Lift

11-71C The contribution of viscous effects to lift is usually negligible for airfoils since the wall shear is parallel to the surfaces of such devices and thus nearly normal to the direction of lift.

11-72C When air flows past a symmetrical airfoil at zero angle of attack, (*a*) the lift will be zero, but (*b*) the drag acting on the airfoil will be nonzero.

11-73C When air flows past a nonsymmetrical airfoil at zero angle of attack, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

11-74C When air flows past a symmetrical airfoil at an angle of attack of 5°, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

11-75C The decrease of lift with an increase in the angle of attack is called *stall*. When the flow separates over nearly the entire upper half of the airfoil, the lift is reduced dramatically (the separation point is near the leading edge). Stall is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. The commercial aircraft are not allowed to fly at velocities near the stall velocity for safety reasons. Airfoils stall at high angles of attack (flow cannot negotiate the curve around the leading edge). If a plane stalls, it loses mush of its lift, and it can crash.

11-76C Both the lift and the drag of an airfoil increase with an increase in the angle of attack, but in general lift increases at a much higher rate than does the drag.

11-77C Flaps are used at the leading and trailing edges of the wings of large aircraft during takeoff and landing to alter the shape of the wings to maximize lift and to enable the aircraft to land or takeoff at low speeds. An aircraft can takeoff or land without flaps, but it can do so at very high velocities, which is undesirable during takeoff and landing.

11-78C Flaps increase both the lift and the drag of the wings. But the increase in drag during takeoff and landing is not much of a concern because of the relatively short time periods involved. This is the penalty we pay willingly to takeoff and land at safe speeds.

11-79C The effect of wing tip vortices is to increase drag (induced drag) and to decrease lift. This effect is also due to the downwash, which causes an effectively smaller angle of attack.

11-80C Induced drag is the additional drag caused by the tip vortices. The tip vortices have a lot of kinetic energy, all of which is wasted and is ultimately dissipated as heat in the air downstream. The induced drag can be reduced by using long and narrow wings.

11-81C When air is flowing past a spherical ball, the lift exerted on the ball will be zero if the ball is not spinning, and it will be nonzero if the ball is spinning about an axis normal to the free stream velocity (no lift will be generated if the ball is spinning about an axis parallel to the free stream velocity).

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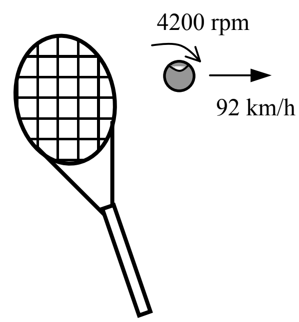
11-82 A tennis ball is hit with a backspin. It is to be determined if the ball will fall or rise after being hit.

Assumptions **1** The outer surface of the ball is smooth enough for Fig. 11-53 to be applicable. **2** The ball is hit horizontally so that it starts its motion horizontally.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s.

Analysis The ball is hit horizontally, and thus it would normally fall under the effect of gravity without the spin. The backspin will generate a lift, and the ball will rise if the lift is greater than the weight of the ball. The lift can be determined from

$$F_L = C_L A \frac{\rho V^2}{2}$$



where A is the frontal area of the ball, which is $A = \pi D^2 / 4$. The regular and angular velocities of the ball are

$$V = (92\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 25.56\ \text{m/s}$$

$$\omega = (4200\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 440\ \text{rad/s}$$

Then,

$$\frac{\omega D}{2V} = \frac{(440\ \text{rad/s})(0.064\ \text{m})}{2(25.56\ \text{m/s})} = 0.551\ \text{rad}$$

From Fig. 11-53, the lift coefficient corresponding to this value is C_L = 0.11. Then the lift acting on the ball is

$$F_L = (0.11)\frac{\pi(0.064\ \text{m})^2}{4}\frac{(1.184\ \text{kg/m}^3)(25.56\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 0.14\ \text{N}$$

The weight of the ball is

$$W = mg = (0.057\ \text{kg})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 0.56\ \text{N}$$

which is more than the lift. Therefore, the ball will **drop** under the combined effect of gravity and lift due to spinning after hitting, with a net force of 0.56 - 0.14 = 0.42 N.

Discussion The Reynolds number for this problem is

$$\text{Re}_L = \frac{VD}{\upsilon} = \frac{(25.56\text{m/s})(0.064\ \text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 1.05\times10^5$$

which is close enough to 6×10^4 for which Fig. 11-53 is prepared. Therefore, the result should be close enough to the actual answer.

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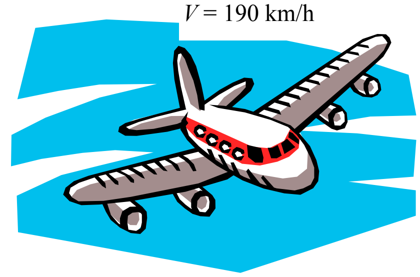
11-83 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the weight of the aircraft is increased by 20% as a result of overloading is to be determined. √

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$



We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and area remain constant, the ratio of the velocities of the overloaded and fully loaded aircraft becomes

$$\frac{V_2}{V_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} \quad \rightarrow \quad V_2 = V_1 \sqrt{\frac{W_2}{W_1}}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$V_2 = V_1 \sqrt{\frac{1.2W_1}{W_1}} = (190 \text{ km/h})\sqrt{1.2} = \mathbf{208\ km/h}$$

Discussion A similar analysis can be performed for the effect of the variations in density, lift coefficient, and planform area on the takeoff velocity.

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11-84 The takeoff speed and takeoff time of an aircraft at sea level are given. The required takeoff speed, takeoff time, and the additional runway length required at a higher elevation are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane and the planform area remain constant. **3** The acceleration of the aircraft during takeoff remains constant.

Properties The density of standard air is ρ_1 = 1.225 kg/m^3 at sea level, and ρ_2 = 1.048 kg/m^3 at 1600 m altitude.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

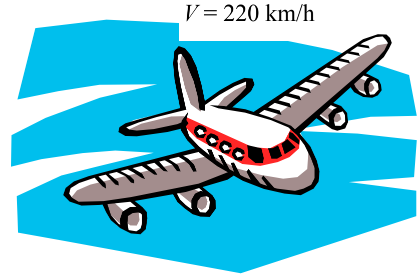
We note that the takeoff speed is inversely proportional to the square root of air density. When the weight, lift coefficient, and area remain constant, the ratio of the speeds of the aircraft at high altitude and at sea level becomes

$$\frac{V_2}{V_1} = \frac{\sqrt{2W/\rho_2 C_L A}}{\sqrt{2W/\rho_1 C_L A}} = \frac{\sqrt{\rho_1}}{\sqrt{\rho_2}} \quad \rightarrow \quad V_2 = V_1\sqrt{\frac{\rho_1}{\rho_2}} = (220\text{ km/h})\sqrt{\frac{1.225}{1.048}} = \mathbf{238\ km/h}$$

Therefore, the takeoff velocity of the aircraft at higher altitude is 238 km/h.

(*b*) The acceleration of the aircraft at sea level is

$$a = \frac{\Delta V}{\Delta t} = \frac{220\text{ km/h - 0}}{15\text{ s}}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 4.074\text{ m/s}^2$$



which is assumed to be constant both at sea level and the higher altitude. Then the takeoff time at the higher altitude becomes

$$a = \frac{\Delta V}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta V}{a} = \frac{238\text{ km/h - 0}}{4.074\text{ m/s}^2}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = \mathbf{16.2\ s}$$

(*c*) The additional runway length is determined by calculating the distance traveled during takeoff for both cases, and taking their difference:

$$L_1 = \tfrac{1}{2} a t_1^2 = \tfrac{1}{2}(4.074\text{ m/s}^2)(15\text{ s})^2 = 458\text{ m}$$

$$L_2 = \tfrac{1}{2} a t_2^2 = \tfrac{1}{2}(4.074\text{ m/s}^2)(16.2\text{ s})^2 = 535\text{ m}$$

$$\Delta L = L_2 - L_1 = 535 - 458 = \mathbf{77\ m}$$

Discussion Note that altitude has a significant effect on the length of the runways, and it should be a major consideration on the design of airports. It is interesting that a 1.2 second increase in takeoff time increases the required runway length by about 100 m.

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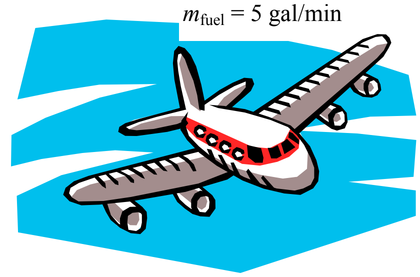
11-85E The rate of fuel consumption of an aircraft while flying at a low altitude is given. The rate of fuel consumption at a higher altitude is to be determined for the same flight velocity.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the drag coefficient of the plane and the planform area remain constant. **3** The velocity of the aircraft and the propulsive efficiency remain constant. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is ρ_1 = 0.05648 lbm/ft^3 at 10,000 ft, and ρ_2 = 0.02866 lbm/ft^3 at 30,000 ft altitude.

