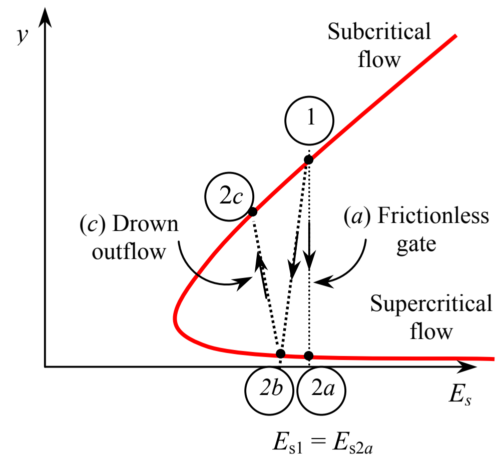
Flow Control and Measurement in Channels

13-89C

On the figure, diagram 1-2a is for frictionless gate, 1-2b is for sluice gate with free outflow, and 1-2b-2c is for sluice gate with drown outflow, including the hydraulic jump back to subcritical flow.



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13-90C For sluice gates, the discharge coefficient C_d is defined as the ratio of the actual velocity through the gate to the maximum velocity as determined by the Bernoulli equation for the idealized frictionless flow case, for which C_d = 1. Typical values of C_d for sluice gates with free outflow are in the range of 0.55 to 0.60.

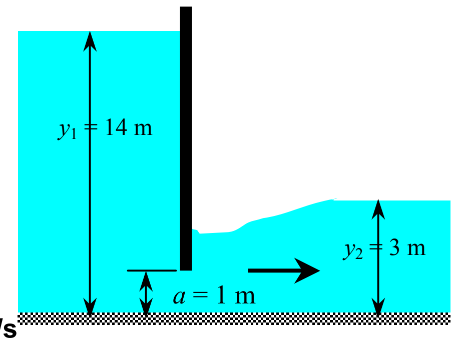
13-91C The operation of broad crested weir is based on blocking the flow in the channel with a large rectangular block, and establishing critical flow over the block. Then the flow rate can be determined by measuring flow depths.

13-92C In the case of subcritical flow, the flow depth y will decrease during flow over the bump.

13-93C When the specific energy reaches its minimum value, the flow is critical, and the flow at this point is said to be choked. If the bumper height is increased even further, the flow remains critical and thus choked. The flow will not become supercritical.

13-94C A sharp-crested weir is a vertical plate placed in a channel that forces the fluid to flow through an opening to measure the flow rate. They are characterized by the shape of the opening. For example, a weir with a triangular opening is referred to as a triangular weir.

13-95 Water is released from a reservoir through a sluice gate into an open channel. For specified flow depths, the rate of discharge is to be determined. √EES



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

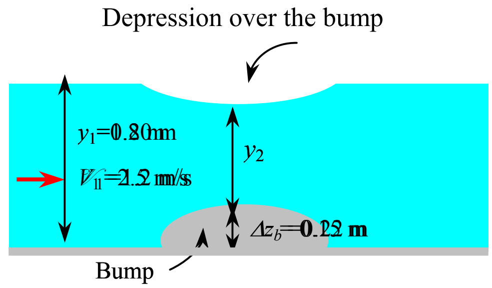
$$\frac{y_1}{a} = \frac{14\text{ m}}{1\text{ m}} = 14 \quad \text{and} \quad \frac{y_2}{a} = \frac{3\text{ m}}{1\text{ m}} = 3$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.59$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.59\,(5\text{ m})(1\text{ m})\sqrt{2\,(9.81\text{ m/s}^2)(14\text{ m})} = \mathbf{48.9\ m^3/s}$$

Discussion Discharge coefficient is the same as free flow because of small depth ratio after the gate. So, the flow rate would not change if it were not drowned.

13-96 Water flowing in a horizontal open channel encounters a bump. It will be determined if the flow over the bump is choked. √EES



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2.5 \text{ m/s}}{\sqrt{(9.81 \text{ m}^2/\text{s})(1.2 \text{ m})}} = 0.729$$

$$y_c = \left(\frac{\dot{V}^2}{g b^2}\right)^{1/3} = \left(\frac{(b y_1 V_1)^2}{g b^2}\right)^{1/3} = \left(\frac{y_1^2 V_1^2}{g}\right)^{1/3} = \left(\frac{(1.2 \text{ m})^2 (2.5 \text{ m/s})^2}{9.81 \text{ m/s}^2}\right)^{1/3} = 0.972 \text{ m}$$

The flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream, over the bump, and critical specific energy is

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.2 \text{ m}) + \frac{(2.5 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.52 \text{ m}$$

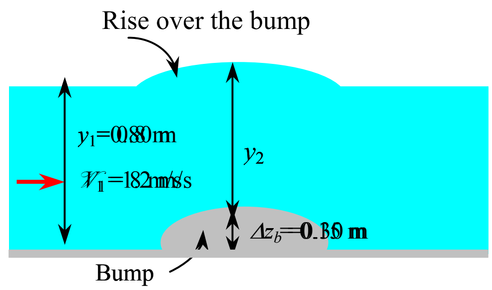
$$E_{s2} = E_{s1} - \Delta z_b = 1.52 - 0.22 = 1.30 \text{ m}$$

$$E_c = \frac{3}{2} y_c = 1.46 \text{ m}$$

We have an interesting situation: The calculations show that $E_{s2} < E_c$. That is, the specific energy of the fluid decreases below the level of energy at the critical point, which is the minimum energy, and this is impossible. Therefore, the flow at specified conditions cannot exist. The flow is **choked** when the specific energy drops to the minimum value of 1.46 m, which occurs at a bump-height of $\Delta z_{b,\max} = E_{s1} - E_c = 1.52 - 1.46 = 0.06 \text{ m}$.

Discussion A bump-height over 6 cm results in a reduction in the flow rate of water, or a rise of upstream water level. Therefore, a 22-cm high bump alters the upstream flow. On the other hand, a bump less than 6 cm high will not affect the upstream flow.

13-97 Water flowing in a horizontal open channel encounters a bump. The change in the surface level over the bump and the type of flow (sub- or supercritical) over the bump are to be determined. √EES



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{8\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m^2/s})(0.8\ \mathrm{m})}} = 2.856$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(0.8\ \mathrm{m})^2(8\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s^2}}\right)^{1/3} = 1.61\ \mathrm{m}$$

The upstream flow is supercritical since Fr > 1, and the flow depth increases over the bump. The upstream, over the bump, and critical specific energy are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (0.8\ \mathrm{m}) + \frac{(8\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 4.06\ \mathrm{m}$$

$$E_{s2} = E_{s1} - \Delta z_b = 4.06 - 0.30 = 3.76\ \ \mathrm{m}$$

$$E_c = \frac{3}{2}y_c = 2.42\ \ \mathrm{m}$$

The flow depth over the bump can be determined from

$$y_2^3 - (E_{s1} - \Delta z_b)y_2^2 + \frac{V_1^2}{2g}y_1^2 = 0 \quad \rightarrow \quad y_2^3 - (4.06 - 0.30\ \mathrm{m})y_2^2 + \frac{(8\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})}(0.80\ \mathrm{m})^2 = 0$$

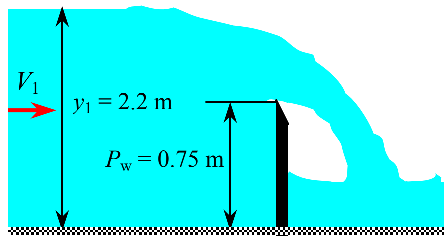
Using an equation solver, the physically meaningful root of this equation is determined to be 0.846 m. Therefore, there is a rise of

$$\text{Rise over bump} = y_2 - y_1 + \Delta z_b = 0.846 - 0.80 + 0.30 = \mathbf{0.346\ m}$$

over the surface relative to the upstream water surface. The specific energy decreases over the bump from, 4.06 to 3.76 m, but it is still over the minimum value of 2.42 m. Therefore, the flow over the bump is still **supercritical**.

Discussion The actual value of surface rise may be different than the 4.6 cm because of the frictional effects that are neglected in the analysis.

13-98 The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined. √EES



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 2.2 - 0.75 = 1.45 \text{ m}$$

The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.45 \text{ m}}{0.75 \text{ m}} = 0.771$$

The condition $H/P_w < 2$ is satisfied since 1.45/0.75 = 1.93. Then the water flow rate through the channel becomes

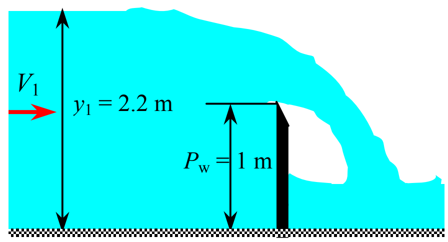
$$\begin{aligned}\dot{V}_{\text{rec}} &= C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2} \\ &= (0.7714)\frac{2}{3}(4 \text{ m})\sqrt{2(9.81 \text{ m/s}^2)}(1.45 \text{ m})^{3/2} \\ &= \mathbf{15.9\ m^3/s}\end{aligned}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{15.9 \text{ m}^3/\text{s}}{(4 \text{ m})(2.2 \text{ m})} = 1.81 \text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(1.81 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.167 \text{ m}$$

This is 11.5% of the weir head, which is significant. When the upstream velocity head is considered, the flow rate becomes 18.1 m^3/s, which is about 14 percent higher than the value determined above. Therefore, it is good practice to consider the upstream velocity head unless the weir height P_w is very large relative to the weir head H.

13-99 The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined. √EES



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 2.2 - 1.0 = 1.2 \text{ m}$$

The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.2 \text{ m}}{1.0 \text{ m}} = 0.7056$$

The condition $H/P_w < 2$ is satisfied since 1.2/1.0 = 1.20. Then the water flow rate through the channel becomes

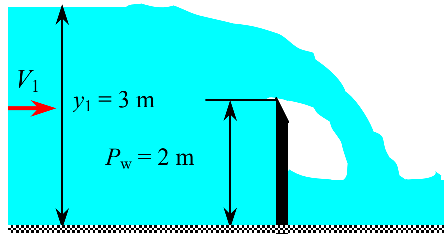
$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}$$
$$= (0.7056)\frac{2}{3}(4 \text{ m})\sqrt{2(9.81 \text{ m/s}^2)}(1.2 \text{ m})^{3/2}$$
$$= \mathbf{11.0\ m^3/s}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{11.0 \text{ m}^3\text{/s}}{(4 \text{ m})(2.2 \text{ m})} = 1.25 \text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(1.25 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.079 \text{ m}$$

This is 6.6% of the weir head, which may be significant. When the upstream velocity head is considered, the flow rate becomes 11.9 m^3/s, which is about 8 percent higher than the value determined above. Therefore, it is good practice to consider the upstream velocity head unless the weir height P_w is very large relative to the weir head H.

13-100 Water flowing over a sharp-crested rectangular weir is discharged into a channel where uniform flow conditions are established. The maximum slope of the downstream channel to avoid hydraulic jump is to be determined. √EES



Sharp-crested rectangular weir

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Properties Manning coefficient for an open channel of unfinished concrete is n = 0.014 (Table 13-1).

