot{V}_3\frac{V_3^2}{2}+\dot{V}_3 g\times 2.58\frac{V_3^2}{2g} = \dot{V}_3\frac{(7.33\dot{V}_3)^2}{2}+\dot{V}_3\times 2.58\frac{(7.33\dot{V}_3)^2}{2} = 96.2\dot{V}_3^3$$

Solving for $\dot{V}_3$ and substituting the numerical values gives

$$\dot{V}_3 = \left(\dot{V}_2\frac{V_2^2}{2\times 96.2}\right)^{1/3} = \left(1.2\frac{8.80^2}{2\times 96.2}\right)^{1/3} = \mathbf{0.78\ ft^3/s}$$

Discussion Note that the flow rate decreased considerably for the same fan power input, as expected. We could also solve this problem by solving for the useful fan power first,

$$\dot{W}_{\text{fan, u}} = \rho\dot{V}_2\frac{V_2^2}{2} = (0.06843\text{ lbm/ft}^3)(1.2\text{ ft}^3/\text{s})\frac{(8.80\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = 0.13\text{ W}$$

Therefore, the fan supplies 0.13 W of useful mechanical power when the drier is running.

8-87 Hot water in a water tank is circulated through a loop made of cast iron pipes at a specified average velocity. The required power input for the recirculating pump is to be determined.

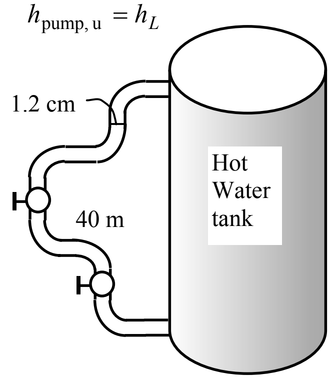
Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

Properties The density and dynamic viscosity of water at 60°C are ρ = 983.3 kg/m^3, μ = 0.467×10^{-3} kg/m·s. The roughness of cast iron pipes is 0.00026 m. The loss coefficient is K_L = 0.9 for a threaded 90° smooth bend and K_L = 0.2 for a fully open gate valve.

Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

where



$$h_L=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are

$$\dot{V}=VA_c=V(\pi D^2/4)=(2.5\text{ m/s})[\pi(0.012\text{ m})^2/4]=2.83\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.012\text{ m}}=0.0217$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.0217}{3.7}+\frac{2.51}{63200\sqrt{f}}\right)$$

It gives f = 0.05075. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.05075)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=55.8\text{ m}$$

$$\Delta P=\Delta P_L=\rho g h_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(55.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=538\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.83\times10^{-4}\text{ m}^3/\text{s})(538\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{0.217\text{ kW}}$$

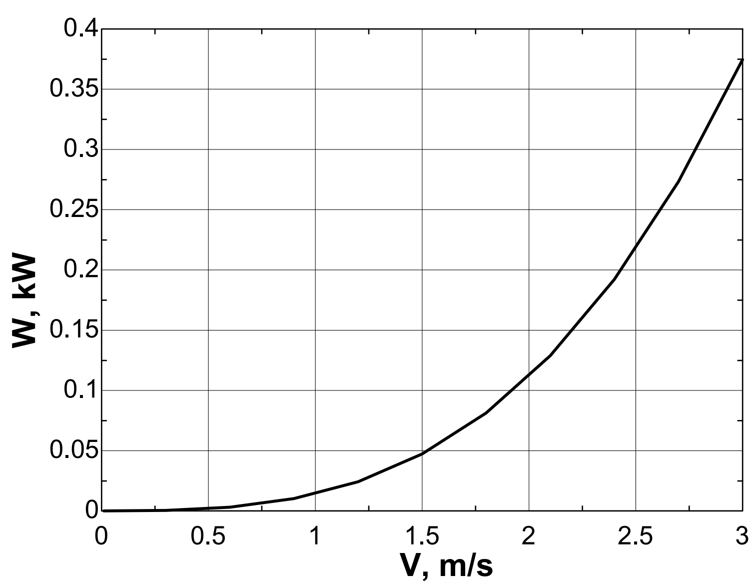
Therefore, the required power input of the recirculating pump is 0.217 kW

Discussion It can be shown that the required pumping power input for the recirculating pump is 0.210 kW when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.

8-88 In Prob. 8-87, the effect of average flow velocity on the power input to the recirculating pump for the same constant flow rate is to be investigated by varying the velocity from 0 to 3 m/s in increments of 0.3 m/s.

```
g=9.81
rho=983.3
nu=mu/rho
mu=0.000467
D=0.012
L=40
KL=6*0.9+2*0.2
Eff=0.7
Ac=pi*D^2/4
Vdot=V*Ac
eps=0.00026
rf=eps/D
"Reynolds number"
Re=V*D/nu
1/sqrt(f)=-2*log10(rf/3.7+2.51/(Re*sqrt(f)))
DP=(f*L/D+KL)*rho*V^2/2000 "kPa"
W=Vdot*DP/Eff "kW"
HL=(f*L/D+KL)*(V^2/(2*g))
```

V, m/s	W_{pump}, kW	ΔP_L, kPa	Re
0.0	0.0000	0.0	0
0.3	0.0004	8.3	7580
0.6	0.0031	32.0	15160
0.9	0.0103	71.0	22740
1.2	0.0243	125.3	30320
1.5	0.0472	195.0	37900
1.8	0.0814	279.9	45480
2.1	0.1290	380.1	53060
2.4	0.1922	495.7	60640
2.7	0.2733	626.6	68220
3.0	0.3746	772.8	75800



