Chapter 14 Turbomachinery

General Problems

14-1C

Solution We are to discuss energy producing and energy absorbing devices.

Analysis A more common term for an energy producing turbomachine is a **turbine**. Turbines extract energy from the moving fluid, and convert that energy into useful mechanical energy in the surroundings, usually in the form of a rotating shaft. Thus, the phrase "energy producing" is from a frame of reference of the fluid – the fluid loses energy as it drives the turbine, producing energy to the surroundings. On the other hand, a more common term for an energy absorbing turbomachine is a **pump**. Pumps absorb mechanical energy from the surroundings, usually in the form of a rotating shaft, and increase the energy of the moving fluid. Thus, the phrase "energy absorbing" is from a frame of reference of the fluid – the fluid gains or absorbs energy as it flows through the pump.

Discussion From the frame of reference of the surroundings, a pump absorbs energy from the surroundings, while a turbine produces energy to the surroundings. Thus, you may argue that the terminology also holds for the frame of reference of the surroundings. This alternative explanation is also acceptable.

14-2C

Solution We are to discuss the differences between fans, blowers, and compressors.

Analysis A *fan* is a gas pump with relatively l**ow pressure rise** and **high flow rate**. A ***blower*** is a gas pump with relatively **moderate to high pressure rise** and **moderate to high flow rate**. A ***compressor*** is a gas pump designed to deliver a very **high pressure rise**, typically at **low to moderate flow rates**.

Discussion The boundaries between these three types of pump are not always clearly defined.

14-3C

Solution We are to list examples of fans, blowers, and compressors.

Analysis Common examples of fans are **window fans, ceiling fans, fans in computers and other electronics equipment, radiator fans in cars**, etc. Common examples of blowers are **leaf blowers, hair dryers, air blowers in furnaces and automobile ventilation systems**. Common examples of compressors are **tire pumps, refrigerator and air conditioner compressors**.

Discussion Students should come up with a diverse variety of examples.

14-4C

Solution We are to discuss the difference between a positive-displacement turbomachine and a dynamic turbomachine.

Analysis **A *positive-displacement turbomachine* is a device that contains a closed volume; energy is transferred to the fluid (pump) or from the fluid (turbine) via movement of the boundaries of the closed volume**. On the other hand, **a *dynamic turbomachine* has no closed volume; instead, energy is transferred to the fluid (pump) or from the fluid (turbine) via rotating blades**. Examples of positive-displacement pumps include well **pumps, hearts, some aquarium pumps, and pumps designed to release precise volumes of medicine**. Examples of positive-displacement turbines include **water meters and gas meters in the home**. Examples of dynamic pumps include **fans, centrifugal blowers, airplane propellers, centrifugal water pumps** (like in a car engine), etc. Examples of dynamic turbines include **windmills, wind turbines, turbine flow meters**, etc.

Discussion Students should come up with a diverse variety of examples.

14-5C

Solution We are to discuss the difference between brake horsepower and water horsepower, and then discuss pump efficiency.

Analysis In turbomachinery terminology, **brake horsepower is the power actually delivered to the pump through the shaft**. (One may also call it "shaft power".) On the other hand, **water horsepower is the useful portion of the brake horsepower that is actually delivered to the fluid**. Water horsepower is always less than brake horsepower; hence **pump efficiency is defined as the ratio of water horsepower to brake horsepower**.

Discussion For a turbine, efficiency is defined in the opposite way, since brake horsepower is *less* than water horsepower.

14-6C

Solution We are to discuss the difference between brake horsepower and water horsepower, and then discuss turbine efficiency.

Analysis In turbomachinery terminology, **brake horsepower is the power actually delivered by the turbine to the shaft.** (One may also call it "shaft power".) On the other hand, **water horsepower is the power extracted from the water flowing through the turbine**. Water horsepower is always greater than brake horsepower; because of inefficiencies; hence **turbine efficiency is defined as the ratio of brake horsepower to water horsepower**.

Discussion For a pump, efficiency is defined in the opposite way, since brake horsepower is *greater* than water horsepower.

14-7C

Solution We are to explain the "extra" term in the Bernoulli equation in a rotating reference frame.

Analysis A rotating reference frame is not an inertial reference frame. When we move outward in the radial direction, the absolute velocity at this location is faster due to the rotating body, since v_θ is equal to ωr. When solving a turbomachinery problem in a rotating reference frame, we use the *relative* fluid velocity (velocity relative to the rotating reference frame). Thus, **in order for the Bernoulli equation to be physically correct, we must subtract the absolute velocity of the rotating body so that the equation applies to an inertial reference frame. This accounts for the "extra" term**.

Discussion The Bernoulli equation is the same physical equation in either the absolute or the rotating reference frame, but it is more convenient to use the form with the extra term in turbomachinery applications.

14-8

Solution We are to determine how the average speed at the outlet compares to the average speed at the inlet of a water pump.

Assumptions **1** The flow is steady (in the mean). **2** The water is incompressible.

Analysis Conservation of mass requires that the mass flow rate in equals the mass flow rate out. Thus,

Conservation of mass: $$\dot{m}_{\text{in}} = \rho_{\text{in}} V_{\text{in}} A_{\text{in}} = \dot{m}_{\text{out}} = \rho_{\text{out}} V_{\text{out}} A_{\text{out}}$$

or, since the cross-sectional area is proportional to the square of diameter,

$$V_{\text{out}} = V_{\text{in}} \frac{\rho_{\text{in}}}{\rho_{\text{out}}} \left(\frac{D_{\text{in}}}{D_{\text{out}}} \right)^2 = V_{\text{in}} \left(\frac{D_{\text{in}}}{D_{\text{out}}} \right)^2 \qquad (1)$$

(a) For the case where $D_{\text{out}} < D_{\text{in}}$, V_{out} must be **greater than** V_{in}.

(b) For the case where $D_{\text{out}} = D_{\text{in}}$, V_{out} must be **equal to** V_{in}.

(c) For the case where $D_{\text{out}} > D_{\text{in}}$, V_{out} must be **less than** V_{in}.

Discussion A pump does not necessarily increase the speed of the fluid passing through it. In fact, the average speed through the pump can actually *decrease*, as it does here in part (c).

14-9

Solution For an air compressor with equal inlet and outlet areas, and with both density and pressure increasing, we are to determine how the average speed at the outlet compares to the average speed at the inlet.

Assumptions **1** The flow is steady.

Analysis Conservation of mass requires that the mass flow rate in equals the mass flow rate out. The cross-sectional areas of the inlet and outlet are the same. Thus,

Conservation of mass: $$\dot{m}_{\text{in}} = \rho_{\text{in}} V_{\text{in}} A_{\text{in}} = \dot{m}_{\text{out}} = \rho_{\text{out}} V_{\text{out}} A_{\text{out}}$$

or

$$V_{\text{out}} = V_{\text{in}} \frac{\rho_{\text{in}}}{\rho_{\text{out}}} \qquad (1)$$

Since $\rho_{\text{in}} < \rho_{\text{out}}$, V_{out} must be **less than** V_{in}.

Discussion A compressor does not necessarily increase the speed of the fluid passing through it. In fact, the average speed through the pump can actually *decrease*, as it does here.

Pumps

14-10C

Solution We are to list and define the three categories of dynamic pumps.

Analysis The three categories are: **Centrifugal flow pump** – fluid enters axially (in the same direction as the axis of the rotating shaft) in the center of the pump, but is discharged radially (or tangentially) along the outer radius of the pump casing. **Axial-flow pump** – fluid enters and leaves axially, typically only along the outer portion of the pump because of blockage by the shaft, motor, hub, etc. **Mixed-flow pump** – intermediate between centrifugal and axial, with the flow entering axially, not necessarily in the center, but leaving at some angle between radially and axially.

Discussion There are also some non-rotary dynamic pumps, such as jet pumps and electromagnetic pumps, that are not discussed in this text.

14-11C

Solution

(*a*) *False*: Actually, backward-inclined blades yield the highest efficiency.
(*b*) *True*: The pressure rise is higher, but at the cost of less efficiency.
(*c*) *True*: In fact, this is the primary reason for choosing forward-inclined blades.
(*d*) *False*: Actually, the opposite is true – a pump with forward-inclined blades usually has more blades, but they are usually smaller.

14-12C

Solution We are to choose which pump location is better and explain why.

Analysis The two systems are identical except for the location of the pump (and some minor differences in pipe layout). The overall length of pipe, number of elbows, elevation difference between the two reservoir free surfaces, etc. are the same. **Option (a) is better because it has the pump at a lower elevation, increasing the net positive suction head, and lowering the possibility of pump cavitation**. In addition, the length of pipe from the lower reservoir to the pump inlet is smaller in Option (a), and there is one less elbow between the lower reservoir and the pump inlet, thereby decreasing the head loss upstream of the pump – both of which also increase *NPSH*, and reduce the likelihood of cavitation.

Discussion Another point is that if the pump is not self-priming, Option (b) may run into start-up problems if the free surface of the lower reservoir falls below the elevation of the pump inlet. Since the pump in Option (a) is below the reservoir, self-priming is not an issue.

14-13C

Solution We are to define and discuss *NPSH* and $NPSH_{\text{required}}$.

Analysis Net positive suction head (*NPSH*) is defined as **the difference between the pump's inlet stagnation pressure head and the vapor pressure head**,

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{\text{pump inlet}} - \frac{P_v}{\rho g}$$

We may think of *NPSH* as the actual or available net positive suction head. On the other hand, required net positive suction head ($NPSH_{\text{required}}$) is defined as the **minimum NPSH necessary to avoid cavitation in the pump**. As long as the actual *NPSH* is greater than $NPSH_{\text{required}}$, there should be no cavitation in the pump.

Discussion Although *NPSH* and $NPSH_{\text{required}}$ are measured at the pump inlet, cavitation (if present) happens somewhere inside the pump, typically on the suction surface of the rotating pump impeller blades.

14-14C

Solution

(*a*) *True*: As volume flow rate increases, not only does $NPSH_{\text{required}}$ increase, but the available *NPSH* decreases, increasing the likelihood that *NPSH* will drop below $NPSH_{\text{required}}$ and cause cavitation to occur.
(*b*) *False*: $NPSH_{\text{required}}$ is *not* a function of water temperature, although available *NPSH* does depend on water temperature.
(*c*) *False*: Available *NPSH* actually *decreases* with increasing water temperature, making cavitation more likely to occur.
(*d*) *False*: Actually, warmer water causes cavitation to be *more likely*. The best way to think about this is that warmer water is already closer to its boiling point, so cavitation is more likely to happen in warm water than in cold water.

14-15C

Solution We are to explain why dissimilar pumps should not be arranged in series or in parallel.

Analysis Arranging dissimilar pumps in series can create problems because the volume flow rate through each pump must be the same, but the overall pressure rise is equal to the pressure rise of one pump plus that of the other. If the pumps have widely different performance curves, the smaller pump may be forced to operate beyond its free delivery flow rate, whereupon it acts like a head *loss*, reducing the total volume flow rate. Arranging dissimilar pumps in parallel can create problems because the overall pressure rise must be the same, but the net volume flow rate is the sum of that through each branch. If the pumps are not sized properly, the smaller pump may not be able to handle the large head imposed on it, and the flow in its branch could actually be *reversed*; this would inadvertently reduce the overall pressure rise. In either case, the power supplied to the smaller pump would be wasted.

Discussion If the pumps are not significantly dissimilar, a series or parallel arrangement of the pumps might be wise.

14-16C

Solution

(*a*) *True*: The maximum volume flow rate occurs when the net head is zero, and this "free delivery" flow rate is typically much higher than that at the BEP.

(*b*) *True*: By definition, there is no flow rate at the shutoff head. Thus the pump is not doing any useful work, and the efficiency must be zero.

(*c*) *False*: Actually, the net head is typically greatest near the shutoff head, at zero volume flow rate, not near the BEP.

(*d*) *True*: By definition, there is no head at the pump's free delivery. Thus, the pump is working against no "resistance", and is therefore not doing any useful work, and the efficiency must be zero.

14-17C

Solution We are to discuss ways to improve the cavitation performance of a pump, based on the equation that defines *NPSH*.

Analysis *NPSH* is defined as

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{\text{pump inlet}} - \frac{P_v}{\rho g} \qquad (1)$$

To avoid cavitation, *NPSH* must be increased as much as possible. For a given liquid at a given temperature, the vapor pressure head (last term on the right side of Eq. 1) is constant. Hence, the only way to increase *NPSH* is to increase the stagnation pressure head at the pump inlet. We list several ways to increase the available *NPSH*: (1) **Lower the pump or raise the inlet reservoir level. (2) Use a larger diameter pipe upstream of the pump. (3) Re-route the piping system such that fewer minor losses (elbows, valves, etc.) are encountered upstream of the pump. (4) Shorten the length of pipe upstream of the pump. (5) Use a smoother pipe. (6) Use elbows, valves, inlets, etc. that have smaller minor loss coefficients**. Suggestion (1) raises *NPSH* by increasing the hydrostatic component of pressure at the pump inlet. Suggestions (2) through (6) raise *NPSH* by lowering the irreversible head losses, thereby increasing the pressure at the pump inlet.

Discussion By definition, when the available *NPSH* falls below the required *NPSH*, the pump is prone to cavitation, which should be avoided if at all possible.

14-18C

Solution

(*a*) *False*: Since the pumps are in series, the volume flow rate through each pump must be the same: $\dot{V} = \dot{V}_1 = \dot{V}_2$.

(*b*) *True*: The net head increases by H_1 through the first pump, and then by H_2 through the second pump. The overall rise in net head is thus the sum of the two.

(*c*) *True*: Since the pumps are in parallel, the total volume flow rate is the sum of the individual volume flow rates.

(*d*) *False*: For pumps in parallel, the change in pressure from the upstream junction to the downstream junction is the same regardless of which parallel branch is under consideration. Thus, even though the volume flow rate may not be the same in each branch, the net head must be the same: $H = H_1 = H_2$.

14-19C

Solution We are to label several items on the provided plot.

Analysis The figure is re-drawn as Fig. 1, and the requested items are labeled.

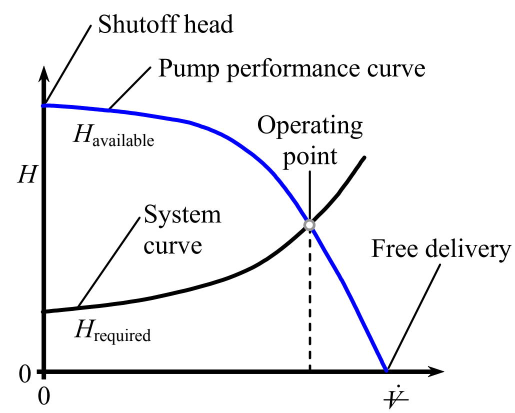


FIGURE 1
Pump net head versus pump capacity, with labels.

Discussion Also labeled are the available net head, corresponding to the pump performance curve, and the required net head, corresponding to the system curve. The intersection of these two curves is the operating point of the pump.

14-20

Solution We are to determine which free surface is at higher elevation, and justify our answer with the energy equation.

Analysis It is simplest to consider zero-flow conditions ($\dot{V} = 0$), at which we see that the required net head is positive. This implies that, even when there is no flow between the two tanks, the pump would need to provide some net head just to overcome the pressure differences. Since there is no flow, pressure differences can come only from gravity. Hence, **the outlet tank's free surface must be higher than that of the inlet tank**. Mathematically, we apply the energy equation in head form between the inlet tank's free surface (1) and the outlet tank's free surface (2),

Energy equation at zero flow conditions:

$$H_{\text{required}} = h_{\text{pump,u}} = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 \cancel{V_2^2} - \alpha_1 \cancel{V_1^2}}{2g} + \left(z_2 - z_1\right) + \cancel{h_{\text{turbine}}} + \cancel{h_{L,\text{total}}} \qquad (1)$$

Since both free surfaces are at atmospheric pressure, $P_1 = P_2 = P_{\text{atm}}$, and the first term on the right side of Eq. 1 vanishes. Furthermore, since there is no flow, $V_1 = V_2 = 0$, and the second term vanishes. There is no turbine in the control volume, so the second-to-last term is zero. Finally, there are no irreversible head losses since there is no flow, and the last term is also zero. Equation 1 reduces to

$$H_{\text{required}} = h_{\text{pump}} = \left(z_2 - z_1\right) \qquad (2)$$

Since H_{required} is positive on Fig. P14-19 at $\dot{V} = 0$, the quantity $(z_2 - z_1)$ must also be positive by Eq. 2. Thus we have shown mathematically that **the outlet tank's free surface is higher in elevation than that of the inlet tank**.

Discussion If the reverse were true (outlet tank free surface lower than inlet tank free surface), H_{required} at $\dot{V} = 0$ would be *negative*, implying that the pump would need to supply enough *negative* net head to hold back the natural tendency of the water to flow from higher to lower elevation. In reality, the pump would not be able to do this unless it were spun backwards.

14-21

Solution We are to discuss what would happen to the pump performance curve, the system curve, and the operating point if the free surface of the outlet tank were raised to a higher elevation.

Analysis The pump is the same pump regardless of the locations of the inlet and outlet tanks' free surfaces; thus, **the pump performance curve does not change**. The energy equation is

$$H_{\text{required}} = h_{\text{pump,u}} = \frac{P_2 - P_1}{\rho g} + \frac{\alpha_2 V_2^{\,2} - \alpha_1 V_1^{\,2}}{2g} + \left(z_2 - z_1\right) + \cancel{h_{\text{turbine}}} + h_{L,\text{total}} \qquad (1)$$

Since the only thing that changes is the elevation difference, Eq. 1 shows that H_{required} shifts up as $(z_2 - z_1)$ increases. Thus, **the system curve rises linearly with elevation increase**. A plot of H versus $\dot{V}$ is shown in Fig. 1, and the new operating point is labeled. Because of the upward shift of the system curve, **the operating point moves to a lower value of volume flow rate**.

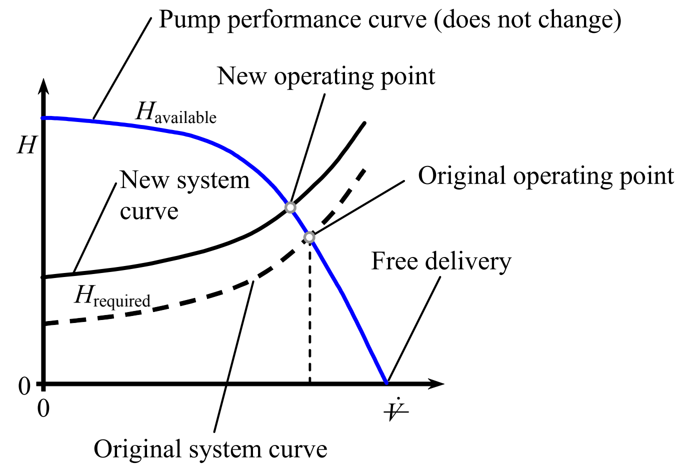


FIGURE 1
Pump net head versus pump capacity for the case in which the elevation of the outlet is increased, causing the system curve to shift upwards.

Discussion The shift of operating point to lower $\dot{V}$ agrees with our physical intuition. Namely, as we raise the elevation of the outlet, the pump has to do more work to overcome gravity, and we expect the flow rate to decrease accordingly.

14-22

Solution We are to discuss what would happen to the pump performance curve, the system curve, and the operating point if a valve changes from 100% to 50% open.

Analysis The pump is the same pump regardless of the locations of the inlet and outlet tanks' free surfaces; thus, **the pump performance curve does not change**. The energy equation is

$$H_{\text{required}} = h_{\text{pump,u}} = \frac{P_2 - P_1}{\rho g} + \frac{\alpha_2 V_2^{\ 2} - \alpha_1 V_1^{\ 2}}{2g} + \left(z_2 - z_1\right) + \cancel{h_{\text{turbine}}} + h_{L,\text{total}} \quad (1)$$

Since both free surfaces are open to the atmosphere, the pressure term vanishes. Since both V_1 and V_2 are negligibly small at the free surface (the tanks are large), the second term on the right also vanishes. The elevation difference $(z_2 - z_1)$ does not change, and so the only term in Eq. 1 that is changed by closing the valve is the irreversible head loss term. We know that the minor loss associated with a valve increases significantly as the valve is closed. Thus, **the system curve** (the curve of H_{required} versus $\dot{V}$) **increases more rapidly with volume flow rate (has a larger slope) when the valve is partially closed**. A sketch of H versus $\dot{V}$ is shown in Fig. 1, and the new operating point is labeled. Because of the higher system curve, ***the operating point moves to a lower value of volume flow rate,*** as indicated on Fig. 1. I.e., **the volume flow rate decreases**.

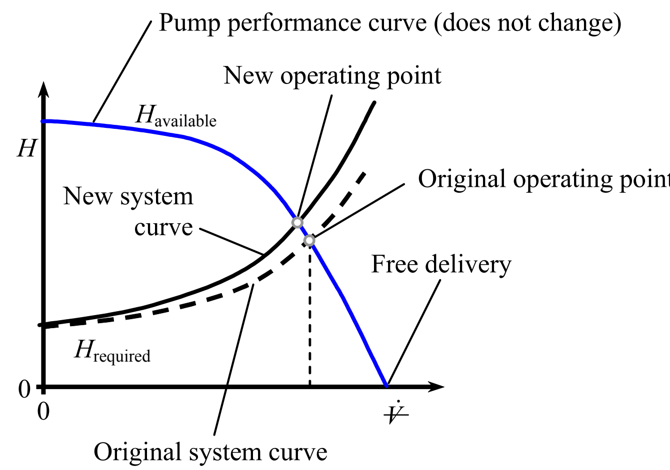


FIGURE 1
Pump net head versus pump capacity for the case in which a valve is partially closed, causing the system curve to rise more rapidly with volume flow rate.

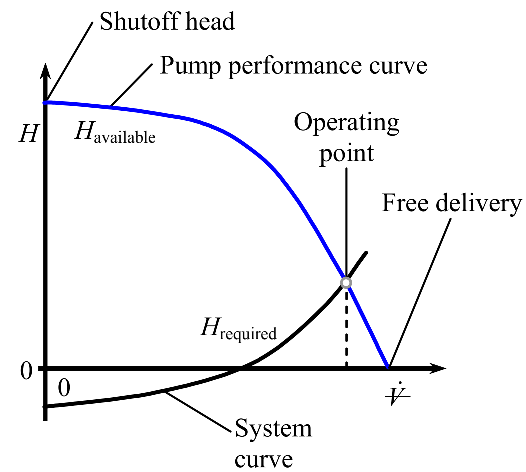
Discussion The shift of operating point to lower $\dot{V}$ agrees with our physical intuition. Namely, as we close the valve somewhat, the pump has to do more work to overcome the losses, and we expect the flow rate to decrease accordingly.

14-23

Solution We are to create a qualitative plot of pump net head versus pump capacity.

Analysis The result is shown in Fig. 1, and the requested items are labeled. Also labeled are the available net head, corresponding to the pump performance curve, and the required net head, corresponding to the system curve. The intersection of these two curves is the operating point of the pump. Note that since the elevation of the outlet is lower than that of the free surface of the inlet tank, the required net head must be *negative* at zero flow rate conditions, as sketched, implying that the pump holds back the natural tendency of the water to flow from higher to lower elevation. Only at higher flow rates does the system curve rise to positive values of $H_{required}$.

FIGURE 1
Pump net head versus pump capacity, with labels.



Discussion A real pump cannot produce negative net head at zero volume flow rate unless its blades are spun in the opposite direction than that for which they are designed.

14-24

Solution We are to estimate the volume flow rate through a piping system.

Assumptions **1** Since the reservoir is large, the flow is nearly steady. **2** The water is incompressible. **3** The water is at room temperature. **4** The flow in the pipe is fully developed and turbulent, with $\alpha = 1.05$.

Properties The density and viscosity of water at $T = 20^{o}C$ are 998.0 kg/m^3 and 1.002×10^{-3} kg/m·s respectively.

Analysis By definition, at free delivery conditions, the net head across the pump is zero. Thus, there is no loss or gain of pressure across the pump, and we can essentially ignore it in the calculations here. We apply the head form of the steady energy equation from location 1 to location 2,

$$H_{\text{required}} = h_{\text{pump}} = 0 = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 V_2^2 - \cancel{V_1^2}}{2g} + \left(z_2 - z_1\right) + \cancel{h_{\text{turbine}}} + h_{L,\text{total}} \tag{1}$$

where the pressure term vanishes since the free surface at location 1 and at the exit (location 2) are both open to the atmosphere. The inlet velocity term disappears since V_1 is negligibly small at the free surface. Thus, Eq. 1 reduces to a balance between supplied potential energy head ($z_1 - z_2$), kinetic energy head at the exit $\alpha_2 V_2^2/2g$, and irreversible head losses,

$$\left(z_1 - z_2\right) = \frac{\alpha_2 V_2^2}{2g} + h_{L,\text{total}} \tag{2}$$

The total irreversible head loss in Eq. 2 consists of both major and minor losses. We split the minor losses into those associated with the mean velocity V through the pipe, and the minor loss associated with the contraction, based on exit velocity V_2,

$$\left(z_1 - z_2\right) = \frac{\alpha_2 V_2^2}{2g} + \frac{V^2}{2g}\left(f\frac{L}{D} + \sum_{\text{pipe}} K_L\right) + \frac{V_2^2}{2g} K_{L,\text{contraction}} \tag{3}$$

where $\sum_{\text{pipe}} K_L$ = 0.50 + 2(2.4) + 3(0.90) = 8.0, and $K_{L,\text{contraction}}$ = 0.15.