Analysis The weir head is

$$H = y_1 - P_w = 3.0\text{ m} - 2.0\text{ m} = 1.0\text{ m}$$

The condition $H/P_w < 2$ is satisfied since 1.0/2.0 = 0.5. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.0\text{ m}}{2.0\text{ m}} = 0.6429$$

Then the water flow rate through the channel per meter width (i.e., taking b = 1 m) becomes

$$\begin{aligned}\dot{V}_{\text{rec}} &= C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}\\ &= (0.6429)\frac{2}{3}(1\text{ m})\sqrt{2(9.81\text{ m/s}^2)}(1.0\text{ m})^{3/2}\\ &= 1.898\text{ m}^3/\text{s}\end{aligned}$$

To avoid hydraulic jump, we must avoid supercritical flow in the channel. Therefore, the bottom slope should not be higher than the critical slope, in which case the flow depth becomes the critical depth,

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(1.898\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(1\text{ m}^2)}\right)^{1/3} = 0.7162\text{ m}$$

Noting that the hydraulic radius of a wide channel is equal to the flow depth, the bottom slope is determined from the Manning equation to be

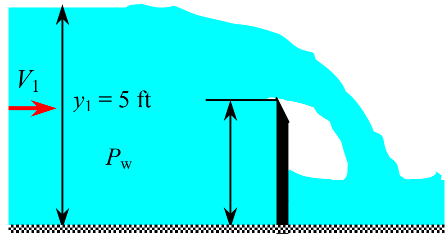
$$\dot{V} = \frac{a}{n}A_c R_h^{2/3}S_0^{1/2} \quad \rightarrow \quad 1.898\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.014}(0.7162\times 1\text{ m}^2)(0.7162\text{ m})^{2/3}S_0^{1/2}$$

It gives the slope to be $S_0 = 0.00215$. Therefore,

$S_{0,\max}$ = **0.00215**

Discussion. For a bottom slope smaller than calculated value, downstream channel would have a mild slope, that will force the flow to remain subcritical.

13-101E The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For specified upper limits of flow rate and flow depth, the appropriate height of the weir is to be determined. √EES



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 5 - P_w$$

The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{5 - P_w}{P_w}$$

The water flow rate through the channel can be expressed as

$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}$$

Substituting the known quantities,

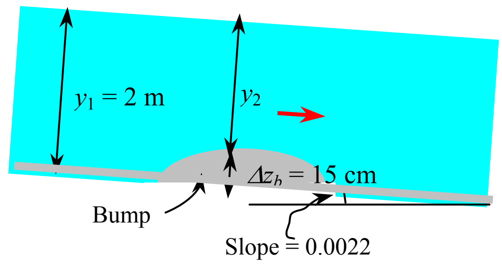
$$150\ \text{ft}^3/\text{s} = \left(0.598 + 0.0897\frac{5 - P_w}{P_w}\right)\frac{2}{3}(10\ \text{ft})\sqrt{2(32.2\ \text{ft/s}^2)}(5 - P_w)^{3/2}$$

It gives the weir height to be

$$P_w = \mathbf{2.46\ ft}$$

Discussion Nonlinear equations of this kind can be solved easily using equation solvers like EES.

13-102 The flow of water in a wide channel with a bump is considered. The flow rate of water without the bump and the effect of the bump on the flow rate for the case of a flat surface are to be determined. √EES



Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** The channel is sufficiently wide so that the end effects are negligible. **5** Frictional effects during flow over the bump are negligible.

Properties Manning coefficient for an open channel of unfinished concrete is n = 0.014 (Table 13-1).

Analysis For a wide channel, the hydraulic radius is equal to the flow depth, and thus $R_h = 2$ m. Then the flow rate *before the bump* per m width (i.e., $b = 1$ m) can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\ \mathrm{m}^{1/3}/s}{0.014}(1\times 2\ \mathrm{m}^2)(2\ \mathrm{m})^{2/3}(0.0022)^{1/2} = 10.64\ \mathrm{m}^3/\mathrm{s}$$

The average flow velocity is

$$V = \frac{\dot{V}}{A_c} = \frac{10.64\ \mathrm{m}^3/\mathrm{s}}{1\times 2\ \mathrm{m}^2} = 5.32\ \mathrm{m/s}$$

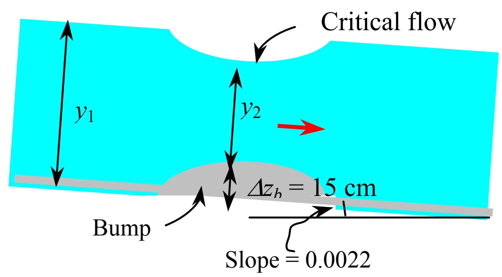
When a bump is placed, it is said that the flow depth remains the same and there is no rise/drop, and thus $y_2 = y_1 - \Delta z_b$. But the energy equation is given as

$$E_{s2} = E_{s1} - \Delta z_b \quad \rightarrow \quad y_2 + \frac{V_2^2}{2g} = y_1 + \frac{V_1^2}{2g} - \Delta z_b \quad \rightarrow \quad \frac{V_2^2}{2g} = \frac{V_1^2}{2g}$$

since $y_2 = y_1 - \Delta z_b$, and thus $V_1 = V_2$. But from the continuity equation $y_2 V_2 = y_1 V_1$, this is possible only if the flow depth over the bump remains constant, i.e., $y_1 = y_2$, which is a contradiction since y_2 cannot be equal to both y_1 and $y_1 - \Delta z_b$ while Δz_b remains nonzero. Therefore, the second part of the problem can have **no solution** since it is physically impossible.

Discussion Note that sometimes it is better to investigate whether there is really a solution before spending a lot of time trying to find a solution.

13-103 Uniform subcritical water flow of water in a wide channel with a bump is considered. For critical flow over the bump, the flow rate of water and the flow depth over the bump are to be determined. √EES



Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** The channel is sufficiently wide so that the end effects are negligible. **5** Frictional effects during flow over the bump are negligible.

Properties Manning coefficient for an open channel of unfinished concrete is n = 0.014 (Table 13-1).

Analysis Let subscript 1 denote the upstream conditions (uniform flow) in the channel, and 2 denote the critical conditions over the bump. For a wide channel, the hydraulic radius is equal to the flow depth, and thus $R_h = y_1$. Then the flow rate per m width (i.e., b = 1 m) can be determined from Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\,\text{m}^{1/3}/s}{0.014} y_1 (y_1)^{2/3} (0.0022)^{1/2} = 3.350 y_1^{5/3}\ \text{m}^3/\text{s}$$

The critical depth corresponding to this flow rate is (note that b = 1 m),

$$y_2 = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(3.350 y_1^{5/3})^2}{g}\right)^{1/3} = \left(\frac{11.224 y_1^{10/3}}{9.81\,\text{m/s}^2}\right)^{1/3} = 1.046 y_1^{10/9}$$

The average flow velocity is $V_1 = \dot{V}/A_c = 3.350 y_1^{5/3}/y_1 = 3.350 y_1^{2/3}$ m/s . Also,

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{(3.350 y_1^{2/3})^2}{2(9.81\,\text{m/s}^2)} = y_1 + 0.5720 y_1^{4/3}$$

$$E_{s2} = E_c = \frac{3}{2} y_c = \frac{3}{2}(1.046 y_1^{10/9}) = 1.569 y_1^{10/9}$$

Substituting these two relations into $E_{s2} = E_{s1} - \Delta z_b$ where $\Delta z_b = 0.15$ m gives

$$1.569 y_1^{10/9} = y_1 + 0.5720 y_1^{4/3} - 0.15$$

Using an equation solver such as EES or an iterative approach, the flow depth upstream is determined to be

$$y_1 = 2.947\ \text{m}$$

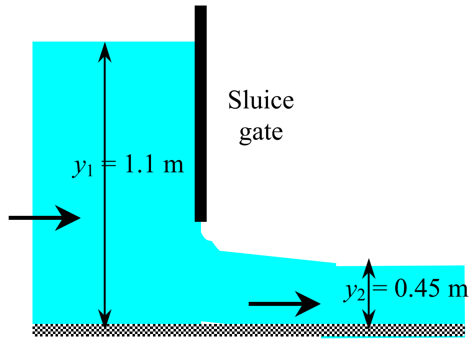
Then the flow rate and the flow depth over the bump becomes

$$\dot{V} = 3.350 y_1^{5/3} = 3.350(2.947)^{5/3} = \mathbf{20.3\ m^3/s}$$

$$y_2 = y_c = 1.046 y_1^{10/9} = 1.046(2.947)^{10/9} = \mathbf{3.48\ m}$$

Discussion Note that when critical flow is established and the flow is "Choked", the flow rate calculations become very easy, and it required minimal measurements. Also, $V_1 = 3.350(2.947)^{2/3} = 6.89$ m/s and $\text{Fr}_1 = V_1/\sqrt{gy_1} = (6.89\,\text{m/s})/\sqrt{(9.81\,\text{m}^2/\text{s})(2.947\,\text{m})} = 1.28$, and thus the upstream flow is supercritical.

13-104 A sluice gate is used to control the flow rate of water in a channel. For specified flow depths upstream and downstream from the gate, the flow rate of water and the downstream Froude number are to be determined. √EES



Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** Frictional effects associated with sluice gate are negligible. **4** The channel is horizontal.

Analysis When frictional effects are negligible and the flow section is horizontal, the specific energy remains constant, $E_{s1} = E_{s2}$. Then,

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \quad \rightarrow \quad 1.1\text{ m} + \frac{\dot{V}^2}{2(9.81\text{ m/s}^2)[(5\text{ m})(1.1\text{ m})]^2} = 0.45\text{ m} + \frac{\dot{V}^2}{2(9.81\text{ m/s}^2)[(5\text{ m})(0.45\text{ m})]^2}$$

Solving for the flow rate gives

$$\dot{V} = \mathbf{8.806\ m^3/s}$$

The downstream velocity and Froude number are

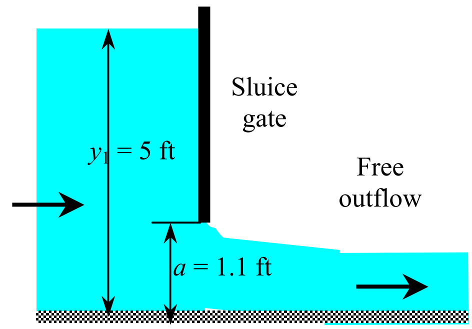
$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{8.806\text{ m}^3\text{/s}}{(5\text{ m})(0.45\text{ m})} = 3.914\text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{3.914\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.45\text{ m})}} = \mathbf{1.86}$$

Discussion The actual values will be somewhat different because of the frictional effects.

13-105E Water is released from a reservoir through a sluice gate with free outflow. For specified flow depths, the flow rate per unit width and the downstream Froude number are to be determined. √EES

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient (for drowned outflow, we also need to know y_2/a and thus the flow depth y_2 downstream the gate)

$$\frac{y_1}{a} = \frac{5\text{ ft}}{1.1\text{ ft}} = 4.55$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.55$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.55\,(1\text{ ft})(1.1\text{ ft})\sqrt{2\,(32.2\text{ ft/s}^2)(5\text{ ft})} = \mathbf{10.9\ ft^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth past the gate and the Froude number are determined to be

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 5\text{ ft} + \frac{(10.9\text{ ft/s}^2)^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(5\text{ ft})]^2} = 5.074\text{ ft}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \;\rightarrow\; y_2 + \frac{(10.9\text{ ft/s}^2)^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(y_2)]^2} = 5.074\text{ ft}$$

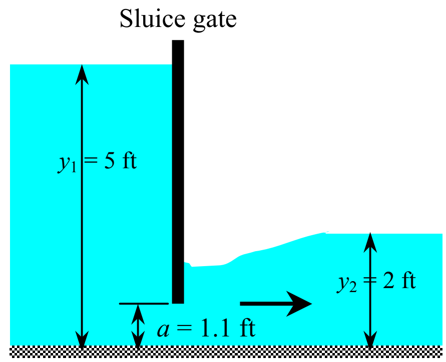
It gives $y_2 = 0.643$ ft as the physically meaningful root (positive and less than 5 ft). Then,

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{10.9\text{ ft}^3\text{/s}}{(1\text{ ft})(0.643\text{ ft})} = 16.9\text{ ft/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{16.9\text{ ft/s}}{\sqrt{(32.2\text{ ft/s}^2)(0.643\text{ ft})}} = \mathbf{3.71}$$

Discussion In actual gates some frictional losses are unavoidable, and thus the actual velocity and Froude number downstream will be lower.

