Review Problems

12-131 A leak develops in an automobile tire as a result of an accident. The initial mass flow rate of air through the leak is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow of air through the hole is isentropic.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K. The specific heat ratio of air at room temperature is k = 1.4.

Analysis The absolute pressure in the tire is

$$P = P_{\text{gage}} + P_{\text{atm}} = 220 + 94 = 314 \text{ kPa}$$

The critical pressure is, from Table 12-2,

$$P^* = 0.5283 P_0 = (0.5283)(314 \text{ kPa}) = 166 \text{ kPa} > 94 \text{ kPa}$$

Therefore, the flow is choked, and the velocity at the exit of the hole is the sonic speed. Then the flow properties at the exit becomes

$$\rho_0 = \frac{P_0}{RT_0} = \frac{314 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 3.671 \text{ kg/m}^3$$

$$\rho^* = \rho_0 \left(\frac{2}{k+1}\right)^{1/(k-1)} = (3.671 \text{ kg/m}^3)\left(\frac{2}{1.4+1}\right)^{1/(1.4-1)} = 2.327 \text{ kg/m}^3$$

$$T^* = \frac{2}{k+1} T_0 = \frac{2}{1.4+1}(298 \text{ K}) = 248.3 \text{ K}$$

$$V = c = \sqrt{kRT^*} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)(248.3 \text{ K})} = 315.9 \text{ m/s}$$

Then the initial mass flow rate through the hole becomes

$$\dot{m} = \rho A V = (2.327 \text{ kg/m}^3)[\pi (0.004 \text{ m})^2/4](315.9 \text{ m/s}) = 0.00924 \text{ kg/s} = \mathbf{0.554\ kg/min}$$

Discussion The mass flow rate will decrease with time as the pressure inside the tire drops.

12-132 The thrust developed by the engine of a Boeing 777 is about 380 kN. The mass flow rate of gases through the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow of combustion gases through the nozzle is isentropic. **3** Choked flow conditions exist at the nozzle exit. **4** The velocity of gases at the nozzle inlet is negligible.

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K, and it can also be used for combustion gases. The specific heat ratio of combustion gases is $k = 1.33$.

Analysis The velocity at the nozzle exit is the sonic speed, which is determined to be

$$V = c = \sqrt{kRT} = \sqrt{(1.33)(0.287\ \text{kJ/kg}\cdot\text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)(295\ \text{K})} = 335.6\ \text{m/s}$$

Noting that thrust F is related to velocity by $F = \dot{m}V$, the mass flow rate of combustion gases is determined to be

$$\dot{m} = \frac{F}{V} = \frac{380{,}000\ \text{N}}{335.6\ \text{m/s}}\left(\frac{1\ \text{kg.m/s}^2}{1\ \text{N}}\right) = \mathbf{1132\ kg/s}$$

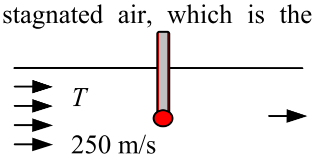
Discussion The combustion gases are mostly nitrogen (due to the 78% of N_2 in air), and thus they can be treated as air with a good degree of approximation.

12-133 A stationary temperature probe is inserted into an air duct reads 85°C. The actual temperature of air is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The stagnation process is isentropic.

Properties The specific heat of air at room temperature is $c_p = 1.005$ kJ/kg·K.

Analysis The air that strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature. The actual air temperature is determined from



$$T = T_0 - \frac{V^2}{2c_p} = 85°\text{C} - \frac{(250\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{53.9°C}$$

Discussion Temperature rise due to stagnation is very significant in high-speed flows, and should always be considered when compressibility effects are not negligible.

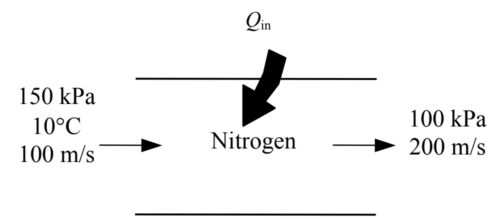
12-134 Nitrogen flows through a heat exchanger. The stagnation pressure and temperature of the nitrogen at the inlet and the exit states are to be determined.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow of nitrogen through the heat exchanger is isentropic.

Properties The properties of nitrogen are c_p = 1.039 kJ/kg·K and k = 1.4.

Analysis The stagnation temperature and pressure of nitrogen at the inlet and the exit states are determined from

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 10°\text{C} + \frac{(100\text{ m/s})^2}{2\times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.8°C}$$



$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (150\text{ kPa})\left(\frac{288.0\text{ K}}{283.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{159\ kPa}$$

From the energy balance relation $E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$ with $w = 0$

$$q_{\text{in}} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + \Delta pe^{\nearrow 0}$$

$$150\text{ kJ/kg} = (1.039\text{ kJ/kg}\cdot°\text{C})(T_2 - 10°\text{C}) + \frac{(200\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

$$T_2 = 139.9°\text{C}$$

and

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 139.9°\text{C} + \frac{(200\text{ m/s})^2}{2\times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{159°C}$$

$$P_{02} = P_2\left(\frac{T_{02}}{T_2}\right)^{k/(k-1)} = (100\text{ kPa})\left(\frac{432.3\text{ K}}{413.1\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{117\ kPa}$$

Discussion Note that the stagnation temperature and pressure can be very different than their thermodynamic counterparts when dealing with compressible flow.

12-135 An expression for the speed of sound based on van der Waals equation of state is to be derived. Using this relation, the speed of sound in carbon dioxide is to be determined and compared to that obtained by ideal gas behavior.

Properties The properties of CO_2 are R = 0.1889 kJ/kg·K and k = 1.279 at T = 50°C = 323.2 K.

Analysis Van der Waals equation of state can be expressed as $P = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$.

Differentiating, $\left(\dfrac{\partial P}{\partial v}\right)_T = \dfrac{RT}{(v-b)^2} + \dfrac{2a}{v^3}$

Noting that $\rho = 1/v \longrightarrow d\rho = -dv/v^2$, the speed of sound relation becomes

$$c^2 = k\left(\frac{\partial P}{\partial r}\right)_T = v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

Substituting,

$$c^2 = \frac{v^2 kRT}{(v-b)^2} - \frac{2ak}{v}$$

Using the molar mass of CO_2 (M = 44 kg/kmol), the constant a and b can be expressed per unit mass as

$$a = 0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^2 \quad \text{and} \qquad b = 9.70\times10^{-4}\ \text{m}^3/\text{kg}$$

The specific volume of CO_2 is determined to be

$$200\ \text{kPa} = \frac{(0.1889\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323.2\ \text{K})}{\text{v} - 0.000970\ \text{m}^3/\text{kg}} - \frac{2\times0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^2}{v^2} \rightarrow v = 0.300\ \text{m}^3/\text{kg}$$

Substituting,

$$\text{c} = \left(\left(\frac{(0.300\ \text{m}^3/\text{kg})^2(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})}{(0.300-0.000970\ \text{m}^3/\text{kg})^2} - \frac{2(0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^3)(1.279)}{(0.300\ \text{m}^3/\text{kg})^2}\right)\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kPa}\cdot\text{m}^3/\text{kg}}\right)\right)^{1/2}$$
$$= \mathbf{271\ m/s}$$

If we treat CO_2 as an ideal gas, the speed of sound becomes

$$c = \sqrt{kRT} = \sqrt{(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{279\ \ m/s}$$

Discussion Note that the ideal gas relation is the simplest equation of state, and it is very accurate for most gases encountered in practice. At high pressures and/or low temperatures, however, the gases deviate from ideal gas behavior, and it becomes necessary to use more complicated equations of state.

12-136 The equivalent relation for the speed of sound is to be verified using thermodynamic relations.

Analysis The two relations are $c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s$ and $c^2 = k\left(\dfrac{\partial P}{\partial \rho}\right)_T$

From $r = 1/v \quad \longrightarrow \quad dr = -dv/v^2$. Thus,

$$c^2 = \left(\frac{\partial P}{\partial r}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\frac{\partial T}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial v}\right)_s$$

From the cyclic rule,

$$(P,T,s):\left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial s}\right)_P\left(\frac{\partial s}{\partial P}\right)_T = -1 \longrightarrow \left(\frac{\partial P}{\partial T}\right)_s = -\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T$$

$$(T,v,s):\left(\frac{\partial T}{\partial v}\right)_s\left(\frac{\partial v}{\partial s}\right)_T\left(\frac{\partial s}{\partial T}\right)_v = -1 \longrightarrow \left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v$$

Substituting,

$$c^2 = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial T}{\partial s}\right)_v\left(\frac{\partial P}{\partial v}\right)_T$$

Recall that

$$\frac{c_p}{T} = \left(\frac{\partial s}{\partial T}\right)_P \quad \text{and} \quad \frac{c_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v$$

Substituting,

$$c^2 = -v^2\left(\frac{c_p}{T}\right)\left(\frac{T}{c_v}\right)\left(\frac{\partial P}{\partial v}\right)_T = -v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

Replacing $-dv/v^2$ by dρ,

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

Discussion Note that the differential thermodynamic property relations are very useful in the derivation of other property relations in differential form.

12-137 For ideal gases undergoing isentropic flows, expressions for P/P^*, T/T^*, and ρ/ρ^* as functions of k and Ma are to be obtained.

Analysis Equations 12-18 and 12-21 are given to be $\dfrac{T_0}{T} = \dfrac{2+(k-1)\mathrm{Ma}^2}{2}$ and $\dfrac{T^*}{T_0} = \dfrac{2}{k+1}$

Multiplying the two, $\left(\dfrac{T_0}{T}\dfrac{T^*}{T_0}\right) = \left(\dfrac{2+(k-1)\mathrm{Ma}^2}{2}\right)\left(\dfrac{2}{k+1}\right)$

Simplifying and inverting, $\dfrac{T}{T^*} = \dfrac{k+1}{2+(k-1)\mathrm{Ma}^2}$ (1)

From $\dfrac{P}{P^*} = \left(\dfrac{T}{T^*}\right)^{k/(k-1)} \longrightarrow \dfrac{P}{P^*} = \left(\dfrac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)}$ (2)

From $\dfrac{\rho}{\rho^*} = \left(\dfrac{\rho}{\rho^*}\right)^{k/(k-1)} \longrightarrow \dfrac{\rho}{\rho^*} = \left(\dfrac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)}$ (3)

Discussion Note that some very useful relations can be obtained by very simple manipulations.