By conservation of mass,

$$VA = V_2 A_2 \qquad \rightarrow \qquad V_2 = V\frac{A}{A_2} = V\left(\frac{D}{D_2}\right)^2 \tag{4}$$

Substitution of Eq. 4 into Eq. 3 yields

$$\left(z_1 - z_2\right) = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum_{\text{pipe}} K_L + \left(\frac{D}{D_2}\right)^4 \left(\alpha_2 + K_{L,\text{contraction}}\right)\right) \tag{5}$$

Equation 5 is an implicit equation for V since the Darcy friction factor is a function of Reynolds number Re = $\rho VD/\mu$, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is V = 1.911 m/s, or to three significant digits, V = 1.91 m/s, from which the volume flow rate is

$$\dot{V} = V\frac{\pi D^2}{4} = \left(1.911 \text{ m/s}\right)\frac{\pi\left(0.020 \text{ m}\right)^2}{4} = 6.01\times10^{-4} \text{ m}^3\text{/s} \tag{6}$$

In more common units, $\dot{V}$ = **36.0 Lpm** (liters per minute). The Reynolds number is 3.81×10^4.

Discussion Since there is no net head across the pump at free delivery conditions, the pump could be removed (inlet and outlet pipes connected together without the pump), and the flow rate would be the same. Another way to think about this is that the pump's efficiency is zero at the free delivery operating point, so it is doing no useful work.

14-25

Solution We are to calculate the volume flow rate through a piping system in which the pipe is rough.

Assumptions **1** Since the reservoir is large, the flow is nearly steady. **2** The water is incompressible. **3** The water is at room temperature. **4** The flow in the pipe is fully developed and turbulent, with $\alpha = 1.05$.

Properties The density and viscosity of water at $T = 20^{o}$C are 998.0 kg/m^3 and 1.002×10^{-3} kg/m·s respectively.

Analysis The relative pipe roughness is ε/D = (0.050 cm)/(2.0 cm) = 0.025 (very rough, as seen on the Moody chart). The calculations are identical to that of Problem 14-24, except for the pipe roughness. The result is V = 1.597 m/s, or to three significant digits, V = 1.60 m/s, from which the volume flow rate is 5.02×10^{-4} m^3/s, or $\dot{V}$ = **30.1 Lpm**. The Reynolds number is 3.18×10^4. The volume flow rate is lower by about 16%. This agrees with our intuition, since pipe roughness leads to more pressure drop at a given flow rate.

Discussion If the calculations of Problem 14-24 are done on a computer, it is trivial to change ε for the present calculations.

14-26

Solution For a given pump and piping system, we are to calculate the volume flow rate and compare with that calculated for problem 14-24.

Assumptions **1** Since the reservoir is large, the flow is nearly steady. **2** The water is incompressible. **3** The water is at room temperature. **4** The flow in the pipe is fully developed and turbulent, with $\alpha = 1.05$.

Properties The density and viscosity of water at $T = 20^{\circ}\text{C}$ are 998.0 kg/m^3 and 1.002×10^{-3} kg/m·s respectively.

Analysis The calculations are identical to those of Problem 14-24 except that the pump's net head is not zero as in Problem 14-24, but instead is given in the problem statement. At the operating point, we match $H_{\text{available}}$ to H_{required}, yielding

$$H_{\text{available}} = H_{\text{required}} \quad \rightarrow$$

$$H_0 - a\dot{V}^2 = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum_{\text{pipe}} K_L + \left(\frac{D}{D_2}\right)^4 \left(\alpha_2 + K_{L,\text{contraction}}\right)\right) - \left(z_1 - z_2\right) \tag{1}$$

We re-write the second term on the left side of Eq. 1 in terms of average pipe velocity V instead of volume flow rate, since $\dot{V} = V\pi D^2/4$, and solve for V,

$$V = \sqrt{\frac{H_0 + \left(z_1 - z_2\right)}{\frac{1}{2g}\left(f\frac{L}{D} + \sum_{\text{pipe}} K_L + \left(\frac{D}{D_2}\right)^4 \left(\alpha_2 + K_{L,\text{contraction}}\right)\right) + a\frac{\pi^2 D^4}{16}}} \tag{2}$$

Equation 2 is an implicit equation for V since the Darcy friction factor is a function of Reynolds number Re = $\rho VD/\mu$, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is V = 2.846 m/s, from which the volume flow rate is

$$\dot{V} = V\frac{\pi D^2}{4} = \left(2.846 \text{ m/s}\right)\frac{\pi\left(0.020 \text{ m}\right)^2}{4} = 8.942\times 10^{-4}\ \frac{\text{m}^3}{\text{s}} \tag{3}$$

In more common units, $\dot{V}$ = **53.6 Lpm**. This represents an increase of about 49% compared to the flow rate of Problem 14-24. This agrees with our expectations – adding a pump in the line produces a higher flow rate.

Discussion Although there was a pump in Problem 14-24 as well, it was operating at free delivery conditions, implying that it was not contributing anything to the flow – that pump could be removed from the system with no change in flow rate. Here, however, the net head across the pump is about 5.34 m, implying that it is contributing useful head to the flow (in addition to the gravity head already present).

14-27E

Solution We are to calculate pump efficiency and estimate the BEP conditions.

Properties The density of water at 77°F is 62.24 lbm/ft^3.

Analysis (*a*) Pump efficiency is

Pump efficiency:
$$\eta_{\text{pump}} = \frac{\rho g \dot{V} H}{bhp} \tag{1}$$

We show the second row of data (at $\dot{V}$ = 4.0 gpm) as an example – the rest are calculated in a spreadsheet for convenience,

$$\eta_{\text{pump}} = \frac{\left(62.24\ \text{lbm/ft}^3\right)\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)\left(4.0\ \frac{\text{gal}}{\text{min}}\right)\left(18.5\ \text{ft}\right)}{0.064\ \text{hp}}\left(\frac{0.1337\ \text{ft}^3}{\text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$\times\left(\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}\right)\left(\frac{\text{hp}\cdot\text{s}}{550\ \text{ft}\cdot\text{lbf}}\right) = 0.292 = 29.2\%$$

TABLE 1
Pump performance data for water at 77°F.

$\dot{V}$ (gpm)	*H* (ft)	*bhp* (hp)	η_{pump} (%)
0.0	19.0	0.06	**0.0**
4.0	18.5	0.064	**29.2**
8.0	17.0	0.069	**49.7**
12.0	14.5	0.074	**59.3**
16.0	10.5	0.079	**53.6**
20.0	6.0	0.08	**37.8**
24.0	0	0.078	**0.0**

The results for all rows are shown in Table 1.

(*b*) The best efficiency point (BEP) occurs at approximately the fourth row of data: $\dot{V}^*$ = **12.0 gpm, *H** = 14.5 ft of head, *bhp** = 0.074 hp, and η_{pump}^* = 59.3%**.

Discussion A more precise BEP could be obtained by curve-fitting the data, as in Problem 14-29.

14-28

Solution We are to convert the pump performance data to metric units and calculate pump efficiency.

Properties The density of water at T = 20°C is 998.0 kg/m^3.

Analysis The conversions are straightforward, and the results are shown in Table 1. A sample calculation of the pump efficiency for the second row of data is shown below:

$$\eta_{\text{pump}} = \frac{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(15.1\ \text{L/min}\right)\left(5.64\ \text{m}\right)}{47.7\ \text{W}}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$\times\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right) = 0.292 = 29.2\%$$

TABLE 1
Pump performance data for water at 77°F.

$\dot{V}$ (Lpm)	*H* (m)	*bhp* (W)	η_{pump} (%)
0.0	5.79	44.7	**0.0**
15.1	5.64	47.7	**29.2**
30.3	5.18	51.5	**49.7**
45.4	4.42	55.2	**59.3**
60.6	3.20	58.9	**53.6**
75.7	1.83	59.7	**37.8**
90.9	0.00	58.2	**0.0**

The pump efficiency data are identical to those of Problem 14-27, as they must be, regardless of the system of units.

Discussion If the calculations of Problem 14-27 are done on a computer, it is trivial to convert to metric units in the present calculations.

CD-EES 14-29E

Solution We are to generate least-squares polynomial curve fits of a pump's performance curves, plot the curves, and calculate the BEP.

Properties The density of water at 77°F is 62.24 lbm/ft³.

Analysis The efficiencies for each data point in Table P14-27 were calculated in Problem 14-27. We use Regression anaylsis to generate the least-squares fits. The equation and coefficients for *H* are

$$H = H_0 - a\dot{V}^2 \qquad H_0 = 19.0774 \text{ ft} \qquad a = 0.032996 \text{ ft/gpm}^2$$

Or, to three digits of precision, $H_0 = 19.1$ ft $\quad a = 0.0330$ ft/gpm^2

The equation and coefficients for *bhp* are

$$bhp = bhp_0 + a_1\dot{V} + a_2\dot{V}^2 \qquad bhp_0 = 0.0587 \text{ hp}$$
$$a_1 = 0.00175 \text{ hp/gpm} \qquad a_2 = -3.72\times10^{-5} \text{ hp/gpm}^2$$

The equation and coefficients for η_{pump} are

$$\eta_{\text{pump}} = \eta_{\text{pump},0} + a_1\dot{V} + a_2\dot{V}^2 + a_3\dot{V}^3 \qquad \eta_{\text{pump},0} = 0.0523\%$$
$$a_1 = 8.21 \text{ \%/gpm} \qquad a_2 = -0.210 \text{ \%/gpm}^2 \qquad a_3 = -0.00546 \text{ \%/gpm}^3$$

The tabulated data are plotted in Fig. 1 as symbols only. The fitted data are plotted on the same plots as lines only. The agreement is excellent.

The best efficiency point is obtained by differentiating the curve-fit expression for η_{pump} with respect to volume flow rate, and setting the derivative to zero (solving the resulting quadratic equation for $\dot{V}^*$),

$$\frac{d\eta_{\text{pump}}}{d\dot{V}} = a_1 + 2a_2\dot{V} + 3a_3\dot{V}^2 = 0 \qquad \rightarrow \qquad \dot{V}^* = 12.966 \text{ GPM} \approx 13.0 \text{ gpm}$$

At this volume flow rate, the curve-fitted expressions for *H*, *bhp*, and η_{pump} yield the operating conditions at the best efficiency point (to three digits each):

BEP: $\dot{V}^* = 13.0$ gpm, $\quad H^* = 13.5$ ft, $\quad bhp^* = 0.0752$ hp, $\quad \eta^* = 59.2\%$

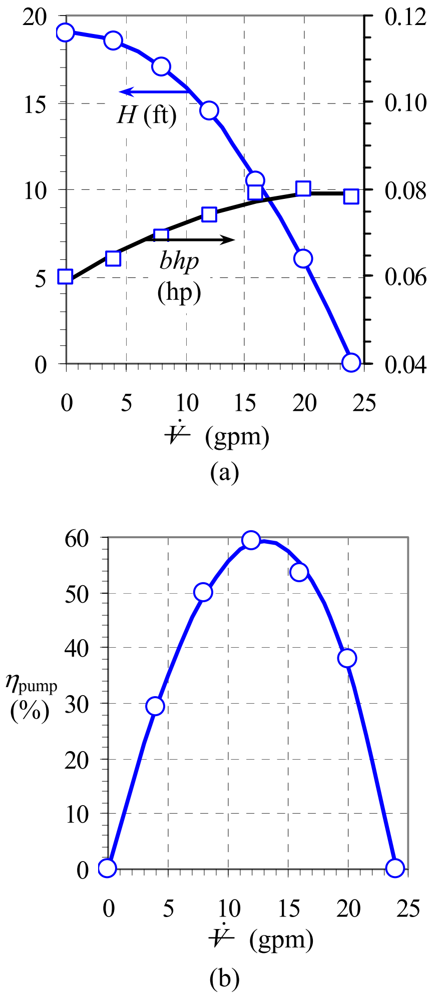


FIGURE 1
Pump performance curves: (a) *H* and *bhp* versus $\dot{V}$, and (b) η_{pump} versus $\dot{V}$.

Discussion This BEP is more precise than that of Problem 14-27 because of the curve fit. The other root of the quadratic is negative – obviously not the correct choice.

14-30E

Solution For a given pump and system requirement, we are to estimate the operating point.

Assumptions **1** The flow is steady. **2** The water is at 77°F and is incompressible.

Analysis The operating point is the volume flow rate at which $H_{\text{required}} = H_{\text{available}}$. We set the given expression for H_{required} to the curve fit expression of Problem 14-29, $H_{\text{available}} = H_0 - a\dot{V}^2$, and obtain

Operating point:

$$\dot{V} = \sqrt{\frac{H_0 - (z_2 - z_1)}{a+b}} = \sqrt{\frac{19.0774\text{ ft} - 15.5\text{ ft}}{(0.032996 + 0.00986)\text{ ft/gpm}^2}} = \mathbf{9.14\ gpm}$$

At this volume flow rate, the net head of the pump is **16.3 ft**.

Discussion At this operating point, the flow rate is lower than that at the BEP.

14-31

Solution We are to calculate pump efficiency and estimate the BEP conditions.

Properties The density of water at 20°C is 998.0 kg/m³.

Analysis (*a*) Pump efficiency is

Pump efficiency:

$$\eta_{\text{pump}} = \frac{\rho g \dot{V} H}{bhp} \qquad (1)$$

We show the second row of data (at $\dot{V}$ = 6.0 Lpm) as an example – the rest are calculated in a spreadsheet for convenience,

$$\eta_{\text{pump}} = \frac{(998.0\text{ kg/m}^3)(9.81\text{ m/s}^2)(6.0\text{ L/min})(46.2\text{ m})}{142\text{ W}}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$
$$\times\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right) = 0.319 = 31.9\%$$

The results for all rows are shown in Table 1.

(*b*) The best efficiency point (BEP) occurs at approximately the fourth row of data: $\dot{V}$* = **18.0 Lpm,** H* = **36.2 m of head,** bhp* = **164. W**, and η_{pump}* = **64.8%**.

Discussion A more precise BEP could be obtained by curve-fitting the data, as in the following exercise.

TABLE 1
Pump performance data for water at 20°C.

$\dot{V}$ (Lpm)	*H* (m)	*bhp* (W)	η_{pump} (%)
0.0	47.5	133	**0.0**
6.0	46.2	142	**31.9**
12.0	42.5	153	**54.4**
18.0	36.2	164	**64.8**
24.0	26.2	172	**59.7**
30.0	15.0	174	**42.2**
36.0	0.0	174	**0.0**

14-32

Solution We are to generate least-squares polynomial curve fits of a pump's performance curves, plot the curves, and calculate the BEP.

Properties The density of water at 20°C is 998.0 kg/m³.

Analysis The efficiencies for each data point in Table P14-31 were calculated in Problem 14-31. We use Regression anaylsis to generate the least-squares fits. The equation and coefficients for H are

$$H = H_0 - a\dot{V}^2 \qquad H_0 = 47.6643\ \text{m} \qquad a = 0.0366453\ \text{m/Lpm}^2$$

Or, to three significant digits, $H_0 = 47.7$ m $\qquad a = 0.0366$ m/Lpm²

The equation and coefficients for *bhp* are

$$bhp = bhp_0 + a_1\dot{V} + a_2\dot{V}^2 \qquad bhp_0 = 131.\ \text{W}$$
$$a_1 = 2.37\ \text{W/Lpm} \qquad a_2 = -0.0317\ \text{W/Lpm}^2$$

The equation and coefficients for η_{pump} are

$$\eta_{\text{pump}} = \eta_{\text{pump},0} + a_1\dot{V} + a_2\dot{V}^2 + a_3\dot{V}^3 \qquad \eta_{\text{pump},0} = 0.152\%$$
$$a_1 = 5.87\ \%/\text{Lpm} \qquad a_2 = -0.0905\ \%/\text{Lpm}^2 \qquad a_3 = -0.00201\ \%/\text{Lpm}^3$$

The tabulated data are plotted in Fig. 1 as symbols only. The fitted data are plotted on the same plots as lines only. The agreement is excellent.

The best efficiency point is obtained by differentiating the curve-fit expression for η_{pump} with respect to volume flow rate, and setting the derivative to zero (solving the resulting quadratic equation for $\dot{V}^*$),

$$\frac{d\eta_{\text{pump}}}{d\dot{V}} = a_1 + 2a_2\dot{V} + 3a_3\dot{V}^2 = 0 \qquad \rightarrow \qquad \dot{V}^* = 19.6\ \text{Lpm}$$

At this volume flow rate, the curve-fitted expressions for H, bhp, and η_{pump} yield the operating conditions at the best efficiency point (to three digits each):

BEP: $\dot{V}^* = 19.6$ Lpm, $\quad H^* = 33.6$ m, $\quad bhp^* = 165.$ W, $\quad \eta^* = 65.3\%$

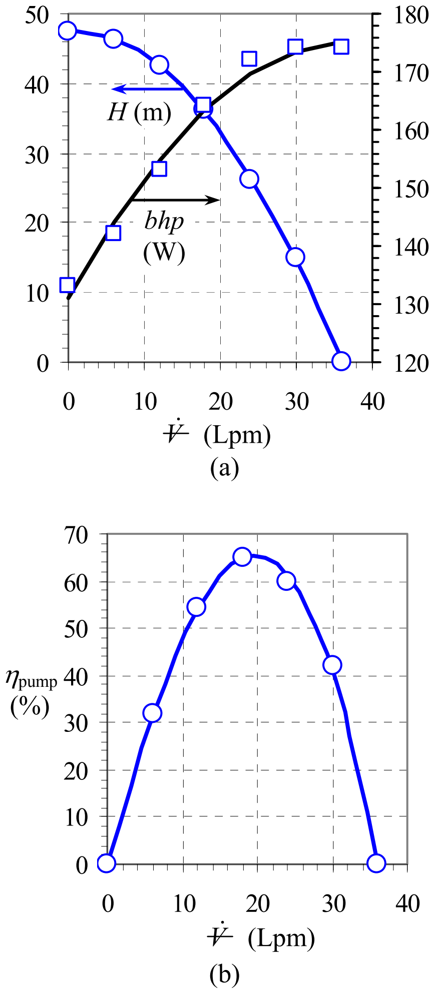


FIGURE 1
Pump performance curves: (a) H and *bhp* versus $\dot{V}$, and (b) η_{pump} versus $\dot{V}$.

Discussion This BEP is more precise than that of Problem 14-31 because of the curve fit. The other root of the quadratic is negative – obviously not the correct choice.

14-33

Solution For a given pump and system requirement, we are to estimate the operating point.

Assumptions **1** The flow is steady. **2** The water is at 20°C and is incompressible.

Analysis The operating point is the volume flow rate at which $H_{required}$ = $H_{available}$. We set the given expression for $H_{required}$ to the curve fit expression of Problem 14-32, $H_{available} = H_0 - a\dot{V}^2$, and obtain

Operating point:

$$\dot{V} = \sqrt{\frac{H_0 - (z_2 - z_1)}{a+b}} = \sqrt{\frac{47.6643\text{ m} - 10.0\text{ m}}{(0.0366453 + 0.0185)\text{ m/Lpm}^2}} = \mathbf{26.1\ Lpm}$$

Discussion At this operating point, the flow rate is higher than that at the BEP.

14-34

Solution We are to perform a regression analysis to estimate the shutoff head and free delivery of a pump, and then we are to determine if this pump is adequate for the system requirements.

Assumptions **1** The water is incompressible. **2** The water is at room temperature.

Properties The density of water at T = 20°C is 998.0 kg/m³.

Analysis (a) We perform a regression analysis, and obtain $\boldsymbol{H_0}$ **= 23.9 m** and $\boldsymbol{a}$ **= 0.00642 m/Lpm²**. The curve fit is reasonable, as seen in Fig. 1. **The shutoff head is estimated as 23.9 m of water column**. At the pump's free delivery, the net head is zero. Setting $H_{available}$ to zero in Eq. 1 gives

Free delivery:

$$\dot{V}_{max}^{\ 2} = \frac{H_0}{a} \quad \rightarrow \quad \dot{V}_{max} = \sqrt{\frac{H_0}{a}} = \sqrt{\frac{23.9\text{ m}}{0.00642\text{ m/(Lpm)}^2}} = 61.0\text{ Lpm}$$

The free delivery is estimated as **61.0 Lpm**.

(b) At the required operating conditions, $\dot{V}$ = 57.0 Lpm, and the net head is converted to meters of water column for analysis,

Required operating head:

$$H_{required} = \frac{(\Delta P)_{required}}{\rho g}$$

$$= \frac{5.8\text{ psi}}{(998.\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{6{,}894.8\text{ N/m}^2}{\text{psi}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) = 4.08\text{ m}$$

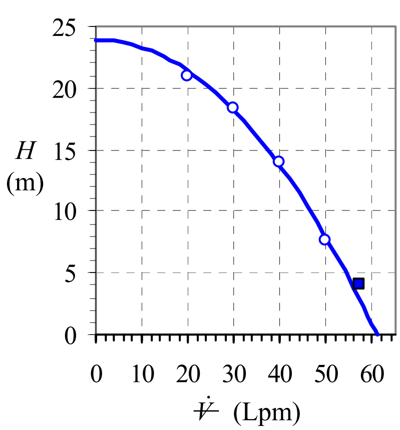


FIGURE 1
Tabulated data (circles) and curve-fitted data (line) for $H_{available}$ versus $\dot{V}$ for the given pump. The filled, square data point is the required operating point.

As seen in Fig. 1, this operating point lies *above* the pump performance curve. Thus, **this pump is not quite adequate for the job at hand**.

Discussion The operating point is also very close to the pump's free delivery, and therefore the pump efficiency would be low even if it *could* put out the required head.

14-35E

Solution We are to find the units of coefficient a, write $\dot{V}_{max}$ in terms of H_0 and a, and calculate the operating point of the pump.

Assumptions **1** The flow is steady. **2** The water is incompressible.

Analysis (a) Solving the given expression for a gives

Coefficient a:
$$a = \frac{H_0 - H_{available}}{\dot{V}^2} \rightarrow \boxed{\text{units of } a = \frac{\text{ft}}{\text{gpm}^2}} \qquad (1)$$

(b) At the pump's free delivery, the net head is zero. Setting $H_{available}$ to zero in the given expression gives

Free delivery:
$$\dot{V}_{max}^{\ 2} = \frac{H_0}{a} \rightarrow \boxed{\dot{V}_{max} = \sqrt{\frac{H_0}{a}}} \qquad (2)$$

(c) The operating point is obtained by matching the pump's performance curve to the system curve. Equating these gives

$$H_{available} = H_0 - a\dot{V}^2 = H_{required} = \left(z_2 - z_1\right) + b\dot{V}^2 \qquad (3)$$

After some algebra, Eq. 3 reduces to

Operating point capacity:
$$\boxed{\dot{V}_{operating} = \sqrt{\frac{H_0 - \left(z_2 - z_1\right)}{a+b}}} \qquad (4)$$

and the net pump head at the operating point is obtained by plugging Eq. 4 into the given expression,

Operating point pump head:
$$\boxed{H_{operating} = \frac{H_0 b + a\left(z_2 - z_1\right)}{a+b}} \qquad (5)$$

Discussion Equation 4 reveals that H_0 must be greater than elevation difference ($z_2 - z_1$) in order to have a valid operating point. This agrees with our intuition, since the pump must be able to overcome the gravitational head between the tanks.

14-36

Solution We are to calculate the operating point of a given pipe/pump system.

Assumptions **1** The water is incompressible. **2** The flow is steady since the reservoirs are large. **3** The water is at room temperature.

Properties The density and viscosity of water at T = 20°C are 998.0 kg/m^3 and 1.002 × 10^{-3} kg/m·s respectively.

Analysis The operating point is obtained by matching the pump's performance curve to the system curve,

Operating point:
$$H_{\text{available}} = H_0 - a\dot{V}^2 = H_{\text{required}} = (z_2 - z_1) + b\dot{V}^2$$

from which we solve for the volume flow rate (capacity) at the operating point,

$$\dot{V}_{\text{operating}} = \sqrt{\frac{H_0 - (z_2 - z_1)}{a+b}} = \sqrt{\frac{5.30\text{ m} - 3.52\text{ m}}{(0.0453 + 0.0261)\text{ m/Lpm}^2}} = \mathbf{4.99\ Lpm}$$

and for the net pump head at the operating point,

$$H_{\text{operating}} = \frac{H_0 b + a(z_2 - z_1)}{a+b} = \frac{(5.30\text{ m})(0.0261\text{ m}) + (0.0453\text{ m})(3.52\text{ m})}{(0.0453\text{ m}) + (0.0261\text{ m})} = \mathbf{4.17\ m}$$

Discussion The water properties ρ and μ are not needed because the system curve (H_{required} versus $\dot{V}$) is provided here.

14-37E

Solution For a given pump and system, we are to calculate the capacity.

Assumptions **1** The water is incompressible. **2** The flow is nearly steady since the reservoirs are large. **3** The water is at room temperature.

Properties The kinematic viscosity of water at $T = 68^{\circ}$F is 1.055×10^{-5} ft^2/s.