Analysis When an aircraft cruises steadily (zero acceleration) at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force. Also, power is force times velocity (distance per unit time), and thus the propulsive power required to overcome drag is equal to the thrust times the cruising velocity. Therefore,

Cruising
m_{fuel} = 5 gal/min



$$\dot{W}_{\text{propulsive}} = \text{Thrust} \times V = F_D V = C_D A \frac{\rho V^2}{2} V = C_D A \frac{\rho V^3}{2}$$

The propulsive power is also equal to the product of the rate of fuel energy supplied (which is the rate of fuel consumption times the heating value of the fuel, $\dot{m}_{\text{fuel}}\text{HV}$) and the propulsive efficiency. Then,

$$\dot{W}_{\text{prop}} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV} \quad \rightarrow \quad C_D A \frac{\rho V^3}{2} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV}$$

We note that the rate of fuel consumption is proportional to the density of air. When the drag coefficient, the wing area, the velocity, and the propulsive efficiency remain constant, the ratio of the rates of fuel consumptions of the aircraft at high and low altitudes becomes

$$\frac{\dot{m}_{\text{fuel},2}}{\dot{m}_{\text{fuel},1}} = \frac{C_D A \rho_2 V^3 / 2\eta_{\text{prop}}\ \text{HV}}{C_D A \rho_1 V^3 / 2\eta_{\text{prop}}\ \text{HV}} = \frac{\rho_2}{\rho_1} \quad \rightarrow \quad \dot{m}_{\text{fuel},2} = \dot{m}_{\text{fuel},1} \frac{\rho_2}{\rho_1} = (5\ \text{gal/min}) \frac{0.02866}{0.05648} = \mathbf{2.54\ gal/min}$$

Discussion Note the fuel consumption drops by half when the aircraft flies at 30,000 ft instead of 10,000 ft altitude. Therefore, large passenger planes routinely fly at high altitudes (usually between 30,000 and 40,000 ft) to save fuel. This is especially the case for long flights.

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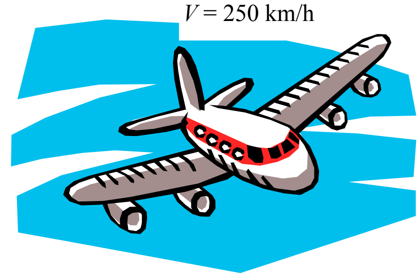
11-86 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the aircraft has 100 empty seats is to be determined. √EES

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same. **3** A passenger with luggage has an average mass of 140 kg.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

Takeoff
V = 250 km/h



We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and wing area remain constant, the ratio of the velocities of the under-loaded and fully loaded aircraft becomes

$$\frac{V_2}{V_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} = \frac{\sqrt{m_2 g}}{\sqrt{m_1 g}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad \rightarrow \quad V_2 = V_1 \sqrt{\frac{m_2}{m_1}}$$

where

$$m_2 = m_1 - m_{\text{unused capacity}} = 400{,}000 \text{ kg} - (140 \text{ kg/passanger}) \times (100 \text{ passengers}) = 386{,}000 \text{ kg}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$V_2 = V_1 \sqrt{\frac{m_2}{m_1}} = (250 \text{ km/h}) \sqrt{\frac{386{,}000}{400{,}000}} = \mathbf{246 \text{ km/h}}$$

Discussion Note that the effect of empty seats on the takeoff velocity of the aircraft is small. This is because the most weight of the aircraft is due to its empty weight (the aircraft itself rather than the passengers themselves and their luggage.)

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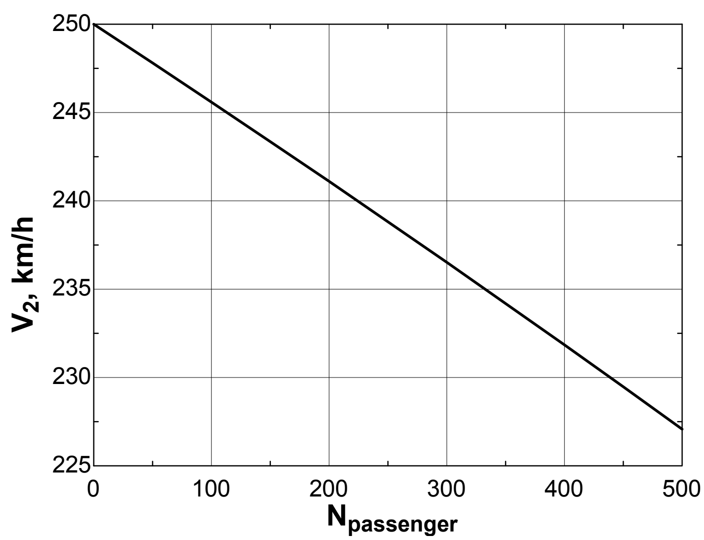
11-87 Problem 11-86 is reconsidered. The effect of passenger count on the takeoff speed of the aircraft as the number of passengers varies from 0 to 500 in increments of 50 is to be investigated.

```
m_passenger=140 "kg"
m1=400000 "kg"
m2=m1-N_passenger*m_passenger
V1=250 "km/h"
V2=V1*SQRT(m2/m1)
```

Passenger count	$m_{\text{airplane},1}$, kg	$m_{\text{airplane},1}$, kg	V_{takeoff}, m/s
0	400000	400000	250.0
50	400000	393000	247.8
100	400000	386000	245.6
150	400000	379000	243.3
200	400000	372000	241.1
250	400000	365000	238.8
300	400000	358000	236.5
350	400000	351000	234.2
400	400000	344000	231.8
450	400000	337000	229.5
500	400000	330000	227.1



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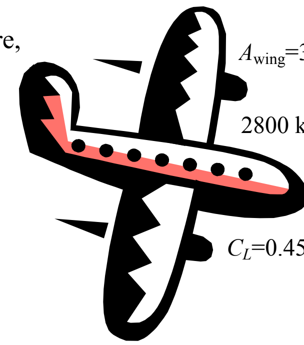
11-88 The wing area, lift coefficient at takeoff settings, the cruising drag coefficient, and total mass of a small aircraft are given. The takeoff speed, the wing loading, and the required power to maintain a constant cruising speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225\ \text{kg/m}^3$.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

Substituting, the takeoff speed is determined to be

$$V_{\text{takeoff}} = \sqrt{\frac{2mg}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(2800\ \text{kg})(9.81\ \text{m/s}^2)}{(1.225\ \text{kg/m}^3)(0.45)(30\ \text{m}^2)}}$$
$$= 57.6\ \text{m/s} = \mathbf{207\ km/h}$$

(*b*) Wing loading is the average lift per unit planform area, which is equivalent to the ratio of the lift to the planform area of the wings since the lift generated during steady cruising is equal to the weight of the aircraft. Therefore,

$$F_{\text{loading}} = \frac{F_L}{A} = \frac{W}{A} = \frac{(2800\ \text{kg})(9.81\ \text{m/s}^2)}{30\ \text{m}^2} = \mathbf{916\ N/m^2}$$

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho V^2}{2} = (0.035)(30\ \text{m}^2)\frac{(1.225\ \text{kg/m}^3)(300/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4.466\ \text{kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (4.466\ \text{kN})(300/3.6\ \text{m/s})\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{372\ kW}$$

Therefore, the engines must supply 372 kW of propulsive power to overcome the drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

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11-89 The total mass, wing area, cruising speed, and propulsive power of a small aircraft are given. The lift and drag coefficients of this airplane while cruising are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air at an altitude of 4000 m is $\rho = 0.819$ kg/m^3.

Analysis Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity. Also, when the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. Then,

$$\dot{W}_{\text{prop}} = \text{Thrust} \times \text{Velocity} = F_D V \quad \rightarrow \quad F_D = \frac{\dot{W}_{\text{prop}}}{V} = \frac{190\text{ kW}}{280/3.6\text{ m/s}}\left(\frac{1000\text{ N}\cdot\text{m/s}}{1\text{ kW}}\right) = 2443\text{ N}$$

Then the drag coefficient becomes

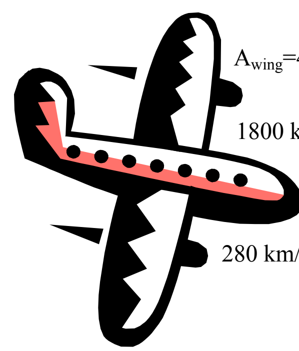
$$F_D = C_D A\frac{\rho V^2}{2} \quad \rightarrow \quad C_D = \frac{2F_D}{\rho A V^2} = \frac{2(2443\text{ N})}{(0.819\text{ kg/m}^3)(42\text{ m}^2)(280/3.6\text{ m/s})^2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{0.0235}$$

An aircraft cruises at constant altitude when lift equals the total weight. Therefore,

$$W = F_L = \tfrac{1}{2}C_L \rho V^2 A \quad \rightarrow \quad C_L = \frac{2W}{\rho V^2 A} = \frac{2(1800\text{ kg})(9.81\text{ m/s}^2)}{(0.819\text{ kg/m}^3)(42\text{ m}^2)(280/3.6\text{ m/s})^2} = \mathbf{0.17}$$

Therefore, the drag and lift coefficients of this aircraft during cruising are 0.0235 and 0.17, respectively, with a C_L/C_D ratio of 7.2.