12-138 It is to be verified that for the steady flow of ideal gases $dT_0/T = dA/A + (1\text{-Ma}^2)\, dV/V$. The effect of heating and area changes on the velocity of an ideal gas in steady flow for subsonic flow and supersonic flow are to be explained.

Analysis We start with the relation $\dfrac{V^2}{2} = c_p(T_0 - T)$, (1)

Differentiating, $V\,dV = c_p(dT_0 - dT)$ (2)

We also have $\dfrac{d\rho}{\rho} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$ (3)

and $\dfrac{dP}{\rho} + V\,dV = 0$ (4)

Differentiating the ideal gas relation $P = \rho RT$, $\dfrac{dP}{P} = \dfrac{d\rho}{\rho} + \dfrac{dT}{T} = 0$ (5)

From the speed of sound relation, $c^2 = kRT = (k-1)c_pT = kP/\rho$ (6)

Combining Eqs. (3) and (5), $\dfrac{dP}{P} - \dfrac{dT}{T} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$ (7)

Combining Eqs. (4) and (6), $\dfrac{dP}{\rho} = \dfrac{dP}{kP/c^2} = -VdV$

or, $\dfrac{dP}{P} = -\dfrac{k}{C^2}V\,dV = -k\dfrac{V^2}{C^2}\dfrac{dV}{V} = -k\text{Ma}^2\dfrac{dV}{V}$ (8)

Combining Eqs. (2) and (6), $dT = dT_0 - V\dfrac{dV}{c_p}$

or, $\dfrac{dT}{T} = \dfrac{dT_0}{T} - \dfrac{V^2}{C_pT}\dfrac{dV}{V} = \dfrac{dT}{T} = \dfrac{dT_0}{T} - \dfrac{V^2}{C^2/(k-1)}\dfrac{dV}{V} = \dfrac{dT_0}{T} - (k-1)\text{Ma}^2\dfrac{dV}{V}$ (9)

Combining Eqs. (7), (8), and (9), $-(k-1)\text{Ma}^2\dfrac{dV}{V} - \dfrac{dT_0}{T} + (k-1)\text{Ma}^2\dfrac{dV}{V} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$

or, $\dfrac{dT_0}{T} = \dfrac{dA}{A} + \left[-k\text{Ma}^2 + (k-1)\text{Ma}^2 + 1\right]\dfrac{dV}{V}$

Thus, $\dfrac{dT_0}{T} = \dfrac{dA}{A} + (1-\text{Ma}^2)\dfrac{dV}{V}$ (10)

Differentiating the steady-flow energy equation $q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$

$$\delta q = c_p dT_0 \qquad (11)$$

Eq. (11) relates the stagnation temperature change dT_0 to the net heat transferred to the fluid. Eq. (10) relates the velocity changes to area changes dA, and the stagnation temperature change dT_0 or the heat transferred.

(*a*) When Ma < 1 (subsonic flow), the fluid will accelerate if the duck converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$). The fluid will decelerate if the duck converges ($dA < 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$).

(*b*) When Ma > 1 (supersonic flow), the fluid will accelerate if the duck diverges ($dA > 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$). The fluid will decelerate if the duck converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$).

12-139 A Pitot-static probe measures the difference between the static and stagnation pressures for a subsonic airplane. The speed of the airplane and the flight Mach number are to be determined.

Assumptions **1** Air is an ideal gas with a constant specific heat ratio. **2** The stagnation process is isentropic.

Properties The properties of air are R = 0.287 kJ/kg·K and k = 1.4.

Analysis The stagnation pressure of air at the specified conditions is

$$P_0 = P + \Delta P = 70.109 + 22 = 92.109 \text{ kPa}$$

Then,

$$\frac{P_0}{P} = \left(1 + \frac{(k-1)\text{Ma}^2}{2}\right)^{k/k-1} \longrightarrow \frac{92.109}{70.109} = \left(1 + \frac{(1.4-1)\text{Ma}^2}{2}\right)^{1.4/0.4}$$

It yields **Ma = 0.637**

The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(268.65 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 328.5 \text{ m/s}$$

Thus,

$$V = \text{Ma} \times c = (0.637)(328.5 \text{ m/s}) = \mathbf{209 \text{ m/s}}$$

Discussion Note that the flow velocity can be measured in a simple and accurate way by simply measuring pressure.

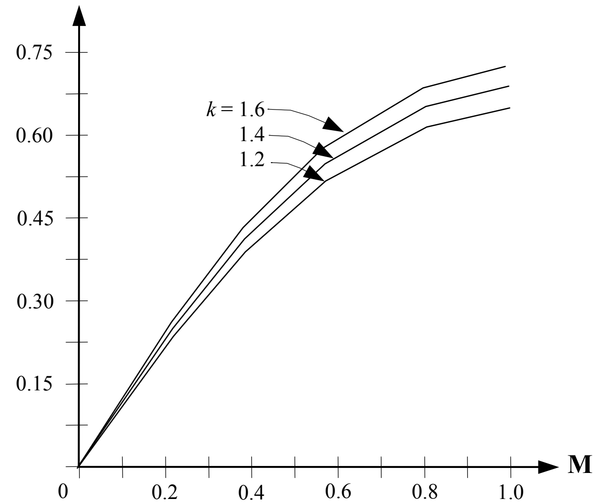
12-140 The mass flow parameter $\dot{m}\sqrt{RT_0}/(AP_0)$ versus the Mach number for k = 1.2, 1.4, and 1.6 in the range of $0 \le \text{Ma} \le 1$ is to be plotted.

Analysis The mass flow rate parameter $(\dot{m}\sqrt{RT_0})/P_0 A$ can be expressed as

$$\frac{\dot{m}\sqrt{RT_0}}{P_0 A} = \text{Ma}\sqrt{k}\left(\frac{2}{2+(k-1)M^2}\right)^{(k+1)/2(k-1)}$$

Thus,

Ma	***k* = 1.2**	***k* = 1.4**	***k* = 1.6**
0.0	0	0	0
0.1	0.1089	0.1176	0.1257
0.2	0.2143	0.2311	0.2465
0.3	0.3128	0.3365	0.3582
0.4	0.4015	0.4306	0.4571
0.5	0.4782	0.5111	0.5407
0.6	0.5411	0.5763	0.6077
0.7	0.5894	0.6257	0.6578
0.8	0.6230	0.6595	0.6916
0.9	0.6424	0.6787	0.7106
1.0	0.6485	0.6847	0.7164



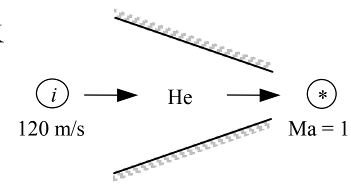
Discussion Note that the mass flow rate increases with increasing Mach number and specific heat ratio. It levels off at Ma = 1, and remains constant (choked flow).

12-141 Helium gas is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.



$$T_0 = T_i + \frac{V_i^2}{2c_p} = 500\text{ K} + \frac{(120\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 501.4\text{ K}$$

and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (0.8\text{ MPa})\left(\frac{501.4\text{ K}}{500\text{ K}}\right)^{1.667/(1.667-1)} = 0.806\text{ MPa}$$

The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (501.4\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{376\text{ K}}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.806\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.393\text{ MPa}}$$

The speed of sound and the Mach number at the nozzle inlet are

$$c_i = \sqrt{kRT}_i = \sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(500\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1316\text{ m/s}$$

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{120\text{ m/s}}{1316\text{ m/s}} = 0.0912$$

The ratio of the entrance-to-throat area is

$$\begin{aligned}\frac{A_i}{A^*} &= \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]} \\ &= \frac{1}{0.0912}\left[\left(\frac{2}{1.667+1}\right)\left(1+\frac{1.667-1}{2}(0.0912)^2\right)\right]^{2.667/(2\times 0.667)} \\ &= 6.20\end{aligned}$$

Then the ratio of the throat area to the entrance area becomes

$$\frac{A^*}{A_i} = \frac{1}{6.20} = \mathbf{0.161}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

12-142 Helium gas enters a nozzle with negligible velocity, and is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The entrance velocity is negligible.

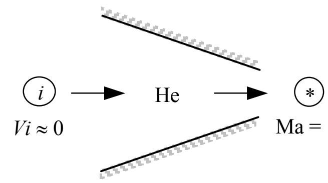
Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667.

Analysis We treat helium as an ideal gas with k = 1.667. The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *.

The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i = 500 \text{ K}$$

$$P_0 = P_i = 0.8 \text{ MPa}$$



The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (500 \text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{375\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.8 \text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.390\ MPa}$$

The ratio of the nozzle inlet area to the throat area is determined from

$$\frac{A_i}{A^*} = \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]}$$

But the Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

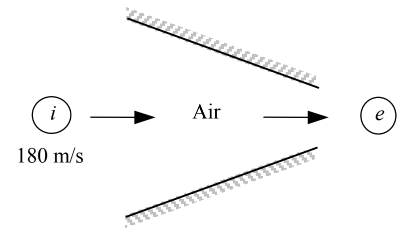
12-143 Air enters a converging nozzle. The mass flow rate, the exit velocity, the exit Mach number, and the exit pressure-stagnation pressure ratio versus the back pressure-stagnation pressure ratio for a specified back pressure range are to be calculated and plotted.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, c_p = 1.005 kJ/kg·K, and k = 1.4.