13-106E Water is released from a reservoir through a drowned sluice gate into an open channel. For specified flow depths, the rate of discharge is to be determined. √EES



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

$$\frac{y_1}{a} = \frac{5 \text{ ft}}{1.1 \text{ ft}} = 4.55 \quad \text{and} \quad \frac{y_2}{a} = \frac{3.3 \text{ ft}}{1.1 \text{ ft}} = 3$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.44$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.44\,(1 \text{ ft})(1.1 \text{ ft})\sqrt{2\,(32.2 \text{ ft/s}^2)(5 \text{ ft})} = \mathbf{8.69\ ft^3/s}$$

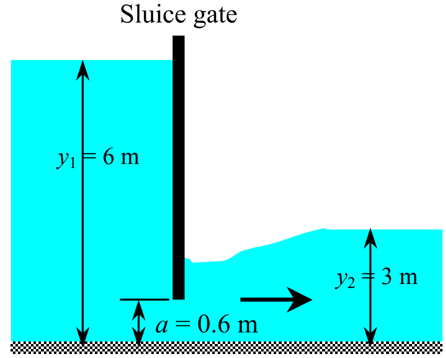
Then the Froude number downstream the gate becomes

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{8.69 \text{ ft}^3\text{/s}}{(1 \text{ ft})(3.3 \text{ ft})} = 2.63 \text{ ft/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{2.63 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(3.3 \text{ ft})}} = \mathbf{0.255}$$

Discussion Note that the flow past the gate becomes subcritical when the outflow is drowned.

13-107 Water is released from a lake through a drowned sluice gate into an open channel. For specified flow depths, the rate of discharge through the gate is to be determined. √EES



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

$$\frac{y_1}{a} = \frac{6\text{ m}}{0.6\text{ m}} = 10 \quad \text{and} \quad \frac{y_2}{a} = \frac{3\text{ m}}{0.6\text{ m}} = 5$$

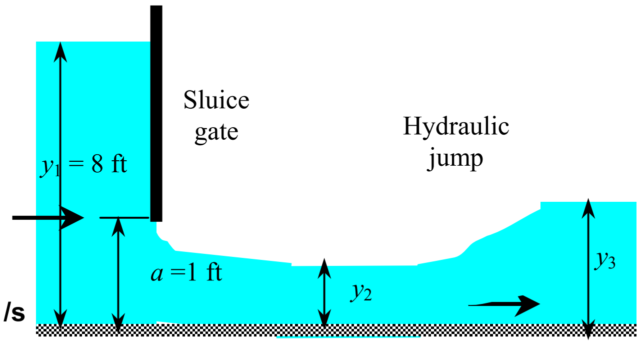
The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.48$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.48(5\text{ m})(0.6\text{ m})\sqrt{2(9.81\text{ m/s}^2)(6\text{ m})} = \mathbf{15.6\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate through sluice gates by measuring 3 flow depths only.

13-108E Water discharged through a sluice gate undergoes a hydraulic jump. The flow depth and velocities before and after the jump and the fraction of mechanical energy dissipated are to be determined. √EES

Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** Frictional effects associated with sluice gate are negligible. **4** The channel is horizontal.



Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient,

$$\frac{y_1}{a} = \frac{8\text{ ft}}{1\text{ ft}} = 8$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.58$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.58\,(1\text{ ft})(1\text{ ft})\sqrt{2\,(32.2\text{ ft/s}^2)(8\text{ ft})} = \mathbf{13.16\ ft^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth past the gate and the Froude number are determined to be

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 8\text{ ft} + \frac{(13.16\text{ ft}^3/\text{s})^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(8\text{ ft})]^2} = 8.042\text{ ft}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \;\rightarrow\; y_2 + \frac{(13.16\text{ ft}^3/\text{s})^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(y_2)]^2} = 8.042\text{ ft}$$

It gives y_2 **= 0.601** ft as the physically meaningful root (positive and less than 8 ft). Then,

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{13.16\text{ ft}^3/\text{s}}{(1\text{ ft})(0.601\text{ ft})} = \mathbf{21.9\ ft/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{21.9\text{ ft/s}}{\sqrt{(32.2\text{ ft/s}^2)(0.601\text{ ft})}} = 4.97$$

Then the flow depth and velocity after the jump (state 3) become

$$y_3 = 0.5y_2\left(-1+\sqrt{1+8\text{Fr}_2^2}\right) = 0.5(0.601\text{ ft})\left(-1+\sqrt{1+8\times 4.97^2}\right) = \mathbf{3.94\ ft}$$

$$V_3 = \frac{y_2}{y_3}V_2 = \frac{0.601\text{ ft}}{3.94\text{ ft}}(21.9\text{ ft/s}) = \mathbf{3.34\ ft/s}$$

The head loss and the fraction of mechanical energy dissipated during the jump are

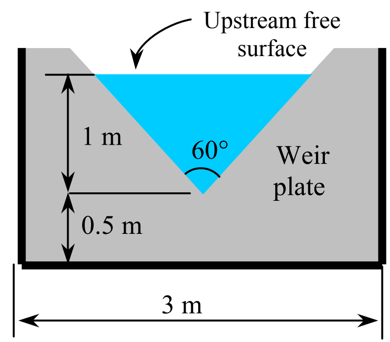
$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (0.601\text{ ft}) - (3.94\text{ ft}) + \frac{(21.9\text{ ft/s})^2 - (3.34\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 3.93\text{ ft}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{3.93\text{ ft}}{(0.601\text{ ft})(1+4.97^2/2)} = \mathbf{0.488}$$

Discussion Note that almost half of the mechanical energy of the fluid is dissipated during hydraulic jump.

13-109 The flow rate of water in an open channel is to be measured with a sharp-crested triangular weir. For a given flow depth upstream the weir, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Properties The weir discharge coefficient is given to be 0.60.

Analysis The discharge rate of water is determined directly from

$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

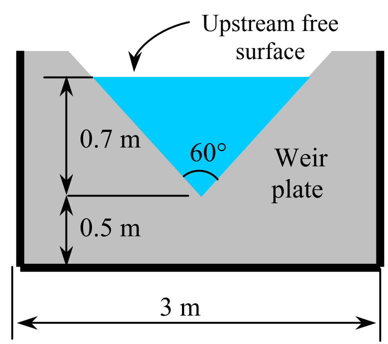
where C_{wd} = 0.60, θ = 60°, and H = 1 m. Substituting,

$$\dot{V} = (0.60)\frac{8}{15}\tan\left(\frac{60°}{2}\right)\sqrt{2(9.81\text{ m/s}^2)}(1\text{ m})^{5/2} = \mathbf{0.818\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate in a channel by measuring a single flow depth. Triangular weirs are best-suited to measure low discharge rates as they are more accurate than the other weirs for small heads.

13-110 The flow rate of water in an open channel is to be measured with a sharp-crested triangular weir. For a given flow depth upstream the weir, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Properties The weir discharge coefficient is given to be 0.60.

Analysis The discharge rate of water is determined directly from

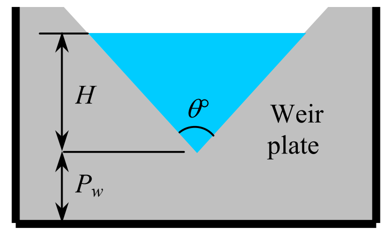
$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

where C_{wd} = 0.60, θ = 60°, and H = 0.7 m. Substituting,

$$\dot{V} = (0.60)\frac{8}{15}\tan\left(\frac{60°}{2}\right)\sqrt{2(9.81\text{ m/s}^2)}(0.7\text{ m})^{5/2} = \mathbf{0.335\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate in a channel by measuring a single flow depth. Triangular weirs are best-suited to measure low discharge rates as they are more accurate than the other weirs for small heads.

13-111 The notch angle of a sharp-crested triangular weir used to measure the discharge rate of water from a lake is reduced by half. The percent reduction in the discharge rate is to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The water depth in the lake and the weir discharge coefficient remain unchanged.

Analysis The discharge rate through a triangular weir is given as

$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

Therefore, the discharge rate is proportional to the tangent of the half notch angle, and the ratio of discharge rates is calculated to be

$$\dot{V} = \frac{\dot{V}_{50°}}{\dot{V}_{100°}} = \frac{\tan(50°/2)}{\tan(100°/2)} = 0.391$$

When the notch angle is reduced by half, the discharge rate drops to 39.1% of the original level. Therefore, the percent reduction in the discharge rate is

Percent reduction = 1-0.391 =0.609= **60.9%**

Discussion Note that triangular weirs with small notch angles can be used to measure small discharge rates while weirs with large notch angles can be used to measure for large discharge rates.

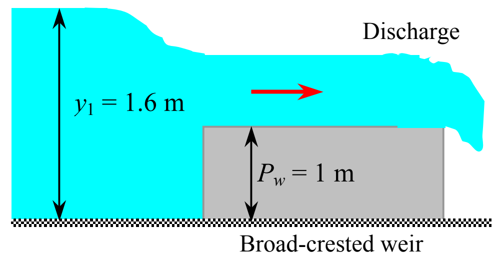
13-112 The flow rate in an open channel is to be measured using a broad-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 1.6 - 1.0 = 0.6\text{ m}$$

The discharge coefficient of the weir is



$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1 + H/P_w}} = \frac{0.65}{\sqrt{1 + (0.6\text{ m})/(1.0\text{ m})}} = 0.5139$$

Then the water flow rate through the channel becomes

$$\dot{V}_{\text{rec}} = C_{wd,\text{broad}} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$
$$= (0.5139)(5\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}(0.6\text{ m})^{3/2}$$
$$= \mathbf{2.04\ m^3/s}$$

The minimum flow depth above the weir is the critical depth, which is determined from

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.04\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(5\text{ m})^2}\right)^{1/3} = \mathbf{0.257\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{2.04\text{ m}^3/\text{s}}{(5\text{ m})(1.6\text{ m})} = 0.255\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.255\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.0033\text{ m}$$

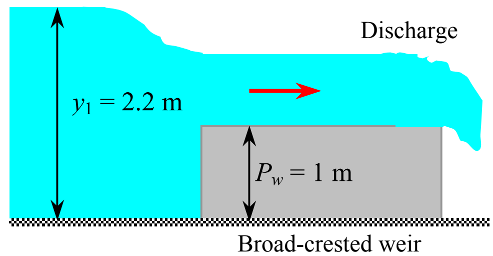
This is 0.3% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$, the flow rate becomes 2.05 m^3/s, which is practically identical to the value determined above.

13-113 The flow rate in an open channel is to be measured using a broad-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 2.2 - 1.0 = 1.2\text{ m}$$



The discharge coefficient of the weir is

$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1+H/P_w}} = \frac{0.65}{\sqrt{1+(1.2\text{ m})/(1.0\text{ m})}} = 0.4382$$

Then the water flow rate through the channel becomes

$$\dot{V}_{\text{rec}} = C_{wd,\text{broad}}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

$$= (0.4382)(5\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}\,(1.2\text{ m})^{3/2}$$

$$= \mathbf{4.91\ m^3/s}$$

The minimum flow depth above the weir is the critical depth, which is determined from

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(4.91\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(5\text{ m})^2}\right)^{1/3} = \mathbf{0.462\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{4.91\text{ m}^3/\text{s}}{(5\text{ m})(2.2\text{ m})} = 0.446\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.446\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.010\text{ m}$$

This is 0.8% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$, the flow rate becomes 4.97 m^3/s, which is practically identical to the value determined above.