Analysis We apply the energy equation in head form between the inlet reservoir's free surface (1) and the outlet reservoir's free surface (2),

$$H_{\text{required}} = h_{\text{pump,u}} = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 \cancel{V_2^2} - \alpha_1 \cancel{V_1^2}}{2g} + (z_2 - z_1) + \cancel{h_{\text{turbine}}} + h_{L,\text{total}} \qquad (1)$$

Since both free surfaces are at atmospheric pressure, $P_1 = P_2 = P_{\text{atm}}$, and the first term on the right side of Eq. 1 vanishes. Furthermore, since there is no flow, $V_1 = V_2 = 0$, and the second term also vanishes. There is no turbine in the control volume, so the second-to-last term is zero. Finally, the irreversible head losses are composed of both major and minor losses, but the pipe diameter is constant throughout. Equation 1 therefore reduces to

$$H_{\text{required}} = (z_2 - z_1) + h_{L,\text{total}} = (z_2 - z_1) + \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (2)$$

The dimensionless roughness factor is $\varepsilon/D = 0.0011/1.20 = 9.17 \times 10^{-4}$, and the sum of all the minor loss coefficients is

$$\sum K_L = 0.5 + 2.0 + 6.8 + (3 \times 0.34) + 1.05 = 11.37$$

The pump/piping system operates at conditions where the available pump head equals the required system head. Thus, we equate the given expression and Eq. 2 to find the operating point,

$$H_{\text{available}} = H_{\text{required}} \quad \rightarrow \quad H_0 - a\frac{\pi^2 D^4}{16}V^2 = (z_2 - z_1) + \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (3)$$

where we have written the volume flow rate in terms of average velocity through the pipe,

Volume flow rate in terms of average velocity: $$\dot{V} = V\frac{\pi D^2}{4} \qquad (4)$$

Equation 3 is an implicit equation for V since the Darcy friction factor f is a function of Reynolds number Re $= \rho VD/\mu = VD/\nu$, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is $V = 1.80$ ft/s, from which the volume flow rate is $\dot{V}$ = **6.34 gpm**. The Reynolds number is 1.67×10^4.

Discussion We verify our results by comparing $H_{\text{available}}$ (given) and H_{required} (Eq. 2) at this flow rate: $H_{\text{available}} = 24.4$ ft and $H_{\text{required}} = 24.4$ ft.

14-38E

Solution We are to plot $H_{required}$ and $H_{available}$ versus $\dot{V}$, and indicate the operating point.

Analysis We use the equations of Problem 14-37, with the same constants and parameters, to generate the plot (Fig. 1). The operating point is the location where the two curves intersect. The values of H and $\dot{V}$ at the operating point match those of Problem 14-37, as they should.

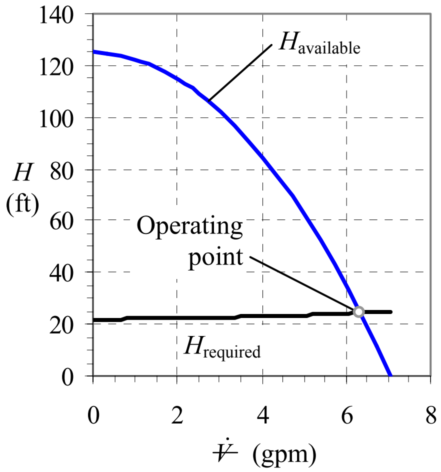


FIGURE 1

$H_{available}$ and $H_{required}$ versus $\dot{V}$ for a piping system with pump; the operating point is also indicated, where the two curves meet.

Discussion A plot like this, in fact, is an alternate method of obtaining the operating point. In this case, the curve of $H_{required}$ is fairly flat, indicating that the majority of the required pump head is attributed to elevation change, while a small fraction is attributed to major and minor head losses through the piping system.

14-39E

Solution We are to re-calculate volume flow rate for a piping system with a much longer pipe, and we are to compare with the previous results.

Analysis All assumptions, properties, dimensions, and parameters are identical to those of Problem 14-38, except that total pipe length L is longer. We repeat the calculations and find that V = 1.68 ft/s, from which the volume flow rate is $\dot{V}$ = **5.93 gpm**, and the net head of the pump is 37.0 ft. The Reynolds number for the flow in the pipe is 1.56×10^4. **The volume flow rate has decreased by about 6.5%**.

Discussion The decrease in volume flow rate is smaller than we may have suspected. This is because the majority of the pump work goes into raising the elevation of the water. In addition, as seen in Fig. 1 of Problem 14-38, the pump performance curve is quite steep near these flow rates – a significant change in required net head leads to a much less significant change in volume flow rate.

14-40E

Solution We are to perform a regression analysis to translate tabulated pump performance data into an analytical expression, and then use this expression to predict the volume flow rate through a piping system.

Assumptions **1** The water is incompressible. **2** The flow is nearly steady since the reservoirs are large. **3** The water is at room temperature.

Properties For water at $T = 68^\circ$F, $\mu = 6.572 \times 10^{-4}$ lbm/ft·s, and $\rho = 62.31$ lbm/ft^3, from which $\nu = 1.055 \times 10^{-5}$ ft^2/s.

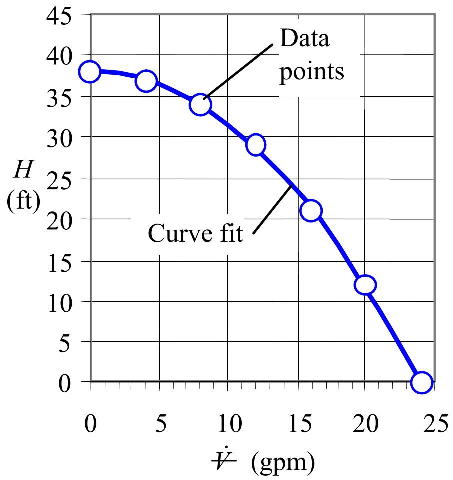


FIGURE 1
Tabulated data (symbols) and curve-fitted data (line) for $H_{available}$ versus $\dot{V}$ for the proposed pump.

Analysis (*a*) We perform a regression analysis, and obtain $\boldsymbol{H_0}$ **= 38.15 ft** and ***a*** **= 0.06599 ft/gpm^2**. The curve fit is very good, as seen in Fig. 1.

(*b*) We repeat the calculations of Problem 14-37 with the new pump performance coefficients, and find that $V = 3.29$ ft/s, from which the volume flow rate is $\dot{V}$ = **11.6 gpm**, and the net head of the pump is 29.3 ft. The Reynolds number for the flow in the pipe is 3.05×10^4. **The volume flow rate has increased by about 83%**. **Paul is correct** – this pump performs much better, nearly doubling the flow rate.

(*c*) A plot of net head versus volume flow rate is shown in Fig. 2.

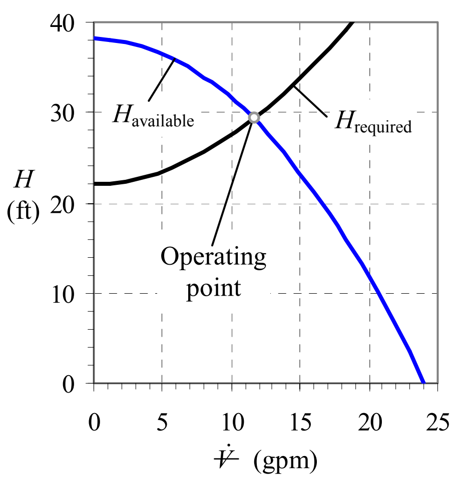


FIGURE 2
$H_{available}$ and $H_{required}$ versus $\dot{V}$ for a piping system with pump; the operating point is also indicated, where the two curves meet.

Discussion This pump is more appropriate for the piping system at hand.

14-41

Solution For a given pump and system, we are to calculate the capacity.

Assumptions **1** The water is incompressible. **2** The flow is nearly steady since the reservoirs are large. **3** The water is at room temperature.

Properties The density and viscosity of water at T = 20°C are 998.0 kg/m^3 and 1.002 × 10^{-3} kg/m·s respectively.

Analysis We apply the energy equation in head form between the inlet reservoir's free surface (1) and the outlet reservoir's free surface (2),

$$H_{\text{required}} = h_{\text{pump,u}} = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 \cancel{V_2^2} - \alpha_1 \cancel{V_1^2}}{2g} + (z_2 - z_1) + \cancel{h_{\text{turbine}}} + h_{L,\text{total}} \quad (1)$$

Since both free surfaces are at atmospheric pressure, $P_1 = P_2 = P_{\text{atm}}$, and the first term on the right side of Eq. 1 vanishes. Furthermore, since there is no flow, $V_1 = V_2 = 0$, and the second term also vanishes. There is no turbine in the control volume, so the second-to-last term is zero. Finally, the irreversible head losses are composed of both major and minor losses, but the pipe diameter is constant throughout. Equation 1 therefore reduces to

$$H_{\text{required}} = (z_2 - z_1) + h_{L,\text{total}} = (z_2 - z_1) + \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \quad (2)$$

The dimensionless roughness factor is

$$\frac{\varepsilon}{D} = \frac{0.25 \text{ mm}}{2.03 \text{ cm}}\left(\frac{1 \text{ cm}}{10 \text{ mm}}\right) = 0.0123$$

The sum of all the minor loss coefficients is

$$\sum K_L = 0.5 + 17.5 + (5 \times 0.92) + 1.05 = 23.65$$

The pump/piping system operates at conditions where the available pump head equals the required system head. Thus, we equate the given expression and Eq. 2 to find the operating point,

Operating point:

$$H_{\text{available}} = H_{\text{required}} \quad \rightarrow \quad H_0 - a\frac{\pi^2 D^4}{16}V^2 = (z_2 - z_1) + \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \quad (3)$$

where we have written the volume flow rate in terms of average velocity through the pipe,

$$\dot{V} = V\frac{\pi D^2}{4}$$

Equation 3 is an implicit equation for V since the Darcy friction factor f is a function of Reynolds number Re = $\rho VD/\mu$, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is V = 0.59603 ≈ 0.596

m/s, from which the volume flow rate is $\dot{V}$ = **11.6 Lpm**. The Reynolds number is 1.21×10^4.

Discussion We verify our results by comparing $H_{available}$ (given) and $H_{required}$ (Eq. 2) at this flow rate: $H_{available}$ = 15.3 m and $H_{required}$ = 15.3 m.

14-42

Solution We are to plot $H_{required}$ and $H_{available}$ versus $\dot{V}$, and indicate the operating point.

Analysis We use the equations of Problem 14-41, with the same constants and parameters, to generate the plot (Fig. 1). The operating point is the location where the two curves intersect. The values of H and $\dot{V}$ at the operating point match those of Problem 14-41, as they should.

Discussion A plot like this, in fact, is an alternate method of obtaining the operating point.

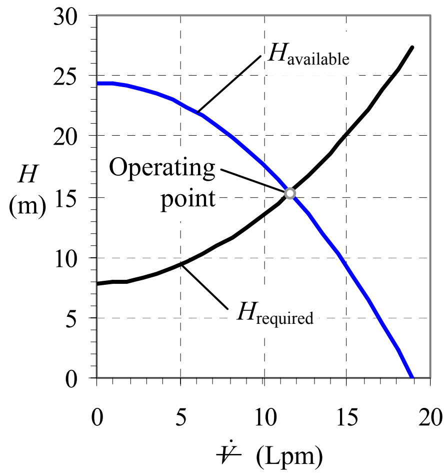


FIGURE 1
$H_{available}$ and $H_{required}$ versus $\dot{V}$ for a piping system with pump; the operating point is also indicated, where the two curves meet.

14-43

Solution We are to re-calculate volume flow rate for a piping system with a smaller elevation difference, and we are to compare with the previous results.

Analysis All assumptions, properties, dimensions, and parameters are identical to those of Problem 14-42, except that the elevation difference between reservoir surfaces ($z_2 - z_1$) is smaller. We repeat the calculations and find that V = 0.682 m/s, from which the volume flow rate is $\dot{V}$ = **13.2 Lpm**, and the net head of the pump is 12.5 m. The Reynolds number for the flow in the pipe is 1.38×10^4. **The volume flow rate has increased by about 14%**.

Discussion The increase in volume flow rate is modest. This is because only about half of the pump work goes into raising the elevation of the water – the other half goes into overcoming irreversible losses.

14-44

Solution We are to perform a regression analysis to translate tabulated pump performance data into an analytical expression, and then use this expression to predict the volume flow rate through a piping system.

Assumptions **1** The water is incompressible. **2** The flow is nearly steady since the reservoirs are large. **3** The water is at room temperature.

Properties The density and viscosity of water at T = 20°C are 998.0 kg/m^3 and 1.002 × 10^{-3} kg/m·s respectively.

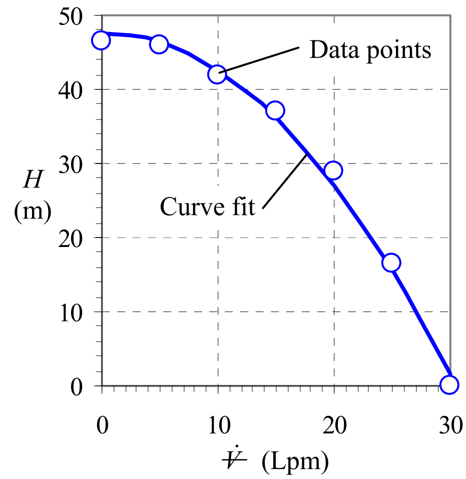


FIGURE 1
Tabulated data (symbols) and curve-fitted data (line) for $H_{\text{available}}$ versus $\dot{V}$ for the proposed pump.

Analysis (*a*) We perform a regression analysis, and obtain $\boldsymbol{H_0}$ **= 47.6 m** and $\boldsymbol{a}$ **= 0.05119 m/Lpm^2**. The curve fit is reasonable, as seen in Fig. 1.

(*b*) We repeat the calculations of Problem 14-41 with the new pump performance coefficients, and find that V = 1.00 m/s, from which the volume flow rate is $\dot{V}$ = **19.5 Lpm**, and the net head of the pump is 28.3 m. The Reynolds number for the flow in the pipe is 2.03 × 10^4. **The volume flow rate has increased by about 69%. April's goal has *not* been reached**. She will need to search for an even stronger pump.

(*c*) A plot of net head versus volume flow rate is shown in Fig. 2.

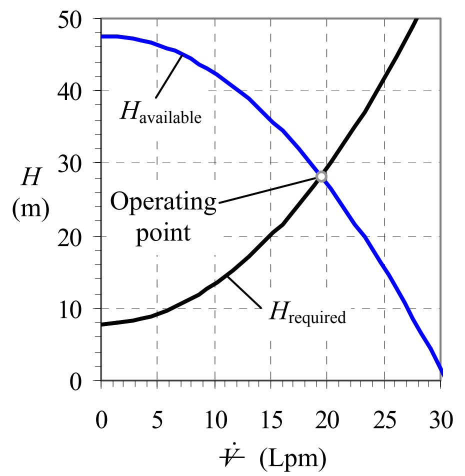


FIGURE 2
$H_{\text{available}}$ and H_{required} versus $\dot{V}$ for a piping system with pump; the operating point is also indicated, where the two curves meet.

Discussion As is apparent from Fig. 2, the required net head increases rapidly with increasing volume flow rate. Thus, doubling the flow rate would require a significantly heftier pump.

14-45

Solution We are to calculate the volume flow rate when the pipe diameter of a piping/pump system is doubled.

Analysis The analysis is identical to that of Problem 14-41 except for the diameter change. The calculations yield V = 0.19869 ≈ 0.199 m/s, from which the volume flow rate is $\dot{V}$ = **15.4 Lpm**, and the net head of the pump is 8.25 m. The Reynolds number for the flow in the pipe is 8.03 × 10^3. **The volume flow rate has increased by about 33%**. This agrees with our intuition since irreversible head losses go down significantly by increasing pipe diameter.

Discussion The gain in volume flow rate is significant because the irreversible head losses contribute to about half of the total pump head requirement in the original problem.

14-46

Solution We are to compare Reynolds numbers for a pipe flow system – the second case having a pipe diameter twice that of the first case.

Properties The density and viscosity of water at T = 20°C are 998.0 kg/m^3 and 1.002 × 10^{-3} kg/m·s respectively.

Analysis From the results of the two problems, the Reynolds number of the first case is

Case 1 *(D* = 2.03 cm*)*:

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\left(998\ \mathrm{kg/m^3}\right)\left(0.59603\ \mathrm{m/s}\right)\left(0.0203\ \mathrm{m}\right)}{1.002\times10^{-3}\ \mathrm{kg/m^3}} = 1.21\times10^4$$

and that of the second case is

Case 2 *(D* = 4.06 cm*)*:

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\left(998\ \mathrm{kg/m^3}\right)\left(0.19869\ \mathrm{m/s}\right)\left(0.0406\ \mathrm{m}\right)}{1.002\times10^{-3}\ \mathrm{kg/m^3}} = 0.803\times10^4$$

Thus, **the Reynolds number of the larger diameter pipe is *smaller* than that of the smaller diameter pipe**. This may be somewhat surprising, but since average pipe velocity scales as the inverse of pipe diameter squared, Reynolds number increases linearly with pipe diameter due to the D in the numerator, but decreases quadratically with pipe diameter due to the V in the numerator. The net effect is a decrease in Re with pipe diameter when $\dot{V}$ is the same. In this problem, $\dot{V}$ increases somewhat as the diameter is doubled, but not enough to increase the Reynolds number.

Discussion At first glance, most people would think that Reynolds number increases as both diameter and volume flow rate increase, but this is not always the case.

14-47

Solution We are to compare the volume flow rate in a piping system with and without accounting for minor losses.

Analysis The analysis is identical to that of Problem 14-41, except we ignore all the minor losses. The calculations yield V = 0.604 m/s, from which the volume flow rate is $\dot{V}$ = **11.7 Lpm**, and the net head of the pump is 15.1 m. The Reynolds number for the flow in the pipe is 1.22 × 10^4. **The volume flow rate has increased by about 1.3%**. Thus, **minor losses are nearly negligible in this calculation**. This agrees with our intuition since the pipe is very long.

Discussion Since the Colebrook equation is accurate to at most 5%, a 1.3% change is well within the error. Nevertheless, it is not excessively difficult to include the minor losses, especially when solving the problem on a computer.

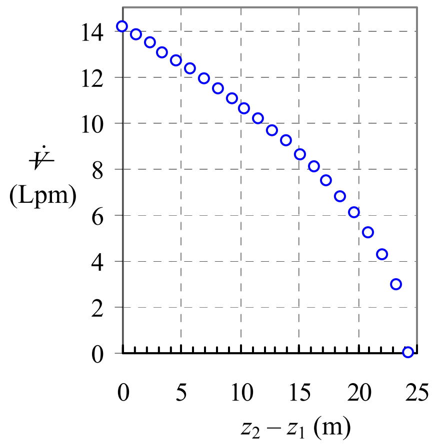


FIGURE 1
The dependence of volume flow rate $\dot{V}$ on elevation difference $z_2 - z_1$.

14-48

Solution We are to examine how increasing ($z_2 - z_1$) affects the volume flow rate of water pumped by the water pump.

Assumptions **1** The flow at any instant of time is still considered quasi-steady, since the surface level of the upper reservoir rises very slowly. **2** The minor losses, dimensions, etc., fluid properties, and all other assumptions are identical to those of Problem 14-41 except for the elevation difference ($z_2 - z_1$).

Analysis We repeat the calculations of Problem 14-41 for several values of ($z_2 - z_1$), ranging from 0 to H_0, the shutoff head of the pump, since above the shutoff head, the pump cannot overcome the elevation difference. **The volume flow rate is zero at the shutoff head of the pump**. The data are plotted in Fig. 1. As expected, the volume flow rate decreases as ($z_2 - z_1$) increases, starting at a maximum flow rate of about 14.1 Lpm when there is no elevation difference, and reaching zero (no flow) when ($z_2 - z_1$) = H_0 = 24.4 m. **The curve is *not* linear**, since neither the Darcy friction factor nor the pump performance curve are linear. If ($z_2 - z_1$) were increased beyond H_0, the pump would not be able to handle the elevation difference. Despite its valiant efforts, with blades spinning as hard as they could, **the water would flow backwards through the pump**.

Discussion You may wish to think of the backward-flow through the pump as a case in which the pump efficiency is *negative*. In fact, at ($z_2 - z_1$) = H_0, the pump could be replaced by a closed valve to keep the water from draining from the upper reservoir to the lower reservoir.

14-49E

Solution We are to estimate the operating point of a given fan and duct system.

Assumptions **1** The flow is steady. **2** The concentration of contaminants in the air is low; the fluid properties are those of air alone. **3** The air is at standard temperature and pressure (STP), and is incompressible. **4** The air flowing in the duct is turbulent with α = 1.05.

Properties For air at STP (T = 77°F, P = 14.696 psi = 2116.2 lbf/ft^2), $\mu = 1.242 \times 10^{-5}$ lbm/ft·s, ρ = 0.07392 lbm/ft^3, and $\nu = 1.681 \times 10^{-4}$ ft^2/s. The density of water at STP (for conversion to inches of water head) is 62.24 lbm/ft^3.

Analysis We apply the steady energy equation along a streamline from point 1 in the stagnant air region in the room to point 2 at the duct outlet,

Required net head:
$$H_{\text{required}} = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 V_2^2 - \alpha_1 \cancel{V_1^2}}{2g} + \cancel{(z_2 - z_1)} + h_{L,\text{total}} \qquad (1)$$

At point 1, P_1 is equal to P_{atm}, and at point 2, P_2 is also equal to P_{atm} since the jet discharges into the outside air on the roof of the building. Thus the pressure terms cancel out in Eq. 1. We ignore the air speed at point 1 since it is chosen (wisely) far enough away from the hood inlet so that the air is nearly stagnant. Finally, the elevation difference is neglected for gases. Equation 1 reduces to

$$H_{\text{required}} = \frac{\alpha_2 V_2^{\,2}}{2g} + h_{L,\text{total}} \qquad (2)$$

The total head loss in Eq. 2 is a combination of major and minor losses, and depends on volume flow rate. Since the duct diameter is constant,

Total irreversible head loss:

$$h_{L,\text{total}} = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (3)$$

The required net head of the fan is thus

$$H_{\text{required}} = \left(\alpha_2 + f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (4)$$

To find the operating point, we equate $H_{\text{available}}$ and H_{required}, being careful to keep consistent units. Note that the required head in Eq. 4 is expressed naturally in units of equivalent column height of the pumped fluid, which is air in this case. However, the available net head (given) is in terms of equivalent water column height. We convert constants H_0 and a to inches of *air* column for consistency by multiplying by the ratio of water density to air density,

$$H_{0,\text{ inch water}}\rho_{\text{water}} = H_{0,\text{ inch air}}\rho_{\text{air}} \qquad \rightarrow \qquad H_{0,\text{ inch air}} = H_{0,\text{ inch water}}\frac{\rho_{\text{water}}}{\rho_{\text{air}}}$$

and similarly,

$$a_{(\text{inch air})/\text{SCFM}^2} = a_{(\text{inch water})/\text{SCFM}^2}\frac{\rho_{\text{water}}}{\rho_{\text{air}}}$$

We re-write the given expression in terms of average duct velocity rather than volume flow rate,

Available net head:

$$H_{\text{available}} = H_0 - a\frac{\pi^2 D^4}{16}V^2 \qquad (5)$$

again taking care to keep consistent units. Equating Eqs. 4 and 5 yields

Operating point:

$$H_{\text{available}} = H_{\text{required}} \qquad \rightarrow \qquad H_0 - a\frac{\pi^2 D^4}{16}V^2 = \left(\alpha_2 + f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (6)$$

The dimensionless roughness factor is ε/D = 0.0059/9.06 = 6.52 × 10^{-4}, and the sum of all the minor loss coefficients is

Minor losses:

$$\sum K_L = 4.6 + (3\times 0.21) + 1.8 = 7.03$$

Note that there is no minor loss associated with the exhaust, since point 2 is taken at the exit plane of the duct, and does not include irreversible losses associated with the turbulent jet. Equation 6 is an implicit equation for V since the Darcy friction factor is a function of Reynolds number Re = $\rho VD/\mu$ = VD/ν, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is V = 16.8 ft/s, from which the volume flow rate is $\dot{V}$ = **452 SCFM**. The Reynolds number is 7.63 × 10^4.

Discussion We verify our results by comparing $H_{\text{available}}$ (Eq. 1) and H_{required} (Eq. 5) at this flow rate: $H_{\text{available}}$ = 0.566 inches of water and H_{required} = 0.566 inches of water.

14-50E

Solution We are to calculate the value of $K_{L,\text{ damper}}$ such that the volume flow rate through the duct decreases by 50%.

Analysis All assumptions and properties are the same as those of Problem 14-49. We set the volume flow rate to $\dot{V}$ = 226 SCFM, one-half of the previous result, and solve for $K_{L,\text{ damper}}$. The result is $\boldsymbol{K_{L,\text{ damper}} = 112}$, significantly higher than the value of 1.8 for the fully open case.

Discussion Because of the nonlinearity of the problem, we cannot simply double the damper's loss coefficient in order to decrease the flow rate by a factor of two. Indeed, the minor loss coefficient must be increased by a factor of more than 60. If a computer was used for the calculations of Problem 14-49, the solution here is most easily obtained by trial and error.

14-51E

Solution We are to estimate the volume flow rate at the operating point without accounting for minor losses, and then we are to compare with the previous results.

Analysis All assumptions and properties are the same as those of Problem 14-49, except that we ignore all minor losses (we set $\Sigma K_L = 0$). The resulting volume flow rate at the operating point is $\boldsymbol{\dot{V}}$ **= 503 SCFM**, approximately 11% higher than for the case with minor losses taken into account. In this problem, minor losses are indeed "minor", although they are not negligible. We should not be surprised at this result, since there are several minor losses, and the duct is not extremely long (L/D is only 45.0).

Discussion An error of 11% may be acceptable in this type of problem. However, since it is not difficult to account for minor losses, especially if the calculations are performed on a computer, it is wise not to ignore these terms.

14-52

Solution We are to estimate the operating point of a given fan and duct system.

Assumptions **1** The flow is steady. **2** The concentration of contaminants in the air is low; the fluid properties are those of air alone. **3** The air is at 25°C and 101,300 Pa, and is incompressible. **4** The air flowing in the duct is turbulent with $\alpha = 1.05$.