Analysis The stagnation properties remain constant throughout the nozzle since the flow is isentropic. They are determined from

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 400\text{ K} + \frac{(180\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 416.1\text{ K}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (900\text{ kPa})\left(\frac{416.1\text{ K}}{400\text{ K}}\right)^{1.4/(1.4-1)} = 1033.3\text{ kPa}$$

The critical pressure is determined to be

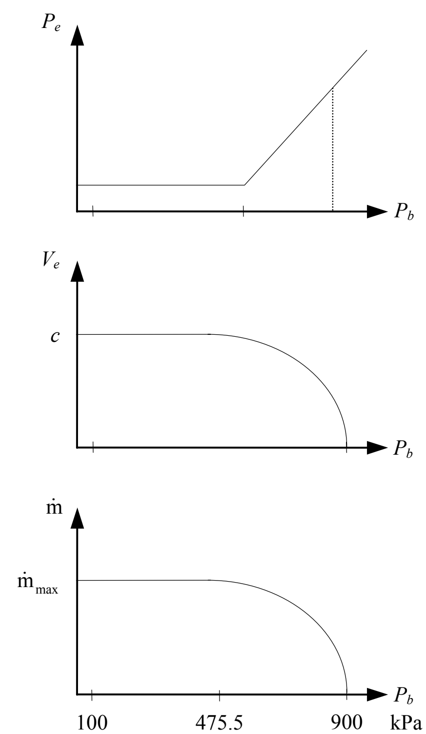
$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1033.3\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 545.9\text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$P_e = P_b$ for $P_b \geq 545.9$ kPa

$P_e = P^* = 545.9$ kPa for $P_b < 545.9$ kPa (choked flow)

Thus the back pressure will not affect the flow when 100 < P_b < 545.9 kPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from



Temperature $T_e = T_0\left(\dfrac{P_e}{P_0}\right)^{(k-1)/k} = (416.1\text{ K})\left(\dfrac{P_e}{1033.3}\right)^{0.4/1.4}$

Velocity $V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005\text{ kJ/kg}\cdot\text{K})(416.1 - T_e)\left(\dfrac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$

Speed of sound $c_e = \sqrt{kRT_e} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})\left(\dfrac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$

Mach number $\text{Ma}_e = V_e / c_e$

Density $\rho_e = \dfrac{P_e}{RT_e} = \dfrac{P_e}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$

Mass flow rate $\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001\text{ m}^2)$

The results of the calculations can be tabulated as

P_b, kPa	P_b, P_0	P_e, kPa	P_b, P_0	T_e, K	V_e, m/s	Ma	ρ_e, kg/m^3	$\dot{m}$, kg / s
900	0.871	900	0.871	400.0	180.0	0.45	7.840	0
800	0.774	800	0.774	386.8	162.9	0.41	7.206	1.174
700	0.677	700	0.677	372.3	236.0	0.61	6.551	1.546
600	0.581	600	0.581	356.2	296.7	0.78	5.869	1.741
545.9	0.528	545.9	0.528	333.3	366.2	1.00	4.971	1.820
500	0.484	545.9	0.528	333.2	366.2	1.00	4.971	1.820
400	0.387	545.9	0.528	333.3	366.2	1.00	4.971	1.820
300	0.290	545.9	0.528	333.3	366.2	1.00	4.971	1.820
200	0.194	545.9	0.528	333.3	366.2	1.00	4.971	1.820
100	0.097	545.9	0.528	333.3	366.2	1.00	4.971	1.820

12-144 Steam enters a converging nozzle. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure for a specified back pressure range are to be plotted.

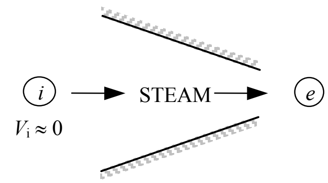
Assumptions **1** Steam is to be treated as an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The ideal gas properties of steam are R = 0.462 kJ/kg.K, c_p = 1.872 kJ/kg.K, and k = 1.3.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Since the flow is isentropic, they remain constant throughout the nozzle,

$$P_0 = P_{\rm i} = 6 \text{ MPa}$$

$$T_0 = T_i = 700 \text{ K}$$



The critical pressure is determined from to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (6\text{ MPa})\left(\frac{2}{1.3+1}\right)^{1.3/0.3} = 3.274\text{ MPa}$$

Then the pressure at the exit plane (throat) will be

$P_e = P_b$ for $P_b \geq 3.274$ MPa

P_e = P* = 3.274 MPa for $P_b < 3.274$ MPa (choked flow)

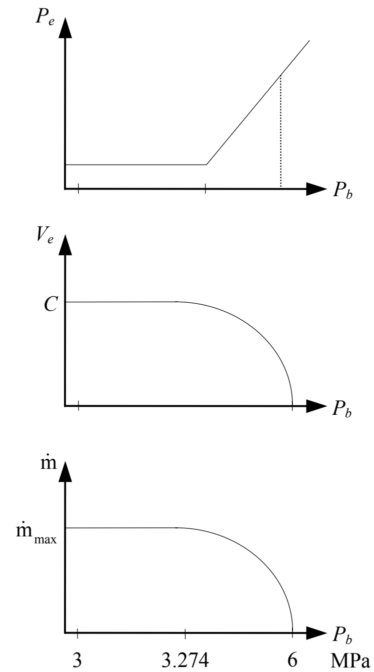
Thus the back pressure will not affect the flow when 3 < P_b < 3.274 MPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from

Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (700\text{ K})\left(\frac{P_e}{6}\right)^{0.3/1.3}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.872\text{ kJ/kg}\cdot\text{K})(700 - T_e)\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.462\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.0008\text{ m}^2)$$



The results of the calculations can be tabulated as follows:

P_b, MPa	P_e, MPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	ṁ, kg / s
6.0	6.0	700	0	18.55	0
5.5	5.5	686.1	228.1	17.35	3.166
5.0	5.0	671.2	328.4	16.12	4.235
4.5	4.5	655.0	410.5	14.87	4.883
4.0	4.0	637.5	483.7	13.58	5.255
3.5	3.5	618.1	553.7	12.26	5.431
3.274	3.274	608.7	584.7	11.64	5.445
3.0	3.274	608.7	584.7	11.64	5.445

12-145 An expression for the ratio of the stagnation pressure after a shock wave to the static pressure before the shock wave as a function of k and the Mach number upstream of the shock wave is to be found.

Analysis The relation between P_1 and P_2 is

$$\frac{P_2}{P_1} = \frac{1+k\mathrm{Ma}_2^2}{1+k\mathrm{Ma}_1^2} \longrightarrow P_2 = P_1\left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)$$

Substituting this into the isentropic relation

$$\frac{P_{02}}{P_1} = \left(1+(k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

Then,

$$\frac{P_{02}}{P_1} = \left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)\left(1+(k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

where

$$\mathrm{Ma}_2^2 = \frac{\mathrm{Ma}_1^2+2/(k-1)}{2k\mathrm{Ma}_2^2/(k-1)-1}$$

Substituting,

$$\frac{P_{02}}{P_1} = \left(\frac{(1+k\mathrm{Ma}_1^2)(2k\mathrm{Ma}_1^2-k+1)}{k\mathrm{Ma}_1^2(k+1)-k+3}\right)\left(1+\frac{(k-1)\mathrm{Ma}_1^2/2+1}{2k\mathrm{Ma}_1^2/(k-1)-1}\right)^{k/(k-1)}$$

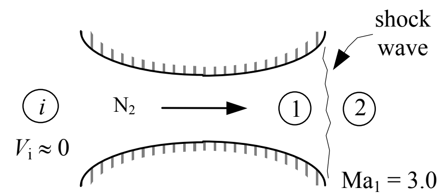
12-146 Nitrogen entering a converging-diverging nozzle experiences a normal shock. The pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock are to be determined. The results are to be compared to those of air under the same conditions.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of nitrogen are R = 0.297 kJ/kg·K and k = 1.4.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Assuming the flow before the shock to be isentropic,

$$P_{01} = P_i = 700\text{ kPa}$$
$$T_{01} = T_i = 300\text{ K}$$



Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (300\text{ K})\left(\frac{2}{2+(1.4-1)3^2}\right) = 107.1\text{ K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{k/(k-1)} = (700\text{ kPa})\left(\frac{107.1}{300}\right)^{1.4/0.4} = 19.06\text{ kPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 3.0$ we read

$$\text{Ma}_2 = \mathbf{0.4752},\ \frac{P_{02}}{P_{01}} = 0.32834,\ \frac{P_2}{P_1} = 10.333,\text{ and } \frac{T_2}{T_1} = 2.679$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.32834P_{01} = (0.32834)(700\text{ kPa}) = \mathbf{230\ kPa}$$
$$P_2 = 10.333P_1 = (10.333)(19.06\text{ kPa}) = \mathbf{197\ kPa}$$
$$T_2 = 2.679T_1 = (2.679)(107.1\text{ K}) = \mathbf{287\ K}$$

The velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.4752)\sqrt{(1.4)(0.297\text{ kJ/kg}\cdot\text{K})(287\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{164\ m/s}$$

Discussion For **air** at specified conditions k = 1.4 (same as nitrogen) and R = 0.287 kJ/kg·K. Thus the only quantity which will be different in the case of air is the velocity after the normal shock, which happens to be 161.3 m/s.

12-147 The diffuser of an aircraft is considered. The static pressure rise across the diffuser and the exit area are to be determined.

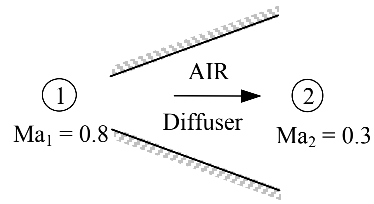
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the diffuser is steady, one-dimensional, and isentropic. **3** The diffuser is adiabatic.

Properties Air properties at room temperature are R = 0.287 kJ/kg·K, c_p = 1.005 kJ/kg·K, and k = 1.4.

