Chapter 13
OPEN-CHANNEL FLOW

Classification, Froude Number, and Wave Speed

13-1C Open-channel flow is the flow of liquids in channels open to the atmosphere or in partially filled conduits, and is characterized by the presence of a liquid-gas interface called the free surface, whereas internal flow is the flow of liquids or gases that completely fill a conduit.

13-2C Flow in a channel is driven naturally by gravity. Water flow in a river, for example, is driven by the elevation difference between the source and the sink. The flow rate in an open channel is established by the dynamic balance between gravity and friction. Inertia of the flowing fluid also becomes important in unsteady flow.

13-3C The free surface coincides with the hydraulic grade line (HGL), and the pressure is constant along the free surface.

13-4C No, the slope of the free surface is not necessarily equal to the slope of the bottom surface even during steady fully developed flow.

13-5C The flow in a channel is said to be *uniform* if the flow depth (and thus the average velocity) remains constant. Otherwise, the flow is said to be *nonuniform* or *varied*, indicating that the flow depth varies with distance in the flow direction. Uniform flow conditions are commonly encountered in practice in long straight sections of channels with constant slope and constant cross-section.

13-6C In open channels of constant slope and constant cross-section, the fluid accelerates until the head loss due to frictional effects equals the elevation drop. The fluid at this point reaches its terminal velocity, and uniform flow is established. The flow remains uniform as long as the slope, cross-section, and the surface roughness of the channel remain unchanged. The flow depth in uniform flow is called the *normal depth* y_n, which is an important characteristic parameter for open-channel flows.

13-7C The presence of an obstruction in a channel such as a gate or a change in slope or cross-section causes the flow depth to vary, and thus the flow to become varied or nonuniform. The varied flow is called *rapidly varied flow* (RVF) if the flow depth changes markedly over a relatively short distance in the flow direction (such as the flow of water past a partially open gate or shortly before a falls), and *gradually varied flow* (GVF) if the flow depth changes gradually over a long distance along the channel.

13-8C The hydraulic radius R_h is defined as the ratio of the cross-sectional flow area A_c and the wetted perimeter p. That is, $R_h = A_c/p$. Knowing the hydraulic radius, the hydraulic diameter is determined from $D_h = 4R_h$.

13-9C Knowing the average flow velocity and flow depth, the Froude number is determined from $\mathrm{Fr} = V / \sqrt{gy}$. Then the flow is classified as

Fr < 1	Subcritical or tranquil flow
Fr = 1	Critical flow
Fr > 1	Supercritical or rapid flow

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13-10C Froude number is a dimensionless number that governs the character of flow in open channels. It is defined as $\mathrm{Fr} = V / \sqrt{gy}$ where g is the gravitational acceleration, V is the mean fluid velocity at a cross-section, and L_c is the characteristic length which is taken to be the flow depth y for wide rectangular channels. It represents the ratio of inertia forces to viscous forces in channel flow. The Froude number is also the ratio of the flow speed to wave speed, Fr = V/c_o.

13-11C The flow depth corresponding to a Froude number of Fr = 1 is the critical depth, and it is determined from $V = \sqrt{gy_c}$ or $y_c = V^2 / g$.

13-12C Yes to both questions.

13-13 The flow of water in a wide channel is considered. The speed of a small disturbance in flow for two different flow depths is to be determined for both water and oil.

Assumptions The distance across the wave is short and thus friction at the bottom surface and air drag at the top are negligible,

Analysis Surface wave speed can be determined directly from the relation $c_0 = \sqrt{gh}$.

(a) $c_0 = \sqrt{gh} = \sqrt{(9.81\,\mathrm{m/s^2})(0.1\,\mathrm{m})} = \mathbf{0.990\ m/s}$

(b) $c_0 = \sqrt{gh} = \sqrt{(9.81\,\mathrm{m/s^2})(0.8\,\mathrm{m})} = \mathbf{2.80\ m/s}$

Therefore, a disturbance in the flow will travel at a speed of 0.990 m/s in the first case, and 2.80 m/s in the second case.

Discussion. Note that wave speed depends on the water depth, and the wave speed increases as the water depth increases as long as the water remains shallow. Results would not change if the fluid were oil, because the wave speed depends only on the fluid depth.

13-14 Water flows uniformly in a wide rectangular channel. For given flow depth and velocity, it is to be determined whether the flow is laminar or turbulent, and whether it is subcritical or supercritical.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 20ºC are ρ = 998.0 kg/m^3 and μ = 1.002×10^{-3} kg/m·s.

Analysis (a) The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V y}{\mu} = \frac{(998.0\ \mathrm{kg/m^3})(2\ \mathrm{m/s})(0.2\ \mathrm{m})}{1.002\times10^{-3}\ \mathrm{kg/m\cdot s}} = 3.984\times10^5$$

which is greater than the critical value of 500. Therefore, the flow is **turbulent**.

(*b*) The Froude number is

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{2\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.2\ \mathrm{m})}} = 1.43$$

which is greater than 1. Therefore, the flow is **supercritical**.

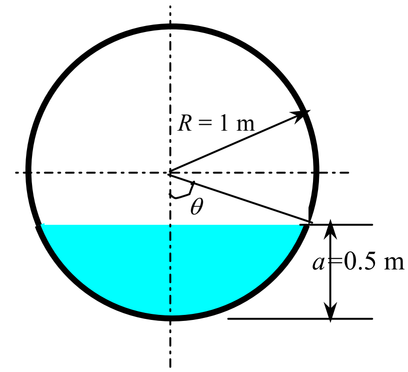
Discussion. The result in (*a*) is expected since almost all open channel flows are turbulent. Also, hydraulic radius for a wide rectangular channel approaches the water depth *y* as the ratio *y*/b approaches zero.

13-15 Water flow in a partially full circular channel is considered. For given water depth and average velocity, the hydraulic radius, Reynolds number, and the flow regime are to be determined.

Assumptions Flow is uniform.

Properties The density and dynamic viscosity of water at 20ºC are ρ = 998.0 kg/m^3 and μ = 1.002×10^{-3} kg/m·s.

Analysis From geometric considerations,



$$\cos\theta = \frac{R-a}{R} = \frac{1-0.5}{1} = 0.5 \quad \rightarrow \quad \theta = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$

Then the hydraulic radius becomes

$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R = \frac{\pi/3 - \sin(\pi/3)\cos(\pi/3)}{2\pi/3}(1\,\text{m}) = \mathbf{0.293\ m}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V R_h}{\mu} = \frac{(998.0\,\text{kg/m}^3)(2\,\text{m/s})(0.293\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}} = \mathbf{5.84\times10^5}$$

which is greater than the critical value of 500. Therefore, the flow is turbulent.

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$A_c = R^2(\theta - \sin\theta\cos\theta) = (1\,\text{m})^2[\pi/3 - \sin(\pi/3)\cos(\pi/3)] = 0.6142\,\text{m}^2$$

$$y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2R\sin\theta} = \frac{0.6142\,\text{m}^2}{2(1\,\text{m})\sin 60°} = 0.3546\ \text{m}$$

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2\,\text{m/s}}{\sqrt{(9.81\,\text{m/s}^2)(0.3546\,\text{m})}} = 1.07$$

which is greater than 1. Therefore, the flow is **supercritical** (although, very close to critical).

Discussion Note that if the maximum flow depth were used instead of the hydraulic depth, the result would be subcritical flow, which is not true.

13-16 Water flows uniformly in a wide rectangular channel. For given values of flow depth and velocity, it is to be determined whether the flow is subcritical or supercritical.

Assumptions **1** The flow is uniform. **2** The channel is wide and thus the side wall effects are negligible.

Analysis The Froude number is

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.08\ \mathrm{m})}} = 4.51$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion Note that the Froude Number is not function of any temperature-dependent properties, and thus temperature.

13-17 Rain water flows on a concrete surface. For given values of flow depth and velocity, it is to be determined whether the flow is subcritical or supercritical.

Assumptions **1** The flow is uniform. **2** The thickness of water layer is constant.

Analysis The Froude number is

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{1.3\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.02\ \mathrm{m})}} = 2.93$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion Note that this water layer will undergo a hydraulic jump when the ground slope decreases or becomes adverse.

13-18E Water flows uniformly in a wide rectangular channel. For given flow depth and velocity, it is to be determined whether the flow is laminar or turbulent, and whether it is subcritical or supercritical.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 70ºF are ρ = 62.30 lbm/ft^3 and μ = 6.556×10^{-4} lbm/ft·s.