Properties For air at 25°C, μ = 1.849 × 10^{-5} kg/m·s, ρ = 1.184 kg/m^3, and ν = 1.562 × 10^{-5} m^2/s. The density of water at STP (for conversion to water head) is 997.0 kg/m^3.

Analysis We apply the steady energy equation along a streamline from point 1 in the stagnant air region in the room to point 2 at the duct outlet,

$$H_{\text{required}} = \cancel{\frac{P_2 - P_1}{\rho g}} + \frac{\alpha_2 V_2^{\,2} - \cancel{\alpha_1 V_1^{\,2}}}{2g} + \cancel{(z_2 - z_1)} + h_{L,\text{total}} \qquad (1)$$

At point 1, P_1 is equal to P_{atm}, and at point 2, P_2 is also equal to P_{atm} since the jet discharges into the outside air on the roof of the building. Thus the pressure terms cancel out in Eq. 1. We ignore the air speed at point 1 since it is chosen (wisely) far enough away from the hood inlet so that the air is nearly stagnant. Finally, the elevation difference is neglected for gases. Equation 1 reduces to

Required net head:
$$H_{\text{required}} = \frac{\alpha_2 V_2^{\,2}}{2g} + h_{L,\text{total}} \qquad (2)$$

The total head loss in Eq. 2 is a combination of major and minor losses, and depends on volume flow rate. Since the duct diameter is constant,

Total irreversible head loss:
$$h_{L,\text{total}} = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (3)$$

The required net head of the fan is thus

$$H_{\text{required}} = \left(\alpha_2 + f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g} \qquad (4)$$

To find the operating point, we equate $H_{\text{available}}$ and H_{required}, being careful to keep consistent units. Note that the required head in Eq. 4 is expressed naturally in units of equivalent column height of the pumped fluid, which is air in this case. However, the available net head (given) is in terms of equivalent water column height. We convert constants H_0 and a in Eq. 1 to mm of *air* column for consistency by multiplying by the ratio of water density to air density,

$$H_{0,\text{ mm water}}\,\rho_{\text{water}} = H_{0,\text{ mm air}}\,\rho_{\text{air}} \qquad \rightarrow \qquad H_{0,\text{ mm air}} = H_{0,\text{ mm water}}\frac{\rho_{\text{water}}}{\rho_{\text{air}}}$$

and similarly,

$$a_{(\text{mm air})/\text{LPM}^2} = a_{(\text{mm water})/\text{LPM}^2}\frac{\rho_{\text{water}}}{\rho_{\text{air}}}$$

We re-write the given expression in terms of average duct velocity rather than volume flow rate,

Available net head:
$$H_{\text{available}} = H_0 - a\frac{\pi^2 D^4}{16}V^2 \qquad (5)$$

Equating Eqs. 4 and 5 yields

Operating point:

$$H_{\text{available}} = H_{\text{required}} \qquad \rightarrow \qquad H_0 - a\frac{\pi^2 D^4}{16}\dot{V}^2 = \left(\alpha_2 + f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \tag{6}$$

The dimensionless roughness factor is $\varepsilon/D = 0.15/150 = 1.00 \times 10^{-3}$, and the sum of all the minor loss coefficients is

Minor losses:
$$\sum K_L = 3.3 + (3\times 0.21) + 1.8 + 0.36 + 6.6 = 12.69$$

Note that there is no minor loss associated with the exhaust, since point 2 is taken at the exit plane of the duct, and does not include irreversible losses associated with the turbulent jet. Equation 6 is an implicit equation for V since the Darcy friction factor is a function of Reynolds number Re = $\rho VD/\mu = VD/\nu$, as obtained from either the Moody chart or the Colebrook equation. The solution can be obtained by an iterative method, or through use of a mathematical equation solver like EES. The result is V = 6.71 m/s, from which the volume flow rate is $\dot{V}$ = **7090 Lpm**.

Discussion We verify our results by comparing $H_{\text{available}}$ (given) and H_{required} (Eq. 5) at this flow rate: $H_{\text{available}}$ = 47.4 mm of water and H_{required} = 47.4 mm of water, both of which are equivalent to 40.0 m of air column.

14-53

Solution We are to plot H_{required} and $H_{\text{available}}$ versus $\dot{V}$, and indicate the operating point.

Analysis We use the equations of Problem 14-52, with the same constants and parameters, to generate the plot (Fig. 1). The operating point is the location where the two curves intersect. The values of H and $\dot{V}$ at the operating point match those of Problem 14-52, as they should.

Discussion A plot like this, in fact, is an alternate method of obtaining the operating point. The operating point is at a volume flow rate near the center of the plot, indicating that the fan efficiency is probably reasonably high.

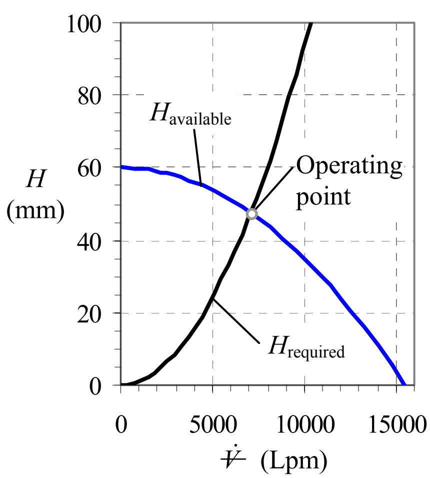


FIGURE 1
$H_{\text{available}}$ and H_{required} versus $\dot{V}$ for a duct system with a fan; the operating point is also indicated, where the two curves meet.

14-54

Solution We are to estimate the volume flow rate at the operating point without accounting for minor losses, and then we are to compare with the previous results.

Analysis All assumptions and properties are the same as those of Problem 14-52, except that we ignore all minor losses (we set $\Sigma K_L = 0$). The resulting volume flow rate at the operating point is $\dot{V}$ **= 10,900 Lpm** (to three significant digits), approximately 54% higher than for the case with minor losses taken into account. In this problem, minor losses are not "minor", and are by no means negligible. Even though the duct is fairly long (L/D is about 163), the minor losses are large, especially those through the damper and the one-way valve.

Discussion An error of 54% is not acceptable in this type of problem. Furthermore, since it is not difficult to account for minor losses, especially if the calculations are performed on a computer, it is wise not to ignore these terms.

14-55

Solution We are to calculate pressure at two locations in a blocked duct system.

Assumptions **1** The flow is steady. **2** The concentration of contaminants in the air is low; the fluid properties are those of air alone. **3** The air is at standard temperature and pressure (STP: 25°C and 101,300 Pa), and is incompressible.

Properties The density of water at 25°C is 997.0 kg/m³.

Analysis Since the air is completely blocked by the one-way valve, there is no flow. Thus, there are no major or minor losses – just a pressure gain across the fan. Furthermore, the fan is operating at its shutoff head conditions. Since the pressure in the room is atmospheric, the gage pressure anywhere in the stagnant air region in the duct between the fan and the one-way valve is therefore equal to H_0 = 60.0 mm of water column. We convert to pascals as follows:

Gage pressure at both locations:

$$P_{\text{gage}} = \rho_{\text{water}} g H_0 = \left(998.0\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.060\ \text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\right) = 587\ \text{Pa}$$

Thus, at either location, the gage pressure is **60.0 mm of water column, or 587 Pa**.

Discussion The answer depends only on the shutoff head of the fan – duct diameter, minor losses, etc, are irrelevant for this case since there is no flow. The fan should not be run for long time periods under these conditions, or it may burn out.

14-56E

Solution For a given pump and piping system we are to estimate the maximum volume flow rate that can be pumped without cavitation.

Assumptions **1** The flow is steady. **2** The water is incompressible. **3** The flow is turbulent.

Properties P_{atm} = 14.696 psi = 2116.2 lbf/ft^2. For water at T = 77^{o}F, μ = 6.002 × 10^{-4} lbm/(ft·s), ρ = 62.24 lbm/ft^3, and P_v = 66.19 lbf/ft^2.

Analysis We apply the steady energy equation in head form along a streamline from point 1 at the reservoir surface to point 2 at the pump inlet,

$$\frac{P_1}{\rho g}+\cancel{\frac{\alpha_1 V_1^2}{2g}}+z_1+\cancel{h_{\text{pump,u}}}=\frac{P_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+\cancel{h_{\text{turbine}}}+h_{L,\text{total}} \quad (1)$$

In Eq. 1 we have ignored the water speed at the reservoir surface ($V_1 \approx 0$). There is no turbine in the piping system. Also, although there is a pump in the system, there is no pump between point 1 and point 2; hence the pump head term also drops out. We solve Eq. 1 for $P_2/(\rho g)$, which is the pump inlet pressure expressed as a head,

Pump inlet pressure head:
$$\frac{P_2}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+\left(z_1-z_2\right)-\frac{\alpha_2 V_2^2}{2g}-h_{L,\text{total}} \quad (2)$$

Note that in Eq. 2, we have recognized that $P_1 = P_{atm}$ since the reservoir surface is exposed to atmospheric pressure.

The available net positive suction head at the pump inlet is obtained from Eq. 14-8. After substitution of Eq. 2, approximating $\alpha_2 \approx 1$, we get

Available NPSH:
$$NPSH=\frac{P_{\text{atm}}-P_v}{\rho g}+\left(z_1-z_2\right)-h_{L,\text{total}} \quad (3)$$

Since we know P_{atm}, P_v, and the elevation difference, all that remains is to estimate the total irreversible head loss through the piping system from the reservoir surface (1) to the pipe inlet (2), which depends on volume flow rate. Since the pipe diameter is constant,

Total irreversible head loss:
$$h_{L,\text{total}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \quad (4)$$

The rest of the problem is most easily solved on a computer. For a given volume flow rate, we calculate speed V and Reynolds number Re. From Re and the known pipe roughness, we use the Moody chart (or the Colebrook equation) to obtain friction factor f. The sum of all the minor loss coefficients is

Minor losses:
$$\sum K_L = 0.5+0.3+6.0=6.8 \quad (5)$$

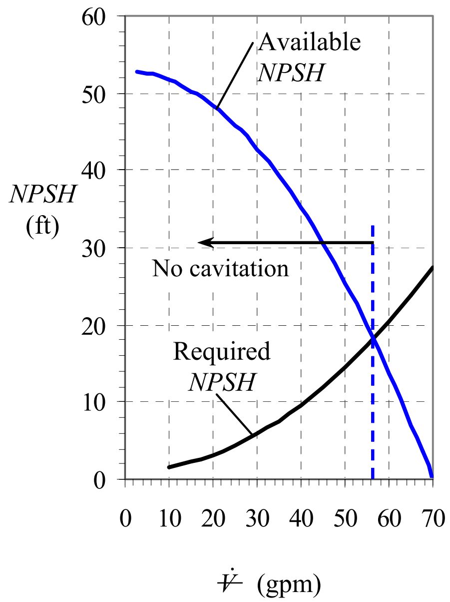


FIGURE 1
Net positive suction head as a function of volume flow rate; cavitation is predicted to occur at flow rates greater than the point where the available and required values of *NPSH* intersect.

where we have not included the minor losses downstream of the pump, since they are irrelevant to the present analysis.

We make one calculation by hand for illustrative purposes. At $\dot{V}$ = 40.0 gpm, the average speed of water through the pipe is

$$V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(40.0 \text{ gal/min})}{\pi(1.2 \text{ in})^2}\left(\frac{231 \text{ in}^3}{\text{gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 11.35 \text{ ft/s} \qquad (6)$$

which produces a Reynolds number of Re = $\rho VD/\mu$ = 1.17 × 10^5. At this Reynolds number, and with roughness factor ε/D = 0, the Colebrook equation yields f = 0.0174. After substitution of the given variables along with f, D, L, and Eqs. 4, 5, and 6 into Eq. 3, we get

$$NPSH = \frac{(2116.2 - 66.19) \text{ lbf/ft}^2}{(62.24 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)}\left(\frac{32.174 \text{ lbm ft}}{\text{s}^2\text{lbf}}\right) + 20.0 \text{ ft}$$
$$-\left(0.0174\frac{12.0 \text{ ft}}{0.10 \text{ ft}} + 6.8\right)\frac{(11.35 \text{ ft/s})^2}{2(32.174 \text{ ft/s}^2)} = 35.1 \text{ ft} \qquad (7)$$

The required net positive suction head is obtained from the given expression. At our example flow rate of 40.0 gpm we see that $NPSH_{\text{required}}$ is about 9.6 ft. Since the actual *NPSH* is much higher than this, we need not worry about cavitation at this flow rate. We use a spreadsheet to calculate *NPSH* as a function of volume flow rate, and the results are plotted in Fig. 1. It is clear from this plot that **cavitation occurs at flow rates above about 56 gallons per minute**.

Discussion Note that $NPSH_{\text{required}}$ rises with volume flow rate, but the actual or available *NPSH* decreases with volume flow rate (Fig. 1).

14-57E

Solution We are to calculate the volume flow rate below which cavitation in a pump is avoided.

Assumptions **1** The flow is steady. **2** The water is incompressible.

Properties P_{atm} = 14.696 psi = 2116.2 lbf/ft^2. For water at T = 150°F, μ = 2.889 × 10^{-4} lbm/ft·s, ρ = 61.19 lbm/ft^3, and P_v = 536.0 lbf/ft^2.

Analysis The procedure is identical to that of Problem 14-57, except for the water properties. The calculations predict that **the pump cavitates at volume flow rates greater than about 53 gpm**. This is somewhat lower than the result of Problem 14-56, as expected, since cavitation occurs more readily in warmer water.

Discussion Note that $NPSH_{\text{required}}$ does not depend on water temperature, but the actual or available *NPSH* decreases with temperature.

14-58

Solution For a given pump and piping system we are to estimate the maximum volume flow rate that can be pumped without cavitation.

Assumptions **1** The flow is steady. **2** The water is incompressible. **3** The flow is turbulent.

Properties Standard atmospheric pressure is P_{atm} = 101.3 kPa. For water at T = 25°C, ρ = 997.0 kg/m^3, $\mu = 8.91\times10^{-4}$ kg/m·s, and P_v = 3.169 kPa.

Analysis We apply the steady energy equation in head form along a streamline from point 1 at the reservoir surface to point 2 at the pump inlet,

Energy equation:

$$\frac{P_1}{\rho g}+\cancel{\frac{\alpha_1 V_1^2}{2g}}+z_1+\cancel{h_{\text{pump,u}}}=\frac{P_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+\cancel{h_{\text{turbine}}}+h_{L,\text{total}} \qquad (1)$$

In Eq. 1 we have ignored the water speed at the reservoir surface ($V_1 \approx 0$). There is no turbine in the piping system. Also, although there is a pump in the system, there is no pump between point 1 and point 2; hence the pump head term also drops out. We solve Eq. 1 for $P_2/(\rho g)$, which is the pump inlet pressure expressed as a head,

Pump inlet pressure head:

$$\frac{P_2}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+\left(z_1-z_2\right)-\frac{\alpha_2 V_2^2}{2g}-h_{L,\text{total}} \qquad (2)$$

Note that in Eq. 2, we have recognized that $P_1 = P_{\text{atm}}$ since the reservoir surface is exposed to atmospheric pressure.

The available net positive suction head at the pump inlet is obtained from Eq. 14-8. After substitution of Eq. 2, and approximating α_2 as 1.0, we get

Available NPSH:

$$NPSH=\frac{P_{\text{atm}}-P_v}{\rho g}+\left(z_1-z_2\right)-h_{L,\text{total}} \qquad (3)$$

Since we know P_{atm}, P_v, and the elevation difference, all that remains is to estimate the total irreversible head loss through the piping system from the reservoir surface (1) to the pipe inlet (2), which depends on volume flow rate. Since the pipe diameter is constant,

Total irreversible head loss:

$$h_{L,\text{total}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \qquad (4)$$

The rest of the problem is most easily solved on a computer. For a given volume flow rate, we calculate speed V and Reynolds number Re. From Re and the known pipe roughness, we use the Moody chart (or the Colebrook equation) to obtain friction factor f. The sum of all the minor loss coefficients is

Sum of minor loss coefficients:

$$\sum K_L = 0.85+0.3=1.15 \qquad (5)$$

where we have not included the minor losses downstream of the pump, since they are irrelevant to the present analysis.

We make one calculation by hand for illustrative purposes. At $\dot{V}$ = 40.0 Lpm, the average speed of water through the pipe is

$$V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(40.0 \text{ L/min})}{\pi(0.024 \text{ m})^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 1.474 \text{ m/s} \quad (6)$$

which produces a Reynolds number of Re = $\rho VD/\mu$ = 3.96 × 10^4. At this Reynolds number, and with roughness factor ε/D = 0, the Colebrook equation yields f = 0.0220. After substitution of the given variables, along with f, D, L, and Eqs. 4, 5, and 6 into Eq. 3, we calculate the available NPSH,

$$NPSH = \frac{(101{,}300 - 3{,}169) \text{ N/m}^2}{(997.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{\text{kg m}}{\text{s}^2\text{N}}\right) - 2.2 \text{ m}$$
$$- \left(0.0220\frac{2.8 \text{ m}}{0.024 \text{ m}} + 1.15\right)\frac{(1.474 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 7.42 \text{ m} \quad (7)$$

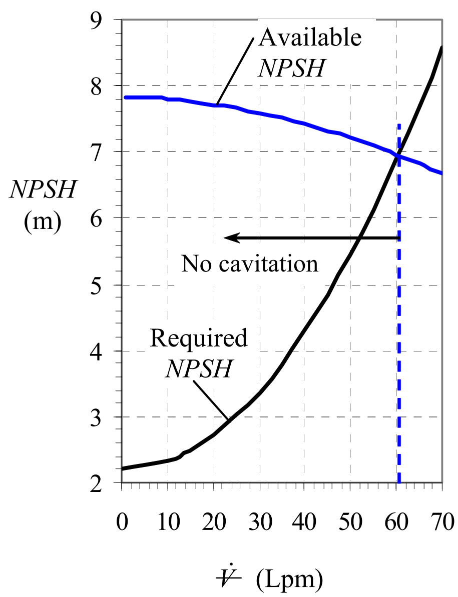


FIGURE 1
Net positive suction head as a function of volume flow rate; cavitation is predicted to occur at flow rates greater than the point where the available and required values of *NPSH* intersect.

The required net positive suction head is obtained from the given expression. At our example flow rate of 40.0 Lpm we see that $NPSH_{\text{required}}$ is about 4.28 m. Since the actual *NPSH* is higher than this, the pump does not cavitate at this flow rate. We use a spreadsheet to calculate *NPSH* as a function of volume flow rate, and the results are plotted in Fig. 1. It is clear from this plot that **cavitation occurs at flow rates above 60.5 liters per minute**.

Discussion Note that $NPSH_{\text{required}}$ rises with volume flow rate, but the actual or available *NPSH* decreases with volume flow rate (Fig. 1).

14-59

Solution We are to calculate the volume flow rate below which cavitation in a pump is avoided, at two temperatures.

Assumptions **1** The flow is steady. **2** The water is incompressible.

Properties Standard atmospheric pressure is P_{atm} = 101.3 kPa. For water at T = 80°C, ρ = 971.9 kg/m^3, μ = 3.55×10^{-4} kg/m·s, and P_v = 47.35 kPa. At T = 90°C, ρ = 965.3 kg/m^3, μ = 3.15×10^{-4} kg/m·s, and P_v = 70.11 kPa.

Analysis The procedure is identical to that of problem 14-58, except for the water properties. The calculations predict that **at *T* = 80°C, the pump cavitates at volume flow rates greater than 28.0 Lpm**. This is substantially lower than the result of Problem 14-58, as expected, since cavitation occurs more readily in warmer water.

At 90°C, the vapor pressure is very high since the water is near boiling (at atmospheric pressure, water boils at 100°C). For this case, the curves of $NPSH_{available}$ and $NPSH_{required}$ do not cross at all (Fig. 1), implying that **the pump cavitates at any flow rate when *T* = 90°C**.

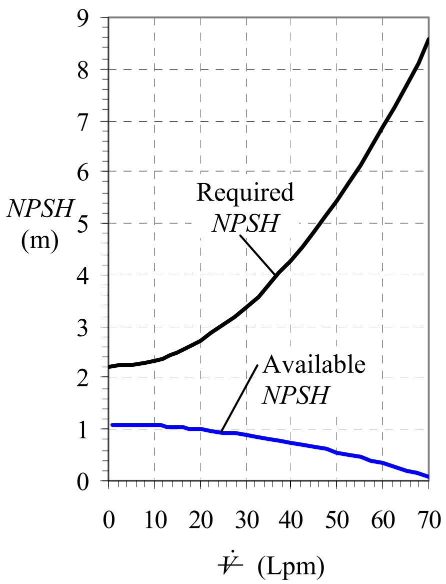


FIGURE 1
Net positive suction head as a function of volume flow rate at T = 90°C; cavitation occurs at any flow rate since the available and required values of *NPSH* do not intersect.

Discussion Note that $NPSH_{required}$ does not depend on water temperature, but the actual or available *NPSH* decreases with temperature.

14-60

Solution We are to calculate the volume flow rate below which cavitation in a pump is avoided, and compare with a previous case with a smaller pipe diameter.

Assumptions **1** The flow is steady. **2** The water is incompressible.

Properties Standard atmospheric pressure is P_{atm} = 101.3 kPa. For water at T = 25°C, ρ = 997.0 kg/m^3, μ = 8.91×10^{-4} kg/m·s, and P_v = 3.169 kPa.

Analysis The analysis is identical to that of Problem 14-58, except that the pipe diameter is 48.0 mm instead of 24.0 mm. Compared to problem 14-58, at a given volume flow rate, the average speed through the pipe decreases by a factor of four since the pipe area increases by a factor of four. The Reynolds number goes down by a factor of two, but the flow is still turbulent (At our sample flow rate of 40.0 Lpm, Re = 1.98 × 10^4). For a smooth pipe at this Reynolds number, f = 0.0260. and the available *NPSH* is 7.81 m, slightly higher than the previous case with the smaller diameter pipe. After repeating the calculations at several flow rates, we find that **the pump cavitates at $\dot{V}$ = 65.5 Lpm**. This represents an increase of about 8.3%. **Cavitation occurs at a higher volume flow rate when the pipe diameter is increased because the irreversible head losses in the piping system upstream of the pump are decreased**.

Discussion If a computer program like EES was used for Problem 14-58, it is a trivial matter to change the pipe diameter and re-do the calculations.

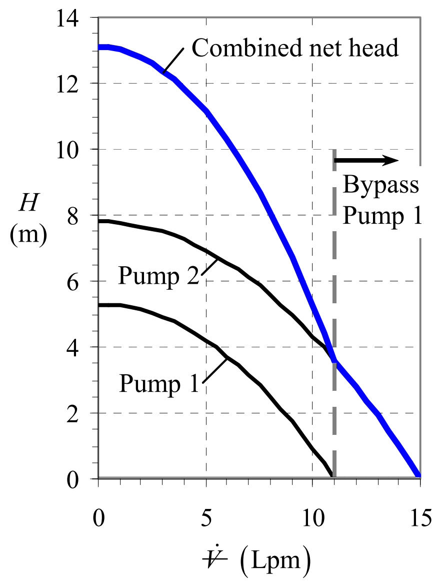


FIGURE 1
Performance of two dissimilar pumps in series.

14-61

Solution We are to calculate the combined shutoff head and free delivery for two pumps in series, and discuss why the weaker pump should be shut off and bypassed above some flow rate.

Assumptions **1** The water is incompressible. **2** The flow is steady.

Analysis The pump performance curves for both pumps individually and for their combination in series are plotted in Fig. 1. At zero flow rate, the shutoff head of the two pumps in series is equal to the sum of their individual shutoff heads: $H_{0,\text{combined}} = H_{0,1} + H_{0,2} = 5.30\text{ m} + 7.80\text{ m}$. Thus **the combined shutoff head is 13.1 m**. At free delivery conditions (zero net head), the free delivery of the two pumps in series is limited to that of the stronger pump, in this case Pump 2: $\dot{V}_{\text{max,combined}} = \text{MAX}(\dot{V}_{\text{max},1}, \dot{V}_{\text{max},2}) = 15.0\text{ Lpm}$. Thus **the combined free delivery is 15.0 Lpm**.

As volume flow rate increases, the combined net pump head is equal to the sum of the net pump heads of the individual pumps, as seen on Fig. 1. However, the free delivery of Pump 1 is

Free delivery, Pump 1:

$$0 = H_0 - a\dot{V}^2 \quad \rightarrow \quad \dot{V}_{\text{max}} = \sqrt{\frac{H_0}{a}} = \sqrt{\frac{5.30\text{ m}}{0.0438\text{ m/(Lpm)}^2}} = 11.0\text{ Lpm}$$

Above this flow rate, Pump 1 no longer contributes to the flow. In fact, it becomes a liability to the system since its net head would be *negative* at flow rates above 11.0 Lpm. For this reason, **it is wise to shut off and bypass Pump 1 above 11.0 Lpm**.

Discussion The free delivery of Pump 2 is 15.0 Lpm. The free delivery of the two-pump system is no better than that of Pump 2 alone, since Pump 1 is shut off and bypassed at flow rates above 11.0 Lpm.

14-62

Solution We are to calculate the free delivery and shutoff head for two pumps in parallel, and discuss why the weaker pump should be shut off and bypassed above some net head.

Assumptions **1** The water is incompressible. **2** The flow is steady.