13-114 The flow rate in an open channel is measured using a broad-crested rectangular weir. For a measured value of minimum flow depth over the weir, the flow rate and the upstream flow dept are to be determined. √EES

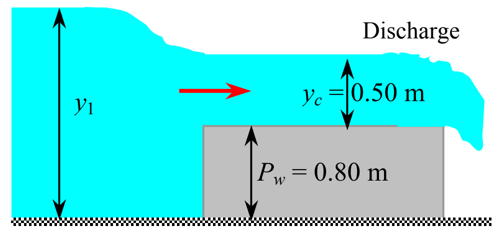
Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow depth over the reaches its minimum value when the flow becomes critical. Therefore, the measured minimum depth is the critical depth y_c. Then the flow rate is determined from the critical depth relation to be

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} \rightarrow \dot{V} = \sqrt{y_c^3 gb^2} = \sqrt{(0.50\text{ m})^3(9.81\text{ m/s}^2)(1\text{ m})^2} = \mathbf{1.11\ m^3/s}$$

This is the flow rate per m width of the channel since we have taken b = 1 m. Disregarding the upstream velocity head and noting that the discharge coefficient of the weir is $C_{wd,\text{broad}} = 0.65/\sqrt{1+H/P_w}$, the flow rate for a broad-crested weir can be expressed as

$$\dot{V}_{\text{rec}} = \frac{0.65}{\sqrt{1+H/\text{P}_w}} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$



Substituting,

$$1.11\text{ m}^3\text{/s} = \frac{0.65\text{ m}}{\sqrt{1+\text{H}/(0.8\text{ m})}}(1\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}\,H^{3/2}$$
$$= \mathbf{4.91\,m^3/s}$$

Its solution is H = 1.40 m. Then the flow depth upstream the weir becomes

$$y_1 = H + P_w = 1.40 + 0.80 = \mathbf{2.20\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{1.11\text{ m}^3\text{/s}}{(1\text{ m})(2.2\text{ m})} = 0.503\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.503\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.013\text{ m}$$

This is 0.9% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$, the flow rate becomes 1.12 m^3/s, which is practically identical to the value determined above.

Review Problems

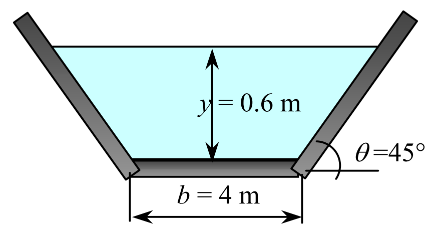
13-115 Water flows uniformly in a trapezoidal channel. For a given flow depth, it is to be determined whether the flow is subcritical or supercritical. √EES

Assumptions The flow is uniform.

Analysis The flow area and the average velocity are

$$A_c = y\frac{(b+b+2y/\tan\theta)}{2} = (0.60\text{ m})\frac{[4+4+2(0.60\text{ m})/\tan 45°]\text{ m}}{2} = 2.76\text{ m}^2$$

$$V = \frac{\dot{V}}{A_c} = \frac{18\text{ m}^3/\text{s}}{2.76\text{ m}^2} = 6.522\text{ m/s}$$



When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{b+2y/\tan\theta} = \frac{2.76\text{ m}^2}{(4+2\times 0.60/\tan 45°)\text{ m}} = 0.5308\text{ m}$$

Then the Froude number becomes

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{6.522\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.5308\text{ m})}} = 2.86$$

which is greater than 1. Therefore, the flow is **supercritical**.

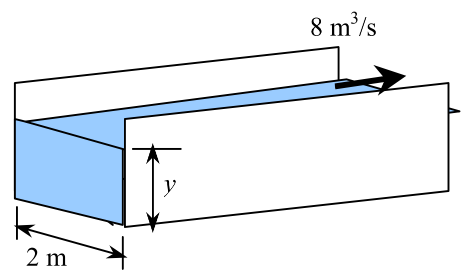
Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

13-116 Water flows in a rectangular channel. The flow depth below which the flow is supercritical is to be determined. √EES

Assumptions The flow is uniform.

Analysis The flow depth below which the flow is super critical is the critical depth y_c determined from

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(8\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(2\text{ m})^2}\right)^{1/3} = \mathbf{1.18\text{ m}}$$



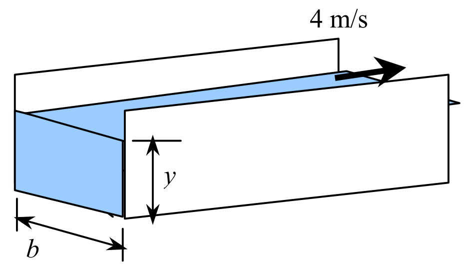
Therefore, flow is **supercritical** for $y < 1.18$ m.

Discussion Note that a flow is more likely to exist as supercritical when the flow depth is low and thus the flow velocity is high.

13-117 Water flows in a canal at a specified average velocity. For various flow depths, it is to be determined whether the flow is subcritical or supercritical. √EES

Assumptions The flow is uniform.

Analysis For each depth, we determine the Froude number and compare it to the critical value of 1:



(***a***) $y = 0.2$ m: $$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.2\ \mathrm{m})}} = 2.86 > 1$$

which is greater than 1. Therefore, the flow is **supercritical**.

(***b***) $y = 2$ m: $$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2\ \mathrm{m})}} = 0.903 < 1$$

which is less than 1. Therefore, the flow is **subcritical**.

(***c***) $y = 1.63$ m: $$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(1.63\ \mathrm{m})}} = 1$$

which is equal to 1. Therefore, the flow is **critical**.

Discussion Note that a flow is more likely to exist as supercritical when the flow depth is low and thus the flow velocity is high. Also, the type of flow can be determined easily by checking Froude number.

13-118 The flow of water in a rectangular channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

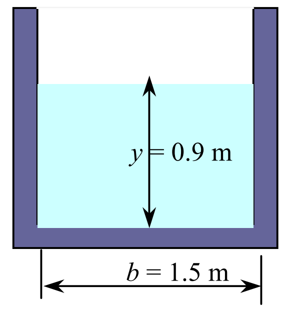
Properties The Manning coefficient is given to be $n = 0.012$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = by = (1.5\text{ m})(0.9\text{ m}) = 1.35\text{ m}^2$$

$$p = 1.5\text{ m} + 2 \times 0.9\text{ m} = 3.3\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{1.35\text{ m}^2}{3.3\text{ m}} = 0.4091\text{ m}$$



Bottom slope of the channel is

$$S_0 = \tan 0.6^\circ = 0.01047$$

Then the flow rate can be determined from Manning's equation to be

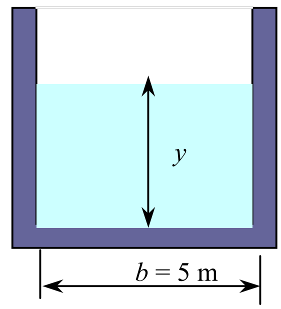
$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.012}(1.35\text{ m}^2)(0.4091\text{ m})^{2/3}(0.01047)^{1/2} = \mathbf{6.34\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-119 The flow of water in a rectangular channel is considered. The effect of bottom slope on the flow rate is to be investigated as the bottom angle varies from 0.5 to 10°.

Assumptions **1** The flow is steady and uniform. **2** Roughness coefficient is constant along the channel.

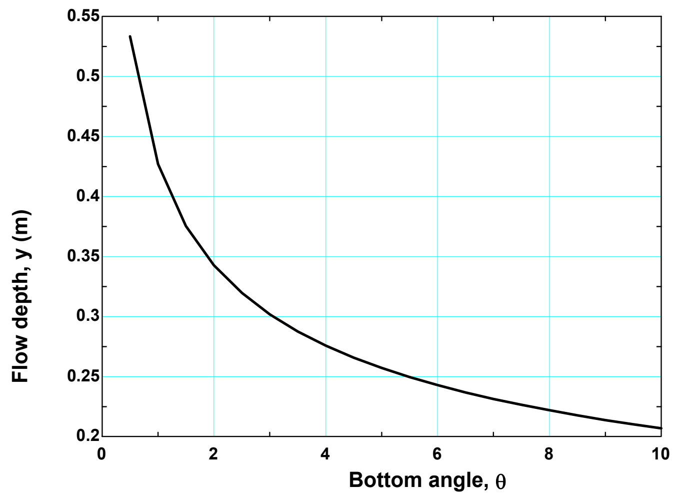
Properties Manning coefficient for an open channel made of finished concrete is n = 0.012 (Table 13-1).




```
a=1
b=5
Vdot = 12 "m3/s"
n=0.012

s=tan(teta)
Ac=b*y
p=b+2*y
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```

Bottom angle, $\theta°$	Flow depth, y, m
0.5	0.533
1.0	0.427
1.5	0.375
2.0	0.343
2.5	0.320
3.0	0.302
3.5	0.287
4.0	0.276
4.5	0.266
5.0	0.257
5.5	0.250
6.0	0.243
6.5	0.237
7.0	0.231
7.5	0.226
8.0	0.222
8.5	0.218
9.0	0.214
9.5	0.210
10.0	0.207



Discussion Note that the flow depth decreases as the bottom angle increases, as expected.

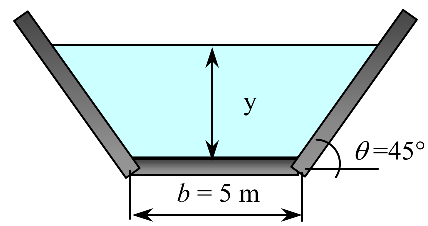
13-120 The flow of water in a rectangular channel is considered. The effect of bottom slope on the flow rate is to be investigated as the bottom angle varies from 0.5 to 10°.

Assumptions **1** The flow is steady and uniform. **2** Roughness coefficient is constant along the channel.

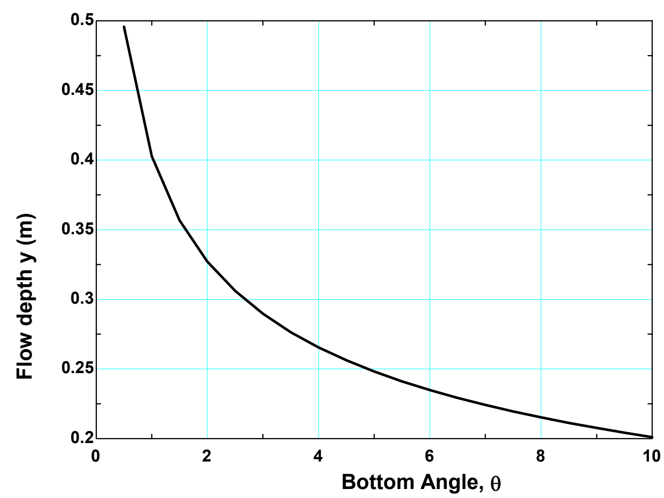
Properties Manning coefficient for an open channel made of finished concrete is n = 0.012 (Table 13-1).