Analysis (*a*) The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V y}{\mu} = \frac{(62.30\ \text{lbm/ft}^3)(6\ \text{ft/s})(0.5\ \text{ft})}{6.556\times10^{-4}\ \text{lbm/ft}\cdot\text{s}} = 2.85\times10^5$$

which is greater than the critical value of 500. Therefore, the flow is **turbulent**.

(*b*) The Froude number is

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{6\ \text{ft/s}}{\sqrt{(32.2\ \text{ft/s}^2)(0.5\ \text{ft})}} = 1.50$$

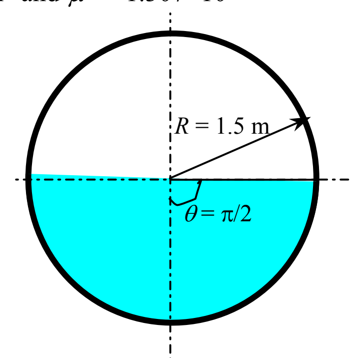
which is greater than 1. Therefore, the flow is **supercritical**.

Discussion. The result in (*a*) is expected since almost all open channel flows are turbulent. Also, hydraulic radius for a wide rectangular channel approaches the water depth *y* as the ratio *y*/b approaches zero.

13-19 Water flows uniformly through a half-full circular channel. For a given average velocity, the hydraulic radius, the Reynolds number, and the flow regime are to be determined.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 10ºC are ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s.



Analysis From geometric considerations, the hydraulic radius is

$$R_h = \frac{A_c}{p} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{1.5\ \text{m}}{2} = \mathbf{0.75\ m}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V R_h}{\mu} = \frac{(999.7\ \text{kg/m}^3)(2.5\ \text{m/s})(0.75\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = \mathbf{1.43\times10^6}$$

which is greater than the critical value of 500. Therefore, the flow is turbulent.

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(1.5\ \text{m})}{4} = 1.178\ \text{m}$$

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2.5\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(1.178\ \text{m})}} = 0.735$$

which is greater than 1. Therefore, the flow is **subcritical**.

Discussion If the maximum flow depth were used instead of the hydraulic depth, the result would still be subcritical flow, but this is not always the case.

13-20 A single wave is initiated in a sea by a strong jolt during an earthquake. The speed of the resulting wave is to be determined.

Assumptions The depth of water is constant,

Analysis Surface wave speed is determined the wave-speed relation to be

$$c_0 = \sqrt{gh} = \sqrt{(9.81\,\text{m/s}^2)(2000\,\text{m})} = \mathbf{140\,m/s}$$

Discussion. Note that wave speed depends on the water depth, and the wave speed increases as the water depth increases. Also, the waves eventually die out because of the viscous effects.

Specific Energy and the Energy Equation

13-21C The specific energy E_s of a fluid flowing in an open channel is the sum of the pressure and dynamic heads of a fluid, and is expressed as $E_s = y + \dfrac{V^2}{2g}$.

13-22C A plot of E_s versus y for constant $\dot{V}$ through a rectangular channel of width b reveals that there are two y values corresponding to a fixed value of E_s: one for subcritical flow and one for supercritical flow. Therefore, the specific energies of water in those two channels can be identical.

13-23C The point of minimum specific energy is the critical point, and thus the first person is right.

13-24C No, we disagree. A plot of E_s versus y for constant $\dot{V}$ reveals that the specific energy decreases as the flow depth increases during supercritical channel flow.

13-25C We agree with the first person since in uniform flow, the flow depth and the flow velocity, and thus the specific energy, remain constant since $E_s = y + V^2/2g$. The head loss is made up by the decline in elevation (the channel is sloped downward in the flow direction).

13-26C The friction slope is related to head loss h_L, and is defined as $S_f = h_L / L$ where L is the channel length. The friction slope becomes equal to the bottom slope when the head loss becomes equal to the elevation drop. That is, $S_f = S_0$ when $h_L = z_1 - z_2$.

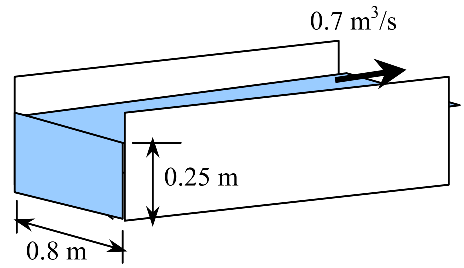
13-27C No, we disagree. The energy line is a distance $E_s = y + V^2/2g$ (total mechanical energy of the fluid) above a horizontal reference datum. When there is no head loss, the energy line is horizontal even when the channel is not. The elevation and velocity heads ($z + y$ and $V^2/2g$) may convert to each other during flow in this case, but their sum remains constant.

13-28C Yes. During steady one-dimensional flow, the total mechanical energy of a fluid at any point of a cross-section is given by $H = z + y + V^2/2g$.

13-29C The total mechanical energy of a fluid at any point of a cross-section is expressed as $H = z + y + V^2/2g$ where y is the flow depth, z is the elevation of the channel bottom, and V is the average flow velocity. It is related to the specific energy of the fluid by $H = z + E_s$.

13-30C The one-dimensional energy equation for open channel flow between an upstream section 1 and downstream section 2 is written as $z_1 + y_1 + \dfrac{V_1^2}{2g} = z_2 + y_2 + \dfrac{V_2^2}{2g} + h_L$ where y is the flow depth, z is the elevation of the channel bottom, and V is the average flow velocity. The head loss h_L due to frictional effects can be determined from $h_L = f \dfrac{L}{R_h} \dfrac{V^2}{8g}$ where f is the average friction factor and L is the length of channel between sections 1 and 2.

13-31 Water flow in a rectangular channel is considered. The character of flow, the flow velocity, and the alternate depth are to be determined.



Assumptions The specific energy is constant.

Analysis The average flow velocity is determined from

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{yb} = \frac{0.7\ \mathrm{m^3/s}}{(0.25\ \mathrm{m})(0.8\ \mathrm{m})} = \mathbf{3.50\ m/s}$$

The critical depth for this flow is

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(0.7\ \mathrm{m^3/s})^2}{(9.81\ \mathrm{m/s^2})(0.8\ \mathrm{m})^2}\right)^{1/3} = 0.427\ \mathrm{m}$$

Therefore, the flow is *supercritical* since the actual flow depth is y = 0.25 m, and $y < y_c$. The specific energy for given conditions is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2 y_1^2} = (0.25\ \mathrm{m}) + \frac{(0.7\ \mathrm{m^3/s})^2}{2(9.81\ \mathrm{m/s^2})(0.8\ \mathrm{m})^2 (0.25\ \mathrm{m})^2} = 0.874\ \mathrm{m}$$

Then the alternate depth is determined from $E_{s1} = E_{s2}$ to be

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2 y_2^2} \quad \rightarrow \quad 0.874\ \mathrm{m} = y_2 + \frac{(0.7\ \mathrm{m^3/s})^2}{2(9.81\ \mathrm{m/s^2})(0.8\ \mathrm{m})^2 y_2^2}$$

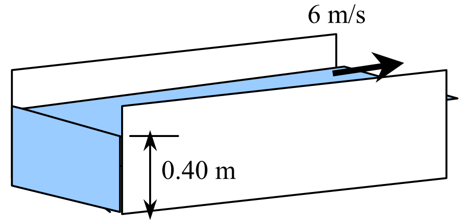
Solving for y_2 gives the alternate depth to be y_2 = **0.82 m**. There are three roots of this equation; one for subcritical, one for supercritical and third one as a negative root. Therefore, if the character of flow is changed from supercritical to subcritical while holding the specific energy constant, the flow depth will rise from 0.25 m to 0.82 m.

Discussion Two alternate depths show two possible flow conditions for a given specific energy. If the energy is not the minimum specific energy, there are two water depths corresponding to subcritical and supercritical states of flow. As an example, these two depths may be observed before and after a sluice gate as alternate depths, if the losses are disregarded.