Discussion The drag and lift coefficient determined are for cruising conditions. The values of these coefficient can be very different during takeoff because of the angle of attack and the wing geometry.



11-90 An airfoil has a given lift-to drag ratio at 0° angle of attack. The angle of attack that will raise this ratio to 80 is to be determined.

Analysis The ratio C_L/C_D for the given airfoil is plotted against the angle of attack in Fig. 11-42. The angle of attack corresponding to $C_L/C_D = 80$ is $\theta = \mathbf{3}$°.

Discussion Note that different airfoils have different C_L/C_D vs. θ charts.

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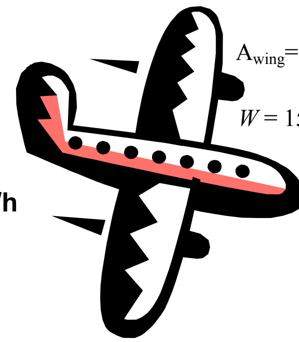
11-91 The wings of a light plane resemble the NACA 23012 airfoil with no flaps. Using data for that airfoil, the takeoff speed at a specified angle of attack and the stall speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225$ kg/m^3. At an angle of attack of 5°, the lift and drag coefficients are read from Fig. 11-45 to be C_L = 0.6 and C_D = 0.015. The maximum lift coefficient is $C_{L,\max}$ = 1.52 and it occurs at an angle of attack of 15°.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$



Substituting, the takeoff speed is determined to be

$$V_{\text{takeoff}} = \sqrt{\frac{2(15{,}000\text{ N})}{(1.225\text{ kg/m}^3)(0.6)(46\text{ m}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 29.8\text{ m/s} = \mathbf{107\ km/h}$$

since 1 m/s = 3.6 km/h. The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$V_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}} = \sqrt{\frac{2(15{,}000\text{ N})}{(1.225\text{ kg/m}^3)(1.52)(46\text{ m}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 18.7\text{ m/s} = \mathbf{67.4\ km/h}$$

Discussion The "safe" minimum velocity to avoid the stall region is obtained by multiplying the stall velocity by 1.2:

$$V_{\text{min,safe}} = 1.2V_{\min} = 1.2 \times (18.7\text{ m/s}) = 22.4\text{ m/s} = 80.8\text{ km/h}$$

Note that the takeoff velocity decreased from 107 km/h at an angle of attack of 5° to 80.8 km/s under stall conditions with a safety margin.

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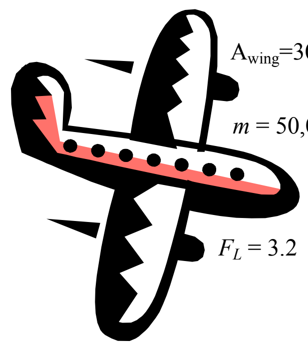
11-92 The mass, wing area, the maximum (stall) lift coefficient, the cruising speed and the cruising drag coefficient of an airplane are given. The safe takeoff speed at sea level and the thrust that the engines must deliver during cruising are to be determined.

Assumptions **1** Standard atmospheric conditions exist **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The takeoff speed is 20% over the stall speed. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is ρ_1 = 1.225 kg/m^3 at sea level, and ρ_2 = 0.312 kg/m^3 at 12,000 m altitude. The cruising drag coefficient is given to be C_D = 0.03. The maximum lift coefficient is given to be $C_{L,\max}$ = 3.2.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}} = \sqrt{\frac{2mg}{\rho C_L A}}$$



The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$V_{\min} = \sqrt{\frac{2mg}{\rho_1 C_{L,\max} A}} = \sqrt{\frac{2(50{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.225\text{ kg/m}^3)(3.2)(300\text{ m}^2)}} = 28.9\text{ m/s} = 104\text{ km/h}$$

since 1 m/s = 3.6 km/h. Then the "safe" minimum velocity to avoid the stall region becomes

$$V_{\min,\text{safe}} = 1.2V_{\min} = 1.2\times(28.9\text{ m/s}) = 34.7\text{ m/s} = \mathbf{125\ km/h}$$

(*b*) When the aircraft cruises steadily at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A\frac{\rho_2 V^2}{2} = (0.03)(300\text{ m}^2)\frac{(0.312\text{ kg/m}^3)(700/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 53.08\text{ kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Ttrust}\times\text{Velocity} = F_D V = (53.08\text{ kN})(700/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{10{,}300\ kW}$$

Therefore, the engines must supply 10,300 kW of propulsive power to overcome drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that act on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

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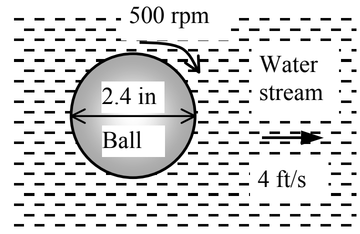
11-93E A spinning ball is dropped into a water stream. The lift and drag forces acting on the ball are to be determined.

Assumptions **1** The outer surface of the ball is smooth enough for Fig. 11-53 to be applicable. **2** The ball is completely immersed in water.

Properties The density and dynamic viscosity of water at 60°F are ρ =62.36 lbm/ft^3 and μ = 2.713 lbm/ft·h =7.536×10^{-4} lbm/ft·s.

Analysis The drag and lift forces can be determined from

$$F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad F_L = C_L A \frac{\rho V^2}{2}$$



where A is the frontal area of the ball, which is $A = \pi D^2 / 4$, and D = 2.4/12 = 0.2 ft. The Reynolds number and the angular velocity of the ball are

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})(0.2 \text{ ft})}{7.536 \times 10^{-4} \text{ ft}^2\text{/s}} = 6.62 \times 10^4$$

$$\omega = (500 \text{ rev/min})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 52.4 \text{ rad/s}$$

and

$$\frac{\omega D}{2V} = \frac{(52.4 \text{ rad/s})(0.2 \text{ ft})}{2(4 \text{ ft/s})} = 1.31 \text{ rad}$$

From Fig. 11-53, the drag and lift coefficients corresponding to this value are C_D = 0.56 and C_L = 0.35. Then the drag and the lift acting on the ball are

$$F_D = (0.56)\frac{\pi(0.2 \text{ ft})^2}{4}\frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = \mathbf{0.27\ lbf}$$

$$F_L = (0.35)\frac{\pi(0.2 \text{ ft})^2}{4}\frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = \mathbf{0.17\ lbf}$$

Discussion The Reynolds number for this problem is 6.62×10^4 which is close enough to 6×10^4 for which Fig. 11-53 is prepared. Therefore, the result should be close enough to the actual answer.

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Review Problems

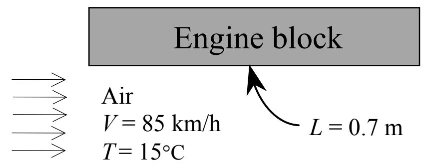
11-94 An automotive engine is approximated as a rectangular block. The drag force acting on the bottom surface of the engine is to be determined. √

Assumptions **1** The air flow is steady and incompressible. **2** Air is an ideal gas. **3** The atmospheric air is calm (no significant winds). **3** The air flow is turbulent over the entire surface because of the constant agitation of the engine block. **4** The bottom surface of the engine is a flat surface, and it is smooth (in reality it is quite rough because of the dirt collected on it).

Properties The density and kinematic viscosity of air at 1 atm and 15°C are ρ = 1.225 kg/m^3 and ν = 1.470×10^{-5} m^2/s.

Analysis The Reynolds number at the end of the engine block is

$$\mathrm{Re}_L = \frac{VL}{\upsilon} = \frac{(85/3.6\ \mathrm{m/s})(0.7\ \mathrm{m})}{1.470\times 10^{-5}\ \mathrm{m^2/s}} = 1.124\times 10^6$$



The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.124\times 10^6)^{1/5}} = 0.004561$$

$$F_D = C_f A\frac{\rho V^2}{2} = (0.004561)[0.6\times 0.7\ \mathrm{m}^2]\frac{(1.225\ \mathrm{kg/m^3})(85/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg.m/s^2}}\right) = \mathbf{0.65\ N}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

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11-95 A fluid flows over a 2.5-m long flat plate. The thickness of the boundary layer at intervals of 0.25 m is to be determined and plotted against the distance from the leading edge for air, water, and oil.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The kinematic viscosity of the three fluids at 1 atm and 20°C are: $\nu = 1.516\times10^{-5}$ m²/s for air , $\nu = \mu/\rho =$ $(1.002\times10^{-3}$ kg/m·s)/(998 kg/m³) $= 1.004\times10^{-6}$ m²/s for water, and $\nu = 9.429\times10^{-4}$ m²/s for oil.