Analysis The pump performance curves for both pumps and for their combination in parallel are plotted in Fig. 1. At zero flow rate, the shutoff head of the two pumps in parallel is equal to that of the stronger pump, in this case Pump 2: $H_{0,\text{combined}} = \text{MAX}(H_{0,1}, H_{0,2}) = 7.80$ m. Thus **the combined shutoff head is 7.80 m**. At free delivery conditions (zero net head), the free delivery of the two pumps in parallel is the sum of their individual free deliveries: $\dot{V}_{\text{max,combined}} = \dot{V}_{\text{max},1} + \dot{V}_{\text{max},2} =$ 11.0 Lpm + 15.0 Lpm. Thus **the combined free delivery is 26.0 Lpm**.

As net head increases, the combined capacity is equal to the sum of the capacities of the individual pumps, as seen on Fig. 1. However, the shutoff head of Pump 1 (5.30 m) is lower than that of Pump 2 (7.80 m). Above the shutoff head of Pump 1, that pump no longer contributes to the flow. In fact, it becomes a liability to the system since it cannot sustain such a high head. If not shut off and blocked, the volume flow rate through Pump 1 would be *negative* at net heads above 7.80 m. For this reason, **it is wise to shut off and block Pump 1 above 7.80 m**.

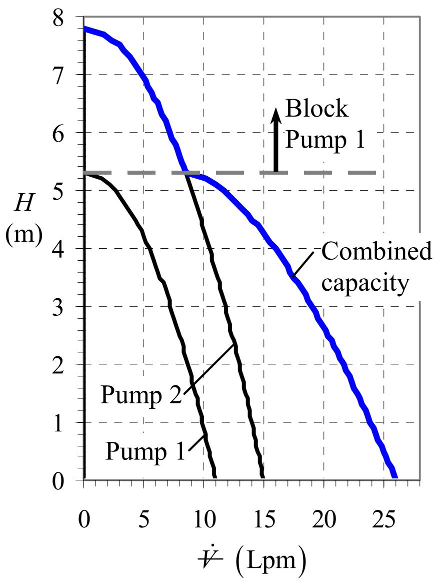


FIGURE 1
Performance of two dissimilar pumps in series.

Discussion The shutoff head of Pump 2 is 7.80 m. The shutoff head of the two-pump system is no better than that of Pump 2 alone, since Pump 1 is shut off and blocked at net heads above 7.80 m.

14-63E

Solution We are to calculate the mass flow rate of slurry through a two-lobe rotary pump for given values of lobe volume and rotation rate.

Assumptions **1** The flow is steady in the mean. **2** There are no leaks in the gaps between lobes or between lobes and the casing. **3** The slurry is incompressible.

Analysis By studying Fig. P14-63, we see that for each 360° rotation of the two counter-rotating shafts (n = 1 rotation), the total volume of pumped fluid is

Closed volume pumped per rotation: $$V_{\text{closed}} = 4V_{\text{lobe}} \qquad (1)$$

The volume flow rate is then calculated from Eq. 14-11,

$$\dot{V} = \dot{n}\frac{V_{\text{closed}}}{n} = (300 \text{ rot/min})\frac{4(0.145 \text{ gal})}{1 \text{ rot}} = 174 \text{ gal/min} \qquad (2)$$

Thus, **the volume flow rate is 174 gpm**.

Discussion If there were leaks in the pump, the volume flow rate would be lower. The fluid density was not used in the calculation of volume flow rate. However, the higher the density, the higher the required shaft torque and brake horsepower.

14-64E

Solution We are to calculate the mass flow rate of slurry through a three-lobe rotary pump for given values of lobe volume and rotation rate.

Assumptions **1** The flow is steady in the mean. **2** There are no leaks in the gaps between lobes or between lobes and the casing. **3** The slurry is incompressible.

Analysis When there are three lobes, three lobe volumes are pumped for each 360° rotation of each rotor (n = 1 rotation). Thus, the total volume of pumped fluid is

Closed volume pumped per rotation: $$V_{\text{closed}} = 6V_{\text{lobe}} \qquad (1)$$

The volume flow rate is then calculated from Eq. 14-11,

$$\dot{V} = \dot{n}\frac{V_{\text{closed}}}{n} = (300 \text{ rot/min})\frac{6(0.087 \text{ gal})}{1 \text{ rot}} = 157 \text{ gal/min} \qquad (2)$$

Thus, **the volume flow rate is 157 gpm**.

Discussion If there were leaks in the pump, the volume flow rate would be lower. This flow rate is slightly lower than that of Problem 14-63. Why? For the same overall diameter, it is clear from geometry that the more lobes, the less the volume per lobe, and the more "wasted" volume inside the pump.

14-65

Solution We are to calculate the volume flow rate of tomato paste through a positive-displacement pump for given values of lobe volume and rotation rate.

Assumptions **1** The flow is steady in the mean. **2** There are no leaks in the gaps between lobes or between lobes and the casing. **3** The fluid is incompressible.

Analysis By studying Fig. 14-27 or 14-30, we can see that for each 360° rotation of the two counter-rotating shafts (n = 1 rotation), the total volume of pumped fluid is

Closed volume pumped per rotation: $$V_{\text{closed}} = 4V_{\text{lobe}} \qquad (1)$$

The volume flow rate is then calculated from Eq. 14-11,

$$\dot{V} = \dot{n}\frac{V_{\text{closed}}}{n} = (400 \text{ rot/min})\frac{4(3.64 \text{ cm}^3)}{1 \text{ rot}} = 5820 \text{ cm}^3\text{/min} \qquad (2)$$

Thus, **the volume flow rate is 5820 cm³/min**.

Discussion If there were leaks in the pump, the volume flow rate would be lower. The fluid density was not used in the calculation of volume flow rate. However, the higher the fluid density, the higher the required shaft torque and brake horsepower.

14-66

Solution We are to calculate the volume flow rate per rotation of a gear pump.

Assumptions **1** The flow is steady in the mean. **2** There are no leaks in the gaps between lobes or between lobes and the casing. **3** The fluid is incompressible.

Analysis From Fig. 14-26c, we count 14 teeth per gear. Thus, for each 360° rotation of each gear (n = 1 rotation), 14·(0.350 cm^3) of fluid is pumped. Sine there are two gears, the total volume of fluid pumped per rotation is 2(14)(0.350 cm^3) = **9.80 cm^3**.

Discussion The actual volume flow rate will be lower than this due to leakage in the gaps.

14-67

Solution We are to calculate the brake horsepower and net head of an idealized centrifugal pump at a given volume flow rate and rotation rate.

Assumptions **1** The flow is steady in the mean. **2** The water is incompressible. **3** The efficiency of the pump is 100% (no irreversible losses).

Properties We take the density of water to be ρ = 998.0 kg/m^3.

Analysis Since the volume flow rate (capacity) is given, we calculate the normal velocity components at the inlet and the outlet using Eq. 14-12,

Normal velocity, inlet: $$V_{1,\mathrm{n}} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.573\ \mathrm{m^3/s}}{2\pi\left(0.120\ \mathrm{m}\right)\left(0.180\ \mathrm{m}\right)} = 4.222\ \mathrm{m/s}$$

$V_1 = V_{1,\mathrm{n}}$, and $V_{1,\mathrm{t}} = 0$, since $\alpha_1 = 0°$. Similarly, $V_{2,\mathrm{n}}$ = 2.714 m/s and

Tangential component of absolute velocity at impeller outlet:
$$V_{2,\mathrm{t}} = V_{2,\mathrm{n}} \tan\alpha_2 = \left(2.714\ \mathrm{m/s}\right)\tan(35°) = 1.900\ \mathrm{m/s}$$

Now we use Eq. 14-17 to predict the net head,

$$H = \frac{\omega}{g}\left(r_2 V_{2,\mathrm{t}} - r_1 \underbrace{\cancel{V_{1,\mathrm{t}}}}_{0}\right) = \frac{78.54\ \mathrm{rad/s}}{9.81\ \mathrm{m/s^2}}\left(0.240\ \mathrm{m}\right)\left(1.900\ \mathrm{m/s}\right) = \mathbf{3.65\ m}$$

Finally, we use Eq. 14-16 to predict the required brake horsepower,

$$bhp = \rho g \dot{V} H = \left(998.\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right)\left(0.573\ \mathrm{m^3/s}\right)\left(3.65\ \mathrm{m}\right)\left(\frac{\mathrm{W\cdot s^3}}{\mathrm{kg\cdot m^2}}\right)$$
$$= \mathbf{20{,}500\ W}$$

Discussion The actual net head delivered to the water will be lower than this due to inefficiencies. Similarly, actual brake horsepower will be higher than that predicted here due to inefficiencies in the pump, friction on the shaft, etc.

14-68

Solution For given flow conditions and stator blade shape at a given radius, we are to design the rotor blade. Specifically, we are to calculate the leading and trailing edge angles of the rotor blade and sketch its shape. We are also to decide how many rotor blades to construct.

Assumptions **1** The air is nearly incompressible. **2** The flow area between hub and tip is constant. **3** Two-dimensional blade row analysis is appropriate.

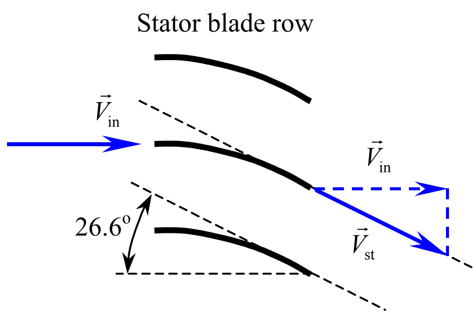


FIGURE 1
Analysis of the stator of the vaneaxial flow fan of Example 14-10 as a two-dimensional cascade of stator blades; absolute reference frame.

Analysis First we analyze flow through the stator from an absolute reference frame, using the two-dimensional approximation of a cascade (blade row) of stator blades as sketched in Fig. 1. Flow enters axially (horizontally), and is turned 26.6° downward. Since the axial component of velocity must remain constant to conserve mass, the magnitude of the velocity leaving the trailing edge of the stator, $\vec{V}_{st}$ is calculated,

$$V_{st} = \frac{V_{in}}{\cos\beta_{st}} = \frac{31.4 \text{ m/s}}{\cos\left(26.6^\circ\right)} = 35.12 \text{ m/s} \qquad (1)$$

The direction of $\vec{V}_{st}$ is assumed to be that of the rotor trailing edge. In other words we assume that the flow turns nicely through the blade row and exits parallel to the trailing edge of the blade, as shown in the sketch.

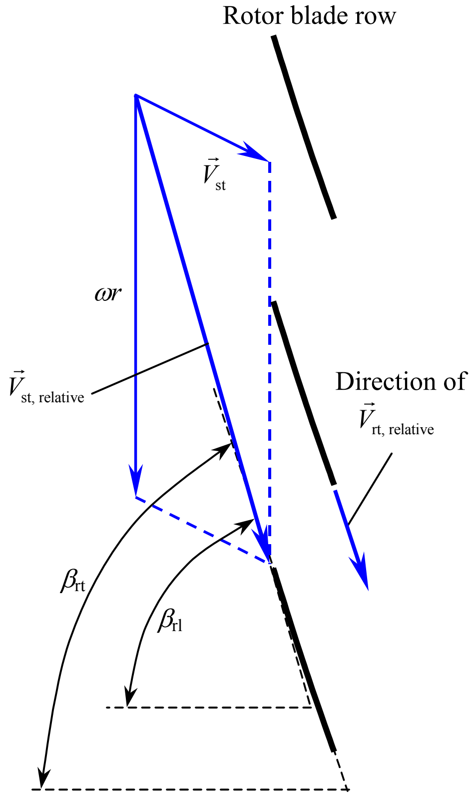


FIGURE 2
Analysis of the stator trailing edge velocity as it impinges on the rotor leading edge; relative reference frame.

We convert $\vec{V}_{\text{st}}$ to the *relative* reference frame moving with the rotor blades. At a radius of 0.50 m, the tangential velocity of the rotor blades is

$$u_\theta = \omega r = \left[\left(1800\ \text{rot/min}\right)\left(\frac{2\pi\ \text{rad}}{\text{rot}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right]\left(0.50\ \text{m}\right) = 94.25\ \text{m/s} \qquad (2)$$

Since the rotor blade row moves upward in Fig. P14-68, we add a *downward* velocity with magnitude given by Eq. 2 to translate $\vec{V}_{\text{st}}$ into the rotating reference frame sketched in Fig. 2. The angle of the leading edge of the rotor, β_{rl}, can be calculated. After some trig,

$$\begin{aligned}\beta_{\text{rl}} &= \arctan\frac{\omega r + V_{\text{in}}\tan\beta_{\text{st}}}{V_{\text{in}}} \\ &= \arctan\frac{94.25\ \text{m/s} + \left(31.4\ \text{m/s}\right)\tan\left(26.6^\circ\right)}{31.4\ \text{m/s}} = 74.06^\circ\end{aligned} \qquad (3)$$

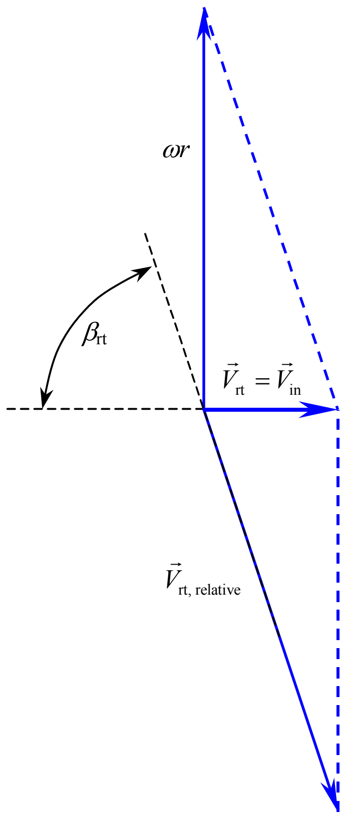


FIGURE 3
Analysis of the rotor trailing edge velocity; absolute reference frame.

The air must now be turned by the rotor blade row in such a way that it leaves the trailing edge of the rotor blade at zero angle (axially – no swirl) from an absolute reference frame. This determines the rotor's trailing edge angle, β_{rt}. Specifically, when we add an *upward* velocity of magnitude ωr (Eq. 2) to the relative velocity exiting the trailing edge of the rotor, $\vec{V}_{rt,\ relative}$, we convert back to the absolute reference frame, and obtain $\vec{V}_{rt}$, the velocity leaving the rotor trailing edge. It is this velocity, $\vec{V}_{rt}$, which must be axial (horizontal). Furthermore, to conserve mass $\vec{V}_{rt}$ must equal $\vec{V}_{in}$ since we are assuming incompressible flow. Working "backwards" we construct $\vec{V}_{rt,\ relative}$ in Fig. 3. Some trigonometry reveals that

$$\beta_{rt} = \arctan\frac{\omega r}{V_{in}} = \arctan\frac{94.25\text{ m/s}}{31.4\text{ m/s}} = 71.57^{\circ} \qquad (4)$$

We conclude that the rotor blade at this radius has a leading edge angle of about **74.1°** (Eq. 3) and a trailing edge angle of about **71.6°** (Eq. 4). A sketch of the rotor blade is provided in Fig. 2; it is clear that the blade is nearly straight, at least at this radius.

Finally, to avoid interaction of the stator blade wakes with the rotor blade leading edges, we choose the number of rotor blades such that it has no common denominator with the number of stator blades. Since there are 18 stator blades, we pick a number like **13**, **17**, or **19** rotor blades. 16 would not be appropriate since it shares a common denominator of 2 with the number 18.

Discussion We can easily repeat the calculation for all radii from hub to tip, completing the design of the entire rotor.

Turbines

14-69C

Solution We are to discuss why turbines have higher efficiencies than pumps.

Analysis There are several reasons for this. First, pumps normally operate at higher rotational speeds than do turbines; therefore, shear stresses and frictional losses are higher. Second, conversion of kinetic energy into flow energy (pumps) has inherently higher losses than does the reverse (turbines). You can think of it this way: Since pressure *rises* across a pump (adverse pressure gradient), but *drops* across a turbine (favorable pressure gradient), boundary layers are less likely to separate in a turbine than in a pump. Third, turbines (especially hydroturbines) are often much larger than pumps, and viscous losses become less important as size increases. Finally, while pumps often operate over a wide range of flow rates, most electricity-generating turbines run within a narrower operating range and at a controlled constant speed; they can therefore be designed to operate very efficiently at those conditions.

Discussion Students' answers should be in their own words.

14-70C

Solution We are to discuss the classification of dynamic pumps and reaction turbines.

Analysis **Dynamic pumps are classified according to the angle at which the flow *exits* the impeller blade** – centrifugal, mixed-flow, or axial. **Reaction turbines, on the other hand, are classified according to the angle that the flow *enters* the runner** – radial, mixed-flow, or axial. This is the main difference between how dynamic pumps and reaction turbines are classified.

Discussion Students' answers should be in their own words.

14-71C

Solution We are to discuss the meaning and purpose of draft tubes.

Analysis **A draft tube is a diffuser that also turns the flow downstream of a turbine. Its purpose is to turn the flow horizontally and recover some of the kinetic energy leaving the turbine runner. If the draft tube is not designed carefully, much of the kinetic energy leaving the runner would be wasted, reducing the overall efficiency of the turbine system**.

Discussion Students' answers should be in their own words.

14-72C

Solution We are to name and describe the two types of dynamic turbine.

Analysis There are two basic types of dynamic turbine – **impulse** and **reaction**. In an impulse turbine, fluid is sent through a nozzle so that most of its available mechanical energy is converted into kinetic energy. The high-speed jet then impinges on bucket-shaped vanes that transfer energy to the turbine shaft. In a reaction turbine, the fluid completely fills the casing, and the runner is rotated by momentum exchange due to pressure differences across the blades, rather than by kinetic energy impingement. Impulse turbines require a higher head, but can operate with a smaller volume flow rate. Reaction turbines can operate with much less head, but require higher volume flow rate.

Discussion Students' answers should be in their own words.

14-73C

Solution We are to discuss reverse swirl in reaction turbines.

Analysis **Reverse swirl is when the runner blades turn the flow so much that the swirl at the runner outlet is in the direction opposite to runner rotation**. Reverse swirl is desirable so that more power is absorbed from the water. We can easily see this from the Euler turbomachine equation,

$$\dot{W}_{\text{shaft}} = \omega \text{T}_{\text{shaft}} = \rho\omega\dot{V}\left(r_2 V_{2,\text{t}} - r_1 V_{1,\text{t}}\right) \qquad (1)$$

Namely, since there is a negative sign on the last term, the shaft power increases if $V_{1,\text{t}}$ is *negative*, in other words, if there is reverse swirl at the runner outlet. If there is too much reverse swirl, a lot of extra kinetic energy gets wasted downstream of the runner.

Discussion A well designed draft tube can recover a good portion of the streamwise kinetic energy of the water leaving the runner. However, the swirling kinetic energy cannot be recovered.

14-74

Solution We are to prove that the maximum power of a Pelton wheel occurs when $\omega r = V_j/2$ (bucket moving at half the jet speed).

Assumptions **1** Frictional losses are negligible, so that the Euler turbomachine equation applies, and the relative exit speed of the jet is the same as its relative inlet speed. **2** The turning angle is sufficient to prevent the exiting fluid from striking the next bucket. **3** The fluid, jet speed, volume flow rate, turning angle, and wheel radius are fixed – rotation rate is the only variable about which we are concerned.

Analysis With the stated assumptions, Eq. 14-40 applies, namely,

Output shaft power:

$$\dot{W}_{\text{shaft}} = \rho\omega r\dot{V}\left(V_j - \omega r\right)\left(1 - \cos\beta\right) \qquad (1)$$

We differentiate Eq. 1 with respect to ω, and set the derivative equal to zero,

Maximum power:

$$\frac{d\dot{W}_{\text{shaft}}}{d\omega} = 0 \quad \rightarrow \quad \frac{d}{d\omega}\left(\omega V_j - \omega^2 r\right) = 0 \quad \rightarrow \quad V_j - 2\omega r = 0 \qquad (2)$$

Solution of Eq. 2 yields the desired result, namely, the maximum power of a Pelton wheel occurs when $\boldsymbol{\omega r = V_j/2}$ (bucket moving at half the jet speed).

Discussion To be sure that we have not identified a *minimum* instead of a maximum, we could substitute some numerical values and plot $\dot{W}_{\text{shaft}}$ versus ω. It turns out that we have indeed found the maximum value of $\dot{W}_{\text{shaft}}$.

14-75

Solution We are to calculate several performance parameters for a Pelton wheel turbine.

Assumptions **1** Frictional losses are negligible, so that the Euler turbomachine equation applies, and the relative exit speed of the jet is the same as its relative inlet speed. **2** The turning angle is sufficient to prevent the exiting fluid from striking the next bucket. **3** The water is at 20°C.

Properties The density of water at T = 20°C is 998.0 kg/m³.

Analysis (*a*) The volume flow rate of the jet is equal to jet area times jet velocity,

$$\dot{V} = V_j \pi D_j^{\,2}/4 = (102\text{ m/s})\pi(0.100\text{ m})^2/4 = \mathbf{0.801\ m^3/s}$$

(*b*) The maximum output shaft power occurs when the bucket moves at half the jet speed ($\omega r = V_j/2$). Thus,

$$\omega = \frac{V_j}{2r} = \frac{102\text{ m/s}}{2(1.83\text{ m})} = 27.87\text{ rad/s}$$

$$\rightarrow \quad \dot{n} = (27.87\text{ rad/s})\left(\frac{\text{rot}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = \mathbf{266\ rpm}$$

(*c*) The ideal shaft power is found from Eq. 14-40,

$$\dot{W}_{\text{ideal}} = \rho\omega r\dot{V}(V_j - \omega r)(1-\cos\beta)$$

$$= (998.0\text{ kg/m}^3)\left(27.87\frac{\text{rad}}{\text{s}}\right)(1.83\text{ m})\left(0.801\frac{\text{m}^3}{\text{s}}\right)\left(\frac{102\text{ m/s}}{2}\right)(1-\cos 165^\circ)$$

$$\times\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right)\left(\frac{1\text{ MW}}{10^6\text{ W}}\right) = 4.09\text{ MW}$$

where we have substituted $\omega r = V_j/2$ for convenience in the calculations. Since the turbine efficiency is given, we calculate the actual output shaft power, or brake horsepower,

$$\dot{W}_{\text{actual}} = bhp = \dot{W}_{\text{ideal}}\eta_{\text{turbine}} = (4.09\text{ MW})(0.82) = \mathbf{3.35\ MW}$$

Discussion At other rotation speeds, the turbine would not be as efficient.

14-76

Solution We are to estimate the ideal power production from a hydroturbine.

Assumptions **1** Frictional losses are negligible. **2** The water is at 20°C.

Properties The density of water at T = 20°C is 998.0 kg/m³.

Analysis The ideal power produced by a hydroturbine is

$$\dot{W}_{\text{ideal}} = \rho g\dot{V}H_{\text{gross}} = \left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.5\ \text{m}^3\text{/s}\right)\left(650\ \text{m}\right)$$

$$\times\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right)\left(\frac{1\ \text{MW}}{10^6\ \text{W}}\right) = \mathbf{9.55\ MW}$$

Discussion If a hydroelectric dam were to be built on this site, the actual power output per turbine would be smaller than this, of course, due to inefficiencies.

14-77E

Solution We are to estimate the power production from a hydroelectric plant.

Properties The density of water at T = 70°F is 62.30 lbm/ft³.

Analysis The ideal power produced by one hydroturbine is

$$\dot{W}_{\text{ideal}} = \rho g\dot{V}H_{\text{gross}} = \left(62.30\ \text{lbm/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left(203{,}000\ \text{gal/min}\right)\left(1065\ \text{ft}\right)$$

$$\times\left(\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}\right)\left(0.1337\frac{\text{ft}^3}{\text{gal}}\right)\left(\frac{1.356\ \text{W}}{\text{ft}\cdot\text{lbf/s}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{MW}}{10^6\ \text{W}}\right) = 40.70\ \text{MW}$$

But inefficiencies in the turbine, the generator, and the rest of the system reduce the actual electrical power output. For each turbine,

$$\dot{W}_{\text{electrical}} = \dot{W}_{\text{ideal}}\eta_{\text{turbine}}\eta_{\text{generator}}\eta_{\text{other}}$$

$$= \left(40.70\ \text{MW}\right)\left(0.952\right)\left(0.945\right)\left(1-0.035\right) = 35.3\ \text{MW}$$

Finally, since there are 12 turbines in parallel, the total power produced is

$$\dot{W}_{\text{total, electrical}} = 12\dot{W}_{\text{electrical}} = 12\left(35.3\ \text{MW}\right) = \mathbf{424\ MW}$$

Discussion A small improvement in any of the efficiencies ends up increasing the power output, and increases the power company's profitability.

14-78

Solution We are to calculate runner blade angles, required net head, and power output for a Francis turbine.

Assumptions **1** The flow is steady. **2** The fluid is water at 20°C. **3** The blades are infinitesimally thin. **4** The flow is everywhere tangent to the runner blades. **5** We neglect irreversible losses through the turbine.

Properties For water at 20°C, $\rho = 998.0$ kg/m^3.