Analysis The inlet velocity is

$$V_1 = \mathrm{Ma}_1 c_1 = M_1\sqrt{kRT_1} = (0.8)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(242.7\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 249.8\ \mathrm{m/s}$$

Then the stagnation temperature and pressure at the diffuser inlet become



$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 242.7 + \frac{(249.8\ \mathrm{m/s})^2}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 273.7\ \mathrm{K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{273.7\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 62.6\ \mathrm{kPa}$$

For an adiabatic diffuser, the energy equation reduces to $h_{01} = h_{02}$. Noting that $h = c_pT$ and the specific heats are assumed to be constant, we have

$$T_{01} = T_{02} = T_0 = 273.7\ \mathrm{K}$$

The isentropic relation between states 1 and 02 gives

$$P_{02} = P_{02} = P_1\left(\frac{T_{02}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{273.72\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 62.61\ \mathrm{kPa}$$

The exit velocity can be expressed as

$$V_2 = \mathrm{Ma}_2 c_2 = \mathrm{Ma}_2\sqrt{kRT_2} = (0.3)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})\,T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 6.01\sqrt{T_2}$$

Thus $$T_2 = T_{02} - \frac{V_2^2}{2c_p} = (273.7) - \frac{6.01^2\,\mathrm{T_2\ m^2/s^2}}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 268.9\ \mathrm{K}$$

Then the static exit pressure becomes

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (62.61\ \mathrm{kPa})\left(\frac{268.9\ \mathrm{K}}{273.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 58.85\ \mathrm{kPa}$$

Thus the static pressure rise across the diffuser is

$$\Delta P = P_2 - P_1 = 58.85 - 41.1 = \mathbf{17.8\ kPa}$$

Also, $$\rho_2 = \frac{P_2}{RT_2} = \frac{58.85\ \mathrm{kPa}}{(0.287\ \mathrm{kPa\cdot m^3/kg\cdot K})(268.9\ \mathrm{K})} = 0.7626\ \mathrm{kg/m^3}$$

$$V_2 = 6.01\sqrt{T_2} = 6.01\sqrt{268.9} = 98.6\ \mathrm{m/s}$$

Thus $$A_2 = \frac{\dot m}{\rho_2 V_2} = \frac{65\ \mathrm{kg/s}}{(0.7626\ \mathrm{kg/m^3})(98.6\ \mathrm{m/s})} = \mathbf{0.864\ m^2}$$

Discussion The pressure rise in actual diffusers will be lower because of the irreversibilities. However, flow through well-designed diffusers is very nearly isentropic.

12-148 Helium gas is accelerated in a nozzle isentropically. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined.

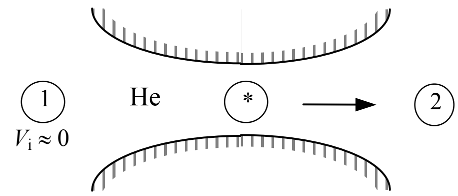
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 2.0769 kJ/kg.K, c_p = 5.1926 kJ/kg.K, and k = 1.667.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 500 \text{ K}$$

$$P_{01} = P_1 = 1.0 \text{ MPa}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 500 \text{ K}$$

$$P_{02} = P_{01} = 1.0 \text{ MPa}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (500\text{ K})\left(\frac{2}{1.667+1}\right) = 375.0\text{ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1.0\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 0.487\text{ MPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{487\text{ kPa}}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(375\text{ K})} = 0.625\text{ kg/m}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(375\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1139.4\text{ m/s}$$

Thus the throat area is

$$A^* = \frac{\dot m}{\rho^* V^*} = \frac{0.25\text{ kg/s}}{(0.625\text{ kg/m}^3)(1139.4\text{ m/s})} = 3.51\times10^{-4}\text{ m}^2 = \mathbf{3.51\,cm^2}$$

At the nozzle exit the pressure is P_2 = 0.1 MPa. Then the other properties at the nozzle exit are determined to be

$$\frac{P_0}{P_2} = \left(1+\frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{1.0\text{ MPa}}{0.1\text{ MPa}} = \left(1+\frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma_2 = 2.130, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (500\text{ K})\left(\frac{2}{2+(1.667-1)\times 2.13^2}\right) = 199.0\text{ K}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{100\text{ kPa}}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(199\text{ K})} = 0.242\text{ kg/m}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.13)\sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(199\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1768.0\text{ m/s}$$

Thus the exit area is

$$A_2 = \frac{\dot m}{\rho_2 V_2} = \frac{0.25\text{ kg/s}}{(0.242\text{ kg/m}^3)(1768\text{ m/s})} = 5.84\times10^{-4}\text{ m}^2 = \mathbf{5.84\ cm^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

12-149E Helium gas is accelerated in a nozzle. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined for the cases of isentropic and 97% efficient nozzles.

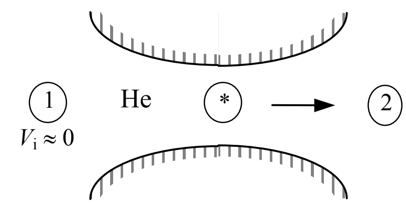
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 0.4961 Btu/lbm·R = 2.6809 psia·ft^3/lbm·R, c_p = 1.25 Btu/lbm·R, and k = 1.667.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 900 \text{ R}$$

$$P_{01} = P_1 = 150 \text{ psia}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 900 \text{ R}$$

$$P_{02} = P_{01} = 150 \text{ psia}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (900\text{ R})\left(\frac{2}{1.667+1}\right) = 674.9\text{ R}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (150\text{ psia})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 73.1\text{ psia}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{73.1\text{ psia}}{(2.6809\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(674.9\text{ R})} = 0.0404\text{ 1bm/ft}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(0.4961\text{ Btu/lbm}\cdot\text{R})(674.9\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 3738\text{ ft/s}$$

and
$$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.2\text{ 1bm/s}}{(0.0404\text{ 1bm/ft}^3)(3738\text{ ft/s})} = \mathbf{0.00132\ ft^2}$$

At the nozzle exit the pressure is P_2 = 15 psia. Then the other properties at the nozzle exit are determined to be

$$\frac{p_0}{p_2} = \left(1+\frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{150\text{ psia}}{15\text{ psia}} = \left(1+\frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma_2 = 2.130, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (900\text{ R})\left(\frac{2}{2+(1.667-1)\times 2.13^2}\right) = 358.1\text{ R}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{15\text{ psia}}{(2.6809\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(358.1\text{ R})} = 0.0156\text{ 1bm/ft}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.13)\sqrt{(1.667)(0.4961\text{ Btu/lbm}\cdot\text{R})(358.1\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 5800\text{ ft/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.2\text{ lbm/s}}{(0.0156\text{ lbm/ft}^3)(5800\text{ ft/s})} = \mathbf{0.00221\ ft^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

12-150 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1}$$

```
k=1.667
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0.0	0	∞	1.0000	1.0000	1.0000
0.1	0.1153	5.6624	0.9917	0.9950	0.9967
0.2	0.2294	2.8879	0.9674	0.9803	0.9868
0.3	0.3413	1.9891	0.9288	0.9566	0.9709
0.4	0.4501	1.5602	0.8782	0.9250	0.9493
0.5	0.5547	1.3203	0.8186	0.8869	0.9230
0.6	0.6547	1.1760	0.7532	0.8437	0.8928
0.7	0.7494	1.0875	0.6850	0.7970	0.8595
0.8	0.8386	1.0351	0.6166	0.7482	0.8241
0.9	0.9222	1.0081	0.5501	0.6987	0.7873
1.0	1.0000	1.0000	0.4871	0.6495	0.7499
1.2	1.1390	1.0267	0.3752	0.5554	0.6756
1.4	1.2572	1.0983	0.2845	0.4704	0.6047
1.6	1.3570	1.2075	0.2138	0.3964	0.5394
1.8	1.4411	1.3519	0.1603	0.3334	0.4806
2.0	1.5117	1.5311	0.1202	0.2806	0.4284
2.2	1.5713	1.7459	0.0906	0.2368	0.3825
2.4	1.6216	1.9980	0.0686	0.2005	0.3424
2.6	1.6643	2.2893	0.0524	0.1705	0.3073
2.8	1.7007	2.6222	0.0403	0.1457	0.2767
3.0	1.7318	2.9990	0.0313	0.1251	0.2499
5.0	1.8895	9.7920	0.0038	0.0351	0.1071
∝	1.9996	∞	0	0	0

12-151 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2 / P_1}{T_2 / T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

```
k=1.667
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	2.0530
1.1	0.9131	1.2625	1.1496	1.0982	0.999	2.3308
1.2	0.8462	1.5500	1.2972	1.1949	0.9933	2.6473
1.3	0.7934	1.8626	1.4413	1.2923	0.9813	2.9990
1.4	0.7508	2.2001	1.5805	1.3920	0.9626	3.3838
1.5	0.7157	2.5626	1.7141	1.4950	0.938	3.8007
1.6	0.6864	2.9501	1.8415	1.6020	0.9085	4.2488
1.7	0.6618	3.3627	1.9624	1.7135	0.8752	4.7278
1.8	0.6407	3.8002	2.0766	1.8300	0.8392	5.2371
1.9	0.6227	4.2627	2.1842	1.9516	0.8016	5.7767
2.0	0.6070	4.7503	2.2853	2.0786	0.763	6.3462
2.1	0.5933	5.2628	2.3802	2.2111	0.7243	6.9457
2.2	0.5814	5.8004	2.4689	2.3493	0.6861	7.5749
2.3	0.5708	6.3629	2.5520	2.4933	0.6486	8.2339
2.4	0.5614	6.9504	2.6296	2.6432	0.6124	8.9225
2.5	0.5530	7.5630	2.7021	2.7989	0.5775	9.6407
2.6	0.5455	8.2005	2.7699	2.9606	0.5442	10.3885
2.7	0.5388	8.8631	2.8332	3.1283	0.5125	11.1659
2.8	0.5327	9.5506	2.8923	3.3021	0.4824	11.9728
2.9	0.5273	10.2632	2.9476	3.4819	0.4541	12.8091
3.0	0.5223	11.0007	2.9993	3.6678	0.4274	13.6750
4.0	0.4905	19.7514	3.3674	5.8654	0.2374	23.9530
5.0	0.4753	31.0022	3.5703	8.6834	0.1398	37.1723
∞	0.4473	∞	3.9985	∞	0	∞

12-152 The critical temperature, pressure, and density of an equimolar mixture of oxygen and nitrogen for specified stagnation properties are to be determined.