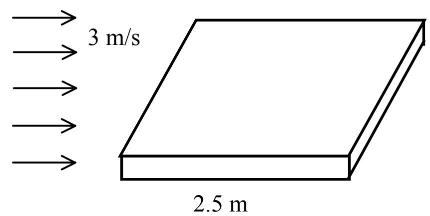
Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

Laminar flow: $\delta_x = \dfrac{4.91x}{\text{Re}_x^{1/2}}$, Turbulent flow: $\delta_x = \dfrac{0.38x}{\text{Re}_x^{1/5}}$

(*a*) AIR: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

$$\text{Re}_x = \frac{Vx}{\upsilon} = \frac{(3\text{ m/s})(0.25\text{ m})}{1.516\times10^{-5}\text{ m}^2/\text{s}} = 0.495\times10^5,$$

$$\delta_x = \frac{5x}{\text{Re}_x^{1/2}} = \frac{4.91\times(0.25\text{ m})}{(0.495\times10^5)^{0.5}} = 5.52\times10^{-3}\text{ m}$$



We repeat calculations for all 0.25 m intervals. The results are:

```
V=3 "m/s"
nu1=1.516E-5 "m2/s, Air"
Re1=x*V/nu1
delta1=4.91*x*Re1^(-0.5) "m, laminar flow"

nu2=1.004E-6 "m2/s, water"
Re2=x*V/nu2
delta2=0.38*x*Re2^(-0.2) "m, turbulent flow"

nu3=9.429E-4 "m2/s, oil"
Re3=x*V/nu3
delta3=4.91*x*Re3^(-0.5) "m, laminar flow"
```

x, cm	Air		Water		Oil	
	Re	δ_x	Re	δ_x	Re	δ_x
0.00	0.000E+00	0.0000	0.000E+00	0.0000	0.000E+00	0.0000
0.25	4.947E+04	0.0055	7.470E+05	0.0064	7.954E+02	0.0435
0.50	9.894E+04	0.0078	1.494E+06	0.0111	1.591E+03	0.0616
0.75	1.484E+05	0.0096	2.241E+06	0.0153	2.386E+03	0.0754
1.00	1.979E+05	0.0110	2.988E+06	0.0193	3.182E+03	0.0870
1.25	2.474E+05	0.0123	3.735E+06	0.0230	3.977E+03	0.0973
1.50	2.968E+05	0.0135	4.482E+06	0.0266	4.773E+03	0.1066
1.75	3.463E+05	0.0146	5.229E+06	0.0301	5.568E+03	0.1152
2.00	3.958E+05	0.0156	5.976E+06	0.0335	6.363E+03	0.1231
2.25	4.453E+05	0.0166	6.723E+06	0.0369	7.159E+03	0.1306
2.50	4.947E+05	0.0175	7.470E+06	0.0401	7.954E+03	0.1376

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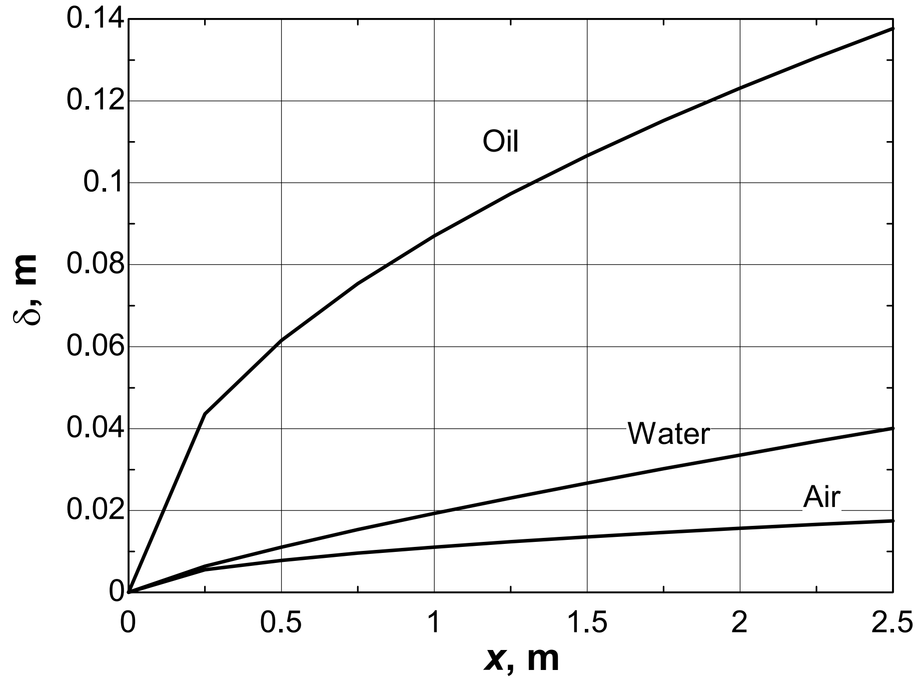

Discussion Note that the flow is laminar for (a) and (c), and turbulent for (b). Also note that the thickness of the boundary layer is very small for air and water, but it is very large for oil. This is due to the high viscosity of oil.

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11-96E The passenger compartment of a minivan is modeled as a rectangular box. The drag force acting on the top and the two side surfaces and the power needed to overcome it are to be determined. √

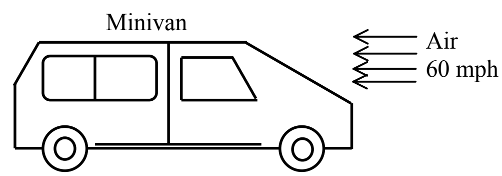
Assumptions **1** The air flow is steady and incompressible. **2** The air flow over the exterior surfaces is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the minivan are flat and smooth (in reality they can be rough). **5** The atmospheric air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.07350$ lbm/ft^3 and $\nu = 0.6110$ ft^2/h $= 1.697\times10^{-4}$ ft^2/s.

Analysis The Reynolds number at the end of the top and side surfaces is

$$\mathrm{Re}_L = \frac{VL}{\upsilon} = \frac{[60\times1.4667 \text{ ft/s}](11 \text{ ft})}{1.697\times10^{-4} \text{ ft}^2/\text{s}} = 5.704\times10^6$$

The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes



$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(5.704\times10^6)^{1/5}} = 0.00330$$

The area of the top and side surfaces of the minivan is

$$A = A_{\text{top}} + 2A_{\text{side}} = 6\times11+2\times3.2\times11 = 136.4 \text{ ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00330\times(136.4 \text{ ft}^2)\frac{(0.074 \text{ lbm/ft}^3)(60\times1.4667 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{4.0\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D V = (4.0 \text{ lbf})(60\times1.4667 \text{ ft/s})\left(\frac{1 \text{ kW}}{737.56 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.48\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

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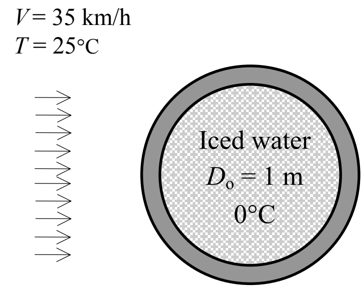
11-97 A large spherical tank located outdoors is subjected to winds. The drag force exerted on the tank by the winds is to be determined.

Assumptions **1** The outer surfaces of the tank are smooth so that Fig. 11-32 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible, and flow around the tank is uniform. **3** Turbulence in the wind is not considered. **4** The effect of any support bars on flow and drag is negligible.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m^3 and $\nu = 1.562\times10^{-5}$ m^2/s.



Analysis Noting that D = 1 m and 1 m/s = 3.6 km/h, the Reynolds number for the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(35/3.6 \text{ m/s})(1\text{ m})}{1.562\times10^{-5}\text{ m}^2/\text{s}} = 6.224\times10^5$$

The drag coefficient for a sphere corresponding to this value is, from Fig. 11-34, C_D = 0.08. Also, the frontal area for flow past a sphere is $A = \pi D^2/4$. Then the drag force becomes

$$F_D = C_D A\frac{\rho V^2}{2} = 0.09[\pi(1\text{ m}^2)/4]\frac{(1.184\text{ kg/m}^3)(35/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{3.5\ N}$$

Discussion Note that the drag coefficient is very low in this case since the flow is turbulent (Re > 2×10^5). Also, it should be kept in mind that wind turbulence may affect the drag coefficient.