```
a=1
b=5
Vdot = 12 "m3/s"
n=0.012

s=tan(teta)
Ac=y*(b+y/tan(45))
p=b+2*y/sin(45)
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```



Bottom angle, $\theta°$	Flow depth, y, m
0.5	0.496
1.0	0.403
1.5	0.357
2.0	0.327
2.5	0.306
3.0	0.290
3.5	0.276
4.0	0.266
4.5	0.256
5.0	0.248
5.5	0.241
6.0	0.235
6.5	0.229
7.0	0.224
7.5	0.220
8.0	0.215
8.5	0.211
9.0	0.208
9.5	0.204
10.0	0.201



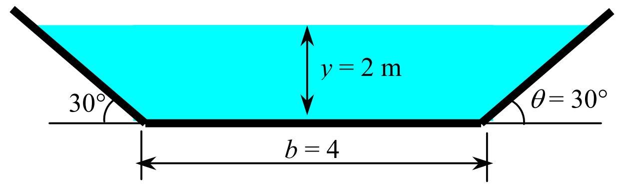
13-121 The flow of water in a trapezoidal channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient for a brick-lined open channel is $n = 0.015$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = y\left(b + \frac{y}{\tan\theta}\right) = (2\text{ m})\left(4\text{ m} + \frac{2\text{ m}}{\tan 30°}\right) = 14.93\text{ m}^2$$



$$p = b + \frac{2y}{\sin\theta} = 4\text{ m} + \frac{2(2\text{ m})}{\sin 30°} = 12\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{14.93\text{ m}^2}{12\text{ m}} = 1.244\text{ m}$$

Bottom slope of the channel is $S_o = 0.001$. Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.015}(14.93\text{ m}^2)(1.244\text{ m})^{2/3}(0.001)^{1/2} = \mathbf{36.4\ m^3/s}$$

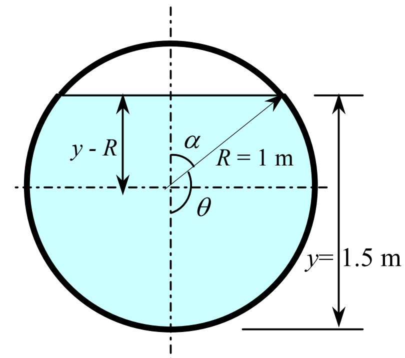
Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-122 The flow of water in a circular open channel is considered. For given flow depth and flow rate, the elevation drop per km length is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient for the steel channel is given to be n = 0.012.

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are



$$\cos\alpha = \frac{y-R}{R} = \frac{1.5-1}{1} = 0.5 \quad \rightarrow \quad \alpha = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$

$$\theta = \pi - \alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3} = 120°$$

$$A_c = R^2(\theta - \sin\theta\cos\theta) = (1\,\text{m})^2[2\pi/3 - \sin(2\pi/3)\cos(2\pi/3)] = 2.527\,\text{m}^2$$

$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R = \frac{2\pi/3 - \sin(2\pi/3)\cos(2\pi/3)}{2\times 2\pi/3}(1\,\text{m}) = 0.6034\,\text{m}$$

Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 12\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.012}(2.527\,\text{m}^2)(0.6034\,\text{m})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.00637. Therefore, the *elevation drop* Δz across a pipe length of L = 1 km must be

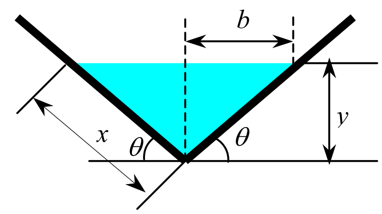
$$\Delta z = S_0 L = 0.00637(1000\,\text{m}) = \mathbf{6.37\,m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-123 The flow of water through a V-shaped open channel is considered. The angle θ the channel makes from the horizontal is to be determined for the case of most efficient flow. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis We let the length of the sidewall of the channel be x. From trigonometry,



$$\sin\theta = \frac{y}{x} \quad \rightarrow \quad y = x\sin\theta$$

$$\cos\theta = \frac{b}{x} \quad \rightarrow \quad b = x\cos\theta$$

Then the cross-sectional area and the perimeter of the flow section become

$$A_c = by = x\cos\theta\sin\theta = \frac{x^2}{2}\sin 2\theta \quad \rightarrow \quad x = \sqrt{\frac{2A_c}{\sin 2\theta}}$$

$$p = 2x = 2\sqrt{\frac{2A_c}{\sin 2\theta}} \quad \rightarrow \quad p = 2\sqrt{2A_c}(\sin 2\theta)^{-1/2}$$

Now we apply the criterion that the best hydraulic cross-section for an open channel is the one with the minimum wetted perimeter for a given cross-section. Taking the derivative of p with respect to θ while holding A_c constant gives

$$\frac{dp}{d\theta} = 2\sqrt{2A_c}\,\frac{d[(\sin 2\theta)^{-1/2}]}{d\theta} = 2\sqrt{2A_c}\,\frac{d[(\sin 2\theta)^{-1/2}]}{d(\sin 2\theta)}\frac{d(\sin 2\theta)}{d\theta} = 2\sqrt{2A_c}\,\frac{-3}{2(\sin 2\theta)^{3/2}}2\cos 2\theta$$

Setting $dp/d\theta = 0$ gives $\cos 2\theta = 0$, which is satisfied when $2\theta = 90°$. Therefore, the criterion for the best hydraulic cross-section for a triangular channel is determined to be

$$\theta = \mathbf{45°}$$

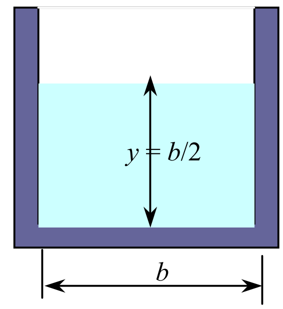
Discussion The procedure outlined above can be used to determine the best hydraulic cross-section for any geometric shape.

13-124E Water is to be transported in a rectangular channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is $n = 0.014$ for channels made of unfinished concrete (Table 13-1).

Analysis For best cross-section of a rectangular cross-section, $y = b/2$. Then $A_c = yb = b^2/2$ and $R_h = b/4$. The flow rate is determined from the Manning equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,



(*a*) Bottom drop of 8 ft per mile:

$$s = (8 \text{ ft})/(5280 \text{ ft}) = 0.001515$$

$$200 \text{ ft}^3/\text{s} = \frac{1.486 \text{ ft}^{1/3}/s}{0.014}(b^2/2)(b/4)^{2/3}(0.001515)^{1/2}$$

It gives b = **7.86 ft,** and $y = b/2 = 3.93$ ft.

(*b*) Bottom drop of 10 ft per mile:

$$s = (10 \text{ ft})/(5280 \text{ ft}) = 0.001894$$

$$200 \text{ ft}^3/\text{s} = \frac{1.486 \text{ ft}^{1/3}/s}{0.014}(b^2/2)(b/4)^{2/3}(0.001894)^{1/2}$$

It gives b = **7.54 ft,** and $y = b/2 = 3.77$ ft.

Discussion. The concept of best cross-section is an important consideration in the design of open channels because it directly affects the construction costs.

13-125E Water is to be transported in a trapezoidal channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is $n = 0.014$ for channels made of unfinished concrete (Table 13-1).

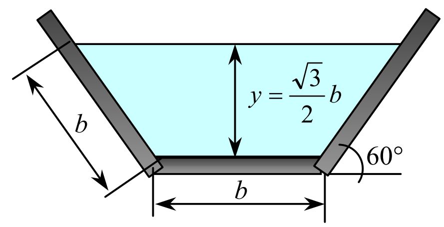
Analysis For best cross-section of a trapezoidal channel of bottom width b, $\theta = 60°$ and $y = b\sqrt{3}/2$. Then,

$A_c = y(b + b\cos\theta) = 0.5\sqrt{3}b^2(1+\cos 60°) = 0.75\sqrt{3}b^2$, $p = 3b$, and $R_h = \dfrac{y}{2} = \dfrac{\sqrt{3}}{4}b$. The flow rate is determined from the Manning equation, $\dot{V} = \dfrac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,

(*a*) Bottom drop of 8 ft per mile:

$$s = (8\text{ ft})/(5280\text{ ft}) = 0.001515$$

$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(0.75\sqrt{3}b^2)(\sqrt{3}b/4)^{2/3}(0.001515)^{1/2}$$



It gives b = **4.79 ft,** and y = 4.15 ft.

(*b*) Bottom drop of 10 ft per mile:

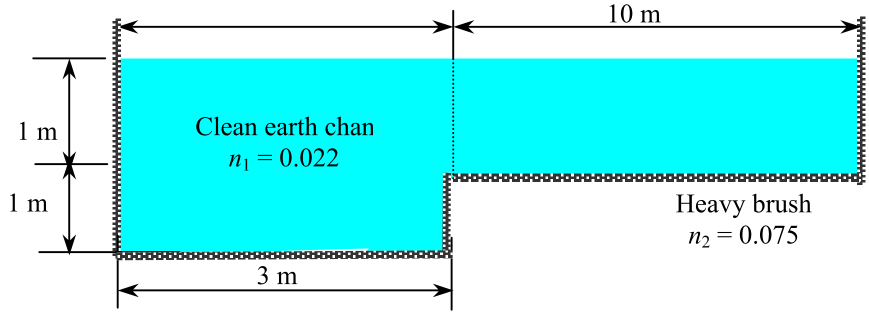
$$s = (10\text{ ft})/(5280\text{ ft}) = 0.001894$$

$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(0.75\sqrt{3}b^2)(\sqrt{3}b/4)^{2/3}(0.001894)^{1/2}$$

It gives b = **4.59 ft,** and y = 3.98 ft.

Discussion. The concept of best cross-section is an important consideration in the design of open channels because it directly affects the construction costs.

13-126 Water is flowing through a channel with nonuniform surface properties. The flow rate through the channel and the effective Manning coefficient are to be determined.



Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The Manning coefficients do not vary along the channel.

Analysis The channel involves two parts with different roughness, and thus it is appropriate to divide the channel into two subsections. The flow rate for each subsection can be determined from the Manning equation, and the total flow rate can be determined by adding them up.
The flow area, perimeter, and hydraulic radius for each subsection and the entire channel are:

Subsection 1: $A_{c1} = 6\text{ m}^2$, $\quad p_1 = 6\text{ m}$, $\quad R_{h1} = \dfrac{A_{c1}}{p_1} = \dfrac{6\text{ m}^2}{6\text{ m}} = 1.00\text{ m}$

Subsection 2: $A_{c2} = 10\text{ m}^2$, $\quad p_2 = 11\text{ m}$, $\quad R_{h2} = \dfrac{A_{c2}}{p_2} = \dfrac{10\text{ m}^2}{11\text{ m}} = 0.909\text{ m}$

Entire channel: $A_c = 16\text{ m}^2$, $\quad p = 17\text{ m}$, $\quad R_h = \dfrac{A_c}{p} = \dfrac{16\text{ m}^2}{17\text{ m}} = 0.941\text{ m}$

Applying the Manning equation to each subsection, the total flow rate through the channel becomes

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \frac{a}{n_1} A_1 R_1^{2/3} S_0^{1/2} + \frac{a}{n_2} A_2 R_2^{2/3} S_0^{1/2}$$

$$= (1\text{ m}^{1/3}/\text{s})\left(\frac{(6\text{ m}^2)\ (1\text{ m})^{2/3}}{0.022} + \frac{(10\text{ m}^2)(0.909\text{ m})^{2/3}}{0.075}\right)(\tan 0.5°)^{1/2}$$

$$= \mathbf{37.2\ m^3/s}$$

Knowing the total flow rate, the effective Manning coefficient for the entire channel can be determined from the Manning equation to be

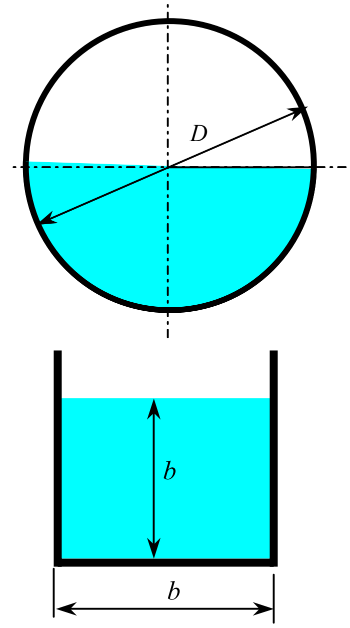
$$n_{\text{eff}} = \frac{aA_c R_h^{2/3} S_0^{1/2}}{\dot{V}} = \frac{(1\text{ m}^{1/3}/\text{s})(16\text{ m}^2)(0.941\text{ m})^{2/3}(0.00873)^{1/2}}{37.2\text{ m}^3/\text{s}} = \mathbf{0.0386}$$

Discussion The effective Manning coefficient n_{eff} of the channel turns out to lie between the two n values, as expected. The weighted average of the Manning coefficient of the channel is $n_{\text{ave}}=(n_1p_1+ n_2p_2)/p$ = 0.056, which is quite different than n_{eff}. Therefore, using a weighted average Manning coefficient for the entire channel may be tempting, but it would not be so accurate.