8-89 Hot water in a water tank is circulated through a loop made of plastic pipes at a specified average velocity. The required power input for the recirculating pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

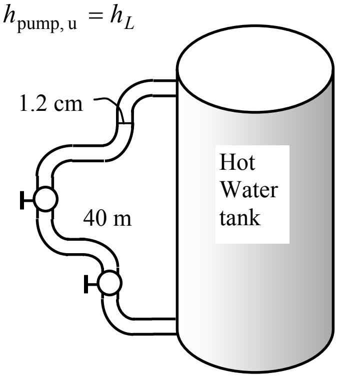
Properties The density and viscosity of water at 60°C are $\rho = 983.3$ kg/m^3, $\mu = 0.467\times10^{-3}$ kg/m·s. Plastic pipes are smooth, and thus their roughness is zero, $\varepsilon = 0$. The loss coefficient is $K_L = 0.9$ for a threaded 90° smooth bend and $K_L = 0.2$ for a fully open gate valve.

Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

where

$$h_L = h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$



since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are

$$\dot{V} = VA_c = V(\pi D^2/4) = (2.5\text{ m/s})[\pi(0.012\text{ m})^2/4] = 2.827\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor corresponding to the relative roughness of zero and this Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{63200\sqrt{f}}\right)$$

It gives $f = 0.0198$. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0198)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=22.9\text{ m}$$

$$\Delta P=\rho g h_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(22.9\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=221\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.827\times10^{-4}\text{ m}^3/\text{s})(221\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{0.0893\text{ kW}}$$

Therefore, the required power input of the recirculating pump is 89.3 W

Discussion It can be shown that the required pumping power input for the recirculating pump is 82.1 W when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.

Flow Rate and Velocity Measurements

8-90C The primary considerations when selecting a flowmeter are cost, size, pressure drop, capacity, accuracy, and reliability.

8-91C The static Pitot tube measures the difference between the stagnation and static pressure, which is the dynamic pressure, which is related to flow velocity by $V = \sqrt{2(P_1 - P_2)/\rho}$. Once the average flow velocity is determined, the flow rate is calculated from $\dot{V} = VA_c$. The Pitot tube is inexpensive, highly reliable since it has no moving parts, it has very small pressure drop, and its accuracy (which is about 3%) is acceptable for most applications.

8-92C The obstruction flow meters measure the flow rate through a pipe by constricting the flow, and measuring the decrease in pressure due to the increase in velocity at constriction site. The flow rate for obstruction flowmeters is expressed as $\dot{V} = A_o C_0 \sqrt{2(P_1 - P_2)/[\rho(1-\beta^4)]}$ where $A_0 = \pi d^2/4$ is the cross-sectional area of the obstruction and $\beta = d/D$ is the ratio of obstruction diameter to the pipe diameter. Of the three types of obstruction flow meters, the orifice meter is the cheapest, smallest, and least accurate, and it causes the greatest head loss. The orifice meter is the most expensive, the largest, the most accurate, and it causes the smallest head loss. The nozzle meter is between the orifice and Venturi meters in all aspects.

8-93C The positive displacement flow meters operate by trapping a certain amount of incoming fluid, displacing it to the discharge side of the meter, and counting the number of such discharge-recharge cycles to determine the total amount of fluid displaced. They are commonly used to meter gasoline, water, and natural gas because they are simple, reliable, inexpensive, and highly accurate even when the flow is unsteady.

8-94C A turbine flowmeter consists of a cylindrical flow section that houses a propeller which is free to rotate, and a sensor that generates a pulse each time a marked point on the propeller passes by to determine the rate of rotation. Turbine flowmeters are relatively inexpensive, give highly accurate results (as accurate as 0.25%) over a wide range of flow rates, and cause a very small head loss.

8-95C A variable-area flowmeter consists of a tapered conical transparent tube made of glass or plastic with a float inside that is free to move. As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other. Variable-area flowmeters are very simple devices with no moving parts (even the float remains stationary during steady operation) and thus very reliable. They are also very inexpensive, and they cause a small head loss.

8-96C A thermal anemometer involves a very small electrically heated sensor which loses heat to the fluid, and the flow velocity is related to the electric current needed to maintain the sensor at a constant temperature. The flow velocity is determined by measuring the voltage applied or the electric current passing through the sensor. A Laser-Doppler Anemometer (LDA) does not have a sensor that intrudes into flow. Instead, it uses two Laser beams that intersect at the point where the flow velocity is to be measured, and it makes use of the frequency shift (the Doppler effect) due to fluid flow to measure velocity.

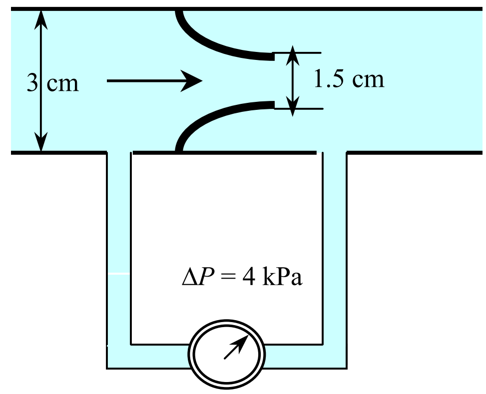
8-97C The Laser Doppler Velocimetry (LDV) measures velocity at a point, but particle image velocimetry (PIV) provides velocity values simultaneously throughout an entire cross-section and thus it is a whole-field technique. The PIV combines the accuracy of LDV with the capability of flow visualization, and provides instantaneous flow field mapping. Both methods are non-intrusive, and both utilize Laser light beams.