13-32 Water flows in a rectangular channel. The specific energy and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis For convenience, we take the channel width to be b = 1 m. Then the volume flow rate and the critical depth for this flow become



$$\dot{V} = VA_c = Vyb = (6\text{ m/s})(0.4\text{ m})(1\text{ m}) = 2.40\text{ m}^3/\text{s}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.40\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(1\text{ m})^2}\right)^{1/3} = \mathbf{0.837\text{ m}}$$

The flow is **supercritical** since the actual flow depth is y = 0.4 m, and $y < y_c$. The specific energy for given conditions is

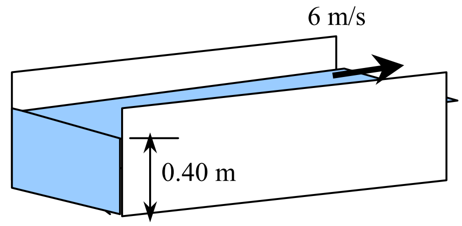
$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2y_1^2} = y_1 + \frac{V^2}{2g} = (0.4\text{ m}) + \frac{(6\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{2.23\text{ m}}$$

Discussion Note that the flow may also exist as subcritical flow at the same value of specific energy,

13-33 Water flows in a rectangular channel. The critical depth, the alternate depth, and the minimum specific energy are to be determined.

Assumptions The channel is sufficiently wide so that the edge effects are negligible.

Analysis For convenience, we take the channel width to be b = 1 m. Then the volume flow rate and the critical depth for this flow become



$$\dot{V} = VA_c = Vyb = (6\text{ m/s})(0.4\text{ m})(1\text{ m}) = 2.40\text{ m}^3/\text{s}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.40\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(1\text{ m})^2}\right)^{1/3} = \mathbf{0.837\text{ m}}$$

(b) The flow is *supercritical* since the actual flow depth is y = 0.4 m, and $y < y_c$. The specific energy for given conditions is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2y_1^2} = y_1 + \frac{V^2}{2g} = (0.4\text{ m}) + \frac{(6\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 2.23\text{ m}$$

Then the alternate depth is determined from $E_{s1} = E_{s2}$ to be

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2y_2^2} \quad \rightarrow \quad 2.23\text{ m} = y_2 + \frac{0.240\text{ m}^3/\text{s}}{y_2^2}$$

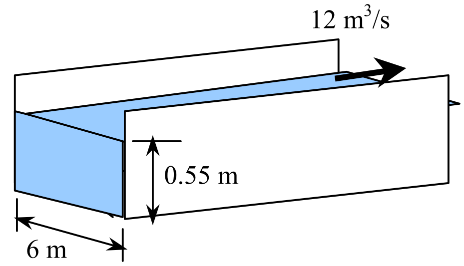
Solving for y_2 gives the alternate depth to be y_2 = **2.17 m**. Therefore, if the character of flow is changed from supercritical to subcritical while holding the specific energy constant, the flow depth will rise from 0.4 m to 2.17 m.

(c) the minimum specific energy is

$$E_{s,\min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{gy_c}{2g} = \frac{3}{2}y_c = \frac{3}{2}(0.837\text{ m}) = \mathbf{1.26\text{ m}}$$

Discussion Note that minimum specific energy is observed when the flow depth is critical

13-34 Water flows in a rectangular channel. The critical depth, the alternate depth, and whether the flow is subcritical or supercritical are to be determined.



Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The critical depth is calculated to be

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(12\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(6\text{ m})^2}\right)^{1/3} = 0.742\text{ m}$$

(*b*) The average flow velocity and the Froude number are

$$V = \frac{\dot{V}}{by} = \frac{12\text{ m}^3/\text{s}}{(6\text{ m})(0.55\text{ m})} = 3.636\text{ m/s}$$

$$\text{Fr}_1 = \frac{V}{\sqrt{gy}} = \frac{3.636\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.55\text{ m})}} = 1.565 > 1$$

which is greater than 1. Therefore, the flow is **supercritical**.

(*c*) Specific energy for this flow is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2y_1^2} = (0.55\text{ m}) + \frac{(12\text{ m}^3/\text{s})^2}{2(9.81\text{ m/s}^2)(6\text{ m})^2(0.55\text{ m})^2} = 1.224\text{ m}$$

Then the alternate depth is determined from $E_{s1}=E_{s2}$,

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2y_2^2} \quad \rightarrow \quad 1.224\text{ m} = y_2 + \frac{(12\text{ m}^3/\text{s})^2}{2(9.81\text{ m/s}^2)(6\text{ m})^2 y_2^2}$$

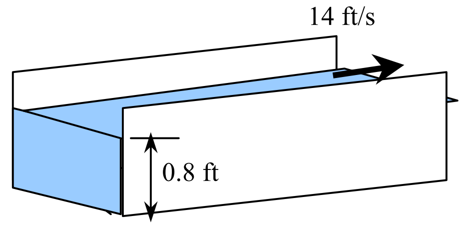
The alternate depth is calculated to be y_2 = **1.03 m** which is the subcritical depth for the same value of specific energy.

Discussion The depths 0.55 m and 1.03 are alternate depths for the given discharge and specific energy. The flow conditions determine which one is observed.

13-35E Water flows in a wide rectangular channel. For specified values of flow depth and average velocity, the Froude number, critical depth, and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The Froude number is



$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{14\ \mathrm{ft/s}}{\sqrt{(32.2\ \mathrm{ft/s^2})(0.8\ \mathrm{ft})}} = \mathbf{2.76}$$

(*b*) The critical depth is calculated to be

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14\ \mathrm{ft/s})^2 (0.8\ \mathrm{ft})^2}{(32.2\ \mathrm{ft/s^2})}\right)^{1/3} = \mathbf{1.57\ ft}$$

(*c*) The flow is **supercritical** since Fr > 1.

For the case of *y* = 0.2 ft:
Replacing 0.8 ft in above calculations by 0.2 ft gives

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{14\ \mathrm{ft/s}}{\sqrt{(32.2\ \mathrm{ft/s^2})(0.2\ \mathrm{ft})}} = \mathbf{5.52}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14\ \mathrm{ft/s})^2 (0.2\ \mathrm{ft})^2}{(32.2\ \mathrm{ft/s^2})}\right)^{1/3} = \mathbf{0.625\ ft}$$

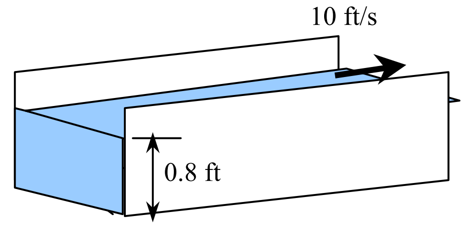
The flow is supercritical in this case also since Fr > 1.

Discussion Note that the value of critical depth depends on flow rate, and it decreases as the flow rate decreases.

13-36E Water flows in a wide rectangular channel. For specified values of flow depth and average velocity, the Froude number, critical depth, and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The Froude number is



$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{10 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(0.8 \text{ ft})}} = \mathbf{1.97}$$

(*b*) The critical depth is calculated to be

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(10 \text{ ft/s})^2 (0.8 \text{ ft})^2}{(32.2 \text{ ft/s}^2)}\right)^{1/3} = \mathbf{1.26 \text{ ft}}$$

(*c*) The flow is **supercritical** since Fr > 1.

For the case of $y = 0.2$ ft:
Replacing 0.8 ft in above calculations by 0.2 ft gives

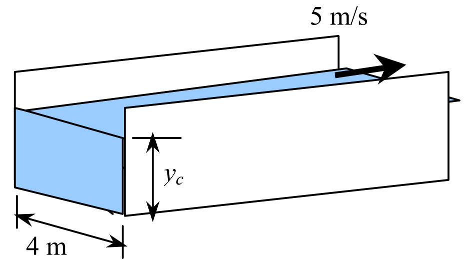
$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{10 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(0.2 \text{ ft})}} = \mathbf{3.94}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14 \text{ ft/s})^2 (0.2 \text{ ft})^2}{(32.2 \text{ ft/s}^2)}\right)^{1/3} = \mathbf{0.50 \text{ ft}}$$

The flow is supercritical in this case also since Fr > 1.

Discussion Note that the value of critical depth depends on flow rate, and it decreases as the flow rate decreases.

13-37 Critical flow of water in a rectangular channel is considered. For a specified average velocity, the flow rate of water is to be determined.



Assumptions The flow is uniform and thus the specific energy is constant.