13-127 Two identical channels, one rectangular of bottom width b and one circular of diameter D, with identical flow rates, bottom slopes, and surface linings are considered. The relation between b and D is to be determined for the case of the flow height $y = b$ and the circular channel is flowing half full.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis The cross-sectional area, perimeter, and hydraulic radius of the rectangular channel are



$$A_c = b^2, \quad p = 3b, \text{ and } \quad R_h = \frac{A_c}{p} = \frac{b^2}{3\text{b}} = \frac{b}{3}$$

Then using the Manning equation, the flow rate can be expressed as

$$\dot{V}_{\text{rec}} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{a}{n} b^2 \left(\frac{b}{3}\right)^{2/3} S_0^{1/2} = \frac{a}{n} S_0^{1/2} \frac{b^{8/3}}{3^{2/3}}$$

The corresponding relations for the semi-circular channel are

$$A_c = \frac{\pi D^2}{8}, \quad p = \frac{\pi D}{2}, \text{ and } \quad R_h = \frac{A_c}{p} = \frac{D}{4}$$

and

$$\dot{V}_{\text{cir}} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{a}{n} \pi \frac{D^2}{8} \left(\frac{D}{4}\right)^{2/3} S_0^{1/2} = \frac{a}{n} S_0^{1/2} \frac{\pi D^{8/3}}{8 \times 4^{2/3}}$$

Setting the flow rates in the two channels equal to each other $\dot{V}_{\text{cir}} = \dot{V}_{\text{rec}}$ gives

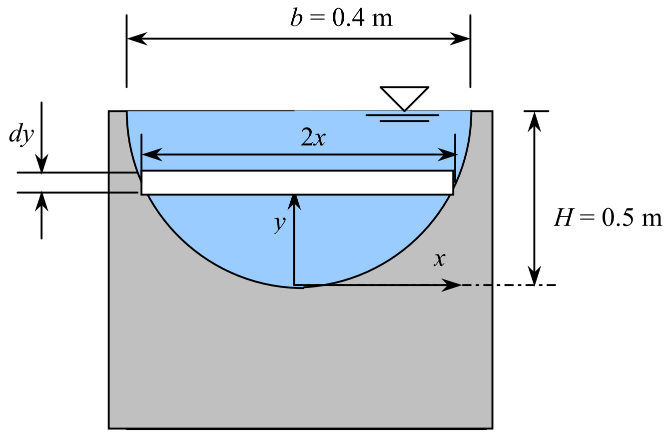
$$\frac{a}{n} S_0^{1/2} \frac{b^{8/3}}{3^{2/3}} = \frac{a}{n} \frac{\pi D^{8/3}}{8 \times 4^{2/3}} S_0^{1/2} \quad \rightarrow \quad \frac{b^{8/3}}{3^{2/3}} = \frac{\pi D^{8/3}}{8 \times 4^{2/3}} \quad \rightarrow \quad \frac{b}{D} = \left(\frac{\pi 3^{2/3}}{8 \times 4^{2/3}}\right)^{3/8} = 0.655$$

Therefore, the desired relation is **b = 0.655D.**

Discussion Note that the wetted perimeters in this case are $p_{\text{rec}} = 3b = 2.0D$ and $p_{\text{cir}} = \pi D/2 = 1.57D$. Therefore, the semi-circular channel is a more efficient channel than the rectangular one.

13-128 The flow of water through a parabolic notch is considered. A relation is to be developed for the flow rate, and its numerical value is to be calculated.

Assumptions **1** The flow is steady. **2** All frictional effects are negligible, and Toricelli's equation can be used for the velocity.



Analysis The notch is parabolic with $y = 0$ at $x = 0$, and thus it can be expressed analytically as $y = Cx^2$. Using the coordinates of the upper right corner, the value of the constant is determined to be $C = y/x^2 = H/(b/2)^2 = 4H/b^2 = 4(0.5\text{ m})/(0.4\text{ m})^2 = 12.5\text{ m}^{-1}$.

A differential area strip can be expressed as

$$dA = 2x dy = 2\sqrt{y/C}\,dy$$

Noting that the flow velocity is $V = \sqrt{2g(H-y)}$, the flow rate through this differential area is

$$VdA = V\left(2\sqrt{y/C}\,dy\right) = \sqrt{2g(H-y)}\,2\sqrt{y/C}\,dy = 2\sqrt{2g/C}\sqrt{y(H-y)}\,dy$$

Then the flow rate through the entire notch is determined by integration to be

$$\dot{V} = \int_A VdA = 2\sqrt{2g/C}\int_{y=0}^{H}\sqrt{y(H-y)}\,dy$$

where [CHECK INTEGRATION]

$$\int_{y=0}^{H}\sqrt{y(H-y)}\,dy = \left[\frac{1}{4}(2y-H)\sqrt{Hy-y^2} + \frac{H^2}{8}Arc\tan\left(\frac{2y-H}{2\sqrt{Hy-y^2}}\right)\right]_0^H = \frac{\pi}{16}H^2$$

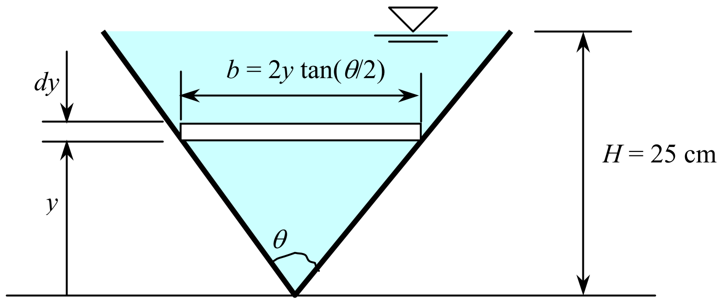
Then the expression for the volume flow rate and its numerical value become

$$\dot{V} = \frac{\pi}{8}\sqrt{\frac{2g}{C}}H^2 = \frac{\pi}{8}\sqrt{\frac{2(9.81\text{ m/s}^2)}{12.5\text{ m}^{-1}}}H^2 = (0.492\text{ m/s})H^2 = (0.492\text{ m/s})(0.5\text{ m})^2 = 0.123\text{ m}^3/\text{s}$$

Discussion Note that a general flow rate equation for parabolic notch would be in the form of $\dot{V} = KH^2$, where $K = C_d\dfrac{\pi}{8}\sqrt{\dfrac{2g}{C}}$ and C_d is the discharge coefficient whose value is determined experimentally to account for nonideal effects.

13-129 The flow of water through a parabolic notch is considered. A relation is to be developed for the flow rate, and its numerical value is to be calculated.

Assumptions **1** The flow is steady. **2** All frictional effects are negligible, and Toricelli's equation can be used for the velocity.



Analysis Consider a differential strip area shown on the sketch. It can be expressed as

$$dA = bdy = 2y\tan(\theta/2)dy$$

Noting that the flow velocity is $V = \sqrt{2g(H-y)}$, the flow rate through this differential area is

$$VdA = V\left(2y\tan(\theta/2)dy\right) = \sqrt{2g(H-y)}\,2y\tan(\theta/2)dy = 2\sqrt{2g}\,\tan(\theta/2)y\sqrt{H-y}dy$$

Then the flow rate through the entire notch is determined by integration to be

$$\dot{V} = \int_A VdA = 2\sqrt{2g}\,\tan(\theta/2)\int_{y=0}^{H} y\sqrt{H-y}dy$$

where

$$\int_{y=0}^{H} y\sqrt{H-y}dy = \left[-\frac{2}{5}y^{5/2} + \frac{2}{3}Hy^{3/2}\right]\Bigg|_0^H = \frac{4}{15}H^{5/2}$$

Then the expression for the volume flow rate and its numerical value become

$$\dot{V} = \frac{8\sqrt{2g}}{15}\tan(\theta/2)H^{5/2} = \frac{8\sqrt{2(9.81\text{ m/s}^2)}}{15}\tan(\theta/2)(0.25)^{5/2} = 0.07382\tan(\theta/2) \quad (\text{m}^3/\text{s})$$

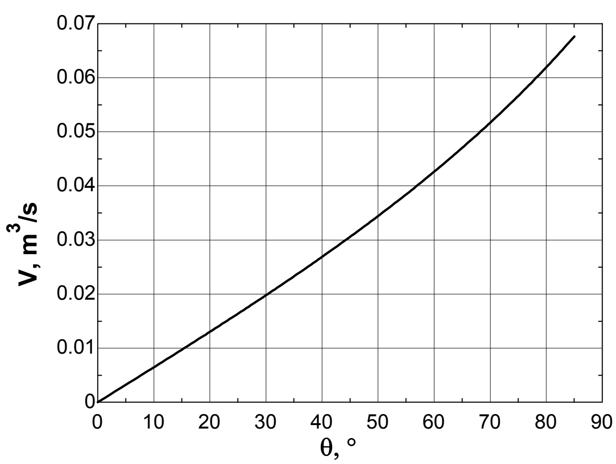
$$\theta = 25°: \quad \dot{V} = 0.07382\tan(25°/2) = 0.0164\text{ m}^3/\text{s}$$

$$\theta = 40°: \quad \dot{V} = 0.07382\tan(40°/2) = 0.0269\text{ m}^3/\text{s}$$

$$\theta = 60°: \quad \dot{V} = 0.07382\tan(60°/2) = 0.0426\text{ m}^3/\text{s}$$

$$\theta = 75°: \quad \dot{V} = 0.07382\tan(75°/2) = 0.0566\text{ m}^3/\text{s}$$

These results are plotted.

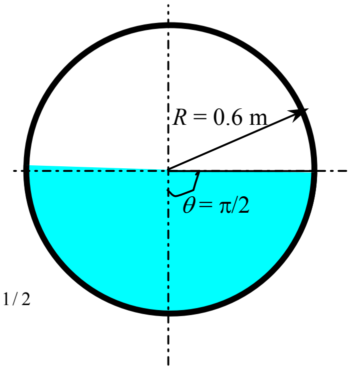


Discussion Note that a general flow rate equation for the V-notch would be in the form of $\dot{V} = K\tan(\theta/2)H^{5/2}$, where $K = C_d 8\sqrt{2g}/15$ and C_d is the discharge coefficient whose value is determined experimentally to account for nonideal effects.