8-98 The flow rate of ammonia is to be measured with flow nozzle equipped with a differential pressure gage. For a given pressure drop, the flow rate and the average flow velocity are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is C_d = 0.96.

Properties The density and dynamic viscosity of ammonia are given to be ρ = 624.6 kg/m^3 and μ = 1.697×10^{-4} kg/m⋅s, respectively.

Analysis The diameter ratio and the throat area of the meter are



$$\beta = d / D = 1.5 / 3 = 0.50$$

$$A_0 = \pi d^2 / 4 = \pi (0.015\text{ m})^2 / 4 = 1.767 \times 10^{-4}\text{ m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (1.767 \times 10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2 \times 4000\text{ N/m}^2}{(624.6\text{ kg/m}^3)((1-0.50^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

$$= \mathbf{0.627 \times 10^{-3}\ m^3/s}$$

which is equivalent to 0.627 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.627 \times 10^{-3}\text{ m}^3/\text{s}}{\pi (0.03\text{ m})^2/4} = \mathbf{0.887\ m/s}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(624.6\text{ kg/m}^3)(0.887\text{ m/s})(0.03\text{ m})}{1.697 \times 10^{-4}\text{ kg/m}\cdot\text{s}} = 9.79 \times 10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(9.79 \times 10^4)^{0.5}} = 0.983$$

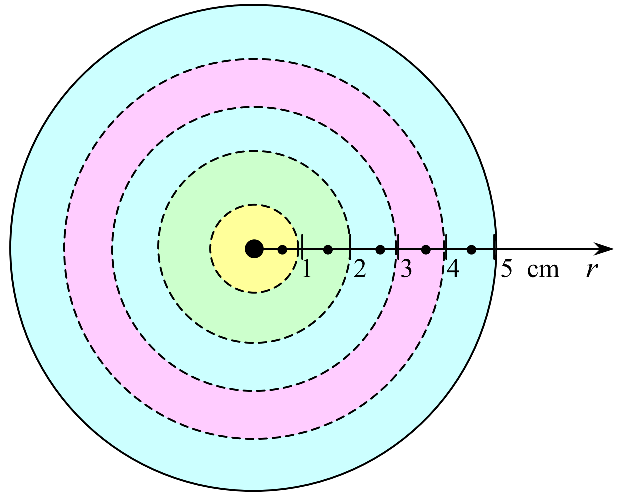
which is about 2% different than the assumed value of 0.96. Using this refined value of C_d, the flow rate becomes 0.642 L/s, which differs from our original result by only 2.4%. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for C_d (which depends on Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

8-99 The flow rate of water through a circular pipe is to be determined by measuring the water velocity at several locations along a cross-section. For a given set of measurements, the flow rate is to be determined.

Assumptions The points of measurements are sufficiently close so that the variation of velocity between points can be assumed to be linear.

Analysis The velocity measurements are given to be

R, cm	V, m/s
0	6.4
1	6.1
2	5.2
3	4.4
4	2.0
5	0.0



The divide the cross-section of the pipe
Into 1-cm thick annual regions, as shown in the figure. Using midpoint velocity values for each section, the flow rate is determined to be

$$\dot{V} = \int_{A_c} V dA_c \cong \sum V\pi(r_{out}^2 - r_{in}^2)$$
$$= \pi\left(\frac{6.4+6.1}{2}\right)(0.01^2 - 0) + \pi\left(\frac{6.1+5.2}{2}\right)(0.02^2 - 0.01^2) + \pi\left(\frac{5.2+4.4}{2}\right)(0.03^2 - 0.02^2)$$
$$+ \pi\left(\frac{4.4+2.0}{2}\right)(0.04^2 - 0.02^2) + \pi\left(\frac{2.0+0}{2}\right)(0.05^2 - 0.04^2)$$
$$= \mathbf{0.0297\ m^3/s}$$

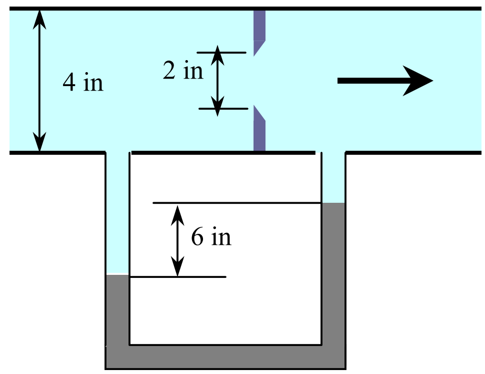
Discussion We can also solve this problem by curve-fitting the given data using a second-degree polynomial, and then performing the integration.

8-100E The flow rate of water is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate, the average velocity, and the head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is $C_d = 0.61$.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 kg/m³ and μ = 7.536×10⁻⁴ lbm/ft·s, respectively. We take the density of mercury to be 847 lbm/ft³.