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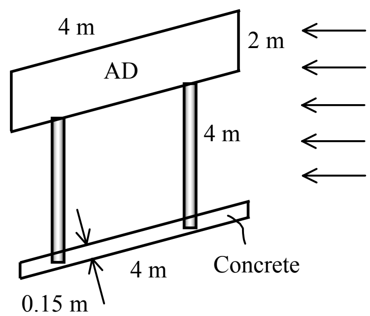
11-98 A rectangular advertisement panel attached to a rectangular concrete block by two poles is to withstand high winds. For a given maximum wind speed, the maximum drag force on the panel and the poles, and the minimum length *L* of the concrete block for the panel to resist the winds are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The wind is normal to the panel (to check for the worst case). **3** The flow is turbulent so that the tabulated value of the drag coefficients can be used.

Properties In turbulent flow, the drag coefficient is C_D = 0.3 for a circular rod, and C_D = 2.0 for a thin rectangular plate (Table 11-2). The densities of air and concrete block are given to be ρ = 1.30 kg/m³ and ρ_c = 2300 kg/m³.

Analysis (*a*) The drag force acting on the panel is



$$F_{D,\text{panel}} = C_D A \frac{\rho V^2}{2}$$

$$= (2.0)(2 \times 4\ \text{m}^2) \frac{(1.30\ \text{kg/m}^3)(150/3.6\ \text{m/s})^2}{2} \left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right)$$

$$= \mathbf{18{,}000\ N}$$

(*b*) The drag force acting on each pole is

$$F_{D,\text{pole}} = C_D A \frac{\rho V^2}{2}$$

$$= (0.3)(0.05 \times 4\ \text{m}^2) \frac{(1.30\ \text{kg/m}^3)(150/3.6\ \text{m/s})^2}{2} \left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right)$$

$$= \mathbf{68\ N}$$

Therefore, the drag force acting on both poles is 68 × 2 = **136 N**. Note that the drag force acting on poles is negligible compared to the drag force acting on the panel.

(*c*) The weight of the concrete block is

$$W = mg = \rho g \boldsymbol{V} = (2300\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(\text{L} \times 4\ \text{m} \times 0.15\ \text{m}) \left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right) = 13{,}540L\ \ \text{N}$$

Note that the resultant drag force on the panel passes through its center, the drag force on the pole passes through the center of the pole, and the weight of the panel passes through the center of the block. When the concrete block is first tipped, the wind-loaded side of the block will be lifted off the ground, and thus the entire reaction force from the ground will act on the other side. Taking the moment about this side and setting it equal to zero gives

$$\sum M = 0 \quad \rightarrow \quad F_{D,\text{panel}} \times (1 + 4 + 0.15) + F_{D,\text{pole}} \times (2 + 0.15) - W \times (L/2) = 0$$

Substituting and solving for *L* gives

$$18{,}000 \times 5.15 + 136 \times 2.15 - 13{,}540L \times L/2 = 0 \quad \rightarrow \quad \text{L} = 3.70\ \text{m}$$

Therefore, the minimum length of the concrete block must be *L* = **3.70**.

Discussion This length appears to be large and impractical. It can be reduced to a more reasonable value by (*a*) increasing the height of the concrete block, (*b*) reducing the length of the poles (and thus the tipping moment), or (*c*) by attaching the concrete block to the ground (through long nails, for example).

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11-99 The bottom surface of a plastic boat is approximated as a flat surface. The friction drag exerted on the bottom surface of the boat by water and the power needed to overcome it are to be determined. √EES

Assumptions **1** The flow is steady and incompressible. **2** The water is calm (no significant currents or waves). **3** The water flow is turbulent over the entire surface because of the constant agitation of the boat. **4** The bottom surface of the boat is a flat surface, and it is smooth.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s.

Analysis The Reynolds number at the end of the bottom surface of the boat is

$$\mathrm{Re}_L = \frac{\rho VL}{\mu} = \frac{(999.1\ \mathrm{kg/m^3})(30/3.6\ \mathrm{m/s})(2\ \mathrm{m})}{1.138\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1.463\times10^7$$

The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.463\times10^7)^{1/5}} = 0.00273$$

$$F_D = C_f A\frac{\rho V^2}{2} = (0.00273)[1.5\times2\ \mathrm{m^2}]\frac{(999.1\ \mathrm{kg/m^3})(30/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg.m/s^2}}\right) = \mathbf{284.1\ N}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\mathrm{drag}} = F_D V = (284.1\ \mathrm{N})(30/3.6\ \mathrm{m/s})\left(\frac{1\ \mathrm{kW}}{1000\ \mathrm{N\cdot m/s}}\right) = \mathbf{2.37\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is relatively small. This is not surprising since the drag force for blunt bodies (including those partially immersed in a liquid) is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

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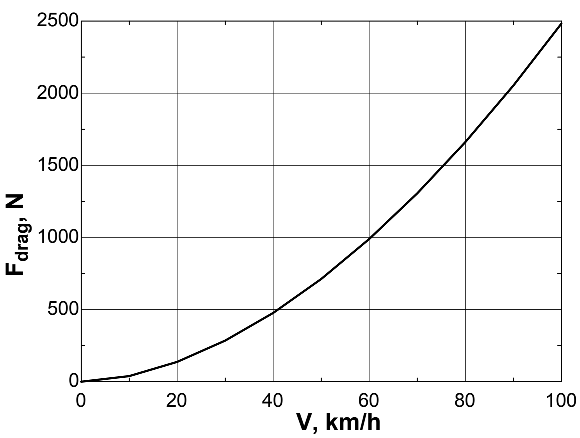
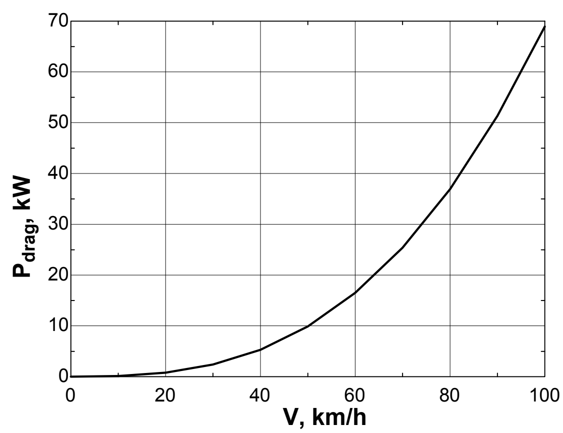
11-100 Problem 11-99 is reconsidered. The effect of boat speed on the drag force acting on the bottom surface of the boat and the power needed to overcome as the boat speed varies from 0 to 100 km/h in increments of 10 km/h is to be investigated.

```
rho=999.1 "kg/m3"
mu=1.138E-3 "m2/s"
V=Vel/3.6 "m/s"
L=2 "m"
W=1.5 "m"
A=L*W

Re=rho*L*V/mu
Cf=0.074/Re^0.2
g=9.81 "m/s2"
F=Cf*A*(rho*V^2)/2 "N"
P_drag=F*V/1000 "kW"
```

V, km/h	Re	C_f	F_{drag}, N	P_{drag}, kW
0	0	0	0	0.0
10	4.877E+06	0.00340	39	0.1
20	9.755E+06	0.00296	137	0.8
30	1.463E+07	0.00273	284	2.4
40	1.951E+07	0.00258	477	5.3
50	2.439E+07	0.00246	713	9.9
60	2.926E+07	0.00238	989	16.5
70	3.414E+07	0.00230	1306	25.4
80	3.902E+07	0.00224	1661	36.9
90	4.390E+07	0.00219	2053	51.3
100	4.877E+07	0.00215	2481	68.9





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11-101E Cruising conditions of a passenger plane are given. The minimum safe landing and takeoff speeds with and without flaps, the angle of attack during cruising, and the power required are to be determined. √

Assumptions **1** The drag and lift produced by parts of the plane other than the wings are not considered. **2** The wings are assumed to be two-dimensional airfoil sections, and the tip effects are neglected. **4** The lift and drag characteristics of the wings can be approximated by NACA 23012 so that Fig. 11-45 is applicable.

Properties The densities of air are 0.075 lbm/ft^3 on the ground and 0.0208 lbm/ft^3 at cruising altitude. The maximum lift coefficients of the wings are 3.48 and 1.52 with and without flaps, respectively (Fig. 11-45).