13-130 Water flows uniformly half-full in a circular channel. For specified flow rate and bottom slope, the Manning coefficient is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are



$$A_c = \frac{\pi R^2}{2} = \frac{\pi\,(0.6\text{ m})^2}{2} = 0.5655\text{ m}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(0.6\text{ m})}{2} = 1.885\text{ m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{0.60\text{ m}}{2} = 0.30\text{ m}$$

Then the Manning coefficient can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 1.25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{n}(0.5655\text{ m}^2)(0.30\text{ m})^{2/3}(0.004)^{1/2}$$

It gives the Manning coefficient to be

$$n = \mathbf{0.013}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(0.6\text{ m})}{4} = 0.4712\text{ m}$$

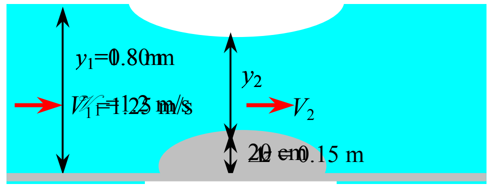
$$V = \frac{\dot{V}}{A_c} = \frac{1.25\ \text{m}^3/\text{s}}{0.5655\ \text{m}^2} = 2.210\text{ m/s}$$

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2.21\text{m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.4721\text{ m})}} = \mathbf{1.03}$$

which is greater than 1. Therefore, the flow is *supercritical*.

Discussion It appears that this channel is made of cast iron or unplaned wood .

13-131 Water flowing in a horizontal open channel encounters a bump. Flow properties over the bump are to be determined.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{1.25\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m}^2/\mathrm{s})(1.8\ \mathrm{m})}} = 0.297$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(1.8\ \mathrm{m})^2(1.25\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s}^2}\right)^{1/3} = 0.802\ \mathrm{m}$$

The upstream flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream, over the bump, and critical specific energy are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.80\ \mathrm{m}) + \frac{(1.25\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s}^2)} = 1.88\ \mathrm{m}$$

The flow depth over the bump can be determined from

$$y_2^3 - (E_{s1} - \Delta z_b)y_2^2 + \frac{V_1^2}{2g}y_1^2 = 0 \quad \rightarrow \quad y_2^3 - (1.88 - 0.20\ \mathrm{m})y_2^2 + \frac{(1.25\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s}^2)}(1.80\ \mathrm{m})^2 = 0$$

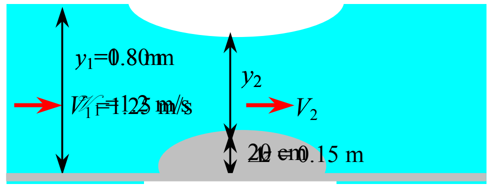
Using an equation solver, the physically meaningful root of this equation is determined to be y_2 = **1.576 m**. Then,

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{1.8\ \mathrm{m}}{1.576\ \mathrm{m}}(1.25\ \mathrm{m/s}) = \mathbf{1.43\ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.428\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s}^2)(1.576\ \mathrm{m})}} = \mathbf{0.363}$$

Discussion The actual values may be somewhat different than those given above because of the frictional effects that are neglected in the analysis.

13-132 Water flowing in a horizontal open channel encounters a bump. The bump height for which the flow over the bump is critical is to be determined.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{1.25\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m^2/s})(1.8\ \mathrm{m})}} = 0.297$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(1.8\ \mathrm{m})^2(1.25\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s^2}}\right)^{1/3} = 0.802\ \mathrm{m}$$

The upstream flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream specific energy is

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.80\ \mathrm{m}) + \frac{(1.25\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 1.88\ \mathrm{m}$$

Noting that the flow over the bump is critical and that $E_{s2} = E_{s1} - \Delta z_b$,

$$E_{s2} = E_c = \frac{3}{2}y_c = \frac{3}{2}(0.802\ \mathrm{m}) = 1.20\ \mathrm{m}$$

and

$$\Delta z_b = E_{s1} - E_{s2} = 1.88 - 1.20 = \mathbf{0.68\ m}$$

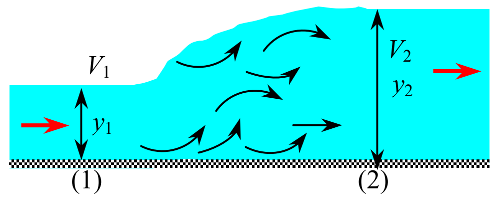
Discussion If a higher bump is used, the flow will remain critical but the flow rate will decrease (the choking effect).

13-133 Water flow through a wide rectangular channel undergoing a hydraulic jump is considered. It is to be shown that the ratio of the Froude numbers before and after the jump can be expressed in terms of flow depths y_1 and y_2 before and after the jump, respectively, as $\mathrm{Fr}_1/\mathrm{Fr}_2 = \sqrt{(y_2/y_1)^3}$.

Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The Froude number for a wide channel of width b and flow depth y is given as

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{\dot{V}/by}{\sqrt{gy}} = \frac{\dot{V}}{by\sqrt{gy}} = \frac{\dot{V}}{b\sqrt{gy^3}}$$



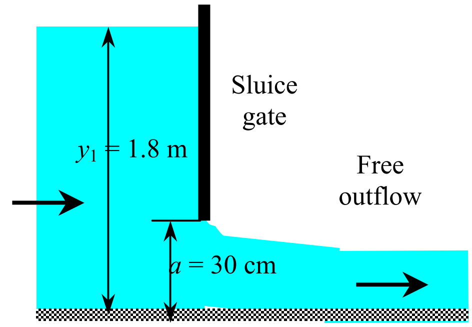
Expressing the Froude number before and after the jump and taking their ratio gives

$$\frac{\mathrm{Fr}_1}{\mathrm{Fr}_2} = \frac{\dot{V}/\left(b\sqrt{gy_1^3}\right)}{\dot{V}/\left(b\sqrt{gy_2^3}\right)} = \frac{\sqrt{gy_2^3}}{\sqrt{gy_1^3}} = \sqrt{\left(\frac{y_2}{y_1}\right)^3}$$

which is the desired result.

Discussion. Using the momentum equation, other relations such as $y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$ can also be developed.

13-134 A sluice gate with free outflow is used to control the flow rate of water. For specified flow depths, the flow rate per unit width and the downstream flow depth and velocity are to be determined. √EES



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient (for drowned outflow, we also need to know y_2/a and thus the flow depth y_2 downstream the gate),

$$\frac{y_1}{a} = \frac{1.8\text{ m}}{0.30\text{ m}} = 6$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d = 0.57. Then the discharge rate per m width becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.57\,(1\text{ m})(0.30\text{ m})\sqrt{2\,(9.81\text{ m/s}^2)(1.8\text{ m})} = \mathbf{1.02\ m^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth and velocity past the gate become

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 1.8\text{ m} + \frac{(1.02\text{ m/s}^2)^2}{2(9.81\text{ m/s}^2)[(1\text{ m})(1.8\text{ m})]^2} = 1.816\text{ m}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \;\rightarrow\; y_2 + \frac{(1.02\text{ m/s}^2)^2}{2(9.81\text{ m/s}^2)[(1\text{ m})(y_2)]^2} = 1.816\text{ m}$$

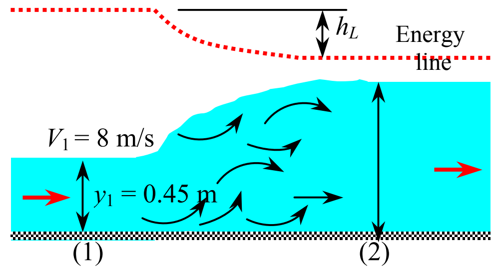
It gives y_2 = **0.179** m for flow depth as the physically meaningful root (positive and less than 1.8 m). Also,

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{1.02\text{ m}^3/\text{s}}{(1\text{ m})(0.179\text{ m})} = \mathbf{5.67\ m/s}$$

Discussion In actual gates some frictional losses are unavoidable, and thus the actual velocity downstream will be lower.

13-135 Water at a specified depth and velocity undergoes a hydraulic jump. The fraction of mechanical energy dissipated is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{8\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.45\ \mathrm{m})}} = 3.808$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(0.45\ \mathrm{m})\left(-1+\sqrt{1+8\times 3.808^2}\right) = 2.209\ \mathrm{m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.45\ \mathrm{m}}{2.209\ \mathrm{m}}(8\ \mathrm{m/s}) = 1.630\ \mathrm{m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.630\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2.209\ \mathrm{m})}} = 0.350$$

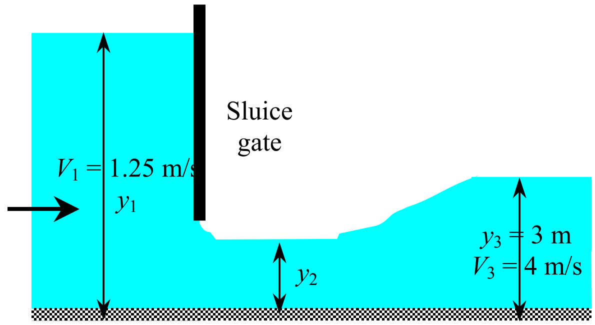
The head loss and the fraction of mechanical energy dissipated during the jump are

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.45\ \mathrm{m}) - (2.209\ \mathrm{m}) + \frac{(8\ \mathrm{m/s})^2 - (1.63\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 1.368\ \mathrm{m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1(1+\mathrm{Fr}_1^2/2)} = \frac{1.368\ \mathrm{m}}{(0.45\ \mathrm{m})(1+3.808^2/2)} = \mathbf{0.369}$$

Discussion Note that almost over one-third of the mechanical energy of the fluid is dissipated during hydraulic jump.

13-136 The flow depth and average velocity of water after a hydraulic jump together with approach velocity to sluice gate are given. The flow rate per m width, the flow depths before and after the gate, and the energy dissipation ratio are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow rate per m width of channel, flow depth before the sluice gate, and the Froude number after the jump is

$$\dot{V} = V_3 A_{c3} = V_3 b y_3 = (4\text{ m/s})(1\text{ m})(3\text{ m}) = \mathbf{12\ m^3/s}$$

$$y_1 = \frac{V_3}{V_1} y_3 = \frac{4\text{ m/s}}{1.25\text{ m/s}}(3\text{ m}) = \mathbf{9.60\ m}$$

$$\text{Fr}_3 = \frac{V_3}{\sqrt{g y_3}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3\text{ m})}} = 0.7373$$

The flow dept, velocity, and Froude number before the jump are

$$y_2 = 0.5 y_3\left(-1+\sqrt{1+8\text{Fr}_3^2}\right) = 0.5(3\text{ m})\left(-1+\sqrt{1+8\times 0.7373^2}\right) = \mathbf{1.969\ m}$$

$$V_2 = \frac{y_3}{y_2} V_3 = \frac{3\text{ m}}{1.969\text{ m}}(4\text{ m/s}) = 6.094\text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{6.094\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.969\text{ m})}} = 1.387$$

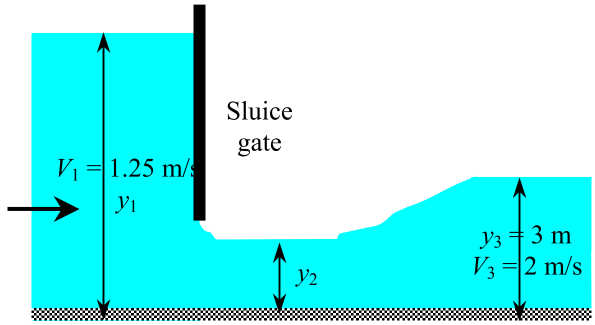
which is greater than 1, and thus the flow before the jump is indeed supercritical. The head loss and the fraction of mechanical energy dissipated during hydraulic jump are

$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (1.969\text{ m}) - (3\text{ m}) + \frac{(6.094\text{ m/s})^2 - (4\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.0463\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{0.0463\text{ m}}{(1.969\text{ m})(1+1.387^2/2)} = \mathbf{0.0120}$$

Discussion Note that this is a "mild" hydraulic jump, and only 1.2% of the mechanical energy is wasted.