Analysis The diameter ratio and the throat area of the orifice are



$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(2/12\text{ ft})^2/4 = 0.02182\text{ ft}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{f}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{f}})gh}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{f}} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (0.02182\text{ ft}^2)(0.61)\sqrt{\frac{2(847/62.36 - 1)(32.2\text{ ft/s}^2)(6/12\text{ ft})}{1-0.50^4}} = \mathbf{0.277\ ft^3/s}$$

The average velocity in the pipe is determined by dividing the flow rate by the crosssectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.277\text{ ft}^3/\text{s}}{\pi(4/12\text{ ft})^2/4} = \mathbf{3.17\ ft/s}$$

The percent permanent pressure (or head) loss for orifice meters is given in Fig. 8-58 for $\beta = 0.5$ to be 74%. Therefore, noting that the density of mercury is 13.6 times that of water,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.74(0.50\text{ ft Hg}) = \mathbf{0.37\ ft\ Hg} = \mathbf{5.03\ ft\ H2O}$$

The head loss between the two measurement sections can also be determined from the energy equation, which simplifies to ($z_1 = z_2$)

$$h_L = \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = \frac{\rho_{Hg} g h_{Hg}}{\rho_f g} - \frac{[(D/d)^4 - 1]V_1^2}{2g} = 13.6 \times 0.50\text{ ft} - \frac{[2^4 - 1](3.17\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 4.46\text{ ft H2O}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36\text{ kg/m}^3)(3.17\text{ ft/s})(4/12\text{ ft})}{7.536\times10^{-4}\text{ lbm/ft}\cdot\text{s}} = 8.744\times10^4$$

Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives $C_d = 0.606$, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 0.275 ft³/s, which differs from our original result by less than 1%. Therefore, it is convenient to analyze orifice meters using the recommended value of $C_d = 0.61$ for the discharge coefficient, and then to verify the assumed value.

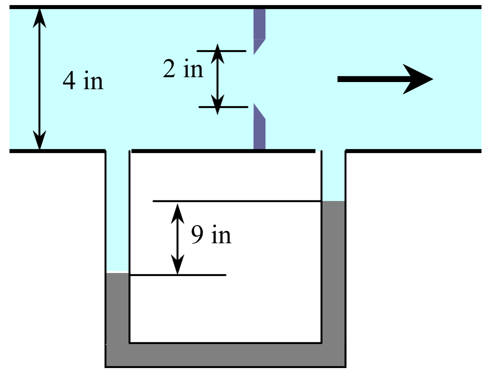
8-101E The flow rate of water is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate, the average velocity, and the head loss are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d = 0.61.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 kg/m^3 and μ = 7.536×10^{-4} lbm/ft·s, respectively. We take the density of mercury to be 847 lbm/ft^3.

Analysis The diameter ratio and the throat area of the orifice are

$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(2/12\text{ ft})^2/4 = 0.02182\text{ ft}^2$$



The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{f}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{f}})gh}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{f}} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (0.02182\text{ ft}^2)(0.61)\sqrt{\frac{2(847/62.36-1)(32.2\text{ ft/s}^2)(9/12\text{ ft})}{1-0.50^4}} = \mathbf{0.339\ ft^3/s}$$

The average velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.339\text{ ft}^3/\text{s}}{\pi(4/12\text{ ft})^2/4} = \mathbf{3.88\ ft/s}$$

The percent permanent pressure (or head) loss for orifice meters is given in Fig. 8-58 for β = 0.5 to be 74%. Therefore, noting that the density of mercury is 13.6 times that of water,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.74(0.75\text{ ft Hg}) = \mathbf{0.555\ ft\ Hg} = \mathbf{7.55\ ft\ H2O}$$

The head loss between the two measurement sections can also be determined from the energy equation, which simplifies to ($z_1 = z_2$)

$$h_L = \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = \frac{\rho_{Hg} g h_{Hg}}{\rho_f g} - \frac{[(D/d)^4 - 1]V_1^2}{2g} = 13.6 \times 0.75\text{ ft} - \frac{[2^4 - 1](3.88\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 6.69\text{ ft H2O}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36\text{ kg/m}^3)(3.88\text{ ft/s})(4/12\text{ ft})}{7.536\times 10^{-4}\text{ lbm/ft}\cdot\text{s}} = 1.070\times 10^5$$

Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

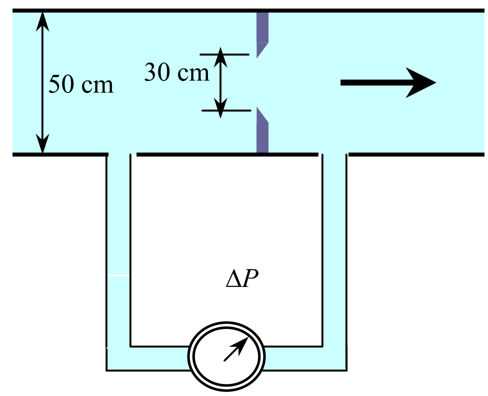
gives C_d = 0.605, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 0.336 ft^3/s, which differs from our original result by less than 1%. Therefore, it is convenient to analyze orifice meters using the recommended value of C_d = 0.61 for the discharge coefficient, and then to verify the assumed value.

8-102 The flow rate of water is measured with an orifice meter. The pressure difference indicated by the orifice meter and the head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is $C_d = 0.61$.