Analysis (*a*) The weight and cruising speed of the airplane are

$$W = mg = (150{,}000\text{ lbm})(32.2\text{ ft/s}^2)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 150{,}000\text{ lbf}$$

$$V = (550\text{ mph})\left(\frac{1.4667\text{ ft/s}}{1\text{ mph}}\right) = 806.7\text{ ft/s}$$

V = 550 mph
m = 150,000 lbm
A_{wing} = 1800 m^2

Minimum velocity corresponding the stall conditions with and without flaps are

$$V_{\min 1} = \sqrt{\frac{2W}{\rho C_{L,\max 1} A}} = \sqrt{\frac{2(150{,}000\text{ lbf})}{(0.075\text{ lbm/ft}^3)(1.52)(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = 217\text{ ft/s}$$

$$V_{\min 2} = \sqrt{\frac{2W}{\rho C_{L,\max 2} A}} == \sqrt{\frac{2(150{,}000\text{ lbf})}{(0.075\text{ lbm/ft}^3)(3.48)(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = 143\text{ ft/s}$$

The "safe" minimum velocities to avoid the stall region are obtained by multiplying these values by 1.2:

Without flaps: $V_{\min 1,\text{safe}} = 1.2V_{\min 1} = 1.2\times(217\text{ ft/s}) = 260\text{ ft/s} = \textbf{178 mph}$

With flaps: $V_{\min 2,\text{safe}} = 1.2V_{\min 2} = 1.2\times(143\text{ ft/s}) = 172\text{ ft/s} = \textbf{117 mph}$

since 1 mph = 1.4667 ft/s. Note that the use of flaps allows the plane to takeoff and land at considerably lower velocities, and thus at a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is determined to be

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{150{,}000\text{ lbf}}{\frac{1}{2}(0.0208\text{ lbm/ft}^3)(806.7\text{ ft/s})^2(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right) = 0.40$$

For the case of no flaps, the angle of attack corresponding to this value of C_L is determined from Fig. 11-45 to be about α = **3.5°**.

(*c*) When aircraft cruises steadily, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 0.40 is C_D = 0.015 (Fig. 11-45). Then the drag force acting on the wings becomes

$$F_D = C_D A\frac{\rho V^2}{2} = (0.015)(1800\text{ ft}^2)\frac{(0.0208\text{ lbm/ft}^3)(806.7\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 5675\text{ lbf}$$

Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust}\times\text{Velocity} = F_D V = (5675\text{ lbf})(806.7\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \textbf{6200 kW}$$

Discussion Note that the engines must supply 6200 kW of power to overcome the drag during cruising. This is the power required to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc).

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11-102 A smooth ball is moving at a specified velocity. The increase in the drag coefficient when the ball spins is to be determined.

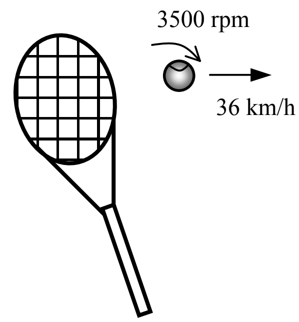
Assumptions **1** The outer surface of the ball is smooth so that Figs. 11-33 and 11-53 can be used to determine the drag coefficient. **2** The air is calm (no winds or drafts).

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s.

Analysis Noting that D = 0.08 m and 1 m/s = 3.6 km/h, the regular and angular velocities of the ball and the Reynolds number are

$$V = (36\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 10\text{ m/s}$$

$$\omega = (3500\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 367\text{ rad/s}$$



$$\text{Re} = \frac{VD}{\nu} = \frac{(10\text{ m/s})(0.08\text{ m})}{1.562\times10^{-5}\text{ m}^2\text{/s}} = 5.122\times10^4$$

and

$$\frac{\omega D}{2V} = \frac{(367\text{ rad/s})(0.08\text{ m})}{2(10\text{ m/s})} = 1.468\text{ rad}$$

Then the drag coefficients for the ball with and without spin are determined from Figs. 11-33 and 11-53 to be:

Without spin:	$C_D = 0.48$	(Fig. 11-34)
With spin:	$C_D = 0.58$	(Fig. 11-53)

Then the increase in the drag coefficient due to spinning becomes

$$\text{Increase in } C_D = \frac{C_{D,\text{spin}} - C_{D,\text{no spin}}}{C_{D,\text{no spin}}} = \frac{0.58-0.48}{0.48} = 0.21 \text{ (or } \mathbf{21\%}\text{)}$$

Therefore, the drag coefficient increases 21% in this case because of spinning.

Discussion Note that the Reynolds number for this problem is 5.122×10^4 which is close enough to 6×10^4 for which Fig. 11-53 is prepared. Therefore, the result obtained should be fairly accurate.

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11-103 The total weight of a paratrooper and its parachute is given. The terminal velocity of the paratrooper in air is to be determined.

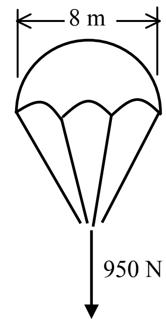
Assumptions **1** The air flow over the parachute is turbulent so that the tabulated value of the drag coefficient can be used. **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low air density.

Properties The density of air is given to be 1.20 kg/m^3. The drag coefficient of a parachute is $C_D = 1.3$.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \qquad \text{where} \qquad F_D = C_D A \frac{\rho_f V^2}{2}, \qquad W = mg = 950\text{ N}, \quad \text{and } F_B \cong 0$$

where $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,



$$C_D A \frac{\rho_f V^2}{2} = W \quad \to \quad C_D \frac{\pi D^2}{4} \frac{\rho_f V^2}{2} = W$$

Solving for V and substituting,

$$V = \sqrt{\frac{8W}{C_D \pi D^2 \rho_f}} = \sqrt{\frac{8(950\text{ N})}{1.3\pi(8\text{ m})^2(1.20\text{ kg/m}^3)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = \mathbf{4.9\ m/s}$$

Therefore, the velocity of the paratrooper will remain constant when it reaches the terminal velocity of 4.9 m/s = 18 km/h.

Discussion The simple analysis above gives us a rough value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air density with altitude, and by considering the uncertainty in the drag coefficient.

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11-104 A fairing is installed to the front of a rig to reduce the drag coefficient. The maximum speed of the rig after the fairing is installed is to be determined.

Assumptions **1** The rig moves steadily at a constant velocity on a straight path in calm weather. **2** The bearing friction resistance is constant. **3** The effect of velocity on the drag and rolling resistance coefficients is negligible. **4** The buoyancy of air is negligible. **5** The power produced by the engine is used to overcome rolling resistance, bearing friction, and aerodynamic drag.

Properties The density of air is given to be 1.25 kg/m^3. The drag coefficient of the rig is given to be C_D = 0.96, and decreases to C_D = 0.76 when a fairing is installed. The rolling resistance coefficient is C_{RR} = 0.05.

Analysis The bearing friction resistance is given to be F_{bearing} = 350 N. The rolling resistance is

$$F_{RR} = C_{RR}W = 0.05(17{,}000\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 8339\text{ N}$$

The maximum drag occurs at maximum velocity, and its value before the fairing is installed is

$$F_{D1} = C_D A\frac{\rho V_1^2}{2} = (0.96)(9.2\text{ m}^2)\frac{(1.25\text{ kg/m}^3)(110/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 5154\text{ N}$$

Power is force times velocity, and thus the power needed to overcome bearing friction, drag, and rolling resistance is the product of the sum of these forces and the velocity of the rig,

$$\begin{aligned}\dot{W}_{\text{total}} &= \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_D + F_{RR})\mathbf{V}\\ &= (350 + 8339 + 5154)(110/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right)\\ &= 423\text{ kW}\end{aligned}$$

The maximum velocity the rig can attain at the same power of 423 kW after the fairing is installed is determined by setting the sum of the bearing friction, rolling resistance, and the drag force equal to 423 kW,

$$\dot{W}_{\text{total}} = \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag2}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_{D2} + F_{RR})V_2 = \left(350 + C_{D2}A\frac{\rho V_2^2}{2} + 5154\right)V_2$$

Substituting the known quantities,

$$(423\text{ kW})\left(\frac{1000\text{ N}\cdot\text{m/s}}{1\text{ kW}}\right) = \left(350\text{ N} + (0.76)(9.2\text{ m}^2)\frac{(1.25\text{ kg/m}^3)V_2^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) + 5154\text{ N}\right)V_2$$

or,

$$423{,}000 = 5504V_2 + 4.37V_2^3$$

Solving it with an equation solver gives V_2 = **36.9 m/s = 133 km/h**.

Discussion Note that the maximum velocity of the rig increases from 110 km/h to 133 km/h as a result of reducing its drag coefficient from 0.96 to 0.76 while holding the bearing friction and the rolling resistance constant.

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11-105 A spherical object is dropped into a fluid, and its terminal velocity is measured. The viscosity of the fluid is to be determined.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is the terminal velocity.

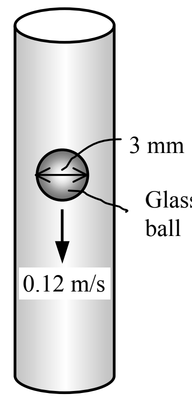
Properties The density of glass ball is given to be ρ_s = 2500 kg/m^3. The density of the fluid is given to be ρ_f = 875 kg/m^3.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \ \text{(Stoke's law)}, \quad W = \rho_s g \mathbf{V}, \quad \text{and} \ F_B = \rho_f g \mathbf{V}$$

Here $\mathbf{V} = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu VD = \rho_s g \mathbf{V} - \rho_f g \mathbf{V} \quad \rightarrow \quad 3\pi\mu VD = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$



Solving for μ and substituting, the dynamic viscosity of the fluid is determined to be

$$\mu = \frac{gD^2(\rho_s - \rho_f)}{18V} = \frac{(9.81\,\text{m/s}^2)(0.003\,\text{m})^2(2500 - 875)\,\text{kg/m}^3}{18(0.12\,\text{m/s})} = \mathbf{0.0664\ kg/m \cdot s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(875\,\text{kg/m}^3)(0.12\,\text{m/s})(0.003\,\text{m})}{0.0664\,\text{kg}\cdot\text{m/s}} = 4.74$$

which is at the order of 1. Therefore, the creeping flow idealization is valid.