Analysis We solve for the normal component of velocity at the inlet,

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{340\ \text{m}^3/\text{s}}{2\pi\left(2.00\ \text{m}\right)\left(0.731\ \text{m}\right)} = 37.0\ \text{m/s} \qquad (1)$$

Using Fig. P14-78 as a guide, the tangential velocity component at the inlet is

$$V_{2,t} = V_{2,n}\tan\alpha_2 = \left(37.0\ \text{m/s}\right)\tan 30^\circ = 21.4\ \text{m/s} \qquad (2)$$

The angular velocity is $\omega = 2\pi\dot{n}/60 = 18.85$ rad/s. We now solve Eq. 14-45 for the runner leading edge angle β_2,

$$\beta_2 = \arctan\left[\frac{V_{2,n}}{\omega r_2 - V_{2,t}}\right] = \arctan\left[\frac{37.0\ \text{m/s}}{\left(18.85\ \text{rad/s}\right)\left(2.00\ \text{m}\right) - 21.4\ \text{m/s}}\right] = \mathbf{66.2^\circ} \qquad (3)$$

Equations 1 through 3 are repeated for the runner outlet, with the following results:

Runner outlet: $V_{1,n} = 17.3\ \text{m/s}, \quad V_{1,t} = 3.05\ \text{m/s}, \quad \beta_1 = \mathbf{36.1^\circ}$ (4)

Using Eqs. 2 and 4, the shaft output power is estimated from the Euler turbomachine equation,

$$\begin{aligned}\dot{W}_{\text{shaft}} &= \rho\omega\dot{V}\left(r_2 V_{2,t} - r_1 V_{1,t}\right) = \left(998.0\ \text{kg/m}^3\right)\left(18.85\ \text{rad/s}\right)\left(340\ \text{m}^3/\text{s}\right)\\ &\quad\times\left[\left(2.00\ \text{m}\right)\left(21.4\ \text{m/s}\right) - \left(1.42\ \text{m}\right)\left(3.05\ \text{m/s}\right)\right]\\ &= 2.46\times10^8\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}\left(\frac{\text{MW}\cdot\text{s}^3}{10^6\ \text{kg}\cdot\text{m}^2}\right) = 246.055\ \text{MW} \cong \mathbf{246\ MW}\end{aligned} \qquad (5)$$

Finally, we calculate the required net head using Eq. 14-44, assuming that η_{turbine} = 100% since we are ignoring irreversibilities,

$$H = \frac{bhp}{\rho g\dot{V}} = \frac{246.055\ \text{MW}}{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(340\ \text{m}^3/\text{s}\right)}\left(\frac{10^6\ \text{kg}\cdot\text{m}^2}{\text{MW}\cdot\text{s}^3}\right) = \mathbf{73.9\ m} \qquad (6)$$

Since the required net head is less than the gross net head, **the design is feasible**.

Discussion This is a preliminary design in which we are neglecting irreversibilities. Actual output power will be lower, and actual required net head will be higher than the values predicted here.

CD-EES 14-79

Solution We are to examine the effect of runner outlet angle α_1 on the required net head and the output power of a hydroturbine.

Assumptions **1** The flow is steady. **2** The fluid is water at 20°C. **3** The blades are infinitesimally thin. **4** The flow is everywhere tangent to the runner blades. **5** We neglect irreversible losses through the turbine.

Properties For water at 20°C, ρ = 998.0 kg/m^3.

Analysis We repeat the calculations of Problem 14-78, but with α_1 varying from -20° to 20°. The results are plotted in Fig. 1. We see that the required net head H and the output brake horsepower *bhp* decrease as α_1 increases. This agrees with our expectations, based on the negative sign on the $V_{1,t}$ term in the Euler turbomachine equation. In other words, we can produce greater power by increasing the reverse swirl, but at the cost of increased required net head. However, when α_1 is smaller than about -9°, the required net head rises above H_{gross}, which is impossible, even with no irreversibilities. Thus, **when α_1 < -9°, the predicted net head and brake horsepower are not feasible – they violate the second law of thermodynamics**.

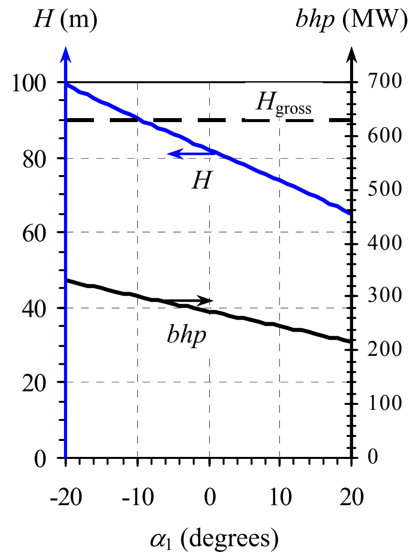


FIGURE 1
Ideal required net head and brake horsepower as functions of runner outlet flow angle for Problem 14-79.

Discussion A small amount of reverse swirl is usually good, but too much reverse swirl is not good.

14-80

Solution We are to calculate flow angles, required net head, and power output for a Francis turbine.

Assumptions **1** The flow is steady. **2** The fluid is water at 20°C. **3** The blades are infinitesimally thin. **4** The flow is everywhere tangent to the runner blades. **5** We neglect irreversible losses through the turbine.

Properties For water at 20°C, ρ = 998.0 kg/m³.

Analysis The angular velocity is $\omega = 2\pi\dot{n}/60$ = 10.47 rad/s. We solve for the normal component of velocity at the inlet,

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{80.0\ \text{m}^3/\text{s}}{2\pi(2.00\ \text{m})(0.85\ \text{m})} = 7.490\ \text{m/s} \tag{1}$$

We then solve Eq. 14-45 for the tangential velocity component at the inlet,

$$V_{2,t} = \omega r_2 - \frac{V_{2,n}}{\tan\beta_2} = (10.47\ \text{rad/s})(2.00\ \text{m}) - \frac{7.490\ \text{m/s}}{\tan 66^\circ} = 17.61\ \text{m/s} \tag{2}$$

From these two components of V_2 in the absolute coordinate system, we calculate the angle α_2 through which the wicket gates should turn the flow,

$$\alpha_2 = \tan^{-1}\frac{V_{2,t}}{V_{2,n}} = \tan^{-1}\frac{17.61\ \text{m/s}}{7.490\ \text{m/s}} = 66.95^\circ \approx \mathbf{67^\circ} \tag{3}$$

In exactly similar fashion, we solve for the velocity components and swirl angle at the runner outlet. We get

Runner outlet:

$$V_{1,n} = 4.664\ \text{m/s},\quad V_{1,t} = -0.3253\ \text{m/s},\quad \alpha_1 = -3.990^\circ \approx \mathbf{-4.0^\circ} \tag{4}$$

Since α_1 is negative, **this turbine operates with a small amount of reverse swirl**.

Using the tangential velocity components of Eqs. 2 and 4, the shaft output power is estimated from the Euler turbomachine equation,

$$\begin{aligned}\dot{W}_{\text{shaft}} &= \rho\omega\dot{V}(r_2V_{2,t} - r_1V_{1,t}) = (998.0\ \text{kg/m}^3)(10.47\ \text{rad/s})(80.0\ \text{m}^3/\text{s})\\ &\quad\times\left[(2.00\ \text{m})(17.61\ \text{m/s}) - (1.30\ \text{m})(-0.3253\ \text{m/s})\right]\\ &= 2.98\times10^7\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}\left(\frac{\text{MW}\cdot\text{s}^3}{10^6\ \text{kg}\cdot\text{m}^2}\right) = \mathbf{29.8\ MW}\end{aligned} \tag{5}$$

Finally, we calculate the required net head using Eq. 14-44, assuming that η_{turbine} = 100% since we are ignoring irreversibilities,

$$H = \frac{bhp}{\rho g\dot{V}} = \frac{29.8\ \text{MW}}{(998.0\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(80.0\ \text{m}^3/\text{s})}\left(\frac{10^6\ \text{kg}\cdot\text{m}^2}{\text{MW}\cdot\text{s}^3}\right) = \mathbf{38.0\ m} \tag{6}$$

Discussion This is a preliminary design in which we are neglecting irreversibilities. Actual output power will be lower, and actual required net head will be higher than the values predicted here. Notice the double negative in the outlet terms of Eq. 5 – reverse swirl leads to greater performance, but requires more head.

14-81E

Solution We are to calculate flow angles, required net head, and power output for a Francis turbine.

Assumptions **1** The flow is steady. **2** The fluid is water at 68°F. **3** The blades are infinitesimally thin. **4** The flow is everywhere tangent to the runner blades. **5** We neglect irreversible losses through the turbine.

Properties For water at 68°F, ρ = 62.32 lbm/ft^3.

Analysis The angular velocity is $\omega = 2\pi\dot{n}/60$ = 12.57 rad/s. The volume flow rate is (4.70 × 10^6 gpm)[ft^3/s / (448.83 gpm)] = 10,470 ft^3/s. We solve for the normal component of velocity at the inlet,

$$V_{2,\mathrm{n}} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{10{,}470\ \mathrm{ft^3/s}}{2\pi(6.60\ \mathrm{ft})(2.60\ \mathrm{ft})} = 97.1\ \mathrm{ft/s} \qquad (1)$$

We then solve Eq. 14-45 for the tangential velocity component at the inlet,

$$V_{2,\mathrm{t}} = \omega r_2 - \frac{V_{2,\mathrm{n}}}{\tan\beta_2} = (12.57\ \mathrm{rad/s})(6.60\ \mathrm{ft}) - \frac{97.12\ \mathrm{ft/s}}{\tan 82^{\circ}} = 69.3\ \mathrm{ft/s} \qquad (2)$$

From these two components of V_2 in the absolute coordinate system, we calculate the angle α_2 through which the wicket gates should turn the flow,

$$\alpha_2 = \tan^{-1}\frac{V_{2,\mathrm{t}}}{V_{2,\mathrm{n}}} = \tan^{-1}\frac{69.3\ \mathrm{ft/s}}{97.1\ \mathrm{ft/s}} = \mathbf{35.5^{o}} \qquad (3)$$

In exactly similar fashion, we solve for the velocity components and swirl angle at the runner outlet. We get

Runner outlet: $$V_{1,\mathrm{n}} = 52.6\ \mathrm{ft/s},\quad V_{1,\mathrm{t}} = 4.49\ \mathrm{ft/s},\quad \alpha_1 = \mathbf{4.9^{o}} \qquad (4)$$

Since α_1 is positive, **this turbine operates with a small amount of forward swirl**.

Using the tangential velocity components of Eqs. 2 and 4, the shaft output power is estimated from the Euler turbomachine equation,

$$\begin{aligned}\dot{W}_{\mathrm{shaft}} &= \rho\omega\dot{V}(r_2 V_{2,\mathrm{t}} - r_1 V_{1,\mathrm{t}}) = (62.32\ \mathrm{lbm/ft^3})(12.57\ \mathrm{rad/s})(10{,}470\ \mathrm{ft^3/s}) \\ &\quad \times\left[(6.60\ \mathrm{ft})(69.3\ \mathrm{ft/s}) - (4.40\ \mathrm{ft})(4.49\ \mathrm{ft/s})\right]\left(\frac{\mathrm{lbf\cdot s^2}}{32.174\ \mathrm{lbm\cdot ft}}\right) \\ &= 1.116\times10^{8}\ \frac{\mathrm{ft\cdot lbf}}{\mathrm{s}}\left(\frac{1.8182\times10^{-3}\ \mathrm{hp\cdot s}}{\mathrm{ft\cdot lbf}}\right) = \mathbf{2.03\times10^{5}\ hp}\end{aligned} \qquad (5)$$

Finally, we calculate the required net head using Eq. 14-44, assuming that η_{turbine} = 100% since we are ignoring irreversibilities,

$$H = \frac{bhp}{\rho g \dot{V}} = \frac{1.116\times10^{8}\ \text{ft}\cdot\text{lbf/s}}{\left(62.32\ \text{lbm/ft}^3\right)\left(32.174\ \text{ft/s}^2\right)\left(10,470\ \text{ft}^3\text{/s}\right)}\left(\frac{32.174\ \text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right) = \mathbf{171\ ft} \quad (6)$$

Discussion This is a preliminary design in which we are neglecting irreversibilities. Actual output power will be lower, and actual required net head will be higher than the values predicted here.

14-82E

Solution We are to calculate the runner blade trailing edge angle such that there is no swirl. At this value of β_1, we are to also calculate the shaft power.

Assumptions **1** The flow is steady. **2** The fluid is water at 68°F. **3** The blades are infinitesimally thin. **4** The flow is everywhere tangent to the runner blades. **5** We neglect irreversible losses through the turbine.

Properties For water at 68°F, ρ = 62.32 lbm/ft^3.

Analysis Using EES or other software, we adjust β_1 by trial and error until α_1 = 0. It turns out that β_1 = **43.6°**, at which $\dot{W}_{\text{shaft}}$ = $\mathbf{2.12 \times 10^5}$ **hp**.

Discussion It turns out that the swirl angle at the runner output is a strong function of β_1 – a small change in β_1 leads to a large change in α_1. The shaft power increases, as expected, since the original swirl angle was positive. The increase in shaft power is less than 5%.

Pump and Turbine Scaling Laws

14-83C

Solution We are to give a definition of "affinity", and explain why the scaling laws are called "affinity laws".

Analysis Among the many definitions of "affinity" is "inherent likeness or agreement", and "...resemblance in general plan or structure". When two pumps or two turbines are geometrically similar and operate under dynamically similar conditions, they indeed have "inherent likeness". Thus, the phrase "affinity laws" is appropriate for the scaling laws of turbomachinery.

Discussion Students will have various definitions, depending on the dictionary they use.

14-84C

Solution

(*a*) *True*: Rotation rate appears with an exponent of 1 in the affinity law for capacity. Thus, the change is linear.

(*b*) *False*: Rotation rate appears with an exponent of 2 in the affinity law for net head. Thus, if the rpm is doubled, the net head increases by a factor of 4.

(*c*) *False*: Rotation rate appears with an exponent of 3 in the affinity law for shaft power. Thus, if the rpm is doubled, the shaft power increases by a factor of 8.

(*d*) *True*: The affinity laws apply to turbines as well as pumps, so this statement is true, as discussed in Part (c).

14-85C

Solution We are to discuss which pump and turbine performance parameters are used as the independent parameter, and explain why

Analysis For pumps, we use C_Q, the **capacity coefficient**, as the independent parameter. The reason is that the goal of a pump is to move fluid from one place to another, and the most important parameter is the pump's capacity (volume flow rate). On the other hand, for most turbines, we use C_P, the **power coefficient**, as the independent parameter. The reason is that the goal of a turbine is to rotate a shaft, and the most important parameter is the turbine's brake horsepower

Discussion There are exceptions. For example, when analyzing a positive-displacement turbine used to measure volume flow rate, capacity is more important than output shaft power, so one might use C_Q, instead of C_P as the independent parameter.

14-86

Solution We are to nondimensionalize a pump performance curve.

Analysis The pump's net head is approximated by the parabolic expression

Pump performance curve: $$H = H_0 - a\dot{V}^2 \qquad (1)$$

where shutoff head H_0 = 24.4 m of water column, coefficient a = 0.0678 m/Lpm^2, available pump head $H_{available}$ is in units of meters of water column, and capacity $\dot{V}$ is in units of liters per minute (Lpm). By definition, head coefficient C_H = $(gH)/(\omega^2 D^2)$, and capacity coefficient C_Q = $(\dot{V})/(\omega D^3)$. To convert, we must be careful with units. For example, we must convert the rotation rate from rpm to radians per second,

Rotational speed: $$\omega = 4200\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rot}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 439.823\frac{\text{rad}}{\text{s}}$$

Sample calculations at 14.0 Lpm are shown below.

Capacity coefficient at 14.0 *Lpm*:

$$C_Q = \frac{\dot{V}}{\omega D^3} = \frac{14.0\text{ L/min}}{(439.823\text{ rad/s})(0.0180\text{ m})^3}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.90966 \cong 0.0910$$

At this flow rate, the net head is obtained from Eq. 1,

Net head at 14.0 *Lpm*:

$$H = 24.4\text{ m} - \left(0.0678\text{ m/Lpm}^2\right)\left(14.0\text{ Lpm}\right)^2 = 11.111\text{ m} \cong 11.1\text{ m}$$

from which the head coefficient can be calculated,

Head coefficient at 14.0 *Lpm*:

$$C_H = \frac{gH}{\omega^2 D^2} = \frac{\left(9.81\text{ m/s}^2\right)\left(11.111\text{ m}\right)}{\left(439.823\text{ rad/s}\right)^2\left(0.0180\text{ m}\right)^2} = 1.7391 \cong 1.74$$

These calculations are repeated for a range of volume flow rates from 0 to $\dot{V}_{max}$. The nondimensionalized pump performance curve is plotted in Fig. 1.

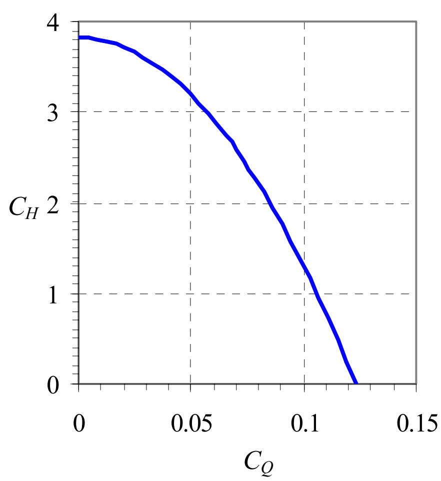


FIGURE 1
Nondimensionalized pump performance curve for Problem 14-86.

Discussion Since radians is a dimensionless unit, the units of C_H and C_Q are unity. In other words, C_H and C_Q are nondimensional parameters, as they must be.

14-87

Solution We are to calculate the specific speed of a water pump, and determine what kind of pump it is.

Analysis First, we use the values of C_H and C_Q calculated in Problem 14-86 at the BEP to calculate the dimensionless form of N_{Sp},

Dimensionless pump specific speed: $$N_{Sp} = \frac{C_Q^{\ 1/2}}{C_H^{\ 3/4}} = \frac{0.090966^{1/2}}{1.7391^{3/4}} = 0.1992$$

Thus, $\boldsymbol{N_{Sp}} = \mathbf{0.199}$, and is dimensionless. From the conversion given in the text, $\boldsymbol{N_{Sp,\mathrm{US}}} = \mathbf{0.1992 \times 2{,}734 = 545}$. Alternatively, we can use the original dimensional data to calculate $N_{Sp,\mathrm{US}}$,

Dimensional pump specific speed in customary US units:

$$N_{Sp,\mathrm{US}} = \frac{(\dot{n},\ \mathrm{rpm})(\dot{V},\ \mathrm{gpm})^{1/2}}{(H,\ \mathrm{ft})^{3/4}} = \frac{(4200\ \mathrm{rpm})\left(14.0\ \mathrm{Lpm}\left(\dfrac{0.2642\ \mathrm{gpm}}{\mathrm{Lpm}}\right)\right)^{1/2}}{\left(11.111\ \mathrm{m}\left(\dfrac{\mathrm{ft}}{0.3048\ \mathrm{m}}\right)\right)^{3/4}} = 545$$

From either the dimensionless or the dimensional pump specific speed, Fig. 14-73 shows that **this is definitely a centrifugal pump**.

Discussion We calculated the dimensional pump specific speed two ways as a check of our algebra, and (fortunately) the results agree.

14-88

Solution We are to nondimensionalize a fan performance curve.

Analysis The fan's net head is approximated by the parabolic expression

Fan performance curve: $$H = H_0 - a\dot{V}^2 \qquad (1)$$

where shutoff head H_0 = 60.0 mm of water column, coefficient $a = 2.50 \times 10^{-7}$ mm/Lpm^2, available fan head $H_{available}$ is in units of mm of water column, and capacity $\dot{V}$ is in units of liters per minute (Lpm). By definition, head coefficient $C_H = (gH)/(\omega^2 D^2)$, and capacity coefficient $C_Q = (\dot{V})/(\omega D^3)$. To convert, we must be careful with units. For example, we must convert the rotation rate from rpm to radians per second,

Rotational speed: $$\omega = 600\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rot}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 62.8319\frac{\text{rad}}{\text{s}}$$

Sample calculations at 13,600 Lpm are shown below.

Capacity coefficient at 13,600 *Lpm*:

$$C_Q = \frac{\dot{V}}{\omega D^3} = \frac{13{,}600\text{ L/min}}{(62.8319\text{ rad/s})(0.300\text{ m})^3}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.13361 \cong 0.134$$

At this flow rate, the net head is obtained from Eq. 1,

$$H = 60.0\text{ mm} - \left(2.50\times10^{-7}\text{ mm/Lpm}^2\right)\left(13{,}600\text{ Lpm}\right)^2 = 13.76\text{ mm H}_2\text{O}$$

from which the head coefficient can be calculated,

Head coefficient at 13,600 *Lpm*:

$$C_H = \frac{gH}{\omega^2 D^2} = \frac{(9.81\text{ m/s}^2)(0.01376\text{ m H}_2\text{O})}{(62.8319\text{ rad/s})^2(0.300\text{ m})^2}\frac{998.\text{ kg/m}^3\,(\text{m air})}{1.184\text{ kg/m}^3\,(\text{m H}_2\text{O})}$$
$$= 0.32023 \cong 0.320$$

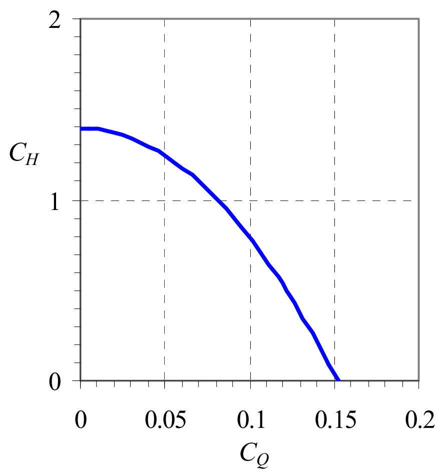


FIGURE 1
Nondimensionalized pump performance curve for Problem 14-88.

Note the ratio of water density to air density to convert the head from water column height to air column height. The calculations are repeated for a range of volume flow rates from 0 to $\dot{V}_{max}$. The nondimensionalized pump performance curve is plotted in Fig. 1.

Discussion Since radians is a dimensionless unit, the units of C_H and C_Q are unity. In other words, C_H and C_Q are nondimensional parameters, as they must be.

14-89

Solution We are to calculate the specific speed of a fan, and determine what kind of fan it is.

Analysis First, we use the values of C_H and C_Q calculated in Problem 14-88 at the BEP (to four significant digits) to calculate the dimensionless form of N_{Sp},

Dimensionless pump specific speed:
$$N_{Sp} = \frac{C_Q^{\ 1/2}}{C_H^{\ 3/4}} = \frac{0.13361^{1/2}}{0.32023^{3/4}} = 0.8587$$

Thus, $\boldsymbol{N_{Sp}}$ **= 0.859**, and is dimensionless. From the conversion given in the text, $\boldsymbol{N_{Sp,\mathrm{US}}}$ **= 0.8587 × 2,734 = 2350**. Alternatively, we can use the original dimensional data to calculate $N_{Sp,\mathrm{US}}$,

Dimensional pump specific speed in customary US units:

$$N_{Sp,\mathrm{US}} = \frac{\left(\dot{n},\ \mathrm{rpm}\right)\left(\dot{V},\ \mathrm{gpm}\right)^{1/2}}{\left(H,\ \mathrm{ft}\right)^{3/4}} = \frac{\left(600\ \mathrm{rpm}\right)\left(13,600\ \mathrm{Lpm}\left(\frac{0.2642\ \mathrm{gpm}}{\mathrm{Lpm}}\right)\right)^{1/2}}{\left(11.60\ \mathrm{m}\left(\frac{\mathrm{ft}}{0.3048\ \mathrm{m}}\right)\right)^{3/4}} = 2350$$

Note that we use H = 11.60 m *of air*, since air is the fluid being pumped here. We calculate H as $H_{\mathrm{air}} = H_{\mathrm{water}}(\rho_{\mathrm{water}}/\rho_{\mathrm{air}})$ = (0.01376 m of water)(998/1.184) = 11.60 m of air. From either the dimensionless or the dimensional pump specific speed, Fig. 14-73 shows that **this is most likely a centrifugal fan** (probably a squirrel cage fan).

Discussion We calculated the dimensional pump specific speed two ways as a check of our algebra, and (fortunately) the results agree.

14-90

Solution We are to calculate the specific speed of a water pump, and determine what kind of pump it is.

Analysis First, we use the values of C_H and C_Q calculated in Example 14-11 at the BEP to calculate the dimensionless form of N_{Sp},

Dimensionless pump specific speed:
$$N_{Sp} = \frac{C_Q^{\ 1/2}}{C_H^{\ 3/4}} = \frac{0.0112^{1/2}}{0.133^{3/4}} = 0.481$$

Thus, $\boldsymbol{N_{Sp}}$ **= 0.481**. From the conversion given in the text, $\boldsymbol{N_{Sp,\mathrm{US}}}$ **= 1310**. From Fig. 14-73, **this is most likely a centrifugal pump**.

Discussion From Fig. 14-73, the maximum efficiency one can expect from a centrifugal pump at this value of N_{Sp} is about 0.87 (87%). The actual pump efficiency is only 81.2% (from Example 14-11), so there is room for improvement in the design.

14-91

Solution We are to decide what king of pump should be designed for given performance criteria.

Analysis We calculate the pump specific speed at the given conditions,

$$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}}$$

$$= \frac{(1200 \text{ rpm})\left(\frac{2\pi}{60} \frac{\text{rad/s}}{\text{rpm}}\right)\left(18.0 \text{ L/min}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right)^{1/2}}{\left((9.81 \text{ m/s}^2)(1.6 \text{ m})\right)^{3/4}} = 0.276$$

From Fig. 14-73, we see that **Len should design a centrifugal pump**. **The maximum pump efficiency is about 0.75 (75%)** (based on Fig. 14-73 again).

Discussion This value of N_{Sp} is, in fact, on the low end of the curve for centrifugal pumps, so a centrifugal pump is the best Len can do, in spite of the low efficiency.

14-92E

Solution We are to decide what king of pump should be designed for given performance criteria.

Properties The density of water at 77°F is 62.24 lbm/ft^3.