Analysis The Froude number must be unity since the flow is critical, And thus $\mathrm{Fr} = V/\sqrt{gy} = 1$. Therefore,

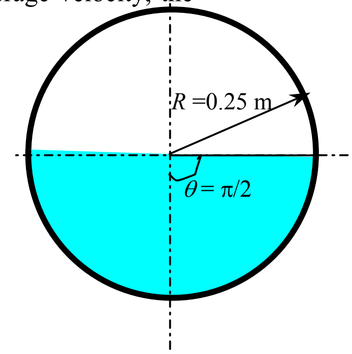
$$y = y_c = \frac{V^2}{g} = \frac{(5\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s}^2} = \mathbf{2.55\ m}$$

Then the flow rate becomes

$$\dot{V} = VA_c = Vby = (5\ \mathrm{m/s})(4\ \mathrm{m})(2.55\ \mathrm{m}) = \mathbf{51.0\ m^3/s}$$

Discussion Critical flow is not a stable type of flow and can be observed for short intervals. Occurrence of critical depth is important as boundary condition most of the time. For example it can be used as a flow rate computation mechanism for a channel ending with a drawdown.

13-38 Water flows uniformly through a half-full circular steel channel. For a given average velocity, the volume flow rate, critical slope, and the critical depth are to be determined.



Assumptions The flow is uniform.

Analysis The volume flow rate is determined from

$$\dot{V} = VA_c = V\frac{\pi R^2}{2} = (2.8\text{ m/s})\frac{\pi(0.25\text{ m})^2}{2} = \mathbf{0.275\ m^3/s}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(0.25\text{ m})}{4} = 0.1963\text{ m}$$

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2.8\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.1963\text{ m})}} = 2.02$$

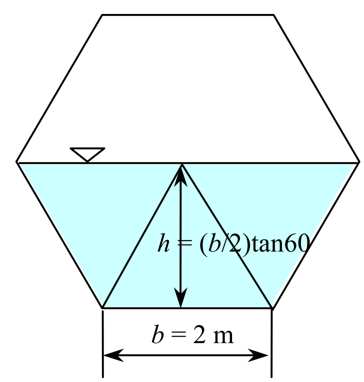
which is greater than 1. Therefore, the flow is **supercritical**.

Discussion Note that if the maximum flow depth were used instead of the hydraulic depth, the result could be different, especially when the Froude number is close to 1.

13-39 Water flows uniformly through a half-full hexagon channel. For a given flow rate, the average velocity and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform.

Analysis (*a*) The flow area is determined from geometric considerations to be



$$A_c = \frac{(b+2b)}{2}\frac{b}{2}\tan 60° = \frac{(2+2\times 2)\text{ m}}{2}\frac{2\text{ m}}{2}\tan 60° = 5.196\ \text{m}^2$$

Then the average velocity becomes

$$V = \frac{\dot{V}}{A_c} = \frac{45\ \text{m}^3/\text{s}}{5.196\ \text{m}^2} = \mathbf{8.66\ m/s}$$

(*b*) When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2b} = \frac{5.196\text{ m}}{2\times 2\text{ m}} = 1.299\text{ m}$$

Then the Froude number becomes

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{8.66\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.299\text{ m})}} = 2.43$$

which is greater than 1. Therefore, the flow is **supercritical**.

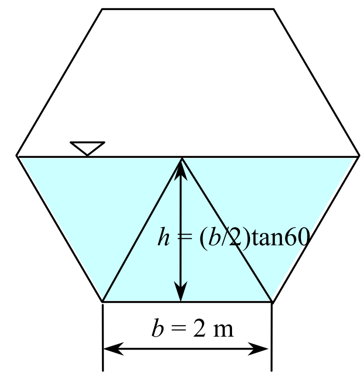
Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

13-40 Water flows uniformly through a half-full hexagon channel. For a given flow rate, the average velocity and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform.

Analysis The flow area is determined from geometric considerations to be

$$A_c = \frac{(b+2b)}{2}\frac{b}{2}\tan 60° = \frac{(2+2\times 2)\text{ m}}{2}\frac{2\text{ m}}{2}\tan 60° = 5.196\ \text{m}^2$$



Then the average velocity becomes

$$V = \frac{\dot{V}}{A_c} = \frac{30\ \text{m}^3/\text{s}}{5.196\ \text{m}^2} = \mathbf{5.77\ m/s}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2b} = \frac{5.196\,\text{m}^2}{2\times 2\,\text{m}} = 1.299\,\text{m}$$

Then the Froude number becomes

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{5.77\,\text{m/s}}{\sqrt{(9.81\,\text{m/s}^2)(1.299\,\text{m})}} = 1.62$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

Uniform Flow and Best Hydraulic Cross Sections

13-41C Flow in a channel is called *uniform flow* if the flow depth (and thus the average flow velocity) remains constant. The flow remains uniform as long as the slope, cross-section, and the surface roughness of the channel remain unchanged.

13-42C The flow depth will **decrease** when the bottom slope is increased.

13-43C Yes, the head loss in uniform flow is $h_L = S_0 L$ since the head loss must equal elevation loss.

13-44C The value of the factor a in SI units is a = 1 m$^{1/3}$/s. Combining the relations $C = \sqrt{8g/f}$ and $C = \frac{a}{n} R_h^{1/6}$ and solving them for n gives the desired relation to be $n = \frac{a}{\sqrt{8g/f}} R_h^{1/6}$. In practice, n is usually determined experimentally.

13-45C It is to be shown that for uniform critical flow, the general critical slope relation $S_c = \frac{gn^2 y_c}{a^2 R_h^{4/3}}$ reduces to $S_c = \frac{gn^2}{a^2 y_c^{1/3}}$ for film flow with $b >> y_c$.

Analysis For critical flow, the flow depth is $y = y_c$. For film flow, the hydraulic radius is $R_h = y = y_c$. Substituting into the critical slope relation gives the desired result,

$$S_c = \frac{gn^2 y_c}{a^2 R_h^{4/3}} = \frac{gn^2 y_c}{a^2 y_c^{4/3}} = \frac{gn^2}{a^2 y_c^{1/3}}$$

13-46C The best hydraulic cross-section for an open channel is the one with the maximum hydraulic radius, or equivalently, the one with the minimum wetted perimeter for a specified cross-section.

13-47C The best hydraulic cross-section for an open channel is a (*a*) **circular** one.

13-48C The best hydraulic cross-section for a rectangular channel is one whose fluid height is (*b*) **half** the channel width.

13-49C The best hydraulic cross-section for a trapezoidal channel of base width b is one for which the length of the side edge of the flow section is b.

13-50C The flow rate in uniform flow is given as $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$, and thus the flow rate is inversely proportional to the Manning coefficient. Therefore, if the Manning coefficient doubles as a result of some algae growth on surfaces while the flow cross-section remains constant, the flow rate will (*d*) **decrease by half**.

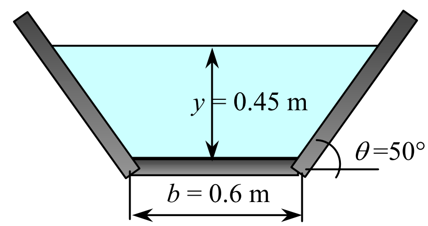
13-51 The flow of water in a trapezoidal finished-concrete channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel of finished concrete is n = 0.012 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = y\left(b + \frac{y}{\tan\theta}\right) = (0.45\ \text{m})\left(0.60\ \text{m} + \frac{0.45\ \text{m}}{\tan 50°}\right) = 0.3724\ \text{m}^2$$



$$p = b + \frac{2y}{\sin\theta} = 0.6\ \text{m} + \frac{2\,(0.45\ \text{m})}{\sin 50°} = 1.775\ \text{m}$$

$$R_h = \frac{A_c}{p} = \frac{0.3724\ \text{m}^2}{1.775\ \text{m}} = 0.2096\ \text{m}$$

Bottom slope of the channel is

$$S_0 = \tan 0.4^\circ = 0.006981$$

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\ \text{m}^{1/3}/s}{0.012}(0.3724\ \text{m}^2)(0.2096\ \text{m})^{2/3}(0.006981)^{1/2} = \mathbf{0.915\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-52 Water flows uniformly half-full in a circular finished-concrete channel. For a given bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

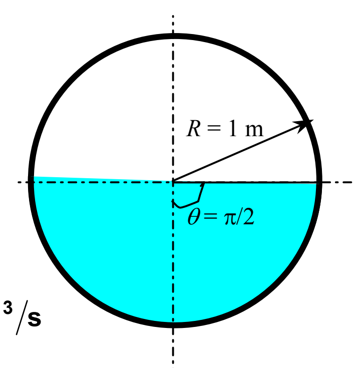
Properties Manning coefficient for an open channel of finished concrete is n = 0.012 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi\,(1\ \text{m})^2}{2} = 1.571\ \text{m}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(1\ \text{m})}{2} = 3.142\ \text{m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{1\ \text{m}}{2} = 0.50\ \text{m}$$



Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\ \text{m}^{1/3}/s}{0.012}(1.571\ \text{m}^2)(0.50\ \text{m})^{2/3}(1.5/1000)^{1/2} = \mathbf{3.19\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-53E Water is to be transported uniformly in a full semi-circular unfinished-concrete channel. For a specified flow rate, the elevation difference across the channel is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

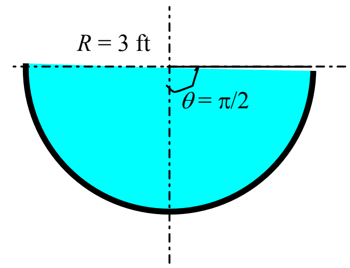
Properties Manning coefficient for an open channel of unfinished concrete is $n = 0.014$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi\,(3\text{ ft})^2}{2} = 14.14\text{ ft}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(3\text{ ft})}{2} = 9.425\text{ ft}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{3\text{ ft}}{2} = 1.50\text{ ft}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow \quad 150\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(14.14\text{ ft}^2)(1.50\text{ ft})^{2/3} S_0^{1/2}$$

It gives the slope to be $S_0 = 0.005817$. Therefore, the *elevation difference* Δz across a pipe length of $L = 1$ mile = 5280 ft must be

$$\Delta z = S_0 L = 0.005817(5280\text{ ft}) = \mathbf{30.7\ ft}$$

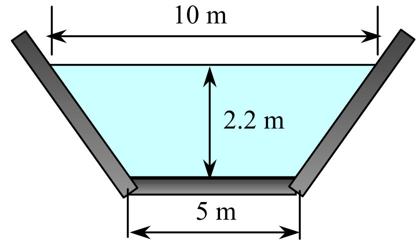
Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-54 Water is to be transported uniformly in a trapezoidal asphalt-lined channel. For a specified flow rate, the required elevation drop per km channel length is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an asphalt-lined open channel is n = 0.016 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are



$$A_c = \frac{10\,\text{m} + 5\,\text{m}}{2}(2.2\,\text{m}) = 16.5\,\text{m}^2$$

$$p = (5\,\text{m}) + 2\sqrt{(2.2\,\text{m})^2 + (2.5\,\text{m})^2} = 11.66\,\text{m}$$

$$R_h = \frac{A_c}{p} = \frac{16.5\,\text{m}^2}{11.66\,\text{m}} = 1.415\,\text{m}$$

Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 120\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.016}(16.5\,\text{m}^2)(1.415\,\text{m})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.008524. Therefore, the *elevation drop* Δz across a pipe length of L = 1 km must be

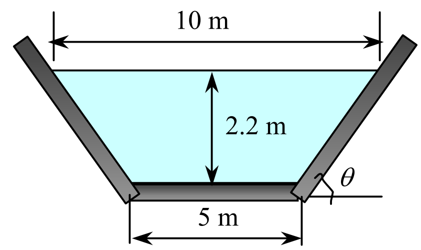
$$\Delta z = S_0 L = 0.008524(1000\,\text{m}) = \mathbf{8.52\,m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-55 The flow of water through the trapezoidal asphalt-lined channel in Problem 13-54 is reconsidered. The maximum flow rate corresponding to a given maximum channel height is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis We denote the flow conditions in the previous problem by subscript 1 and the conditions for the maximum case in this problem by subscript 2. Using the Manning's equation $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$ and noting that the Manning coefficient and the channel slope remain constant, the flow rate in case 2 can be expressed in terms of flow rate in case 1 as



$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{(a/n)A_{c2}R_{h2}^{2/3}}{(a/n)A_{c1}R_{h1}^{2/3}} \quad \rightarrow \quad \dot{V}_2 = \frac{A_{c2}}{A_{c1}}\left(\frac{R_{h2}}{R_{h1}}\right)^{2/3}\dot{V}_1$$

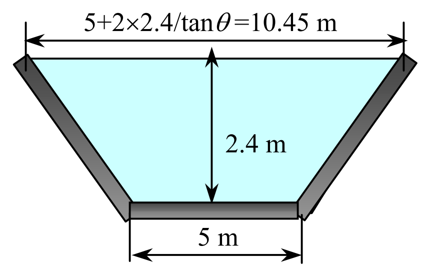
The trapezoid angle is $\tan\theta = 2.2/2.5 = 0.88 \rightarrow \theta = 2.2/2.5 = 41.34°$.

From geometric considerations,

$$A_{c1} = \frac{10\text{ m} + 5\text{ m}}{2}(2.2\text{ m}) = 16.5\text{ m}^2$$

$$p_1 = (5\text{ m}) + 2\sqrt{(2.2\text{ m})^2 + (2.5\text{ m})^2} = 11.66\text{ m}$$

$$R_{h1} = \frac{A_{c1}}{p_1} = \frac{16.5\text{ m}^2}{11.66\text{ m}} = 1.415\text{ m}$$



and

$$A_{c2} = \frac{10.45\text{ m} + 5\text{ m}}{2}(2.4\text{ m}) = 18.54\text{ m}^2$$

$$p_2 = (5\text{ m}) + 2\sqrt{(2.4\text{ m})^2 + (5.45/2\text{ m})^2} = 12.26\text{ m}$$

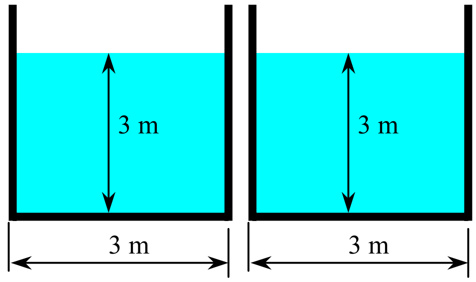
$$R_{h2} = \frac{A_{c2}}{p_2} = \frac{18.54\text{ m}^2}{12.26\text{ m}} = 1.512\text{ m}$$

Substituting,

$$\dot{V}_2 = \frac{A_{c2}}{A_{c1}}\left(\frac{R_{h2}}{R_{h1}}\right)^{2/3}\dot{V}_1 = \frac{18.54\text{ m}^2}{16.5\text{ m}^2}\left(\frac{1.512\text{ m}}{1.415\text{ m}}\right)^{2/3}(120\text{ m}^3/\text{s}) = \mathbf{141\ m^3/s}$$

Discussion Note that a 9% increase in flow depth results in an 18% increase in flow rate.

13-56 The flow of water through two identical channels with square flow sections is considered. The percent increase in flow rate as a result of combining the two channels while the flow depth remains constant is to be determined.



Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis We denote the flow conditions for two separate channels by subscript 1 and the conditions for the combined wide channel by subscript 2. Using the Manning's equation $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$ and noting that the Manning coefficient, channel slope, and the flow area A_c remain constant, the flow rate in case 2 can be expressed in terms of flow rate in case 1 as

$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{(a/n)A_{c2}R_{h2}^{2/3}}{(a/n)A_{c1}R_{h1}^{2/3}} = \left(\frac{R_{h2}}{R_{h1}}\right)^{2/3} = \left(\frac{A_{c2}/p_2}{A_{c1}/p_1}\right)^{2/3} = \left(\frac{p_1}{p_2}\right)^{2/3}$$

where p is the wetted perimeter. Substituting,

$$\frac{\dot{V}_2}{\dot{V}_1} = \left(\frac{p_2}{p_2}\right)^{2/3} = \left(\frac{6\times 3\text{ m}}{4\times 3\text{ m}}\right)^{2/3} = \left(\frac{3}{2}\right)^{2/3} = \mathbf{1.31} \qquad \textbf{(31\% increase)}$$

Discussion This is a very significant increase, and shows the importance of eliminating unnecessary surfaces in flow systems, including pipe flow.

13-51 The flow of water in a trapezoidal channel made of unfinished-concrete is considered. For given flow rate and bottom slope, the flow depth is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

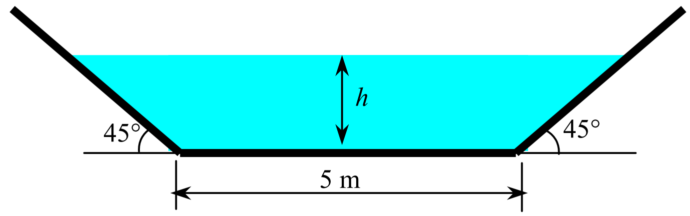
Properties Manning coefficient for an open channel of unfinished concrete is n = 0.014 (Table 13-1).