Assumptions Both oxygen and nitrogen are ideal gases with constant specific heats at room temperature.

Properties The specific heat ratio and molar mass are k = 1.395 and M = 32 kg/kmol for oxygen, and k = 1.4 and M = 28 kg/kmol for nitrogen.

Analysis The gas constant of the mixture is

$$M_m = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5 \times 32 + 0.5 \times 28 = 30 \text{ kg/kmol}$$

$$R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{30 \text{ kg/kmol}} = 0.2771 \text{ kJ/kg}\cdot\text{K}$$

The specific heat ratio is 1.4 for nitrogen, and nearly 1.4 for oxygen. Therefore, the specific heat ratio of the mixture is also 1.4. Then the critical temperature, pressure, and density of the mixture become

$$T^* = T_0\left(\frac{2}{k+1}\right) = (800\text{ K})\left(\frac{2}{1.4+1}\right) = \mathbf{667\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (500\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{264\ kPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{264\text{ kPa}}{(0.2771\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(667\text{ K})} = \mathbf{1.43\ kg/m^3}$$

Discussion If the specific heat ratios k of the two gases were different, then we would need to determine the k of the mixture from $k = C_{p,m}/C_{v,m}$ where the specific heats of the mixture are determined from

$$C_{p,m} = \text{mf}_{O_2} C_{p,O_2} + \text{mf}_{N_2} C_{p,N_2} = (y_{O_2} M_{O_2} / M_m) C_{p,O_2} + (y_{N_2} M_{N_2} / M_m) C_{p,N_2}$$
$$C_{v,m} = \text{mf}_{O_2} C_{v,O_2} + \text{mf}_{N_2} C_{v,N_2} = (y_{O_2} M_{O_2} / M_m) C_{v,O_2} + (y_{N_2} M_{N_2} / M_m) C_{v,N_2}$$

where mf is the mass fraction and y is the mole fraction. In this case it would give

$$C_{p,m} = (0.5 \times 32/30) \times 0.918 + (0.5 \times 28/30) \times 1.039 = 0.974 \text{ kJ/kg.K}$$

$$C_{p,m} = (0.5 \times 32/30) \times 0.658 + (0.5 \times 28/30) \times 0.743 = 0.698 \text{ kJ/kg.K}$$

and k = 0.974/0.698 = 1.40

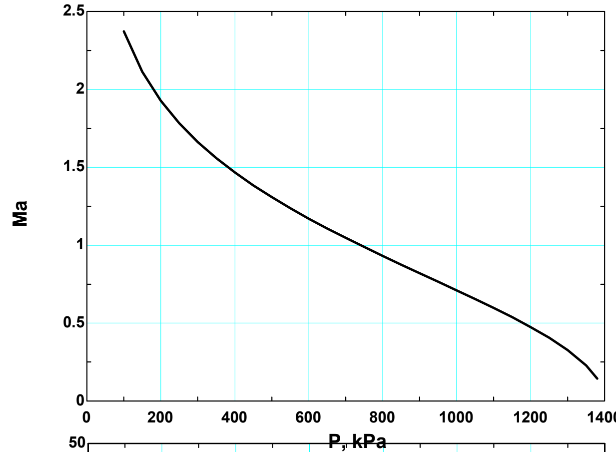
12-153 Using EES (or other) software, the shape of a converging-diverging nozzle is to be determined for specified flow rate and stagnation conditions. The nozzle and the Mach number are to be plotted.

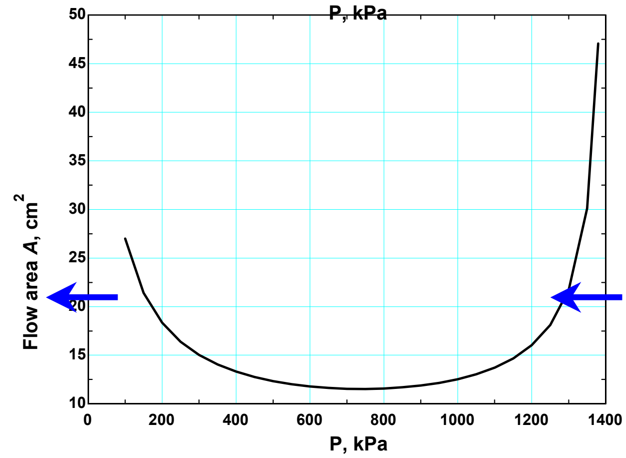
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The specific heat ratio of air at room temperature is 1.4.

Analysis The problems is solved using EES, and the results are tabulated and plotted below.

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
P0=1400 "kPa"
T0=200+273 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
T=T0*(P/P0)^((k-1)/k)
V=SQRT(2*Cp*(T0-T)*1000)
A=m/(rho*V)*10000 "cm2"
C=SQRT(k*R*T*1000)
Ma=V/C
```





Pressure P, kPa	Flow area A, cm^2	Mach number Ma
1400	∞	0
1350	30.1	0.229
1300	21.7	0.327
1250	18.1	0.406
1200	16.0	0.475
1150	14.7	0.538
1100	13.7	0.597
1050	13.0	0.655
1000	12.5	0.710
950	12.2	0.766
900	11.9	0.820
850	11.7	0.876
800	11.6	0.931
750	11.5	0.988
700	11.5	1.047
650	11.6	1.107
600	11.8	1.171
550	12.0	1.237
500	12.3	1.308
450	12.8	1.384
400	13.3	1.467
350	14.0	1.559
300	15.0	1.663
250	16.4	1.784
200	18.3	1.929
150	21.4	2.114
100	27.0	2.373

12-154 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for air and methane.

Properties The specific heat ratio is given to be $k = 1.4$ for air.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1}$$

Air:

```
k=1.4
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.5	1.3646	1.1762	0.2724	0.3950	0.6897
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.5	1.8257	2.6367	0.0585	0.1317	0.4444
3.0	1.9640	4.2346	0.0272	0.0762	0.3571
3.5	2.0642	6.7896	0.0131	0.0452	0.2899
4.0	2.1381	10.7188	0.0066	0.0277	0.2381
4.5	2.1936	16.5622	0.0035	0.0174	0.1980
5.0	2.2361	25.0000	0.0019	0.0113	0.1667
5.5	2.2691	36.8690	0.0011	0.0076	0.1418
6.0	2.2953	53.1798	0.0006	0.0052	0.1220
6.5	2.3163	75.1343	0.0004	0.0036	0.1058
7.0	2.3333	104.1429	0.0002	0.0026	0.0926
7.5	2.3474	141.8415	0.0002	0.0019	0.0816
8.0	2.3591	190.1094	0.0001	0.0014	0.0725
8.5	2.3689	251.0862	0.0001	0.0011	0.0647
9.0	2.3772	327.1893	0.0000	0.0008	0.0581
9.5	2.3843	421.1314	0.0000	0.0006	0.0525
10.0	2.3905	535.9375	0.0000	0.0005	0.0476

12-155 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for air and methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma}\sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

Methane:

```
k=1.3
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5457	0.6276	0.8696
1.5	1.3909	1.1895	0.2836	0.3793	0.7477
2.0	1.6956	1.7732	0.1305	0.2087	0.6250
2.5	1.9261	2.9545	0.0569	0.1103	0.5161
3.0	2.0986	5.1598	0.0247	0.0580	0.4255
3.5	2.2282	9.1098	0.0109	0.0309	0.3524
4.0	2.3263	15.9441	0.0050	0.0169	0.2941
4.5	2.4016	27.3870	0.0024	0.0095	0.2477
5.0	2.4602	45.9565	0.0012	0.0056	0.2105
5.5	2.5064	75.2197	0.0006	0.0033	0.1806
6.0	2.5434	120.0965	0.0003	0.0021	0.1563
6.5	2.5733	187.2173	0.0002	0.0013	0.1363
7.0	2.5978	285.3372	0.0001	0.0008	0.1198
7.5	2.6181	425.8095	0.0001	0.0006	0.1060
8.0	2.6350	623.1235	0.0000	0.0004	0.0943
8.5	2.6493	895.5077	0.0000	0.0003	0.0845
9.0	2.6615	1265.6040	0.0000	0.0002	0.0760
9.5	2.6719	1761.2133	0.0000	0.0001	0.0688
10.0	2.6810	2416.1184	0.0000	0.0001	0.0625

12-156 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for air and methane.

Properties The specific heat ratio is given to be k = 1.4 for air.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

Air:

```
k=1.4
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8929
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.5	0.5130	7.1250	3.3333	2.1375	0.499	8.5261
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
3.5	0.4512	14.1250	4.2609	3.3151	0.2129	16.2420
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
4.5	0.4236	23.4583	4.8119	4.8751	0.0917	26.5387
5.0	0.4152	29.0000	5.0000	5.8000	0.06172	32.6535
5.5	0.4090	35.1250	5.1489	6.8218	0.04236	39.4124
6.0	0.4042	41.8333	5.2683	7.9406	0.02965	46.8152
6.5	0.4004	49.1250	5.3651	9.1564	0.02115	54.8620
7.0	0.3974	57.0000	5.4444	10.4694	0.01535	63.5526
7.5	0.3949	65.4583	5.5102	11.8795	0.01133	72.8871
8.0	0.3929	74.5000	5.5652	13.3867	0.008488	82.8655
8.5	0.3912	84.1250	5.6117	14.9911	0.006449	93.4876
9.0	0.3898	94.3333	5.6512	16.6927	0.004964	104.7536
9.5	0.3886	105.1250	5.6850	18.4915	0.003866	116.6634
10.0	0.3876	116.5000	5.7143	20.3875	0.003045	129.2170