Analysis We calculate the pump specific speed at the given conditions,

$$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}}$$

$$= \frac{(300\text{ rpm})\left(\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rpm}}\right)\left(2{,}500\text{ gal/min}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right)^{1/2}}{\left((32.174\text{ ft/s}^2)(45\text{ ft})\right)^{3/4}} = 0.316$$

From Fig. 14-73, we choose **a centrifugal pump**. **The maximum pump efficiency is about 0.82 (82%)** (based on Fig. 14-73 again).

From the definition of pump efficiency, $bhp = \eta_{\text{pump}} \rho \dot{V} gH$

Brake horsepower required:

$$bhp = \frac{\rho g H \dot{V}}{\eta_{\text{pump}}} = \frac{(62.24\text{ lbm/ft}^3)(32.174\text{ ft/s}^2)(45\text{ ft})(5.57\text{ ft}^3\text{/s})}{0.82}\left(\frac{\text{lbf}\cdot\text{s}^2}{32.174\text{ lbm}\cdot\text{ft}}\right)$$

$$= 19{,}025\text{ ft}\cdot\text{lbf/s}\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lbf/s}}\right) = 34.6\text{ hp}$$

Thus, we expect that **the pump will require about 35 hp to turn the shaft**.

Discussion Most large water pumps are of the centrifugal variety. This problem may also be solved in terms of the dimensional pump specific speed in customary US units: $N_{Sp,\text{US}} = 864$.

14-93

Solution We are to calculate the performance of a modified pump.

Assumptions **1** The modified pump operates at the best efficiency point. **2** Pump diameter and fluid properties remain the same.

Analysis At homologous points, the affinity laws are used to estimate the operating conditions of the modified pump. We let the original pump be Pump A, and the modified pump be Pump B. We get

Volume flow rate:
$$\dot{V}_{\mathrm{B}} = \dot{V}_{\mathrm{A}} \frac{\omega_{\mathrm{B}}}{\omega_{\mathrm{A}}} \underbrace{\left(\frac{D_{\mathrm{B}}}{D_{\mathrm{A}}}\right)^3}_{1} = (18.\ \text{L/min}) \frac{600\ \text{rpm}}{1200\ \text{rpm}} = 9.0\ \text{L/min}$$

and

Net head:
$$H_{\mathrm{B}} = H_{\mathrm{A}} \left(\frac{\omega_{\mathrm{B}}}{\omega_{\mathrm{A}}}\right)^2 \underbrace{\left(\frac{D_{\mathrm{B}}}{D_{\mathrm{A}}}\right)^2}_{1} = (1.6\ \text{m}) \left(\frac{600\ \text{rpm}}{1200\ \text{rpm}}\right)^2 = 0.40\ \text{m}$$

The volume flow rate of the modified pump is 9.0 L/min; the net head is 0.40 m. The pump specific speed of the modified pump is

$$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}} = \frac{(600\ \text{rpm})\left(\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rpm}}\right)\left(9.0\ \text{L/min}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right)^{1/2}}{\left((9.81\ \text{m/s}^2)(0.40\ \text{m})\right)^{3/4}} = 0.276$$

Thus, $\boldsymbol{N_{Sp}}$ **= 0.276**, which is the same as that of the original pump. This is not surprising since the two pumps operate at homologous points.

Discussion When the rpm is cut in half, all else being equal, the volume flow rate of the pump decreases by a factor of two, while the net head decreases by a factor of four. This agrees with the "jingle" of Fig. 14-74. The specific speed of the two pumps must match since they operate at homologous points.

14-94

Solution We are to prove the relationship between turbine specific speed and pump specific speed.

Assumptions **1** The parameters such as net head, volume flow rate, diameter, etc. are defined in similar fashion for the pump and the turbine.

Analysis First we write the definitions of pump specific speed and turbine specific speed,

Pump specific speed:
$$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}} \quad (1)$$

and

Turbine specific speed:
$$N_{St} = \frac{\omega (bhp)^{1/2}}{(\rho)^{1/2}(gH)^{5/4}} \quad (2)$$

After some rearrangement of Eq. 2,

$$N_{St} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}}\left(\frac{bhp}{\rho g H \dot{V}}\right)^{1/2} \quad (3)$$

We recognize the first grouping of terms in Eq. 3 as N_{Sp} and the second grouping of terms as the square root of turbine efficiency η_{turbine}. Thus,

Final relationship:
$$\boxed{N_{St} = N_{Sp}\sqrt{\eta_{\text{turbine}}}} \quad (4)$$

Discussion Since turbine efficiency is typically large (90 to 95% for large hydroturbines), pump specific speed and turbine specific speed are nearly equivalent. Note that Eq. 4 does *not* apply to a pump running backwards as a turbine or vice-versa. Such devices, called *pump-turbines*, are addressed in Problem 14-95.

14-95

Solution We are to prove the relationship between turbine specific speed and pump specific speed for the case of a pump-turbine operating at the same volume flow rate and rotational speed when acting as a pump and as a turbine.

Assumptions **1** The parameters such as net head, volume flow rate, diameter, etc. are defined in similar fashion for the pump and the turbine.

Analysis First we write the definitions of pump specific speed and turbine specific speed,

Pump specific speed:
$$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH_{\text{pump}})^{3/4}} \quad (1)$$

and

Turbine specific speed:
$$N_{St} = \frac{\omega\left(bhp_{\text{turbine}}\right)^{1/2}}{\left(\rho\right)^{1/2}\left(gH_{\text{turbine}}\right)^{5/4}} \qquad (2)$$

Note that we have added subscripts "pump" and "turbine" on net head and brake horsepower since $H_{\text{pump}} \neq H_{\text{turbine}}$ and $bhp_{\text{pump}} \neq bhp_{\text{turbine}}$. After some rearrangement of Eq. 2,

$$N_{St} = \frac{\omega \dot{V}^{1/2}}{\left(gH_{\text{turbine}}\right)^{3/4}}\left(\frac{bhp_{\text{turbine}}}{\rho g H_{\text{turbine}} \dot{V}}\right)^{1/2} = \frac{\omega \dot{V}^{1/2}}{\left(gH_{\text{turbine}}\right)^{3/4}}\left(\eta_{\text{turbine}}\right)^{1/2} \qquad (3)$$

We also write the definitions of pump efficiency and turbine efficiency,

$$\eta_{\text{pump}} = \frac{\rho g \dot{V} H_{\text{pump}}}{bhp_{\text{pump}}} \qquad \eta_{\text{turbine}} = \frac{bhp_{\text{turbine}}}{\rho g \dot{V} H_{\text{turbine}}} \qquad (4)$$

We solve both parts of Eq. 4 for $\dot{V}$ and equate the two, since $\dot{V}$ is the same whether the pump-turbine is operating as a pump or as a turbine. Eliminating $\dot{V}$,

$$H_{\text{turbine}} = \frac{H_{\text{pump}} bhp_{\text{turbine}}}{bhp_{\text{pump}} \eta_{\text{pump}} \eta_{\text{turbine}}} \qquad (5)$$

where ρ and g have also dropped out since they are the same for both cases. We plug Eq. 5 into Eq. 3 and rearrange,

$$N_{St} = \frac{\omega \dot{V}^{1/2}}{\left(gH_{\text{pump}}\right)^{3/4}}\left(\eta_{\text{turbine}}\right)^{1/2}\left(\frac{bhp_{\text{pump}}\eta_{\text{pump}}\eta_{\text{turbine}}}{bhp_{\text{turbine}}}\right)^{3/4} \qquad (6)$$

We recognize the first grouping of terms in Eq. 6 as N_{Sp} and rearrange,

Final relationship:
$$\boxed{N_{St} = N_{Sp}\left(\eta_{\text{turbine}}\right)^{5/4}\left(\eta_{\text{pump}}\right)^{3/4}\left(\frac{bhp_{\text{pump}}}{bhp_{\text{turbine}}}\right)^{3/4}} \qquad (7)$$

An alternate expression is obtained in terms of net heads by substitution of Eq. 5,

Alternate final relationship:
$$\boxed{N_{St} = N_{Sp}\sqrt{\eta_{\text{turbine}}}\left(\frac{H_{\text{pump}}}{H_{\text{turbine}}}\right)^{3/4}} \qquad (8)$$

Discussion It is difficult to achieve high efficiency in a pump-turbine during both the pump duty cycle *and* the turbine duty cycle. To achieve the highest possible efficiency, it is critical that both N_{Sp} and N_{St} are appropriate for the chosen design, e.g. radial flow centrifugal pump and radial-flow Francis turbine.

14-96

Solution We are to apply conversion factors to prove a conversion factor.

Properties We set g = 32.174 ft/s^2 and assume water at density ρ = 1.94 slug/ft^3.

Analysis We convert $N_{St,\mathrm{US}}$ to N_{St} by dividing by $g^{5/4}$ and $\rho^{1/2}$, and then using conversion ratios to cancel all units,

$$N_{St} = \underbrace{\frac{(\omega,\ \text{rot/min})(bhp,\ \text{hp})^{1/2}}{(H,\ \text{ft})^{5/4}}}_{N_{St,\mathrm{US}}} \frac{1}{\left(1.94\ \text{slug/ft}^3\right)^{1/2}\left(32.174\ \text{ft/s}^2\right)^{5/4}}$$

$$\times\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{\text{slug ft}}{\text{s}^2\ \text{lbf}}\right)^{1/2}\left(\frac{550\ \text{ft lbf}}{\text{s hp}}\right)^{1/2}\left(\frac{2\pi\ \text{rad}}{\text{s}}\right) \quad (2)$$

$$N_{St} = 0.02301 N_{St,\mathrm{US}}$$

Finally, the inverse of Eq. 2 yields the desired conversion factor.

Discussion As discussed in the text, some turbomachinery authors do not convert rotations to radians, introducing a confusing factor of 2π into the conversion.

14-97

Solution We are to calculate turbine specific speed for a given hydroturbine and compare to the guidelines of Fig. 14-108.

Assumptions **1** The fluid is water at 20°C.

Properties The density of water at 20°C is ρ = 998.0 kg/m^3.

Analysis First we convert the turbine rotation rate from rpm to radians/s,

Rotational speed:
$$\omega = 180\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rot}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 18.85\ \text{rad/s}$$

With all other parameters already given in SI units, we calculate turbine specific speed,

$$N_{St} = \frac{\omega(bhp)^{1/2}}{(\rho)^{1/2}(gH)^{5/4}} = \frac{(18.85\ \text{rad/s})\left(119\times10^6\ \text{W}\right)^{1/2}}{\left(998\ \text{kg/m}^3\right)^{1/2}\left[\left(9.81\ \text{m/s}^2\right)(105\ \text{m})\right]^{5/4}}\left(\frac{\text{m}^2\text{kg}}{\text{s}^3\cdot\text{W}}\right)^{1/2} = \mathbf{1.12}$$

Comparing to the turbine specific speed guidelines of Fig. 14-108, we see that this turbine should be of the Francis type, which it is.

Discussion In fact, the turbine specific speed of this turbine is close to that which yields the maximum possible efficiency for a Francis turbine.

14-98

Solution We are to calculate turbine specific speed for a given hydroturbine and compare to the guidelines of Fig. 14-108.

Assumptions **1** The fluid is water at 20°C.

Properties The density of water at 20°C is ρ = 998.0 kg/m^3.

Analysis First we convert the turbine rotation rate from rpm to radians/s,

Rotational speed:
$$\omega = 100\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rot}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10.47\text{ rad/s}$$

With all other parameters already given in SI units, we calculate turbine specific speed,

$$N_{St} = \frac{\omega(bhp)^{1/2}}{(\rho)^{1/2}(gH)^{5/4}} = \frac{(10.47\text{ rad/s})(194\times10^6\text{ W})^{1/2}}{(998\text{ kg/m}^3)^{1/2}\left[(9.81\text{ m/s}^2)(54.9\text{ m})\right]^{5/4}}\left(\frac{\text{m}^2\text{kg}}{\text{s}^3\cdot\text{W}}\right)^{1/2} = \mathbf{1.78}$$

Comparing to the turbine specific speed guidelines of Fig. 14-108, we see that this turbine should be of the Francis type, which it is.

Discussion The turbine specific speed of this turbine is close to the crossover point between a Francis turbine and a Kaplan turbine.

14-99

Solution We are to calculate turbine specific speed for a given hydroturbine and compare to the guidelines of Fig. 14-108.

Assumptions **1** The fluid is water at 20°C.

Properties The density of water at 20°C is ρ = 998.0 kg/m^3.

Analysis First we convert the turbine rotation rate from rpm to radians/s,

Rotational speed:
$$\omega = 100\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rot}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10.47\text{ rad/s}$$

With all other parameters already given in SI units, we calculate turbine specific speed,

$$N_{St} = \frac{\omega(bhp)^{1/2}}{(\rho)^{1/2}(gH)^{5/4}} = \frac{(10.47\text{ rad/s})(5.37\times10^6\text{ W})^{1/2}}{(998\text{ kg/m}^3)^{1/2}\left[(9.81\text{ m/s}^2)(9.75\text{ m})\right]^{5/4}}\left(\frac{\text{m}^2\text{kg}}{\text{s}^3\cdot\text{W}}\right)^{1/2} = \mathbf{2.57}$$

Comparing to the turbine specific speed guidelines of Fig. 14-108, we see that this turbine should be of the Kaplan type, which it is.

Discussion In fact, the turbine specific speed of this turbine is close to that which yields the maximum possible efficiency for a Kaplan turbine.

14-100

Solution We are to calculate the specific speed of a turbine, and compare it to the normal range.

Analysis We first calculate the nondimensional form of N_{St},

$$N_{St} = \frac{\omega\left(bhp\right)^{1/2}}{\rho^{1/2}\left(gH\right)^{5/4}}$$

$$= \frac{\left(12.57\text{ rad/s}\right)\left(4.61\times10^{8}\text{ W}\right)^{1/2}}{\left(998.0\text{ kg/m}^3\right)^{1/2}\left[\left(9.81\text{ m/s}^2\right)\left(78.6\text{ m}\right)\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{2.10}$$

From Fig. 14-108, this is on the high end for Francis turbines – the designers may wish to consider a Kaplan turbine instead. From the conversion given in the text, $\boldsymbol{N_{St,\text{US}} = 2.10 \times 43.46 = 91.4}$. Alternatively, we can use the original dimensional data to calculate $N_{St,\text{US}}$,

$$N_{St,\text{US}} = \frac{\left(\dot{n},\text{ rpm}\right)\left(bhp,\text{ hp}\right)^{1/2}}{\left(H,\text{ ft}\right)^{5/4}} = \frac{\left(120\text{ rpm}\right)\left(6.18\times10^{5}\text{ hp}\right)^{1/2}}{\left(78.6\text{ m}\left(\frac{\text{ft}}{0.3048\text{ m}}\right)\right)^{5/4}} = \mathbf{91.3}$$

Note that the actual values of brake horsepower and net head will differ from the values calculated here, because we have neglected irreversible losses; thus, the values of N_{St} and $N_{St,\text{US}}$ may change slightly.

Discussion We calculated the dimensional pump specific speed two ways as a check of our algebra, and (fortunately) the results agree (within round-off error).

14-101

Solution We are to calculate the specific speed of a turbine, and compare it to the normal range.

Analysis We first calculate the nondimensional form of N_{St},

$$N_{St} = \frac{\omega (bhp)^{1/2}}{\rho^{1/2} (gH)^{5/4}}$$

$$= \frac{(10.47 \text{ rad/s})(2.98 \times 10^7 \text{ W})^{1/2}}{(998.0 \text{ kg/m}^3)^{1/2} \left[(9.81 \text{ m/s}^2)(38.05 \text{ m})\right]^{5/4}} \left(\frac{\text{kg} \cdot \text{m}^2}{\text{W} \cdot \text{s}^3}\right)^{1/2} = \mathbf{1.10}$$

From Fig. 14-108, **this is in the range of the typical Francis turbine**. From the conversion given in the text, $\boldsymbol{N_{St,\text{US}}}$ **= 1.10 × 43.46 = 47.9**. Alternatively, we can use the original dimensional data to calculate $N_{St,\text{US}}$,

$$N_{St,\text{US}} = \frac{(\dot{n}, \text{rpm})(bhp, \text{hp})^{1/2}}{(H, \text{ft})^{5/4}} = \frac{(100 \text{ rpm})(4.00 \times 10^4 \text{ hp})^{1/2}}{(124.8 \text{ ft})^{5/4}} = \mathbf{47.9}$$

Note that the actual values of brake horsepower and net head will differ from the values calculated here, because we have neglected irreversible losses; thus, the values of N_{St} and $N_{St,\text{US}}$ may change slightly.

Discussion We calculated the dimensional pump specific speed two ways as a check of our algebra, and the results agree.

14-102E

Solution We are to calculate the specific speed of a turbine, and compare it to the normal range.

Analysis We first calculate the specific speed in customary US units,

$$N_{St,\text{US}} = \frac{(\dot{n}, \text{rpm})(bhp, \text{hp})^{1/2}}{(H, \text{ft})^{5/4}} = \frac{(120 \text{ rpm})(202,700 \text{ hp})^{1/2}}{(170.8 \text{ ft})^{5/4}} = \mathbf{87.5}$$

From Fig. 14-108, **this is on the high end of typical Francis turbines, on the border between Francis and Kaplan turbines**. In nondimensional terms, $\boldsymbol{N_{St,\text{US}}}$ **= 87.5 / 43.46 = 2.01**. Note that the actual values of brake horsepower and net head will differ from the values calculated here, because we have neglected irreversible losses; thus, the values of N_{St} and $N_{St,\text{US}}$ may change slightly.

Discussion The actual values of brake horsepower and net head will differ from the values calculated here, because we have neglected irreversible losses; thus, the values of N_{St} and $N_{St,\text{US}}$ may change slightly.

14-103

Solution We are to calculate the specific speed of a turbine, and compare it to the normal range.

Analysis We first calculate the nondimensional form of N_{St},

$$N_{St} = \frac{\omega (bhp)^{1/2}}{\rho^{1/2} (gH)^{5/4}} = \frac{(18.85 \text{ rad/s})(2.46\times10^8 \text{ W})^{1/2}}{(998.0 \text{ kg/m}^3)^{1/2}\left[(9.81 \text{ m/s}^2)(73.8 \text{ m})\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{2.49}$$

From Fig. 14-108, this is much higher than the typical Francis turbine – **the designers should consider a Kaplan turbine instead**. From the conversion given in the text, $\boldsymbol{N_{St,\mathrm{US}} = 2.49 \times 43.46 = 108}$. Alternatively, we can use the original dimensional data to calculate $N_{St,\mathrm{US}}$,

$$N_{St,\mathrm{US}} = \frac{(\dot{n}, \text{rpm})(bhp, \text{hp})^{1/2}}{(H, \text{ft})^{5/4}} = \frac{(180 \text{ rpm})(3.29\times10^5 \text{ hp})^{1/2}}{\left(73.8 \text{ m}\left(\frac{\text{ft}}{0.3048 \text{ m}}\right)\right)^{5/4}} = \mathbf{108}$$

Note that the actual values of brake horsepower and net head will differ from the values calculated here, because we have neglected irreversible losses; thus, the values of N_{St} and $N_{St,\mathrm{US}}$ may change slightly.

Discussion We calculated the dimensional pump specific speed two ways as a check of our algebra, and the results agree.

14-104

Solution We are to determine a turbine's efficiency and what kind of turbine is being tested.

Properties The density of water at T = 20°C is 998.0 kg/m^3.

Analysis The turbine's efficiency is calculated first,

$$\eta_{\text{turbine}} = \frac{bhp}{\rho g H \dot{V}}$$

$$= \frac{450 \text{ W}}{\left(998.0 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(15.0 \text{ m}\right)\left(17.0 \text{ m}^3/\text{hr}\right)}\left(\frac{3600 \text{ s}}{\text{hr}}\right)\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{64.9\%}$$

After converting 1500 rpm to 157.1 rad/s, we calculate the nondimensional form of turbine specific speed,

$$N_{St} = \frac{\omega\left(bhp\right)^{1/2}}{\rho^{1/2}\left(gH\right)^{5/4}} = \frac{\left(157.1 \text{ rad/s}\right)\left(450 \text{ W}\right)^{1/2}}{\left(998.0 \text{ kg/m}^3\right)^{1/2}\left[\left(9.81 \text{ m/s}^2\right)\left(15.0 \text{ m}\right)\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{0.206}$$

which we convert to customary US units, $\boldsymbol{N_{St,\text{US}}}$ **= 0.206 × 43.46 = 8.94**. This is most likely an **impulse turbine** (e.g., a Pelton wheel turbine).

Discussion We could instead have used the dimensional units to directly calculate the turbine specific speed in customary US units. The result would be the same.

14-105

Solution We are to scale up the model turbine tests to the prototype turbine.

Assumptions **1** The prototype and model are geometrically similar. **2** The tests are conducted under conditions of dynamic similarity. **3** The water is at the same temperature for both the model and the prototype (20°C).

Properties The density of water at T = 20°C is 998.0 kg/m^3.

Analysis We use the turbine scaling laws, starting with the head coefficient, and letting the model be turbine A and the prototype be turbine B,

$$C_{H,\mathrm{A}} = \frac{gH_\mathrm{A}}{\omega_\mathrm{A}{}^2 D_\mathrm{A}{}^2} = \frac{gH_\mathrm{B}}{\omega_\mathrm{B}{}^2 D_\mathrm{B}{}^2} = C_{H,\mathrm{B}} \quad \rightarrow \quad \omega_\mathrm{B} = \omega_\mathrm{A} \frac{D_\mathrm{A}}{D_\mathrm{B}} \sqrt{\frac{H_\mathrm{B}}{H_\mathrm{A}}}$$

$$\rightarrow \quad \omega_\mathrm{B} = \left(1500 \text{ rpm}\right) \frac{1}{5} \sqrt{\frac{50 \text{ m}}{15.0 \text{ m}}} = \mathbf{548\,rpm}$$

We then use the capacity coefficient to calculate the volume flow rate of the prototype,

$$C_{Q,\mathrm{A}} = \frac{\dot{V}_\mathrm{A}}{\omega_\mathrm{A} D_\mathrm{A}{}^3} = \frac{\dot{V}_\mathrm{B}}{\omega_\mathrm{B} D_\mathrm{B}{}^3} = C_{Q,\mathrm{B}} \quad \rightarrow \quad \dot{V}_\mathrm{B} = \dot{V}_\mathrm{A} \frac{\omega_\mathrm{B}}{\omega_\mathrm{A}} \left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^3$$

$$\rightarrow \quad \dot{V}_\mathrm{B} = \left(17.0 \text{ m}^3\text{/hr}\right) \frac{548 \text{ rpm}}{1500 \text{ rpm}} \left(\frac{5}{1}\right)^3 = \mathbf{776\,m^3/hr}$$

Finally, we use the power coefficient to calculate the brake horsepower of the prototype,

$$C_{P,\mathrm{A}} = \frac{bhp_\mathrm{A}}{\omega_\mathrm{A}{}^3 D_\mathrm{A}{}^5} = \frac{bhp_\mathrm{B}}{\omega_\mathrm{B}{}^3 D_\mathrm{B}{}^5} = C_{P,\mathrm{B}} \quad \rightarrow \quad bhp_\mathrm{B} = bhp_\mathrm{A} \left(\frac{\omega_\mathrm{B}}{\omega_\mathrm{A}}\right)^3 \left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^5$$

$$\rightarrow \quad \dot{V}_\mathrm{B} = \left(450 \text{ W}\right) \left(\frac{548 \text{ rpm}}{1500 \text{ rpm}}\right)^3 \left(\frac{5}{1}\right)^5 = \mathbf{68{,}500\,W}$$

Discussion All results are given to 3 significant digits, but we kept several extra digits in the intermediate calculations to reduce round-off errors.

14-106

Solution We are to compare the model and prototype efficiency and turbine specific speed to prove that they operate at homologous points.

Properties The density of water at $T = 20^{\circ}$C is 998.0 kg/m^3.

Analysis The model turbine's efficiency was calculated in Problem 14-104 as 64.9%. We calculate the prototype turbine's efficiency as

$$\eta_{\text{turbine}} = \frac{bhp}{\rho g H \dot{V}}$$

$$= \frac{68{,}500 \text{ W}}{\left(998.0 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(50 \text{ m}\right)\left(776 \text{ m}^3\text{/hr}\right)}\left(\frac{3600 \text{ s}}{\text{hr}}\right)\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{64.9\%}$$

The efficiency of the prototype is the same as that of the model. Similarly, the turbine specific speed of the model turbine was calculated previously as 0.206. After converting 548 rpm to 57.36 rad/s, we calculate the nondimensional form of turbine specific speed for the prototype turbine,

$$N_{St} = \frac{\omega\left(bhp\right)^{1/2}}{\rho^{1/2}\left(gH\right)^{5/4}} = \frac{\left(57.36 \text{ rad/s}\right)\left(68{,}500 \text{ W}\right)^{1/2}}{\left(998.0 \text{ kg/m}^3\right)^{1/2}\left[\left(9.81 \text{ m/s}^2\right)\left(50 \text{ m}\right)\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{0.206}$$

Which is also the same as the previous calculation. Comparing to the results of Problem 14-104, we see that both η_{turbine} and N_{st} match between the model and the prototype. Thus, **the model and the prototype operate at homologous points**.

Discussion Other nondimensional turbine parameters, like head coefficient, capacity coefficient, and power coefficient also match between model and prototype. We could instead have used dimensional units to calculate the turbine specific speed in customary US units.

14-107

Solution We are to estimate the actual efficiency of the prototype, and explain why it is higher than the model efficiency.