Analysis From geometric considerations, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = \frac{5\text{ m} + 5\text{ m} + 2h}{2}h = (5+h)h$$

$$p = (5\text{ m}) + 2h/\sin 45° = 5 + 2.828h$$

$$R_h = \frac{A_c}{p} = \frac{(5+h)h}{5+2h/\sin 45°}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.014}(5+h)h\left(\frac{(5+h)h}{5+2h/\sin 45°}\right)^{2/3}(\tan 1°)^{1/2}$$

It gives the flow depth to be h = **0.685 m**.

Discussion Non-linear equations frequently arise in the solution of open channel flow problems. They are best handled by equation solvers such as EES.

13-51 The flow of water in a weedy excavated trapezoidal channel is considered. For given flow rate and bottom slope, the flow depth is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

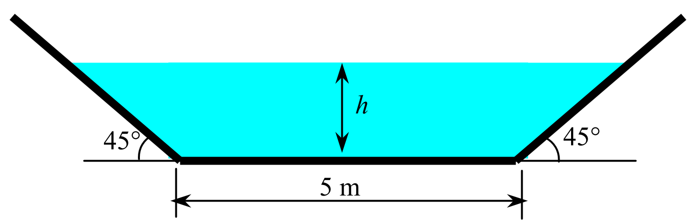
Properties Manning coefficient for the channel is given to be n = 0.030.

Analysis From geometric considerations, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = \frac{5\text{ m} + 5\text{ m} + 2h}{2}h = (5+h)h$$

$$p = (5\text{ m}) + 2h/\sin 45° = 5 + 2.828h$$

$$R_h = \frac{A_c}{p} = \frac{(5+h)h}{5+2h/\sin 45°}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.030}(5+h)h\left(\frac{(5+h)h}{5+2h/\sin 45°}\right)^{2/3}(\tan 1°)^{1/2}$$

It gives the flow depth to be y = **1.07 m**.

Discussion Note that as the Manning coefficient increases because of the increased surface roughness of the channel, the flow depth required to maintain the same flow rate also increases.

13-59 The flow of water in a V-shaped cast iron channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

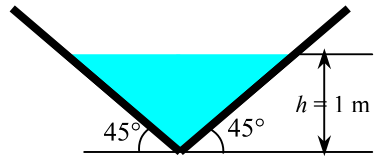
Properties Manning coefficient for an open channel of cast iron is n = 0.013 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{2h \times h}{2} = h^2 = (1\,\text{m})^2 = 1\,\text{m}^2$$

$$p = 2h/\sin\theta = 2(1\,\text{m})/\sin 45° = 2\sqrt{2}\ \text{m}$$

$$R_h = \frac{A_c}{p} = \frac{1\,\text{m}^2}{2\sqrt{2}\ \text{m}} = 0.3536\ \text{m}$$



Bottom slope of the channel is

$$S_0 = \tan 0.5° = 0.008727$$

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\,\text{m}^{1/3}/s}{0.013}(1\,\text{m}^2)(0.3536\,\text{m})^{2/3}(0.008727)^{1/2} = \mathbf{3.59\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-60E The flow of water in a rectangular cast iron channel is considered. For given flow rate and bottom slope, the flow depth is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

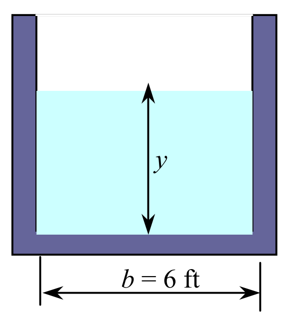
Properties Manning coefficient for a cast iron open channel is n = 0.013 (Table 13-1).

Analysis From geometric considerations, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = by = (6\text{ ft})y = 6y$$

$$p = (6\text{ ft}) + 2y = 6 + 2y$$

$$R_h = \frac{A_c}{p} = \frac{6y}{6+2y}$$



The channel bottom slope is S_0 = 1.5/1000 = 0.0015.

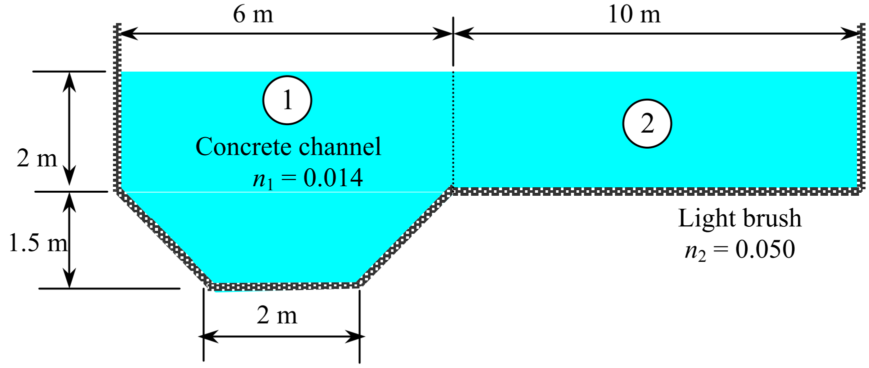
Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 70\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.013}(6y)\left(\frac{6y}{6+2y}\right)^{2/3}(0.0015)^{1/2}$$

It gives the flow depth to be h = **2.24 ft**.

Discussion Non-linear equations frequently arise in the solution of open channel flow problems. They are best handled by equation solvers such as EES.

13-61 Water is flowing through a channel with nonuniform surface properties. The flow rate and the effective Manning coefficient are to be determined.



Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The Manning coefficients do not vary along the channel.

Analysis The channel involves two parts with different roughness, and thus it is appropriate to divide the channel into two subsections. The flow rate for each subsection can be determined from the Manning equation, and the total flow rate can be determined by adding them up.
The flow area, perimeter, and hydraulic radius for each subsection and the entire channel are:

Subsection 1: $A_{c1} = 18\text{ m}^2$, $\quad p_1 = 9\text{ m}$, $\quad R_{h1} = \dfrac{A_{c1}}{p_1} = \dfrac{18\text{ m}^2}{9\text{ m}} = 2.00\text{ m}$

Subsection 2: $A_{c2} = 20\text{ m}^2$, $\quad p_2 = 12\text{ m}$, $\quad R_{h2} = \dfrac{A_{c2}}{p_2} = \dfrac{20\text{ m}^2}{12\text{ m}} = 1.67\text{ m}$

Entire channel: $A_c = 38\text{ m}^2$, $\quad p = 21\text{ m}$, $\quad R_h = \dfrac{A_c}{p} = \dfrac{38\text{ m}^2}{21\text{ m}} = 1.81\text{ m}$

Applying the Manning equation to each subsection, the total flow rate through the channel is determined to be

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \frac{a}{n_1} A_1 R_1^{2/3} S_0^{1/2} + \frac{a}{n_1} A_1 R_1^{2/3} S_0^{1/2}$$

$$= (1\text{ m}^{1/3}/\text{s})\left(\frac{(18\text{ m}^2)\ (2\text{ m})^{2/3}}{0.014} + \frac{(20\text{ m}^2)\ (1.67\text{ m})^{2/3}}{0.05}\right)(0.002)^{1/2}$$

$$= \mathbf{116\ m^3/s}$$

Knowing the total flow rate, the effective Manning coefficient for the entire channel can be determined from the Manning equation to be

$$n_{\text{eff}} = \frac{aA_c R_h^{2/3} S_0^{1/2}}{\dot{V}} = \frac{(1\text{ m}^{1/3}/\text{s})(38\text{ m}^2)(1.81\text{ m})^{2/3}(0.002)^{1/2}}{116\text{ m}^3/\text{s}} = \mathbf{0.0217}$$

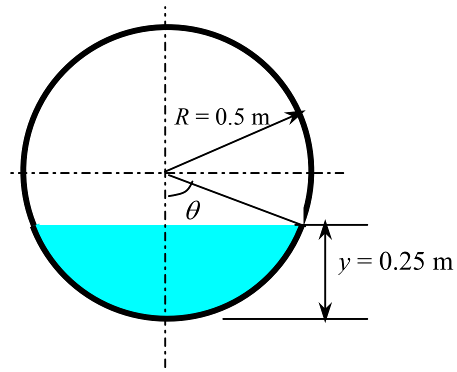
Discussion The effective Manning coefficient n_{eff} of the channel turns out to lie between the two n values, as expected. The weighted average of the Manning coefficient of the channel is $n_{\text{ave}}=(n_1p_1+ n_2p_2)/p$ = 0.035, which is quite different than n_{eff}. Therefore, using a weighted average Manning coefficient for the entire channel may be tempting, but it would not be so accurate.