Discussion Flow separation starts at about Re = 10. Therefore, Stokes law can be used for Reynolds numbers up to this value, but this should be done with care.

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11-106 Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities are to be compared to those predicted by Stoke's law, and the error involved is to be determined.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is terminal velocity.

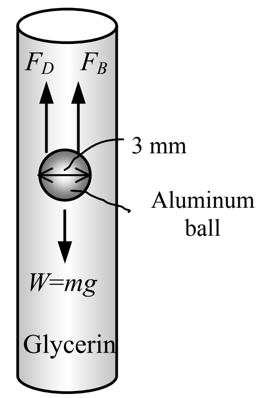
Properties The density of aluminum balls is given to be ρ_s = 2700 kg/m^3. The density and viscosity of glycerin are given to be ρ_f = 1274 kg/m^3 and μ = 1 kg/m·s.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \text{ (Stoke's law)}, \quad W = \rho_s g\boldsymbol{V}, \quad \text{and } F_B = \rho_f g\boldsymbol{V}$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu VD = \rho_s g\boldsymbol{V} - \rho_f g\boldsymbol{V} \quad \rightarrow \quad 3\pi\mu VD = (\rho_s - \rho_f)g\frac{\pi D^3}{6}$$



Solving for the terminal velocity V of the ball gives

$$V = \frac{gD^2(\rho_s - \rho_f)}{18\mu}$$

(*a*) D = 2 mm and V = 3.2 mm/s

$$V = \frac{(9.81\text{ m/s}^2)(0.002\text{ m})^2(2700\text{-}1274)\text{ kg/m}^3}{18(1\text{ kg/ m}\cdot\text{s})} = \mathbf{0.00311\ m/s} = \mathbf{3.11\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{3.2 - 3.11}{3.2} = \mathbf{0.029 \quad or\ 2.9\%}$$

(*b*) D = 4 mm and V = 12.8 mm/s

$$V = \frac{(9.81\text{ m/s}^2)(0.004\text{ m})^2(2700\text{-}1274)\text{ kg/m}^3}{18(1\text{ kg/ m}\cdot\text{s})} = \mathbf{0.0124\ m/s} = \mathbf{12.4\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{12.8 - 12.4}{12.8} = \mathbf{0.029 \quad or\ 2.9\%}$$

(*c*) D = 10 mm and V = 60.4 mm/s

$$V = \frac{(9.81\text{ m/s}^2)(0.010\text{ m})^2(2700\text{-}1274)\text{ kg/m}^3}{18(1\text{ kg/ m}\cdot\text{s})} = \mathbf{0.0777\ m/s} = \mathbf{77.7\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{60.4 - 77.7}{60.4} = \mathbf{-0.287 \quad or\ \text{-}28.7\%}$$

There is a good agreement for the first two diameters. However the error for third one is large. The Reynolds number for each case is

$$(a)\ \text{Re} = \frac{\rho VD}{\mu} = \frac{(1274\text{ kg/m}^3)(0.0032\text{ m/s})(0.002\text{ m})}{1\text{ kg}\cdot\text{m/s}} = 0.008,\ (b)\ \text{Re} = 0.065,\ \text{and}\ \ (c)\ \text{Re} = 0.770.$$

We observe that Re << 1 for the first two cases, and thus the creeping flow idealization is applicable. But this is not the case for the third case.

Discussion If we used the general form of the equation (see next problem) we would obtain V = 59.7 mm/s for part (*c*), which is very close to the experimental result (60.4 mm/s).

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11-107 Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities predicted by general form of Stoke's law, and the error involved are to be determined.

Assumptions **1** The Reynolds number is low (of order 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is terminal velocity.

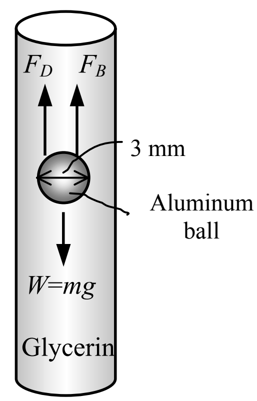
Properties The density of aluminum balls is given to be ρ_s = 2700 kg/m^3. The density and viscosity of glycerin are given to be ρ_f = 1274 kg/m^3 and μ = 1 kg/m·s.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu DV + (9\pi/16)\rho_s V^2 D^2, \quad W = \rho_s g\boldsymbol{V}, \quad \text{and} \quad F_B = \rho_f g\boldsymbol{V}$$

Here $\boldsymbol{V} = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu VD + (9\pi/16)\rho_s V^2 D^2 = (\rho_s - \rho_f)g\frac{\pi D^3}{6}$$



Solving for the terminal velocity V of the ball gives

$$V = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{where} \quad a = \frac{9\pi}{16}\rho_s D^2, \; b = 3\pi\mu D, \text{ and } c = -(\rho_s - \rho_f)g\frac{\pi D^3}{6}$$

(a) D = 2 mm and V = 3.2 mm/s: a = 0.01909, b = 0.01885, c = -0.0000586

$$V = \frac{-0.01885 + \sqrt{(0.01885)^2 - 4\times 0.01909\times(-0.0000586)}}{2\times 0.01909} = \mathbf{0.00310\ m/s = 3.10\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{3.2 - 3.10}{3.2} = \mathbf{0.032 \quad or\ 3.2\%}$$

(b) D = 4 mm and V = 12.8 mm/s: a = 0.07634, b = 0.0377, c = -0.0004688

$$V = \frac{-0.0377 + \sqrt{(0.0377)^2 - 4\times 0.07634\times(-0.0004688)}}{2\times 0.07634} = \mathbf{0.0121\ m/s = 12.1\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{12.8 - 12.1}{12.8} = \mathbf{0.052 \quad or\ 5.2\%}$$

(c) D = 10 mm and V = 60.4 mm/s: a = 0.4771, b = 0.09425, c = -0.007325

$$V = \frac{-0.09425 + \sqrt{(0.09425)^2 - 4\times 0.3771\times(-0.007325)}}{2\times 0.4771} = \mathbf{0.0597\ m/s = 59.7\ mm/s}$$

$$\text{Error} = \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{60.4 - 59.7}{60.4} = \mathbf{0.012 \quad or\ 1.2\%}$$

The Reynolds number for the three cases are

$$(a)\ \text{Re} = \frac{\rho VD}{\mu} = \frac{(1274\ \text{kg/m}^3)(0.0032\ \text{m/s})(0.002\ \text{m})}{1\ \text{kg}\cdot\text{m/s}} = 0.008, \ (b)\ \text{Re} = 0.065, \ \text{and} \ \ (c)\ \text{Re} = 0.770.$$

Discussion There is a good agreement for the third case (case c), but the general Stoke's law increased the error for the first two cases (cases a and b) from 2.9% and 2.9% to 3.2% and 5.2%, respectively. Therefore, the basic form of Stoke's law should be preferred when the Reynolds number is much lower than 1.

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11-108 A spherical aluminum ball is dropped into oil. A relation is to be obtained for the variation of velocity with time and the terminal velocity of the ball. The variation of velocity with time is to be plotted, and the time it takes to reach 99% of terminal velocity is to be determined.

Assumptions **1** The Reynolds number is low (<< 1) so that Stokes law is applicable. **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body.

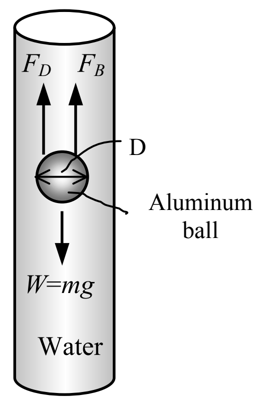
Properties The density of aluminum balls is given to be $\rho_s = 2700$ kg/m^3. The density and viscosity of oil are given to be $\rho_f = 876$ kg/m^3 and $\mu = 0.2177$ kg/m·s.