Properties The density and dynamic viscosity of water are given to be ρ = 998 kg/m^3 and μ = 1.002×10^{-4} kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the orifice are



$$\beta = d / D = 30 / 50 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.30 \text{ m})^2 / 4 = 0.07069 \text{ m}^2$$

For a pressure drop of $\Delta P = P_1 - P_2$ across the orifice plate, the flow rate is expressed as

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

Substituting,

$$0.25 \text{ m}^3/s = (0.07069 \text{ m}^2)(0.61)\sqrt{\frac{2\Delta P}{(998 \text{ kg/m}^3)((1 - 0.60^4)}}$$

which gives the pressure drop across the orifice plate to be

$$\Delta P = 14{,}600 \text{ kg}\cdot\text{m/s}^2 = \mathbf{14.6\ kPa}$$

It corresponds to a water column height of

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{14{,}600 \text{ kg}\cdot\text{m/s}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 1.49 \text{ m}$$

The percent permanent pressure (or head) loss for orifice meters is given in Fig. 8-58 for $\beta = 0.6$ to be 64%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.64(1.49 \text{ m}) = \mathbf{0.95\ m\ H2O}$$

The head loss between the two measurement sections can also be determined from the energy equation, which simplifies to ($z_1 = z_2$)

$$h_L = \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = h_w - \frac{[(D/d)^4 - 1]V_1^2}{2g} = 1.49 \text{ m} - \frac{[(50/30)^4 - 1](1.27 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.94 \text{ m H2O}$$

since $\quad V = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{V}}{\pi D^2/4} = \dfrac{0.250 \text{ m}^3/\text{s}}{\pi (0.50 \text{ m})^2/4} = 1.27 \text{ m/s}$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998 \text{ kg/m}^3)(1.27 \text{ m/s})(0.50 \text{ m})}{1.002 \times 10^{-3} \text{ kg/m}\cdot\text{s}} = 6.32 \times 10^5$$

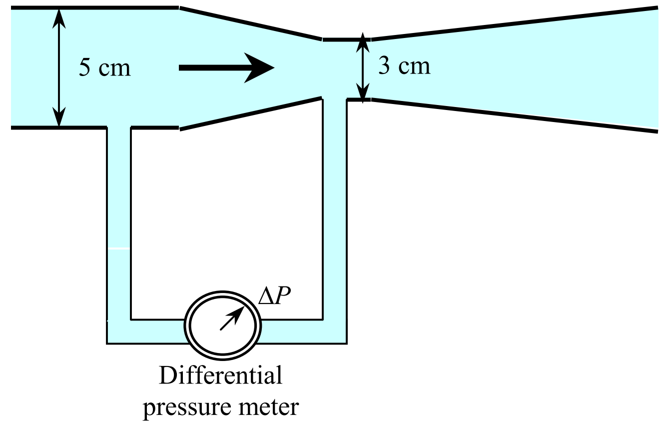
Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives $C_d = 0.605$, which is very close to the assumed value of 0.61.

8-103 A Venturi meter equipped with a differential pressure gage is used to measure to flow rate of water through a horizontal pipe. For a given pressure drop, the volume flow rate of water and the average velocity through the pipe are to be determined.

Assumptions The flow is steady and incompressible.



Properties The density of water is given to be $\rho = 999.1$ kg/m^3. The discharge coefficient of Venturi meter is given to be $C_d = 0.98$.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 3/5 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.03\ \text{m})^2 / 4 = 7.069 \times 10^{-4}\ \text{m}^2$$

Noting that $\Delta P = 5$ kPa $= 5000$ N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (7.069 \times 10^{-4}\ \text{m}^2)(0.98)\sqrt{\frac{2 \times 5000\ \text{N/m}^2}{(999.1\ \text{kg/m}^3)((1-0.60^4)}\left(\frac{1\ \text{kg} \cdot \text{m/s}^2}{1\ \text{N}}\right)}$$

$$= \mathbf{0.00235\ m^3/s}$$

which is equivalent to 2.35 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.00235\ \text{m}^3/\text{s}}{\pi (0.05\ \text{m})^2/4} = \mathbf{1.20\ m/s}$$

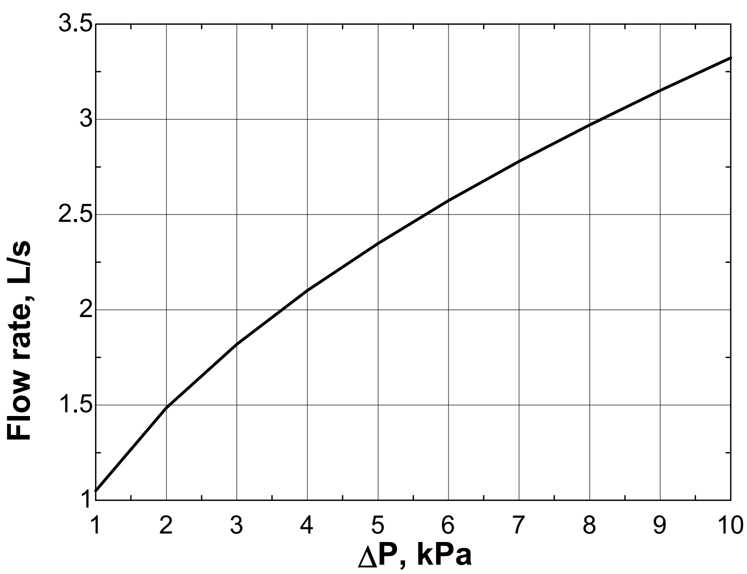
Discussion Note that the flow rate is proportional to the square root of pressure difference across the Venturi meter.