12-157 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for air and methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2 / P_1}{T_2 / T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

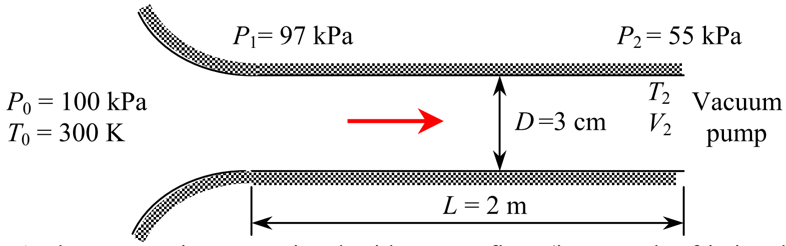
$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

Methane:

```
k=1.3
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8324
1.5	0.6942	2.4130	1.9346	1.2473	0.9261	3.2654
2.0	0.5629	4.3913	2.8750	1.5274	0.7006	5.3700
2.5	0.4929	6.9348	3.7097	1.8694	0.461	8.0983
3.0	0.4511	10.0435	4.4043	2.2804	0.2822	11.4409
3.5	0.4241	13.7174	4.9648	2.7630	0.1677	15.3948
4.0	0.4058	17.9565	5.4118	3.3181	0.09933	19.9589
4.5	0.3927	22.7609	5.7678	3.9462	0.05939	25.1325
5.0	0.3832	28.1304	6.0526	4.6476	0.03613	30.9155
5.5	0.3760	34.0652	6.2822	5.4225	0.02243	37.3076
6.0	0.3704	40.5652	6.4688	6.2710	0.01422	44.3087
6.5	0.3660	47.6304	6.6218	7.1930	0.009218	51.9188
7.0	0.3625	55.2609	6.7485	8.1886	0.006098	60.1379
7.5	0.3596	63.4565	6.8543	9.2579	0.004114	68.9658
8.0	0.3573	72.2174	6.9434	10.4009	0.002827	78.4027
8.5	0.3553	81.5435	7.0190	11.6175	0.001977	88.4485
9.0	0.3536	91.4348	7.0837	12.9079	0.001404	99.1032
9.5	0.3522	101.8913	7.1393	14.2719	0.001012	110.367
10.0	0.3510	112.9130	7.1875	15.7096	0.000740	122.239

12-158 Air enters a constant-area adiabatic duct at a specified state, and leaves at a specified pressure. The mass flow rate of air, the exit velocity, and the average friction factor are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.025.

Analysis Noting that the flow in the nozzle section is isentropic, the Mach number, thermodynamic temperature, and density at the tube inlet become

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} \quad\rightarrow\quad 97\text{ kPa} = (100\text{ kPa})\left(1+\frac{1.4\text{-}1}{2}\mathrm{Ma}_1^2\right)^{-1.4/0.4} \quad\rightarrow\quad \mathrm{Ma}_1 = 0.2091$$

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (300\text{ K})\left(1+\frac{1.4\text{-}1}{2}(0.2091)^2\right)^{-1} = 297.4\text{ K}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{97\text{ kPa}}{(0.287\text{ kJ/kgK})(297.4\text{ K})} = 1.136\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(297.4\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 345.7\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 0.2091(345.7\text{ m/s}) = 72.3\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.136\text{ kg/m}^3)[\pi(0.03\text{ m})^2/4](72.3\text{ m/s}) = \mathbf{0.0581\ kg/s}$$

The Fanno flow functions corresponding to the inlet Mach number are, from Table A-16,

$\mathrm{Ma}_1 = 0.2091$: $(fL^*/D_h)_1$ =13.095 T_1/T^* = 1.1896, P_1/P^* = 5.2173, V_1/V^* = 0.2280

Therefore, $P_1 = 5.2173P^*$. Then the Fanno function P_2/P^* becomes

$$\frac{P_2}{P^*} = \frac{P_2}{P_1/5.2173} = \frac{5.2173(55\text{ kPa})}{97\text{ kPa}} = 2.9583$$

The corresponding Mach number and Fanno flow functions are, from Table A-16,

$\mathrm{Ma}_2 = 0.3655$, $(fL^*/D_h)_1 = 3.0420$, and V_2/V^* =0.3951.

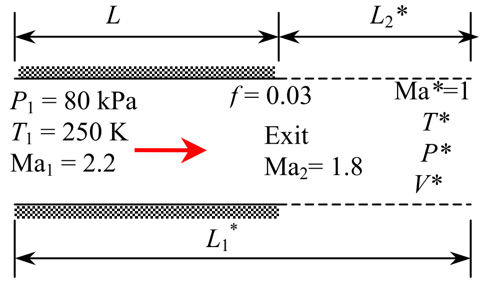
Then the air velocity at the duct exit and the average friction factor become

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.3951}{0.2280} = 1.7329 \quad\rightarrow\quad V_2 = 1.7329V_1 = 1.7329(72.3\text{ m/s}) = \mathbf{125\ m/s}$$

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} \quad\rightarrow\quad 2\text{ m} = (13.095 - 3.042)\frac{0.03\text{ m}}{f} \quad\rightarrow\quad f = \mathbf{0.151}$$

Discussion Note that the mass flow rate and the average friction factor can be determined by measuring static pressure, as in incompressible flow.

12-159 Supersonic airflow in a constant cross-sectional area adiabatic duct is considered. For a specified exit Mach number, the temperature, pressure, and velocity at the duct exit are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.03.

Analysis The inlet velocity is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(250\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 316.9\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2.2(316.9\text{ m/s}) = 697.3\text{ m/s}$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$Ma_1 = 2.2$: $(fL^*/D_h)_1 = 0.3609$ $T_1/T^* = 0.6098$, $P_1/P^* = 0.3549$, $V_1/V^* = 1.7179$

$Ma_2 = 1.8$: $(fL^*/D_h)_2 = 0.2419$ $T_2/T^* = 0.7282$, $P_2/P^* = 0.4741$, $V_2/V^* = 1.5360$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.7282}{0.6098} = 1.1942 \quad \rightarrow \quad T_2 = 1.1942T_1 = 1.1942(250\text{ K}) = \mathbf{299\text{ K}}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.4741}{0.3549} = 1.3359 \quad \rightarrow \quad P_2 = 1.3359P_1 = 1.3359(80\text{ kPa}) = \mathbf{107\text{ kPa}}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1.5360}{1.7179} = 0.8941 \quad \rightarrow \quad V_2 = 0.8941V_1 = 0.8941(697.3\text{ m/s}) = \mathbf{623\text{ m/s}}$$

Discussion The duct length is determined to be

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (0.3609 - 0.2419)\frac{0.04\text{ m}}{0.03} = \mathbf{0.16\text{ m}}$$

Note that it takes a duct length of only 0.16 m for the Mach number to decrease from 2.2 to 1.8. The maximum (or sonic) duct lengths at the inlet and exit states in this case are L_1^* = 0.48 m and L_2^* = 0.32 m. Therefore, the flow would reach sonic conditions if a 0.32-m long section were added to the existing duct.

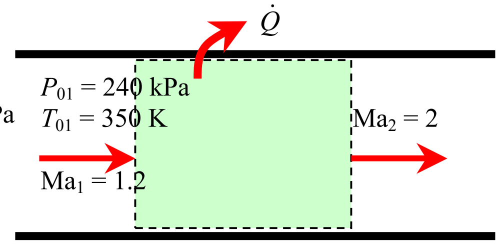
12-160 Air flowing at a supersonic velocity in a duct is accelerated by cooling. For a specified exit Mach number, the rate of heat transfer is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (350\text{ K})\left(1+\frac{1.4-1}{2}1.2^2\right)^{-1} = 271.7\text{ K}$$



$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (240\text{ kPa})\left(1+\frac{1.4-1}{2}1.2^2\right)^{-1.4/0.4} = 98.97\text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{98.97\text{ kPa}}{(0.287\text{ kJ/kgK})(271.7\text{ K})} = 1.269\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(271.7\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 330.4\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.2(330.4\text{ m/s}) = 396.5\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.269\text{ kg/m}^3)[\pi(0.20\text{ m})^2/4](396.5\text{ m/s}) = 15.81\text{ kg/s}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 1.8$: $\quad T_{01}/T_0^* = 0.9787$

$\mathrm{Ma}_2 = 2$: $\quad T_{02}/T_0^* = 0.7934$

Then the exit stagnation temperature is determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.7934}{0.9787} = 0.8107 \quad \rightarrow \quad T_{02} = 0.8107T_{01} = 0.8107(350\text{ K}) = 283.7\text{ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (15.81\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(283.7-350)\text{ K} = \mathbf{-1053\text{ kW}}$$

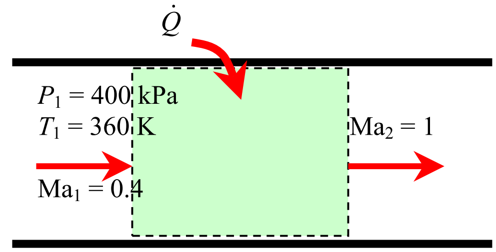
Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 158 K at the exit, which is extremely low. Therefore, the duct may need to be heavily insulated to maintain indicated flow conditions.

12-161 Air flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{400 \text{ kPa}}{(0.287 \text{ kJ/kgK})(360 \text{ K})} = 3.871 \text{ kg/m}^3$$

$$T_{01} = T_1\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right) = (360 \text{ K})\left(1 + \frac{1.4-1}{2}0.4^2\right) = 371.5 \text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(360 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 380.3 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.4(380.3 \text{ m/s}) = 152.1 \text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (3.871 \text{ kg/m}^3)(0.1 \times 0.1 \text{ m}^2)(152.1 \text{ m/s}) = 5.890 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $Ma_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\text{Ma}_1^2[2 + (k-1)\text{Ma}_1^2]}{(1 + k\text{Ma}_1^2)^2} = \frac{(1.4+1)0.4^2[2 + (1.4-1)0.4^2]}{(1 + 1.4 \times 0.4^2)^2} = 0.5290$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.5290} \quad \rightarrow \quad T_{02} = T_{01}/0.5290 = (371.5 \text{ K})/0.5290 = 702.3 \text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5.890 \text{ kg/s})(1.005 \text{ kJ/kg}\cdot\text{K})(702.3 - 371.5) \text{ K} = \mathbf{1958 \text{ kW}}$$

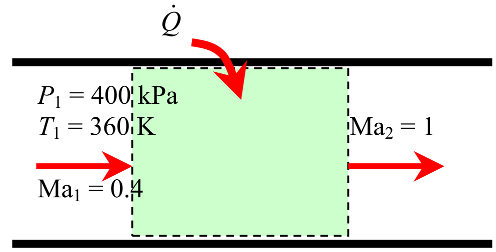
Discussion It can also be shown that $T_2 = 585$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. We can also solve this problem using the Rayleigh function values listed in Table A-15.