Analysis The free body diagram is shown in the figure. The net force acting downward on the ball is the weight of the ball less the weight of the ball and the buoyancy force applied by the fluid,

$$F_{net} = W - F_D - F_B \quad \text{where} \quad F_D = 3\pi\mu DV\,, \quad W = m_s g = \rho_s g\boldsymbol{V}, \quad \text{and} \quad F_B = \rho_f g\boldsymbol{V}$$

where F_D the drag force, F_B the buoyancy force, and W is the weight. Also, $\boldsymbol{V} = \pi D^3/6$ is the volume, m_s is the mass, D is the diameter, and V the velocity of the ball. Applying Newton's second law in the vertical direction,

$$F_{net} = ma \quad \rightarrow \quad m_s g - F_D - F_B = m\frac{dV}{dt}$$



Substituting the drag and buoyancy force relations,

$$\rho_s \frac{\pi D^3}{6} g - 3\pi\mu DV - \rho_f g\frac{\pi D^3}{6} = \rho_s \frac{\pi D^3}{6}\frac{dV}{dt}$$

or,
$$g\left(1 - \frac{\rho_f}{\rho s}\right) - \frac{18\mu}{\rho_s D^2}V = \frac{dV}{dt} \quad \rightarrow \quad a - bV = \frac{dV}{dt}$$

where $a = g(1 - \rho_f / \rho_s)$ and $b = 18\mu/(\rho_s D^2)$. It can be rearranged as

$$\frac{dV}{a - bV} = dt$$

Integrating from $t = 0$ where $V = 0$ to $t = t$ where $V = V$ gives

$$\int_0^V \frac{dV}{a - bV} = \int_0^t dt \quad \rightarrow \quad -\left.\frac{\ln(a - bV)}{b}\right|_0^V = t\,\Big|_0^t \quad \rightarrow \quad \ln\left(\frac{a - bV}{a}\right) = -bt$$

Solving for V gives the desired relation for the variation of velocity of the ball with time,

$$V = \frac{a}{b}\left(1 - e^{-bt}\right) \qquad \text{or} \qquad V = \frac{(\rho_s - \rho_f)gD^2}{18\mu}\left(1 - e^{-\frac{18\mu}{\rho_s D^2}t}\right) \qquad (1)$$

Note that as $t \rightarrow \infty$, it gives the terminal velocity as $V_{\text{terminal}} = \frac{a}{b} = \frac{(\rho_s - \rho_f)gD^2}{18\mu}$ (2)

The time it takes to reach 99% of terminal velocity can to be determined by replacing V in Eq. (1) by $0.99V_{\text{terminal}} = 0.99a/b$. It gives $e^{-bt} = 0.01$ or

$$t_{99\%} = -\frac{\ln(0.01)}{b} = -\frac{\ln(0.01)\rho_s D^2}{18\mu} \qquad (3)$$

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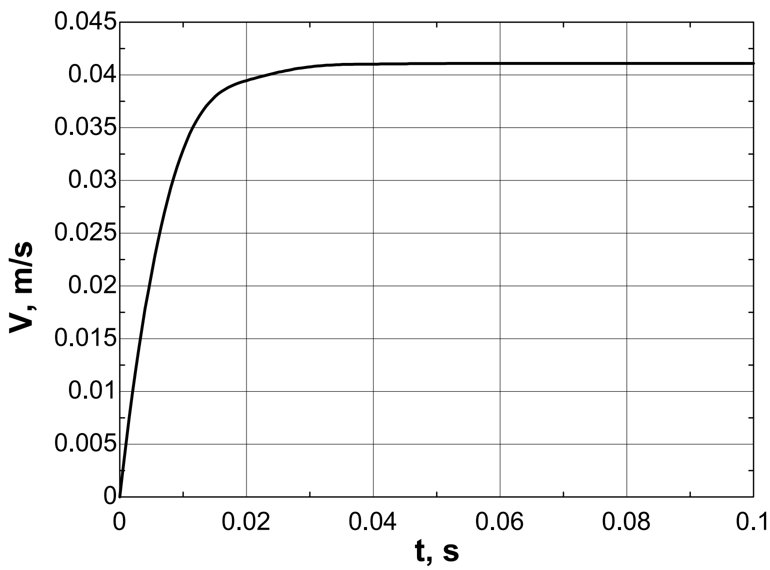
Given values: $D = 0.003$ m, $\rho_f = 876$ kg/m^3, $\mu = 0.2177$ kg/m·s, $g = 9.81$ m/s^2.

Calculation results: Re = 0.50, $a = 6.627$, $b = 161.3$, $t_{99\%}$ = **0.029 s**, and $V_{\text{terminal}} = a/b$ = **0.04 m/s**.

t, s	*V*, m/s
0.00	0.000
0.01	0.033
0.02	0.039
0.03	0.041
0.04	0.041
0.05	0.041
0.06	0.041
0.07	0.041
0.08	0.041
0.09	0.041
0.10	0.041



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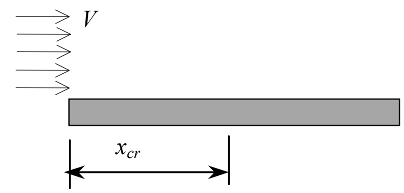
11-109 Engine oil flows over a long flat plate. The distance from the leading edge x_{cr} where the flow becomes turbulent is to be determined, and thickness of the boundary layer over a distance of $2x_{cr}$ is to be plotted.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** The surface of the plate is smooth.

Properties The kinematic viscosity of engine oil at 40°C is $\nu = 2.485\times10^{-4}$ m^2/s.

Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

$$\text{Laminar flow:}\quad \delta_x = \frac{4.91x}{\mathrm{Re}_x^{1/2}}, \qquad \text{Turbulent flow:}\quad \delta_x = \frac{0.38x}{\mathrm{Re}_x^{1/5}}$$



The distance from the leading edge x_{cr} where the flow turns turbulent is determined by setting Reynolds number equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{Vx_{cr}}{\upsilon} \rightarrow x_{cr} = \frac{\mathrm{Re}_{cr}\,\upsilon}{V} = \frac{(5\times10^5)(2.485\times10^{-4}\ \mathrm{m^2/s})}{4\ \mathrm{m/s}} = \mathbf{31.1\,m},$$

Therefore, we should consider flow over 2×31.1 = 62.2 m long section of the plate, and use the laminar relation for the first half, and the turbulent relation for the second part to determine the boundary layer thickness. For example, the Reynolds number and the boundary layer thickness at a distance 2 m from the leading edge of the plate are

$$\mathrm{Re}_x = \frac{Vx}{\upsilon} = \frac{(4\ \mathrm{m/s})(2\ \mathrm{m})}{2.485\times10^{-4}\ \mathrm{m^2/s}} = 32{,}190, \qquad \delta_x = \frac{4.91x}{\mathrm{Re}_x^{1/2}} = \frac{4.91\times(2\ \mathrm{m})}{(32{,}190)^{0.5}} = 0.0547\ \mathrm{m}$$

Calculating the boundary layer thickness and plotting give

x, m	Re	$\delta_{x,\ \text{laminar}}$	$\delta_{x,\ \text{turbulent}}$
0.00	0	0.000	-
5.00	8.05E+04	0.087	-
10.00	1.61E+05	0.122	-
15.00	2.41E+05	0.150	-
20.00	3.22E+05	0.173	-
25.00	4.02E+05	0.194	-
30.00	4.83E+05	0.212	-
35.00	5.63E+05	-	0.941
40.00	6.44E+05	-	1.047
45.00	7.24E+05	-	1.151
50.00	8.05E+05	-	1.252
55.00	8.85E+05	-	1.351
60.00	9.66E+05	-	1.449

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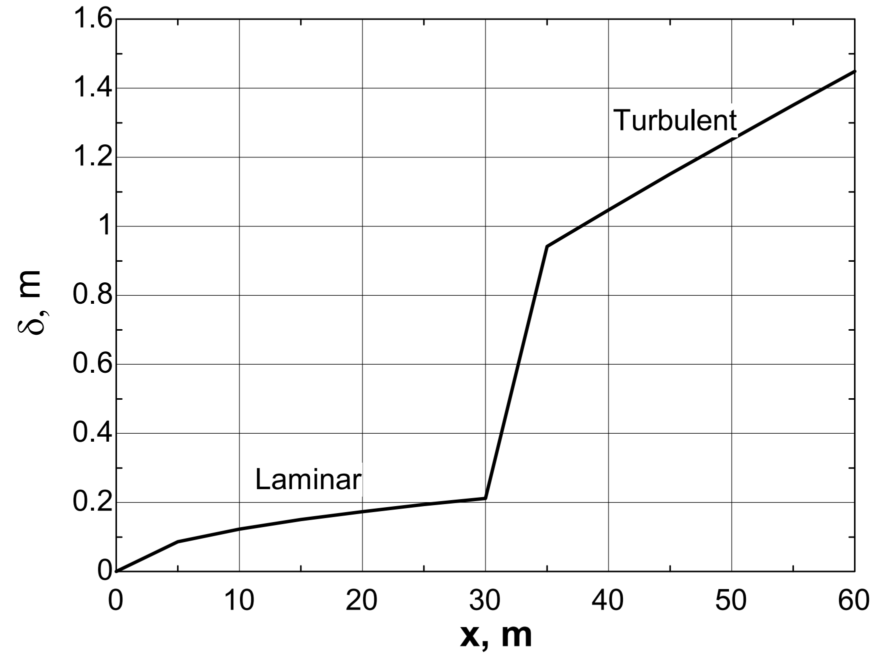

11-110 … 11-113 Design and Essay Problems

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