Analysis We apply the Moody efficiency correction equation,

$$\eta_{\text{turbine, prototype}} \approx 1-\left(1-\eta_{\text{turbine, model}}\right)\left(\frac{D_{\text{model}}}{D_{\text{prototype}}}\right)^{1/5} = 1-\left(1-0.649\right)\left(\frac{1}{5}\right)^{1/5} = \mathbf{74.6\%}$$

This represents an increase of 74.6 – 64.9 = 9.7%. However, as mentioned in the text, we expect only about 2/3 of this increase, or 2(9.7%)/3 = 6.5%. Thus, our best estimate of the actual efficiency of the prototype is

$$\eta_{\text{turbine, prototype}} \approx 64.9+6.5 = \mathbf{71.4\%}$$

There are several reasons for this increase in efficiency: The prototype turbine often operates at high Reynolds numbers that are not achievable in the laboratory. We know from the Moody chart that friction factor decreases with Re, as does boundary layer thickness. Hence, the influence of viscous boundary layers is less significant as turbine size increases, since the boundary layers occupy a less significant percentage of the flow path through the runner. In addition, the relative roughness (ε/D) on the surfaces of the prototype runner blades may be significantly smaller than that on the model turbine unless the model surfaces are micropolished. Finally, large full scale turbines have smaller tip clearances relative to blade diameter; therefore tip losses and leakage are less significant.

Discussion The increase in efficiency between the model and prototype is significant, and helps us to understand why very large hydroturbines can be extremely efficient devices.

14-108

Solution We are to design a new hydroturbine by scaling up an existing hydroturbine. Specifically we are to calculate the new turbine diameter, volume flow rate, and brake horsepower.

Assumptions **1** The new turbine is geometrically similar to the existing turbine. **2** Reynolds number effects and roughness effects are negligible. **3** The new penstock is also geometrically similar to the existing penstock so that the flow entering the new turbine (velocity profile, turbulence intensity, etc.) is similar to that of the existing turbine.

Properties The density of water at 20°C is $\rho = 998.0\ \text{kg/m}^3$.

Analysis Since the new turbine (B) is dynamically similar to the existing turbine (A), we are concerned with only one particular homologous operating point of both turbines, namely the best efficiency point. We solve Eq. 14-38b for D_B,

$$D_B = D_A \sqrt{\frac{H_B}{H_A}}\left(\frac{\dot{n}_A}{\dot{n}_B}\right) = (1.50\ \text{m})\sqrt{\frac{110\ \text{m}}{90.0\ \text{m}}}\left(\frac{150\ \text{rpm}}{120\ \text{rpm}}\right) = 2.0729 \cong \mathbf{2.07\ m}$$

We then solve Eq. 14-38a for $\dot{V}_B$,

$$\dot{V}_B = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \left(162\ \text{m}^3/\text{s}\right)\left(\frac{120\ \text{rpm}}{150\ \text{rpm}}\right)\left(\frac{2.0729\ \text{m}}{1.50\ \text{m}}\right)^3$$
$$= 342.027\frac{\text{m}^3}{\text{s}} \cong \mathbf{342\ \frac{m^3}{s}}$$

Finally, we solve Eq. 14-38c for bhp_B,

$$bhp_B = bhp_A\left(\frac{\rho_B}{\rho_A}\right)\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$
$$= 132\ \text{MW}\left(\frac{998.0\ \text{kg/m}^3}{998.0\ \text{kg/m}^3}\right)\left(\frac{120\ \text{rpm}}{150\ \text{rpm}}\right)^3\left(\frac{2.0729\ \text{m}}{1.50\ \text{m}}\right)^5$$
$$= 340.62\ \text{MW} \cong \mathbf{341\ MW}$$

Discussion To avoid round-off errors in the calculations, we saved several more significant digits for D_B than are reported here.

14-109

Solution We are to compare efficiencies for two geometrically similar turbines, and discuss the Moody efficiency correction.

Analysis We calculate the turbine efficiency for both turbines,

$$\eta_{\text{turbine A}} = \frac{bhp_{\text{A}}}{\rho_{\text{A}} g H_{\text{A}} \dot{V}_{\text{A}}}$$

$$= \frac{132\times10^6\ \text{W}}{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(90\ \text{m}\right)\left(162\ \text{m}^3\text{/s}\right)}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{92.5\%}$$

and

$$\eta_{\text{turbine B}} = \frac{bhp_{\text{B}}}{\rho_{\text{B}} g H_{\text{B}} \dot{V}_{\text{B}}}$$

$$= \frac{340.62\times10^6\ \text{W}}{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(110\ \text{m}\right)\left(342.027\ \text{m}^3\text{/s}\right)}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{92.5\%}$$

As expected, the two efficiencies are the same, since we have assumed dynamic similarity. However, total dynamic similarity may not actually be achieved between the two turbines because of scale effects (larger turbines generally have higher efficiency). The diameter of the new turbine is about 38% greater than that of the existing turbine, so the increase in efficiency due to turbine size should not be very significant. We verify this by using the Moody efficiency correction equation, considering turbine A as the "model" and B as the "prototype",

Efficiency correction:

$$\eta_{\text{turbine, B}} \approx 1-\left(1-\eta_{\text{turbine, A}}\right)\left(\frac{D_{\text{A}}}{D_{\text{B}}}\right)^{1/5} = 1-\left(1-0.925\right)\left(\frac{1.50\ \text{m}}{2.07\ \text{m}}\right)^{1/5} = \mathbf{0.930}$$

or **93.0%**. So, the first-order correction yields a predicted efficiency for the larger turbine that is about half of a percent greater than the smaller turbine. However, as mentioned in the text, we expect only about 2/3 of this increase, or 2(0.5%)/3 = 0.3%. Thus, our best estimate of the actual efficiency of the prototype is

$$\eta_{\text{turbine B}} \approx 92.5 + 0.3 = \mathbf{92.8\%}$$

Thus, our best estimate indicates that the new larger turbine will be slightly more efficient than its smaller brother, but the increase is only about 0.3%.

Discussion If the flow entering the new turbine from the penstock were not similar to that of the existing turbine (e.g., velocity profile and turbulence intensity), we could not expect exact dynamic similarity.

14-110

Solution The turbine specific speed of two dynamically similar turbines is to be compared, and the most likely type of turbine is to be determined.

Properties The density of water at $T = 20^{\circ}$C is $\rho = 998.0$ kg/m^3.

Analysis We calculate the dimensionless turbine specific speed for turbine A,

$$N_{St,\mathrm{A}} = \frac{\omega_\mathrm{A}\left(bhp_\mathrm{A}\right)^{1/2}}{\left(\rho_\mathrm{A}\right)^{1/2}\left(gH_\mathrm{A}\right)^{5/4}}$$

$$= \frac{\left(15.71\ \text{rad/s}\right)\left(132\times10^6\ \text{W}\right)^{1/2}}{\left(998.0\ \text{kg/m}^3\right)^{1/2}\left[\left(9.81\ \text{m/s}^2\right)\left(90.0\ \text{m}\right)\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{1.19}$$

and for turbine B,

$$N_{St,\mathrm{B}} = \frac{\omega_\mathrm{B}\left(bhp_\mathrm{B}\right)^{1/2}}{\left(\rho_\mathrm{B}\right)^{1/2}\left(gH_\mathrm{B}\right)^{5/4}}$$

$$= \frac{\left(12.57\ \text{rad/s}\right)\left(341\times10^6\ \text{W}\right)^{1/2}}{\left(998.0\ \text{kg/m}^3\right)^{1/2}\left[\left(9.81\ \text{m/s}^2\right)\left(110\ \text{m}\right)\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{1.19}$$

We see that the turbine specific speed of the two turbines is the same. Finally, we calculate the turbine specific speed in customary US units,

$$N_{St,\mathrm{US,A}} = N_{St,\mathrm{US,B}} = 43.46 N_{St} = \left(43.46\right)\left(1.19\right) = \mathbf{51.6}$$

From Fig. 14-108, both of these turbines are most likely **Francis turbines**.

Discussion Since turbine A and turbine B operate at homologous points, it is no surprise that their turbine specific speeds are the same. In fact, if they weren't the same, it would be a sure sign of an algebraic or calculation error.

Review Problems

14-111C

Solution

(*a*) *True*: As the gears turn, they direct a closed volume of fluid from the inlet to the outlet of the gear pump.

(*b*) *True* or *False*: Rotary pumps can be either positive displacement or dynamic (an unfortunate use of terminology). As a positive-displacement pump, the rotors direct a closed volume of fluid from the inlet to the outlet of the rotary pump. As a dynamic pump, "rotary pump" is sometimes used in place of the more correct term, "rotodynamic pump".

(*c*) *True*: At a given rotational speed, the volume flow rate of a positive-displacement pump is fairly constant regardless of load because of the fixed closed volume.

(*d*) *False*: Actually, the net head *increases* with fluid viscosity, because high viscosity fluids cannot penetrate the gaps as easily.

14-112C

Solution We are to discuss a water meter from the point of view of a piping system.

Analysis Although a water meter is a type of turbine, when analyzing pipe flow systems, we would think of the water meter as **a type of minor loss in the system**, just as a valve, elbow, etc. would be a minor loss, since there is a pressure drop associated with flow through the water meter.

Discussion In fact, manufacturers of water meters provide minor loss coefficients for their products.

14-113C

Solution We are to discuss the purpose of pump and turbine specific speeds.

Analysis Pump specific speed is used to characterize the operation of a pump at its optimum conditions (best efficiency point), and is useful for **preliminary pump selection**. Likewise, turbine specific speed is used to characterize the operation of a turbine at its optimum conditions (best efficiency point), and is useful for **preliminary turbine selection.**

Discussion Pump specific speed and turbine specific speed are parameters that can be calculated quickly. Based on the value obtained, one can quickly select the type of pump or turbine that is most appropriate for the given application.

14-114C

Solution We are to discuss the definition and usefulness of a pump-turbine.

Analysis A pump-turbine is **a turbomachine that can run both as a *pump* and as a *turbine*** (by running in the opposite direction). A pump-turbine is used by some power plants for **energy storage**; specifically, water is pumped by the pump-turbine during periods of low demand for power, and electricity is generated by the pump-turbine during periods of high demand for power.

Discussion We note that energy is “lost” both ways – when the pump-turbine is acting as a pump, and when it is acting as a turbine. Nevertheless, the energy storage scheme may still be cost-effective and profitable, in spite of the energy losses, because it may enable a power company to delay construction of costly new power-production facilities.

14-115

Solution We are to prove a relationship between two dynamically similar pumps, and discuss its application to turbines.

Assumptions **1** The two pumps are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two pumps are not grossly different from each other in size, capacity, etc.).

Analysis Since the two pumps are dynamically similar, dimensionless pump parameter C_H must be the same for both pumps,

$$C_{H,\mathrm{A}} = \frac{gH_\mathrm{A}}{\omega_\mathrm{A}^2 D_\mathrm{A}^2} = C_{\mathrm{H,B}} = \frac{gH_\mathrm{B}}{\omega_\mathrm{B}^2 D_\mathrm{B}^2} \quad \rightarrow \quad \frac{\omega_\mathrm{A}}{\omega_\mathrm{B}} = \sqrt{\frac{H_\mathrm{A}}{H_\mathrm{B}}}\frac{D_\mathrm{B}}{D_\mathrm{A}}$$

Similarly, dimensionless pump parameter C_Q must be the same for both pumps,

$$C_{Q,\mathrm{A}} = \frac{\dot{V}_\mathrm{A}}{\omega_\mathrm{A} D_\mathrm{A}^3} C_{Q,\mathrm{B}} = \frac{\dot{V}_\mathrm{B}}{\omega_\mathrm{B} D_\mathrm{B}^3} \quad \rightarrow \quad \left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^3 = \frac{\omega_\mathrm{A}}{\omega_\mathrm{B}}\frac{\dot{V}_\mathrm{B}}{\dot{V}_\mathrm{A}}$$

Combining the above two equations yields

$$\left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^3 = \sqrt{\frac{H_\mathrm{A}}{H_\mathrm{B}}}\frac{D_\mathrm{B}}{D_\mathrm{A}}\frac{\dot{V}_\mathrm{B}}{\dot{V}_\mathrm{A}}$$

which reduces to

$$\left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^2 = \sqrt{\frac{H_\mathrm{A}}{H_\mathrm{B}}}\frac{\dot{V}_\mathrm{B}}{\dot{V}_\mathrm{A}} \quad \rightarrow \quad \boxed{D_\mathrm{B} = D_\mathrm{A}\left(\frac{H_\mathrm{A}}{H_\mathrm{B}}\right)^{1/4}\left(\frac{\dot{V}_\mathrm{B}}{\dot{V}_\mathrm{A}}\right)^{1/2}}$$

Thus, we have eliminated the angular velocity as a parameter, and the relationship is proven.

Since turbines follow the same affinity laws as pumps, **the relationship also applies to two dynamically similar turbines**.

Discussion In like manner, we can eliminate other parameters through algebraic manipulations to scale a pump or turbine up or down.

14-116

Solution We are to prove a relationship between two dynamically similar turbines, and discuss its application to pumps.

Assumptions **1** The two turbines are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two turbines are not grossly different from each other in size, capacity, etc.).

Analysis Since the two turbines are dynamically similar, dimensionless turbine parameter C_H must be the same for both turbines,

$$C_{H,\mathrm{A}} = \frac{gH_\mathrm{A}}{\omega_\mathrm{A}^2 D_\mathrm{A}^2} = C_{\mathrm{H,B}} = \frac{gH_\mathrm{B}}{\omega_\mathrm{B}^2 D_\mathrm{B}^2} \quad \rightarrow \quad \frac{\omega_\mathrm{A}}{\omega_\mathrm{B}} = \sqrt{\frac{H_\mathrm{A}}{H_\mathrm{B}}}\frac{D_\mathrm{B}}{D_\mathrm{A}}$$

Similarly, dimensionless turbine parameter C_P must be the same for both turbines,

$$C_{P,\mathrm{A}} = \frac{bhp_\mathrm{A}}{\rho_\mathrm{A}\omega_\mathrm{A}^3 D_\mathrm{A}^5} = C_{Q,\mathrm{B}} = \frac{bhp_\mathrm{B}}{\rho_\mathrm{B}\omega_\mathrm{B}^3 D_\mathrm{B}^5} \quad \rightarrow \quad \left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^5 = \left(\frac{\omega_\mathrm{A}}{\omega_\mathrm{B}}\right)^3 \frac{\rho_\mathrm{A}}{\rho_\mathrm{B}}\frac{bhp_\mathrm{B}}{bhp_\mathrm{A}}$$

Combining the above two equations yields

$$\left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^5 = \left(\frac{H_\mathrm{A}}{H_\mathrm{B}}\right)^{3/2} \left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^3 \frac{\rho_\mathrm{A}}{\rho_\mathrm{B}}\frac{bhp_\mathrm{B}}{bhp_\mathrm{A}}$$

which reduces to

$$\left(\frac{D_\mathrm{B}}{D_\mathrm{A}}\right)^2 = \left(\frac{H_\mathrm{A}}{H_\mathrm{B}}\right)^{3/2} \frac{\rho_\mathrm{A}}{\rho_\mathrm{B}}\frac{bhp_\mathrm{B}}{bhp_\mathrm{A}} \quad \rightarrow \quad \boxed{D_\mathrm{B} = D_\mathrm{A}\left(\frac{H_\mathrm{A}}{H_\mathrm{B}}\right)^{3/4}\left(\frac{\rho_\mathrm{A}}{\rho_\mathrm{B}}\right)^{1/2}\left(\frac{bhp_\mathrm{B}}{bhp_\mathrm{A}}\right)^{1/2}}$$

Thus, we have eliminated the angular velocity as a parameter, and the relationship is proven.

Since pumps follow the same affinity laws as turbines, **the relationship also applies to two dynamically similar pumps**.

Discussion In like manner, we can eliminate other parameters through algebraic manipulations to scale a pump or turbine up or down.

CD-EES 14-117

Solution We are to generate a computer application that uses the affinity laws to design a new pump that is dynamically similar to an existing pump.

Assumptions **1** The two pumps are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two pumps are not grossly different from each other in size, capacity, etc.).

Analysis First, we calculate the brake horsepower for pump A,

$$bhp_A = \frac{\rho_A \dot{V}_A g H_A}{\eta_{\text{pump,A}}}$$
$$= \frac{\left(998.0 \text{ kg/m}^3\right)\left(0.00040 \text{ m}^3\text{/s}\right)\left(9.81 \text{ m/s}^2\right)\left(1.20 \text{ m}\right)}{0.81}\left(\frac{\text{W}\cdot\text{s}^3}{\text{kg}\cdot\text{m}^2}\right) = \mathbf{5.80\ W}$$

We use the affinity laws to calculate the new pump parameters. Using the given data, we first calculate the new pump diameter,

$$D_B = D_A\left(\frac{H_A}{H_B}\right)^{1/4}\left(\frac{\dot{V}_B}{\dot{V}_A}\right)^{1/2} = \left(5.0 \text{ cm}\right)\left(\frac{120 \text{ cm}}{450 \text{ cm}}\right)^{1/4}\left(\frac{2400 \text{ cm}^3\text{/s}}{400 \text{ cm}^3\text{/s}}\right)^{1/2} = \mathbf{8.80\ cm}$$

Knowing D_B, we calculate the rotation rate of the new pump,

$$\dot{n}_B = \dot{n}_A \frac{D_A}{D_B}\left(\frac{H_B}{H_A}\right)^{1/2} = \left(1725 \text{ rpm}\right)\frac{5.0 \text{ cm}}{8.80 \text{ cm}}\left(\frac{450 \text{ cm}}{120 \text{ cm}}\right)^{1/2} = \mathbf{1898\ rpm}$$

Knowing D_B and ω_B, we now calculate the required shaft power,

$$bhp_B = bhp_A \frac{\rho_B}{\rho_A}\left(\frac{D_B}{D_A}\right)^5\left(\frac{\omega_B}{\omega_A}\right)^3$$
$$= \left(5.80 \text{ W}\right)\frac{1226 \text{ kg/m}^3}{998.0 \text{ kg/m}^3}\left(\frac{8.80 \text{ cm}}{5.0 \text{ cm}}\right)^5\left(\frac{1898 \text{ rpm}}{1725 \text{ rpm}}\right)^3 = \mathbf{160\ W}$$

Our answers agree with those given. Now we are confident that we can apply our computer program to other design problems of similar nature, as in Problem 14-118.

Discussion To avoid round-off errors in the calculations, we saved several more significant digits for D_B than are reported here.

14-118

Solution We are to use a computer code and the affinity laws to design a new pump that is dynamically similar to an existing pump.

Assumptions **1** The two pumps are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two pumps are not grossly different from each other in size, capacity, etc.). **3** Both pumps operate at their BEP.

Analysis Using our computer code, D_B = **12.8 cm**, $\dot{n}_B$ = **1924 rpm**, and bhp_B = **235 W**. We calculate pump specific speed for the new pump,

Pump B: $$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}} = \frac{(1924\text{ rpm})\left(\frac{2\pi\text{ rad/s}}{60\text{ rpm}}\right)\left(0.00367\text{ m}^3\text{/s}\right)^{1/2}}{\left((9.81\text{ m/s}^2)(5.70\text{ m})\right)^{3/4}} = \mathbf{0.597}$$

and we repeat the calculation for the existing pump,

Pump A: $$N_{Sp} = \frac{\omega \dot{V}^{1/2}}{(gH)^{3/4}} = \frac{(1500\text{ rpm})\left(\frac{2\pi\text{ rad/s}}{60\text{ rpm}}\right)\left(0.00135\text{ m}^3\text{/s}\right)^{1/2}}{\left((9.81\text{ m/s}^2)(2.10\text{ m})\right)^{3/4}} = \mathbf{0.597}$$

Our answers agree, as they must, since the two pumps are operating at homologous points. From Fig. 14-73, we can see that these are most likely **centrifugal pumps**.

Discussion The pump performance parameters of pump B can be calculated manually to further verify our computer code.

14-119

Solution We are to generate a computer application that uses the affinity laws to design a new turbine that is dynamically similar to an existing turbine.

Assumptions **1** The two turbines are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two turbines are not grossly different from each other in size, capacity, etc.).

Analysis We use the affinity laws to calculate the new turbine parameters. Using the given data, we first calculate the new turbine diameter,

$$D_B = D_A \sqrt{\frac{H_B}{H_A}} \left(\frac{\dot{n}_A}{\dot{n}_B}\right) = (1.40 \text{ m})\sqrt{\frac{95.0 \text{ m}}{80.0 \text{ m}}} \left(\frac{150 \text{ rpm}}{120 \text{ rpm}}\right) = \mathbf{1.91\ m}$$

We then solve Eq. 14-38a for $\dot{V}_B$,

$$\dot{V}_B = \dot{V}_A \left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \left(162 \text{ m}^3/\text{s}\right)\left(\frac{120 \text{ rpm}}{150 \text{ rpm}}\right)\left(\frac{1.91 \text{ m}}{1.40 \text{ m}}\right)^3 = \mathbf{328\ m^3/s}$$

Finally, we solve Eq. 14-38c for bhp_B,

$$bhp_B = bhp_A \left(\frac{\rho_B}{\rho_A}\right)\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

$$= 118 \text{ MW}\left(\frac{998.0 \text{ kg/m}^3}{998.0 \text{ kg/m}^3}\right)\left(\frac{120 \text{ rpm}}{150 \text{ rpm}}\right)^3\left(\frac{1.91 \text{ m}}{1.40 \text{ m}}\right)^5 = \mathbf{283\ MW}$$

Our answers agree with those given. Now we are confident that we can apply our computer program to other design problems of similar nature.

Discussion To avoid round-off errors in the calculations, we saved several more significant digits for D_B than are reported here.

14-120

Solution We are to use a computer program and the affinity laws to design a new turbine that is dynamically similar to an existing turbine.

Assumptions **1** The two pumps are geometrically similar. **2** Reynolds number and roughness effects are not critical in the analysis (the two pumps are not grossly different from each other in size, capacity, etc.).

Analysis Using our computer code, D_B = **2.04 m**, $\dot{V}_B$ = 815 m^3/s, and bhp_B = **577 MW**. We calculate turbine specific speed for the new turbine,

$$N_{St,A} = \frac{\omega_A (bhp_A)^{1/2}}{(\rho_A)^{1/2}(gH_A)^{5/4}}$$
$$= \frac{(25.13 \text{ rad/s})(11.4\times 10^6 \text{ W})^{1/2}}{(998.0 \text{ kg/m}^3)^{1/2}\left[(9.81 \text{ m/s}^2)(22.0 \text{ m})\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{3.25}$$

and for turbine B,

$$N_{St,B} = \frac{\omega_B (bhp_B)^{1/2}}{(\rho_B)^{1/2}(gH_B)^{5/4}}$$
$$= \frac{(21.99 \text{ rad/s})(577\times 10^6 \text{ W})^{1/2}}{(998.0 \text{ kg/m}^3)^{1/2}\left[(9.81 \text{ m/s}^2)(95.0 \text{ m})\right]^{5/4}}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right)^{1/2} = \mathbf{3.25}$$

Our answers agree, as they must, since the two turbines are operating at homologous points. From Fig. 14-108, we see that these are most likely **Kaplan turbines**.

Discussion The turbine parameters of turbine B can be calculated manually to further verify our computer code.

14-121

Solution We are to compare efficiencies for two geometrically similar turbines, and discuss the Moody efficiency correction.

Analysis We calculate the turbine efficiency for both turbines,

$$\eta_{\text{turbine A}} = \frac{bhp_{\text{A}}}{\rho_{\text{A}} g H_{\text{A}} \dot{V}_{\text{A}}}$$

$$= \frac{11.4\times10^{6}\ \text{W}}{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(22.0\ \text{m}\right)\left(69.5\ \text{m}^3\text{/s}\right)}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{76.2\%}$$

and

$$\eta_{\text{turbine B}} = \frac{bhp_{\text{B}}}{\rho_{\text{B}} g H_{\text{B}} \dot{V}_{\text{B}}}$$

$$= \frac{577\times10^{6}\ \text{W}}{\left(998.0\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(95.0\ \text{m}\right)\left(814.6\ \text{m}^3\text{/s}\right)}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{W}\cdot\text{s}^3}\right) = \mathbf{76.2\%}$$

where we have included an extra digit in the intermediate values to avoid round-off error. As expected, the two efficiencies are the same, since we have assumed dynamic similarity. However, total dynamic similarity may not actually be achieved between the two turbines because of scale effects (larger turbines generally have higher efficiency). The diameter of the new turbine is more than twice that of the existing turbine, so the increase in efficiency due to turbine size may be significant. We account for the size increase by using the Moody efficiency correction equation, considering turbine A as the "model" and B as the "prototype",

Efficiency correction:

$$\eta_{\text{turbine, B}} \approx 1-\left(1-\eta_{\text{turbine, A}}\right)\left(\frac{D_{\text{A}}}{D_{\text{B}}}\right)^{1/5} = 1-\left(1-0.762\right)\left(\frac{0.86\ \text{m}}{2.04\ \text{m}}\right)^{1/5} = \mathbf{0.800}$$

or **80.0%**. So, the first-order correction yields a predicted efficiency for the larger turbine that is about four percent greater than the smaller turbine. However, as mentioned in the text, we expect only about 2/3 of this increase, or 2(0.800 – 0.762)/3 = 0.025 or 2.5%. Thus, our best estimate of the actual efficiency of the prototype is

$$\eta_{\text{turbine B}} \approx 76.2 + 2.5 = \mathbf{78.7\%}$$

The higher efficiency of the new larger turbine is significant because an increase in power production of 2.5% can lead to significant profits for the power company.

Discussion If the flow entering the new turbine from the penstock were not similar to that of the existing turbine (e.g., velocity profile and turbulence intensity), we could not expect exact dynamic similarity.