12-162 Helium flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of helium to be $k = 1.667$, $c_p = 5.193$ kJ/kg·K, and $R = 2.077$ kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\text{Ma}_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{400 \text{ kPa}}{(2.077 \text{ kJ/kgK})(360 \text{ K})} = 0.5350 \text{ kg/m}^3$$

$$T_{01} = T_1\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right) = (360 \text{ K})\left(1 + \frac{1.667 - 1}{2}0.4^2\right) = 379.2 \text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.667)(2.077 \text{ kJ/kg}\cdot\text{K})(360 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 1116 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.4(1116 \text{ m/s}) = 446.6 \text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.5350 \text{ kg/m}^3)(0.1\times 0.1 \text{ m}^2)(446.6 \text{ m/s}) = 2.389 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $\text{Ma}_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\text{Ma}_1^2[2+(k-1)\text{Ma}_1^2]}{(1+k\text{Ma}_1^2)^2} = \frac{(1.667+1)0.4^2[2+(1.667-1)0.4^2]}{(1+1.667\times 0.4^2)^2} = 0.5603$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.5603} \quad \rightarrow \quad T_{02} = T_{01}/0.5603 = (379.2 \text{ K})/0.5603 = 676.8 \text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (2.389 \text{ kg/s})(5.193 \text{ kJ/kg}\cdot\text{K})(676.8 - 379.2) \text{ K} = \mathbf{3690 \text{ kW}}$$

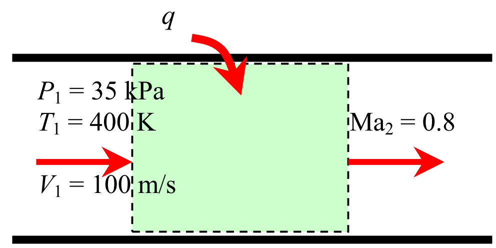
Discussion It can also be shown that $T_2 = 508$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-15 since they are based on $k = 1.4$.

12-163 Air flowing at a subsonic velocity in a duct is accelerated by heating. For a specified exit Mach number, the heat transfer for a specified exit Mach number as well as the maximum heat transfer are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet Mach number and stagnation temperature are



$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(400\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 400.9\ \text{m/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{100\ \text{m/s}}{400.9\ \text{m/s}} = 0.2494$$

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (400\ \text{K})\left(1+\frac{1.4\text{-}1}{2}0.2494^2\right) = 405.0\ \text{K}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.2494$: $\quad T_{01}/T^* = 0.2559$
$\mathrm{Ma}_2 = 0.8$: $\quad T_{02}/T^* = 0.9639$

Then the exit stagnation temperature and the heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.2559} = 3.7667 \rightarrow \quad T_{02} = 3.7667T_{01} = 3.7667(405.0\ \text{K}) = 1526\ \text{K}$$

$$q = c_p(T_{02} - T_{01}) = (1.005\ \text{kJ/kg}\cdot\text{K})(1526 - 405)\ \text{K} = \mathbf{1126\ \ kJ/kg}$$

Maximum heat transfer will occur when the flow is choked, and thus $\mathrm{Ma}_2 = 1$ and thus $T_{02}/T^* = 1$. Then,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.2559} \rightarrow \quad T_{02} = T_{01}/0.2559 = (405\ \text{K})/0.2559 = 1583\ \text{K}$$

$$q_{\max} = c_p(T_{02} - T_{01}) = (1.005\ \text{kJ/kg}\cdot\text{K})(1583 - 405)\ \text{K} = \mathbf{1184\ \ kJ/kg}$$

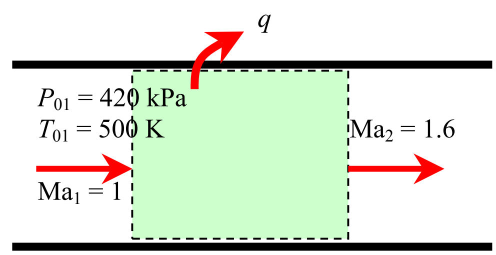
Discussion This is the maximum heat that can be transferred to the gas without affecting the mass flow rate. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease.

12-164 Air flowing at sonic conditions in a duct is accelerated by cooling. For a specified exit Mach number, the amount of heat transfer per unit mass is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis Noting that Ma_1 = 1, the inlet stagnation temperature is



$$T_{01} = T_1\left(1+\frac{k-1}{2}\text{Ma}_1^2\right) = (500\text{ K})\left(1+\frac{1.4-1}{2}1^2\right) = 600\text{ K}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-15):

Ma_1 = 1: $T_{01}/T_0^* = 1$

Ma_2 = 1.6: $T_{02}/T_0^* = 0.8842$

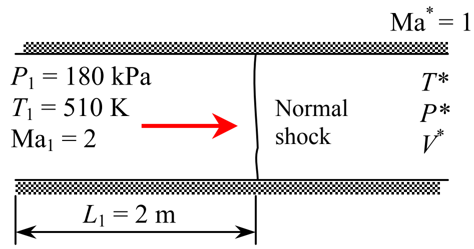
Then the exit stagnation temperature and heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.8842}{1} = 0.8842 \quad \rightarrow \quad T_{02} = 0.8842T_{01} = 0.8842(600\text{ K}) = 530.5\text{ K}$$

$$q = c_p(T_{02}-T_{01}) = (1.005\text{ kJ/kg}\cdot\text{K})(530.5-600)\text{ K} = \mathbf{-69.8\ kJ/kg}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 351 K at the exit

12-165 Combustion gases enter a constant-area adiabatic duct at a specified state, and undergo a normal shock at a specified location. The exit velocity, temperature, and pressure are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties The specific heat ratio and gas constant of combustion gases are given to be k = 1.33 and R = 0.280 kJ/kg·K. The friction factor is given to be f = 0.010.

Analysis The Fanno flow functions corresponding to the inlet Mach number of 2 are calculated from the relations in Table A-16 for k = 1.33 to be

$\text{Ma}_1 = 2$: $(fL^*/D_h)_1 = 0.3402$ $T_1/T^* = 0.7018$, $P_1/P^* = 0.4189$

First we check to make sure that the flow everywhere upstream the shock is supersonic. The required duct length from the inlet L_1^* for the flow to reach sonic conditions is

$$L_1^* = 0.3402\frac{D}{f} = 0.3402\frac{0.10\text{ m}}{0.010} = 3.40\text{ m}$$

which is greater than the actual length 2 m. Therefore, the flow is indeed supersonic when the normal shock occurs at the indicated location. Also, using the actual duct length L_1, we have

$$\frac{fL_1}{D_h} = \frac{(0.010)(2\text{ m})}{0.10\text{ m}} = 0.2000$$

Noting that $L_1 = L_1^* - L_2^*$, the function fL^*/D_h at the exit state and the corresponding Mach number are

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL_1}{D_h} = 0.3402 - 0.2000 = 0.1402 \quad\rightarrow\quad \text{Ma}_2 = 1.476$$

From the relations in Table A-16, at Ma_2 =1.476: $T_2/T^* = 0.8568$, $P_2/P^* = 0.6270$,

Then the temperature, pressure, and velocity before the shock are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.8568}{0.7018} = 1.2209 \quad\rightarrow\quad T_2 = 1.2209T_1 = 1.2209(510\text{ K}) = 622.7\text{ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.6270}{0.4189} = 1.4968 \quad\rightarrow\quad P_2 = 1.4968P_1 = 1.4968(180\text{ kPa}) = 269.4\text{ kPa}$$

The normal shock functions corresponding to a Mach number of 1.476 are, from the relations in Table A-14,

Ma_2 =1.476: $\text{Ma}_3 = 0.7052$, $T_3/T_2 = 1.2565$, $P_3/P_2 = 2.3466$

Then the temperature and pressure after the shock become

$$T_3 = 1.2565T_2 = 1.2565(622.7\text{ K}) = 782.4\text{ K}$$
$$P_3 = 2.3466P_2 = 2.3466(269.4\text{ kPa}) = 632.3\text{ kPa}$$

Sonic conditions exist at the duct exit, and the flow downstream the shock is still Fanno flow. From the relations in Table A-16,

$Ma_3 = 0.7052$: $T_3/T^* = 1.0767$, $P_3/P^* = 1.4713$
$Ma_4 = 1$: $T_4/T^* = 1$, $P_4/P^* = 1$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_4}{T_3} = \frac{T_4/T^*}{T_3/T^*} = \frac{1}{1.0767} \quad \rightarrow \quad T_4 = T_3/1.0767 = (782.4\text{ K})/1.0767 = \mathbf{727\text{ K}}$$

$$\frac{P_4}{P_3} = \frac{P_4/P^*}{P_3/P^*} = \frac{1}{1.4713} \quad \rightarrow \quad P_4 = P_3/1.4713 = (632.3\text{ kPa})/1.4713 = \mathbf{430\text{ kPa}}$$

$$V_4 = \text{Ma}_4 c_4 = (1)\sqrt{kRT_4} = \sqrt{(1.33)(0.280\text{ kJ/kg}\cdot\text{K})(727\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{520\text{ m/s}}$$

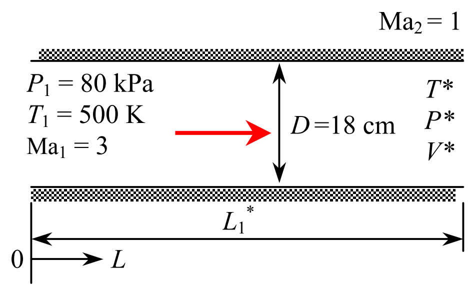
Discussion It can be shown that $L_3^* = 2.13$ m, and thus the total length of this duct is 4.13 m. If the duct is extended, the normal shock will move further upstream, and eventually to the inlet of the duct.

12-166 Choked supersonic airflow in a constant cross-sectional area adiabatic duct is considered. The variation of duct length with Mach number is to be investigated, and the results are to be plotted.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.03.

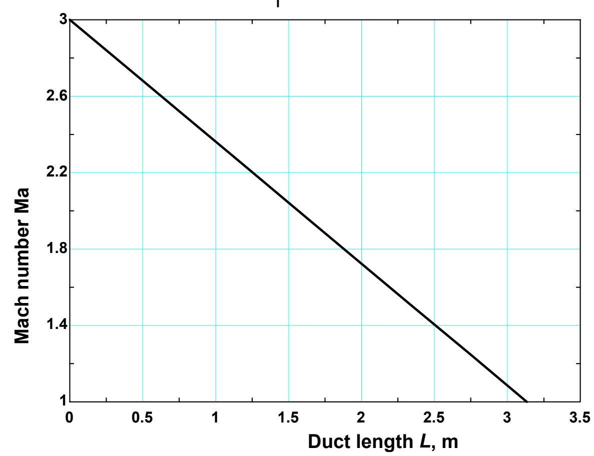
Analysis The flow is choked, and thus Ma_2 =1. Corresponding to the inlet Mach number of Ma_1 = 3 we have, from Table A-16, fL^*/D_h = 0.5222, Therefore, the original duct length is



$$L_1^* = 0.5222\frac{D}{f} = 0.5222\frac{0.18\ \text{m}}{0.03} = 3.13\ \text{m}$$

Repeating the calculations for different Ma_2 as it varies from 3 to 1 results in the following table for the location on the duct from the inlet:

Mach number, Ma	Duct length L, m
3.00	0.00
2.75	0.39
2.50	0.78
2.25	1.17
2.00	1.57
1.75	1.96
1.50	2.35
1.25	2.74
1.00	3.13



EES program:

```
k=1.4
cp=1.005
R=0.287

P1=80
T1=500
Ma1=3
"Ma2=1"
f=0.03
D=0.18

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
```

```
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that the Mach number decreases nearly linearly along the duct.

12-167 Choked subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The effect of duct length on the mass flow rate and the inlet conditions is to be investigated as the duct length is doubled.

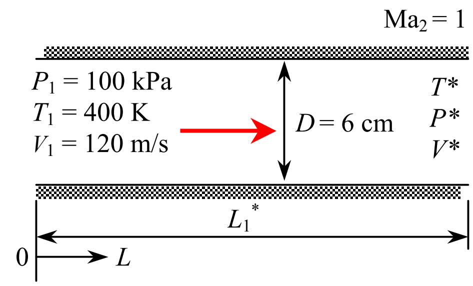
Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.02.

Analysis The flow is choked, and thus Ma_2 =1. The inlet Mach number is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(400\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 400.9\ \mathrm{m/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{120\ \mathrm{m/s}}{400.9\ \mathrm{m/s}} = 0.2993$$



Corresponding to this Mach number we have, from Table A-16, fL^*/D_h = 5.3312, Therefore, the original duct length is

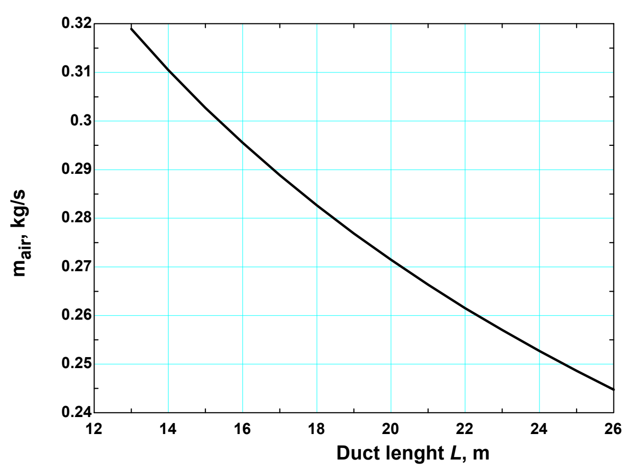
$$L = L_1^* = 5.3312\frac{D}{f} = 5.3312\frac{0.06\ \mathrm{m}}{0.02} = 16.0\ \mathrm{m}$$

Then the initial mass flow rate becomes

$$\rho_1 = \frac{P_1}{RT_1} = \frac{100\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kgK})(400\ \mathrm{K})} = 0.8711\ \mathrm{kg/m^3}$$

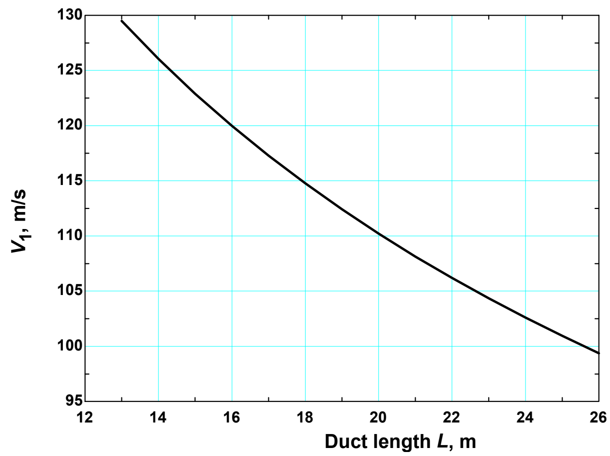
$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.8711\ \mathrm{kg/m^3})[\pi(0.06\ \mathrm{m})^2/4](120\ \mathrm{m/s}) = 0.296\ \mathrm{kg/s}$$

Duct length L, m	Inlet velocity V_1, m/s	Mass flow rate $\dot{m}_{air}$, kg/s
13	129	0.319
14	126	0.310
15	123	0.303
16	120	0.296
17	117	0.289
18	115	0.283
19	112	0.277
20	110	0.271
21	108	0.266
22	106	0.262
23	104	0.257
24	103	0.253
25	101	0.249
26	99	0.245



EES Program:

```
k=1.4
cp=1.005
R=0.287

P1=100
T1=400
"L=26"
Ma2=1
f=0.02
D=0.06

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-
Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-
1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```



Discussion Note that once the flow is choked, any increase in duct length results in a decrease in the mass flow rate and the inlet velocity.

12-168 The flow velocity of air in a channel is to be measured using a Pitot-static probe, which causes a shock wave to occur. For measured values of static pressure before the shock and stagnation pressure and temperature after the shock, the flow velocity before the shock is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady and one-dimensional.

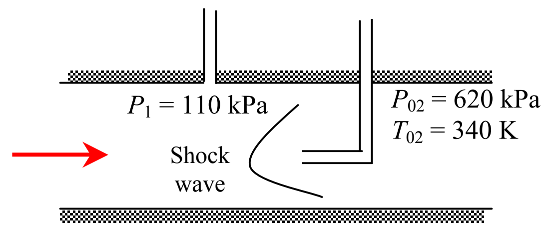
Properties The specific heat ratio of air at room temperature is k= 1.4.

Analysis The nose of the probe is rounded (instead of being pointed), and thus it will cause a bow shock wave to form. Bow shocks are difficult to analyze. But they are normal to the body at the nose, and thus we can approximate them as normal shocks in the vicinity of the probe.

It is given that the static pressure before the shock is P_1 = 110 kPa, and the stagnation pressure and temperature after the shock are P_{02} = 620 kPa, and T_{02} = 340 K. Noting that the stagnation temperature remains constant, we have

$$T_{01} = T_{02} = 340 \text{ K}$$

Also, $\dfrac{P_{02}}{P_1} = \dfrac{620 \text{ kPa}}{110 \text{ kPa}} = 5.6364 \approx 5.64$



The fluid properties after the shock are related to those before the shock through the functions listed in Table A-14.
For $P_{02} / P_1 = 5.64$ we read

$$\text{Ma}_1 = 2.0, \quad \text{Ma}_2 = 0.5774, \quad \frac{P_{02}}{P_{01}} = 0.7209, \quad \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = 2.6667,$$

Then the stagnation pressure and temperature before the shock become

$$P_{01} = P_{02} /0.7209 = (620 \text{ kPa})/0.7209 = 860 \text{ kPa}$$

$$T_1 = T_{01}\left(\frac{P_1}{P_{01}}\right)^{(k-1)/k} = (340 \text{ K})\left(\frac{110 \text{ kPa}}{860 \text{ kPa}}\right)^{(1.4-1)/1.4} = 188.9 \text{ K}$$

The flow velocity before the shock can be determined from V_1 = Ma$_1c_1$, where c_1 is the speed of sound before the shock,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(188.9 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 275.5 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(275.5 \text{ m/s}) = \mathbf{551 \text{ m/s}}$$

Discussion The flow velocity after the shock is $V_2 = V_1/2.6667$ = 551/2.6667 = 207 m/s. Therefore, the velocity measured by a Pitot-static probe would be very different that the flow velocity.

12-169 ··· 12-171 Design and Essay Problems