13-137 The flow depth and average velocity of water after a hydraulic jump together with approach velocity to sluice gate are given. The flow rate per m width, the flow depths before and after the gate, and the energy dissipation ratio are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow rate per m width of channel, flow depth before the sluice gate, and the Froude number after the jump is

$$\dot{V} = V_3 A_{c3} = V_3 b y_3 = (2 \text{ m/s})(1 \text{ m})(3 \text{ m}) = \mathbf{6\ m^3/s}$$

$$y_1 = \frac{V_3}{V_1} y_3 = \frac{2 \text{ m/s}}{1.25 \text{ m/s}}(3 \text{ m}) = \mathbf{4.8\ m}$$

$$\text{Fr}_3 = \frac{V_3}{\sqrt{g y_3}} = \frac{2 \text{ m/s}}{\sqrt{(9.81 \text{ m/s}^2)(3 \text{ m})}} = 0.3687$$

The flow dept, velocity, and Froude number before the jump are

$$y_2 = 0.5 y_3\left(-1+\sqrt{1+8\text{Fr}_3^2}\right) = 0.5(3 \text{ m})\left(-1+\sqrt{1+8\times 0.3687^2}\right) = \mathbf{0.6671\ m}$$

$$V_2 = \frac{y_3}{y_2} V_3 = \frac{3 \text{ m}}{0.6671 \text{ m}}(2 \text{ m/s}) = 8.994 \text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{8.994 \text{ m/s}}{\sqrt{(9.81 \text{ m/s}^2)(0.6671 \text{ m})}} = 3.516$$

which is greater than 1, and thus the flow before the jump is indeed supercritical. The head loss and the fraction of mechanical energy dissipated during hydraulic jump are

$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (0.6671 \text{ m}) - (3 \text{ m}) + \frac{(8.994 \text{ m/s})^2 - (2 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.586 \text{ m}$$

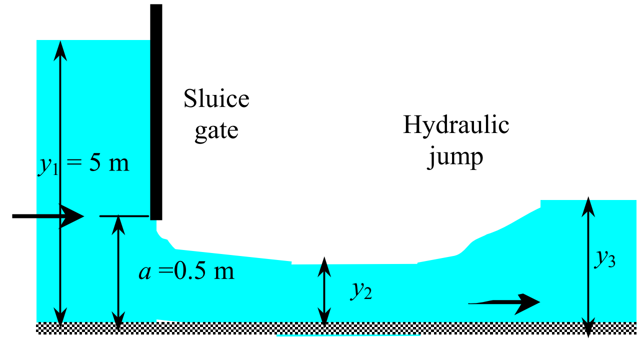
$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{1.586 \text{ m}}{(0.6671 \text{ m})(1+3.516^2/2)} = \mathbf{0.331}$$

Discussion Note that this is a fairly "strong" hydraulic jump, wasting 33.1% of the mechanical energy of the fluid.

13-138 Water from a lake is discharged through a sluice gate into a channel where uniform flow conditions are established, and then undergoes a hydraulic jump. The flow depth, velocity, and Froude number after the jump are to be determined. √EES

Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The effects of channel slope on hydraulic jump are negligible.

Properties The Manning coefficient for an open channel made of finished concrete is $n = 0.012$ (Table 13-1).



Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient,

$$\frac{y_1}{a} = \frac{5\text{ m}}{0.5\text{ m}} = 10$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.58$. Then the discharge rate per m width ($b = 1$ m) becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.58\,(1\text{ m})(0.5\text{ m})\sqrt{2\,(9.81\text{ m/s}^2)(5\text{ m})} = 2.872\text{ m}^3\text{/s}$$

For wide channels, hydraulic radius is the flow depth and thus $R_h = y_2$. Then the flow depth in uniform flow after the gate is determined from the Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 2.872\text{ m}^3\text{/s} = \frac{1\text{ m}^{1/3}\text{ / s}}{0.012}[(1\text{ m})y_2](y_2)^{2/3}\,0.004^{1/2}$$

It gives $y_2 = 0.6948$ m, which is also the flow depth before water undergoes a hydraulic jump. The flow velocity and Froude number in uniform flow are

$$V_2 = \frac{\dot{V}}{b\,y_2} = \frac{2.872\text{ m}^3\text{/s}}{(1\text{ m})\,(0.6948\text{ m})} = 4.134\text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{4.134\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.6948\text{ m})}} = 1.584$$

Then the flow depth, velocity, and Froude number after the jump (state 3) become

$$y_3 = 0.5y_2\left(-1+\sqrt{1+8\text{Fr}_2^2}\right) = 0.5(0.6948\text{ m})\left(-1+\sqrt{1+8\times 1.584^2}\right) = \mathbf{1.25\text{ m}}$$

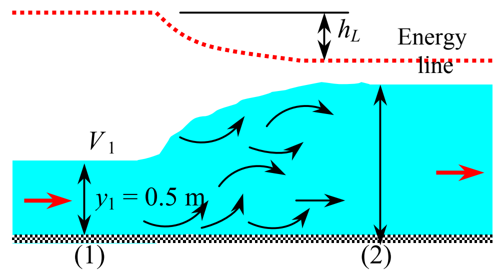
$$V_3 = \frac{y_2}{y_3}V_2 = \frac{0.6948\text{ m}}{1.25\text{ m}}(4.134\text{ m/s}) = \mathbf{2.30\text{ m/s}}$$

$$\text{Fr}_3 = \frac{V_3}{\sqrt{gy_{23}}} = \frac{2.30\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.25\text{ m})}} = \mathbf{0.659}$$

Discussion This is a relatively "mild" jump. It can be shown that the head loss during hydraulic jump is 0.049 m, which corresponds to an energy dissipation ratio of 3.1%.

13-139 Water is discharged from a dam into a wide spillway to reduce the risk of flooding by dissipating a large fraction of mechanical energy via hydraulic jump. For specified flow depths, the velocities before and after the jump, and the mechanical power dissipated per meter with of the spillway are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Properties The density of water is 1000 kg/m³.

Analysis The Froude number and velocity before the jump are

$$\frac{y_2}{y_1} = 0.5\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) \rightarrow \frac{4\text{ m}}{0.5\text{ m}} = 0.5\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$$

which gives $\mathrm{Fr}_1 = 6$. Also, from the definition of Froude number,

$$V_1 = \mathrm{Fr}_1\sqrt{gy_1} = (6)\sqrt{(9.81\text{ m/s}^2)(0.5\text{ m})} = \mathbf{13.3\ m/s}$$

Velocity and Froude number after the jump are

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.5\text{ m}}{4\text{ m}}(13.3\text{ m/s}) = \mathbf{1.66\ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.66\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(4\text{ m})}} = 0.265$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.5\text{ m}) - (4\text{ m}) + \frac{(13.3\text{ m/s})^2 - (1.66\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 5.36\text{ m}$$

The volume and mass flow rates of water per m width are

$$\dot{V} = V_1 A_{c1} = V_1 b y_1 = (13.3\text{ m/s})(1\text{ m})(0.5\text{ m}) = 6.64\text{ m}^3\text{/s}$$

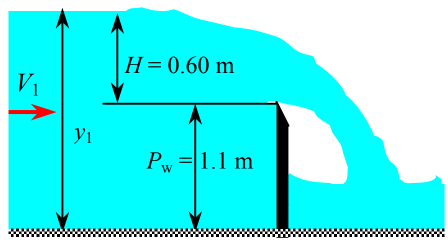
$$\dot{m} = \rho\dot{V} = (1000\text{ kg/m}^3)(6.64\text{ m}^3\text{/s}) = 6640\text{ kg/s}$$

Then the dissipated mechanical power becomes

$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (6640\text{ kg/s})(9.81\text{ m/s}^2)(5.36\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 349\text{ kNm/s} = \mathbf{349\ kW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 349 kW of power in this case.

13-140 The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined. √EES



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is given to be H = 0.60 m. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{0.60\text{ m}}{1.1\text{ m}} = 0.6469$$

The condition $H/P_w < 2$ is satisfied since 0.60/1.1 = 0.55. Then the water flow rate through the channel becomes

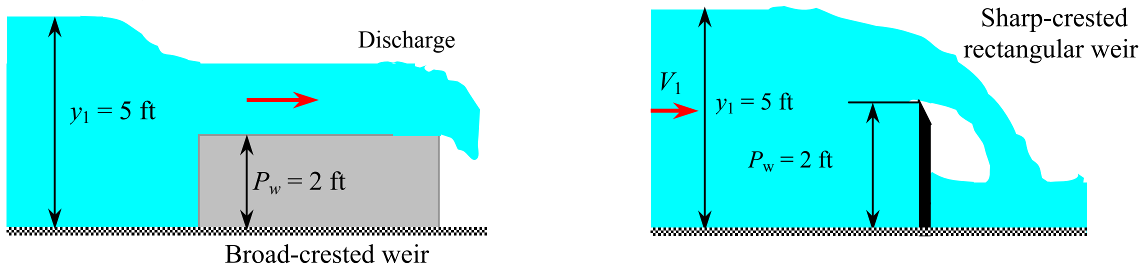
$$\begin{aligned}\dot{V} &= C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}\\ &= (0.6469)\frac{2}{3}(6\text{ m})\sqrt{2(9.81\text{ m/s}^2)}(0.60\text{ m})^{3/2}\\ &= \mathbf{5.33\ m^3/s}\end{aligned}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{5.33\text{ m}^3\text{/s}}{(6\text{ m})(1.70\text{ m})} = 0.522\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.522\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.014\text{ m}$$

This is 2.3% of the weir head, which is negligible. When the upstream velocity head is considered, the flow rate becomes 5.50 m^3/s, which is about 3 percent higher than the value determined above. Therefore, the assumption of negligible velocity head is reasonable in this case.

13-141E The flow rates in two open channels are to be measured using a sharp-crested weir in one and a broad-crested rectangular weir in the other. For identical flow depths, the flow rates through both channels are to be determined. √EES



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 5.0\text{ ft} - 2.0\text{ ft} = 3.0\text{ ft}$$

The condition $H/P_w < 2$ is satisfied since 3.0/2.0 = 1.5. The discharge coefficients of the weirs are

Sharp-crested weir:

$$C_{wd,\text{sharp}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{3.0\text{ ft}}{2.0\text{ ft}} = 0.7326$$

$$\dot{V}_{\text{sharp}} = C_{wd,sharp}\frac{2}{3}b\sqrt{2g}H^{3/2} = (0.7326)\frac{2}{3}(12\text{ ft})\sqrt{2(32.2\text{ ft/s}^2)}(3.0\text{ ft})^{3/2} = \mathbf{244\ m^3/s}$$

Broad-crested weir:

$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1 + H/P_w}} = \frac{0.65}{\sqrt{1 + (3.0\text{ ft})/(2.0\text{ ft})}} = 0.4111$$

$$\dot{V}_{\text{broad}} = C_{wd,\text{broad}}b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2} = (0.4111)(12\text{ ft})(2/3)^{3/2}\sqrt{32.2\text{ ft/s}^2}(3.0\text{ ft})^{3/2} = \mathbf{79.2\ m^3/s}$$

Discussion. Note that the flow rate in the channel with the broad-crested weir is much less than the channel with the sharp-crested weir. Also, if the upstream velocity is taken into consideration, the flow rate would be 270 ft^3/s (11% difference) for the channel with the sharp-crested weir, and 80.3 ft^3/s (1% difference) for the one with broad-crested weir. Therefore, the assumption of negligible dynamic head is not quite appropriate for the channel with the sharp-crested weir.

13-142, 13-143 Design and Essay Problems