8-104 Problem 8-103 is reconsidered. The variation of flow rate as the pressure drop varies from 1 kPa to 10 kPa at intervals of 1 kPa is to be investigated, the results are to be plotted.

```
rho=999.1 "kg/m3"
D=0.05 "m"
d0=0.03 "m"
beta=d0/D
A0=pi*d0^2/4
Cd=0.98

Vol=A0*Cd*SQRT(2*DeltaP*1000/(rho*(1-beta^4)))*1000 "L/s"
```

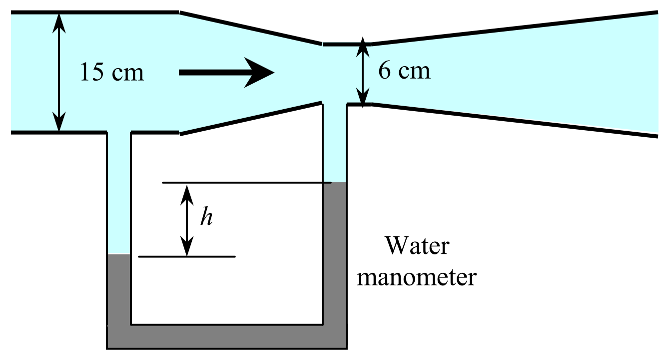
Pressure Drop ΔP, kPa	Flow rate L/s
1	1.05
2	1.49
3	1.82
4	2.10
5	2.35
6	2.57
7	2.78
8	2.97
9	3.15
10	3.32



8-105 A Venturi meter equipped with a water manometer is used to measure to flow rate of air through a duct. For a specified maximum differential height for the manometer, the maximum mass flow rate of air that can be measured is to be determined. .

Assumptions The flow is steady and incompressible.

Properties The density of air is given to be $\rho_{air} = 1.204\ \text{kg/m}^3$. We take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$. The discharge coefficient of Venturi meter is given to be $C_d = 0.98$.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 6/15 = 0.40$$

$$A_0 = \pi d^2/4 = \pi(0.06\ \text{m})^2/4 = 0.002827\ \text{m}^2$$

The pressure drop across the Venturi meter can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_w - \rho_f)gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w - \rho_f)gh}{\rho_f(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w/\rho_{air} - 1)gh}{1-\beta^4}}$$

Substituting and using $h = 0.40$ m, the maximum volume flow rate is determined to be

$$\dot{V} = (0.002827\ \text{m}^2)(0.98)\sqrt{\frac{2(1000/1.204 - 1)(9.81\ \text{m/s}^2)(0.40\ \text{m})}{1 - 0.40^4}} = 0.2265\ \text{m}^3/s$$

Then the maximum mass flow rate this Venturi meter can measure is

$$\dot{m} = \rho\dot{V} = (1.204\ \text{kg/m}^3)(0.2265\ \text{m}^3/\text{s}) = \mathbf{0.273\ kg/s}$$

Also, the average flow velocity in the duct is

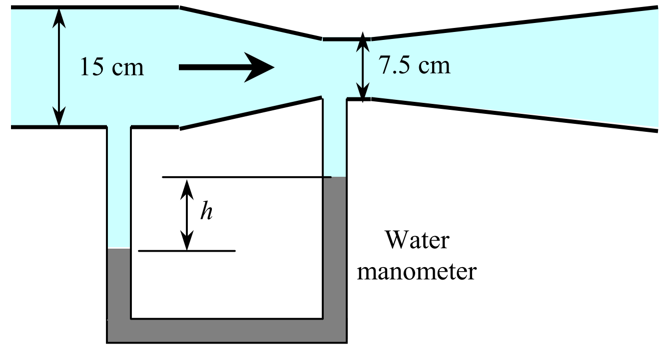
$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2265\ \text{m}^3/\text{s}}{\pi(0.15\ \text{m})^2/4} = 12.8\ \text{m/s}$$

Discussion Note that the maximum available differential height limits the flow rates that can be measured with a manometer.

8-106 A Venturi meter equipped with a water manometer is used to measure to flow rate of air through a duct. For a specified maximum differential height for the manometer, the maximum mass flow rate of air that can be measured is to be determined. .

Assumptions The flow is steady and incompressible.

Properties The density of air is given to be $\rho_{air} = 1.204$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3. The discharge coefficient of Venturi meter is given to be $C_d = 0.98$.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 7.5 / 15 = 0.50$$

$$A_0 = \pi d^2 / 4 = \pi (0.075 \text{ m})^2 / 4 = 0.004418 \text{ m}^2$$

The pressure drop across the Venturi meter can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{w}} - \rho_{\text{f}}) g h$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{w}} - \rho_{\text{f}}) g h}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{w}} / \rho_{\text{air}} - 1) g h}{1-\beta^4}}$$

Substituting and using $h = 0.40$ m, the maximum volume flow rate is determined to be

$$\dot{V} = (0.004418 \text{ m}^2)(0.98) \sqrt{\frac{2(1000/1.204 - 1)(9.81 \text{ m/s}^2)(0.40 \text{ m})}{1 - 0.50^4}} = 0.3608 \text{ m}^3 / s$$

Then the maximum mass flow rate this Venturi meter can measure is

$$\dot{m} = \rho \dot{V} = (1.204 \text{ kg/m}^3)(0.3608 \text{ m}^3\text{/s}) = \mathbf{0.434\ kg/s}$$

Also, the average flow velocity in the duct is

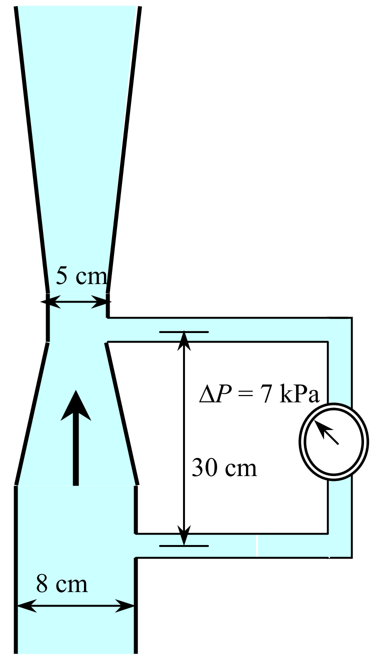
$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2 / 4} = \frac{0.3608 \text{ m}^3 / \text{s}}{\pi (0.15 \text{ m})^2 / 4} = 20.4 \text{ m/s}$$

Discussion Note that the maximum available differential height limits the flow rates that can be measured with a manometer.

8-107 A Venturi meter equipped with a differential pressure gage is used to measure the flow rate of liquid propane through a vertical pipe. For a given pressure drop, the volume flow rate is to be determined.

Assumptions The flow is steady and incompressible.

Properties The density of propane is given to be $\rho = 514.7$ kg/m^3. The discharge coefficient of Venturi meter is given to be $C_d = 0.98$.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 5/8 = 0.625$$

$$A_0 = \pi d^2 / 4 = \pi (0.05 \text{ m})^2 / 4 = 0.001963 \text{ m}^2$$

Noting that ΔP = 7 kPa = 7000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (0.001963 \text{ m}^2)(0.98)\sqrt{\frac{2 \times 7000 \text{ N/m}^2}{(514.7 \text{ kg/m}^3)((1-0.625^4)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)}$$

$$= \mathbf{0.0109 \ m^3/s}$$

which is equivalent to 10.9 L/s. Also, the average flow velocity in the pipe is

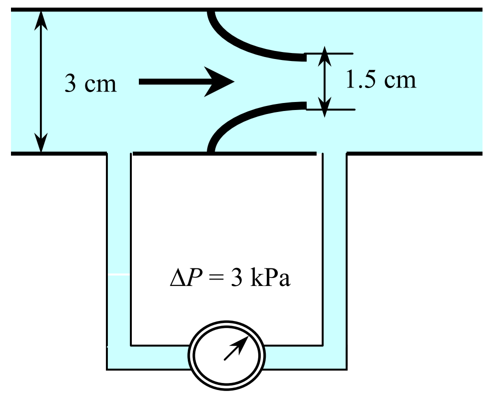
$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.0109 \text{ m}^3/\text{s}}{\pi (0.08 \text{ m})^2 / 4} = 2.17 \text{ m/s}$$

Discussion Note that the elevation difference between the locations of the two probes does not enter the analysis since the pressure gage measures the pressure differential at a specified location. When there is no flow through the Venturi meter, for example, the pressure gage would read zero.

8-108 The flow rate of water is to be measured with flow nozzle equipped with a differential pressure gage. For a given pressure drop, the flow rate, the average flow velocity, and head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is C_d = 0.96.

Properties The density and dynamic viscosity of water are given to be ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m⋅s, respectively.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 1.5/3 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.015\text{ m})^2/4 = 1.767\times10^{-4}\text{ m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d\sqrt{\frac{2(P_1-P_2)}{\rho(1-\beta^4)}}$$

$$= (1.767\times10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2\times3000\text{ N/m}^2}{(999.7\text{ kg/m}^3)((1-0.50^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

$$= \mathbf{0.429\times10^{-3}\ m^3/s}$$

which is equivalent to 0.429 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.429\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.03\text{ m})^2/4} = \mathbf{0.607\ m/s}$$

The water column height corresponding to a pressure drop of 3 kPa is

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{3000\text{ kg}\cdot\text{m/s}^2}{(999.7\text{ kg/m}^3)(9.81\text{ m/s}^2)} = 0.306\text{ m}$$

The percent permanent pressure (or head) loss for nozzle meters is given in Fig. 8-58 for β = 0.5 to be 62%. Therefore,

$$h_L = \text{(Permanent loss fraction)(Total head loss)} = 0.62(0.306\text{ m}) = \mathbf{0.19\ m\ H_2O}$$

The head loss between the two measurement sections can be determined from the energy equation, which simplifies to ($z_1 = z_2$)

$$h_L = \frac{P_1-P_2}{\rho_f g} - \frac{V_2^2-V_1^2}{2g} = h_w - \frac{[(D/d)^4-1]V_1^2}{2g} = 0.306\text{ m} - \frac{[(3/1.5)^4-1](0.607\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.024\text{ m H2O}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(999.7\text{ kg/m}^3)(0.607\text{ m/s})(0.03\text{ m})}{1.307\times10^{-3}\text{ kg/m}\cdot\text{s}} = 1.39\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(1.39\times10^4)^{0.5}} = 0.958$$

which is practically identical to the assumed value of 0.96.

8-109 A kerosene tank is filled with a hose equipped with a nozzle meter. For a specified filling time, the pressure difference indicated by the nozzle meter is to be determined.

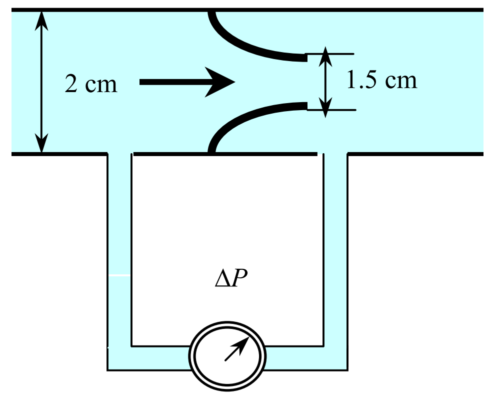
Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is $C_d = 0.96$.

Properties The density of kerosene is given to be $\rho = 820\ \text{kg/m}^3$.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 1.5 / 2 = 0.75$$

$$A_0 = \pi d^2 / 4 = \pi (0.015\ \text{m})^2 / 4 = 1.767 \times 10^{-4}\ \text{m}^2$$



To fill a 16-L tank in 20 s, the flow rate must be

$$\dot{V} = \frac{V_{\text{tank}}}{\Delta t} = \frac{16\ \text{L}}{20\ \text{s}} = 0.8\ \text{L/s}$$

For a pressure drop of $\Delta P = P_1 - P_2$ across the meter, the flow rate is expressed as

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Substituting,

$$0.0008\ \text{m}^3/s = (1.767 \times 10^{-4}\ \text{m}^2)(0.96)\sqrt{\frac{2\Delta P}{(820\ \text{kg/m}^3)((1-0.75^4)}}$$

which gives the pressure drop across the meter to be

$$\Delta P = 6230\ \text{kg} \cdot \text{m/s}^2 = \mathbf{6.23\ kPa}$$

Discussion Note that the flow rate is proportional to the square root of pressure difference across the nozzle meter.

8-110 The flow rate of water is to be measured with flow nozzle equipped with an inverted air-water manometer. For a given differential height, the flow rate and head loss caused by the nozzle meter are to be determined.

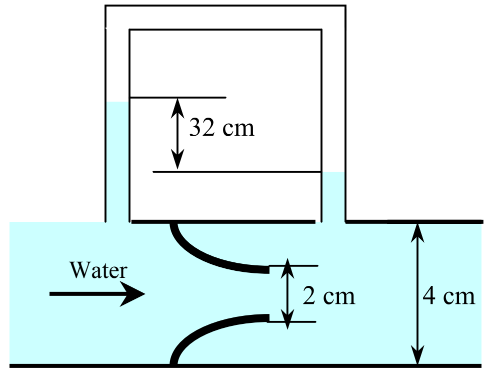
Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is C_d = 0.96.

Properties The density and dynamic viscosity of water are given to be ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.02\text{ m})^2/4 = 3.142\times10^{-4}\text{ m}^2$$



Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d\sqrt{\frac{2(P_1-P_2)}{\rho(1-\beta^4)}} = A_o C_d\sqrt{\frac{2\rho_w gh}{\rho_w(1-\beta^4)}} = A_o C_d\sqrt{\frac{2gh}{1-\beta^4}}$$

$$= (3.142\times10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2(9.81\text{ m/s}^2)(0.32\text{ m})}{1-0.50^4}}$$

$$= \mathbf{0.781\times10^{-3}\ m^3/s}$$

which is equivalent to 0.781 L/s. The average flow velocity in the pipe is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.781\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4} = 0.621\text{ m/s}$$

The percent permanent pressure (or head) loss for nozzle meters is given in Fig. 8-58 for β = 0.5 to be 62%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.62(0.32\text{ m}) = \mathbf{0.20\ m\ H_2O}$$

The head loss between the two measurement sections can be determined from the energy equation, which simplifies to ($z_1 = z_2$)

$$h_L = \frac{P_1-P_2}{\rho_f g} - \frac{V_2^2-V_1^2}{2g} = h_w - \frac{[(D/d)^4-1]V_1^2}{2g} = 0.32\text{ m} - \frac{[(4/2)^4-1](0.621\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.025\text{ m H2O}$$

Discussion The Reynolds number of flow through the pipe is

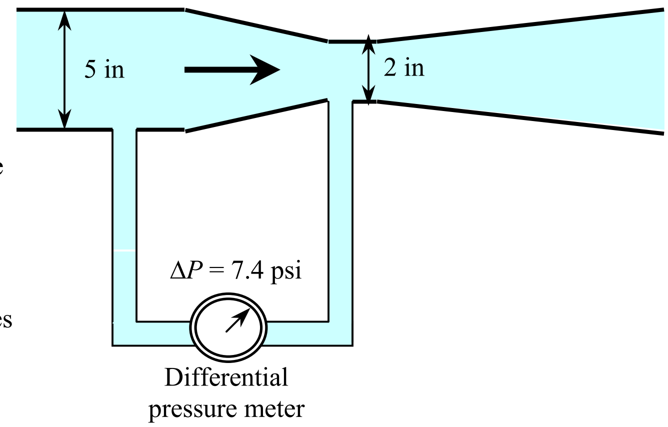
$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(998\text{ kg/m}^3)(0.621\text{ m/s})(0.04\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = 2.47\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(2.47\times10^4)^{0.5}} = 0.968$$

which is almost identical to the assumed value of 0.96.

8-111E A Venturi meter equipped with a differential pressure meter is used to measure to flow rate of refrigerant-134a through a horizontal pipe. For a measured pressure drop, the volume flow rate is to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of R-134a is given to be ρ = 83.31 lbm/ft^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 2/5 = 0.40$$

$$A_0 = \pi d^2 / 4 = \pi (2/12 \text{ ft})^2 / 4 = 0.02182 \text{ ft}^2$$

Noting that ΔP = 7.4 psi = 7.4×144 lbf/ft^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (0.02182 \text{ ft}^2)(0.98)\sqrt{\frac{2 \times 7.4 \times 144 \text{ lbf/ft}^2}{(83.31 \text{ lbm/ft}^3)((1-0.40^4)}\left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right)}$$

$$= \mathbf{0.622 \ ft^3/s}$$

Also, the average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.622 \text{ ft}^3/\text{s}}{\pi (5/12 \text{ ft})^2/4} = \mathbf{4.56 \ ft/s}$$

Discussion Note that the flow rate is proportional to the square root of pressure difference across the Venturi meter.