13-62 Waters flows in a partially filled circular channel made of finished concrete. For a given flow depth and bottom slope, the flow rate is to be determined. √EES

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel made of finished concrete is $n = 0.012$ (Table 13-1).

Analysis From geometric considerations,



$$\cos\theta = \frac{R-y}{R} = \frac{0.5-0.25}{0.5} = 0.5 \qquad \rightarrow \qquad \theta = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$

$$A_c = R^2(\theta - \sin\theta\cos\theta) = (0.5\text{ m})^2[\pi/3 - \sin(\pi/3)\cos(\pi/3)] = 0.1535\text{ m}^2$$

$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R = \frac{\pi/3 - \sin(\pi/3)\cos(\pi/3)}{2\pi/3}(0.5\text{ m}) = 0.1466\text{ m}$$

Then the flow rate can be determined from Manning's equation to be

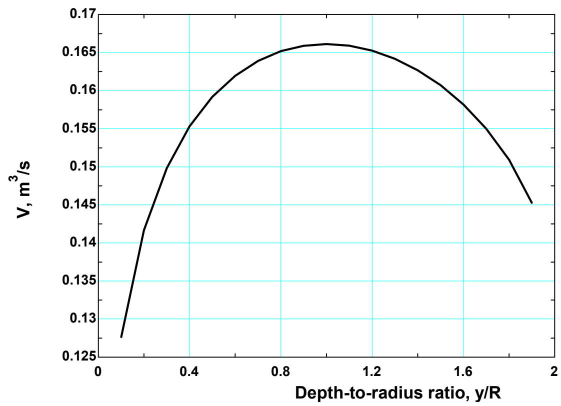
$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.012}(0.1535\text{ m}^2)(0.1466\text{ m})^{2/3}(0.002)^{1/2} = \mathbf{0.159\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

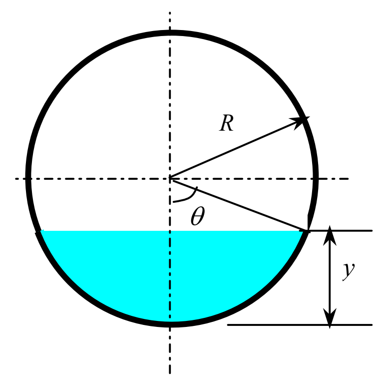
13-63 Problem 13-62 is reconsidered. By varying the flow depth-to-radius ratio from 0.1 to 1.9 for a fixed value of flow area, it is the to be shown that the best hydraulic cross section occurs when the circular channel is half-full, and the results are to be plotted.

Analysis Below is the EES program used to solve the problem:

```
a=1
n=0.012
s=0.002
Ac=0.1536 “Flow area kept constant”
ratio=y/R “This ratio is varied from 0.1 to 1.9”
bdeg=arcsin((R-y)/R)
tetadeg=90-bdeg
teta=tetadeg*2*pi/360
Ac=R^2*(teta-sin(tetadeg)*cos(tetadeg))
p=2*teta*R
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```



Depth-to-radius ratio, y/R	Channel radius, R, m	Flow rate, $\dot{V}$, m^3/s
0.1	1.617	0.1276
0.2	0.969	0.1417
0.3	0.721	0.1498
0.4	0.586	0.1553
0.5	0.500	0.1592
0.6	0.440	0.1620
0.7	0.396	0.1639
0.8	0.362	0.1652
0.9	0.335	0.1659
1.0	0.313	**0.1661**
1.1	0.295	0.1659
1.2	0.279	0.1653
1.3	0.267	0.1642
1.4	0.256	0.1627
1.5	0.247	0.1607
1.6	0.239	0.1582
1.7	0.232	0.1550
1.8	0.227	0.1509
1.9	0.223	0.1453



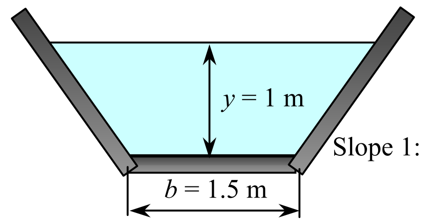
Discussion The depth-to-radius ratio of $y/R = 1$ corresponds to a half-full circular channel, and it is clear from the table and the chart that, for a fixed flow area, the flow rate becomes maximum when the channel is half-full.

13-64 Water is to be transported uniformly in a clean-earth trapezoidal channel. For a specified flow rate, the required elevation drop per km channel length is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for the clean-earth lined open channel is n = 0.022 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are



$$A_c = \frac{(1.5+1.5+2)\ \text{m}}{2}(1\,\text{m}) = 2.5\,\text{m}^2$$

$$p = (1.5\,\text{m}) + 2\sqrt{(1\,\text{m})^2 + (1\,\text{m})^2} = 4.328\,\text{m}$$

$$R_h = \frac{A_c}{p} = \frac{2.5\,\text{m}^2}{4.328\,\text{m}} = 0.5776\,\text{m}$$

Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 8\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.022}(2.5\,\text{m}^2)(0.5776\,\text{m})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.0103. Therefore, the *elevation drop* Δz across a pipe length of L = 1 km must be

$$\Delta z = S_0 L = 0.0103(1000\,\text{m}) = \mathbf{10.3\ m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-65 A water draining system consists of three circular channels, two of which draining into the third one. If all channels are to run half-full, the diameter of the third channel is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** Losses at the junction are negligible.

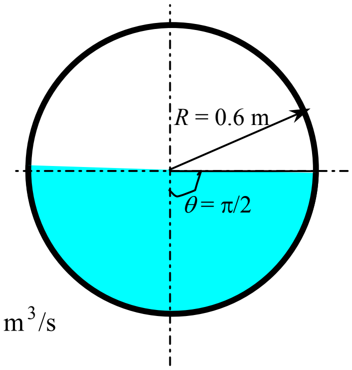
Properties The Manning coefficient for asphalt lined open channels is $n = 0.016$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the two pipes upstream are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi\,(0.6\text{ m})^2}{2} = 0.5655\text{ m}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(0.6\text{ m})}{2} = 1.885\text{ m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{0.6\text{ m}}{2} = 0.30\text{ m}$$



Then the flow rate through the 2 pipes becomes, from Manning's equation,

$$\dot{V} = 2\frac{a}{n}A_c R_h^{2/3} S_0^{1/2} = 2\frac{1\text{ m}^{1/3}/s}{0.016}(0.5655\text{ m}^2)(0.30\text{ m})^{2/3}(0.0015)^{1/2} = 1.227\text{ m}^3\text{/s}$$

The third channel is half-full, and the flow rate through it remains the same. Noting that the flow area is $\pi R^2/2$ and the hydraulic radius is $R/2$, we have

$$1.227\text{ m}^3\text{/s} = \frac{1\text{ m}^{1/3}/s}{0.016}(\pi R^2/2\text{ m}^2)(R/2\text{ m})^{2/3}(0.0015)^{1/2}$$

Solving for R gives $R = 0.778$ m. Therefore, the diameter of the third channel is

D_3 **= 1.56 m**

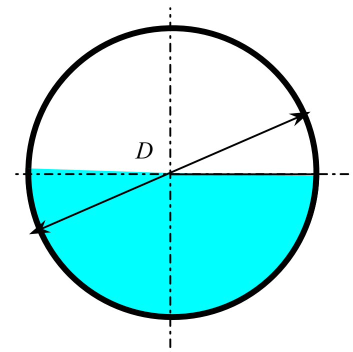
Discussion Note that if the channel diameter were larger, the channel would have been less than half full.

13-66 Waters is transported in an asphalt lined open channel at a specified rate. The dimensions of the best cross-section for various geometric shapes are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient for asphalt lined open channels is $n = 0.016$ (Table 13-1).

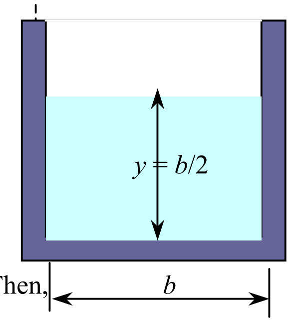
Analysis (*a*) Circular channel of Diameter D: Best cross-section occurs when the channel is half-full, and thus the flow area is $\pi D^2/8$ and the hydraulic radius is $D/4$. Then from Manning's equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,



$$4\ \mathrm{m^3/s} = \frac{1\ \mathrm{m^{1/3}}/s}{0.016}(\pi D^2/8\ \mathrm{m^2})(D/4\ \mathrm{m})^{2/3}(0.0015)^{1/2}$$

It gives D **= 2.42 m.**

(*b*) Rectangular channel of bottom width b: For best cross-section, $y = b/2$. Then $A_c = yb = b^2/2$ and $R_h = b/4$. From Manning equation,

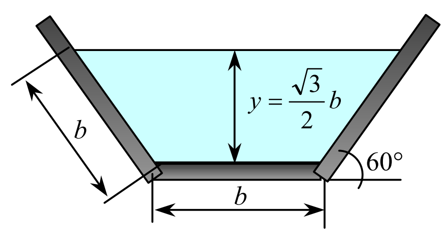


$$4\ \mathrm{m^3/s} = \frac{1\ \mathrm{m^{1/3}}/s}{0.016}(b^2/2\ \mathrm{m^2})(b/4\ \mathrm{m})^{2/3}(0.0015)^{1/2}$$

It gives b **= 2.21 m,** and $y = b/2 =$ **1.11 m.**

(*c*) Trapezoidal channel of bottom width b: For best cross-section, $\theta = 60°$ and $y = b\sqrt{3}/2$. Then, $A_c = y(b + b\cos\theta) = 0.5\sqrt{3}b^2(1+\cos 60°) = 0.75\sqrt{3}b^2$, $p = 3b$, $R_h = \frac{y}{2} = \frac{\sqrt{3}}{4}b$. From Manning equation,

$$4\ \mathrm{m^3/s} = \frac{1\ \mathrm{m^{1/3}}/s}{0.016}(0.75\sqrt{3}b^2\ \mathrm{m^2})(\sqrt{3}b/4\ \mathrm{m})^{2/3}(0.0015)^{1/2}$$



It gives b **= 1.35 m,** and y **= 1.17 m** and θ **= 60°.**

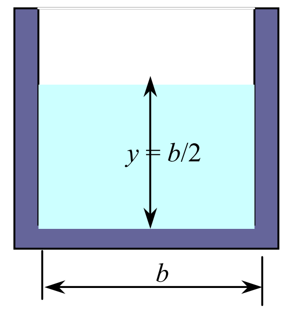
Discussion The perimeters for the circular, rectangular, and trapezoidal channels are 3.80 m, 4.42 m, and 4.05 m, respectively. Therefore, the circular cross-section has the smallest perimeter.

13-67E Water is to be transported in a rectangular channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished and finished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is n = 0.012 and n = 0.014 for finished and unfinished concrete, respectively (Table 13-1).

Analysis For best cross-section of a rectangular cross-section, $y = b/2$. Then $A_c = yb = b^2/2$ and $R_h = b/4$. The flow rate is determined from the Manning equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,



(a) Finished concrete, n = 0.012:

$$800\ \text{ft}^3/\text{s} = \frac{1.486\ \text{ft}^{1/3}/s}{0.012}(b^2/2\ \text{ft}^2)(b/4\ \text{ft})^{2/3}(0.0005)^{1/2}$$

It gives b **= 15.4 ft,** and $y = b/2$ = **7.68 ft**

(b) Unfinished concrete, n = 0.014:

$$800\ \text{ft}^3/\text{s} = \frac{1.486\ \text{ft}^{1/3}/s}{0.014}(b^2/2\ \text{ft}^2)(b/4\ \text{ft})^{2/3}(0.0005)^{1/2}$$

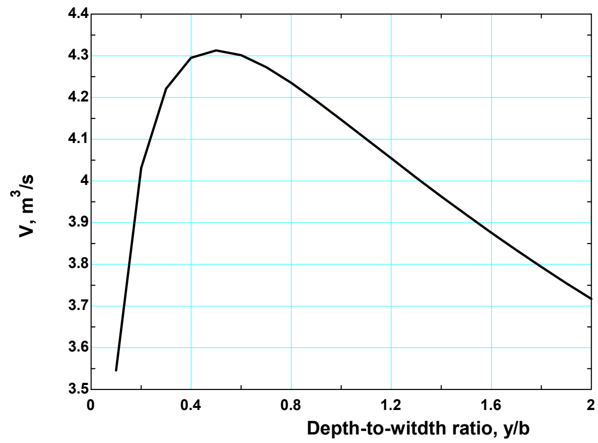
It gives b **= 16.3 ft,** and $y = b/2$ = **8.13 ft**

Discussion Note that channels with rough surfaces require a larger cross-section to transport the same amount of water.

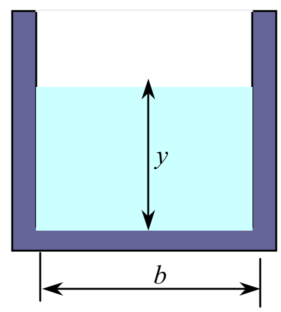
13-68 Uniform flow in an asphalt-lined rectangular channel is considered. By varying the depth-to-width ratio from 0.1 to 2 in increments of 0.1 for a fixed value of flow area, it is the to be shown that the best hydraulic cross section occurs when $y/b = 0.5$, and the results are to be plotted.

Analysis Below is the EES program used to solve the problem:

```
a=1
n=0.016 "Manning coefficient"
s=0.003 "Bottom slope is constant"
Ac=2 "Flow area remains constant at 2 m2"
Ratio=y/b
Ac=b*y
p=b+2*y
Rh=Ac/p "Hydraulic radius"
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s) "Volume flow rate"
```



Depth-to-width ratio, y/b	Channel width, b, m	Flow rate, $\dot{V}$,m^3/s
0.1	4.47	3.546
0.2	3.16	4.031
0.3	2.58	4.221
0.4	2.24	4.295
0.5	2.00	**4.313**
0.6	1.83	4.301
0.7	1.69	4.273
0.8	1.58	4.235
0.9	1.49	4.192
1.0	1.41	4.147
1.1	1.35	4.101
1.2	1.29	4.054
1.3	1.24	4.008
1.4	1.20	3.963
1.5	1.15	3.919
1.6	1.12	3.876
1.7	1.08	3.834
1.8	1.05	3.794
1.9	1.03	3.755
2.0	1.00	3.717



Discussion It is clear from the table and the chart that the depth-to-width ratio of $y/b = 0.5$ corresponds to the best cross-section for an open channel of rectangular cross-section.

Gradually and Rapidly Varied Flows and Hydraulic Jump

13-69C Both uniform and varied flows are steady, and thus neither involves any change with time at a specified location. In uniform flow, the flow depth y and the flow velocity V remain constant whereas in nonuniform or varied flow, the flow depth and velocity vary during flow. In uniform flow, the slope of the energy line is equal to the slope of the bottom surface. Therefore, the friction slope equals the bottom slope, $S_f = S_0$. In varied flow, however, these slopes are different.

13-70C Gradually varied flow (GVF) is characterized by gradual variations in flow depth and velocity (small slopes and no abrupt changes) and a free surface that always remains smooth (no discontinuities or zigzags). Rapidly varied flow (RVF) involves rapid changes in flow depth and velocity. A change in the bottom slope or cross-section of a channel or an obstruction on the path of flow may cause the uniform flow in a channel to become gradually or rapidly varied flow. Relations for the profile of the free surface can be obtained in GVF, but this is not the case for RVF because of the intense agitation.

13-71C Yes, we agree with this claim. Rapidly varied flows occur over a short section of the channel with relatively small surface area, and thus frictional losses associated with wall shear are negligible compared with losses due to intense agitation and turbulence. Losses in GVF, on the other hand, are primarily due to frictional effects along the channel, and should be considered.

13-72C The flow depth y will *decrease* in the flow direction.

13-73C The flow depth y will *increase* in the flow direction.

13-74C The flow depth y will *increase* in the flow direction.

13-75C The flow depth y will *decrease* in the flow direction.

13-76C The flow depth y will *increase* in the flow direction.

13-77C It is impossible for subcritical flow to undergo a hydraulic jump. Such a process would require the head loss h_L to become negative, which is impossible. It would correspond to negative entropy generation, which would be a violation of the second law of thermodynamics. Therefore, the upstream flow must be supercritical ($\mathrm{Fr}_1 > 1$) for hydraulic jump to occur.

13-78C Hydraulic jumps are sometimes often designed in conjunction with stilling basins and spillways of dams in order to waste as much of the mechanical energy as possible to minimize the mechanical energy of the fluid and thus its potential to cause damage. In such cases, a measure of performance of a hydraulic jump is the energy dissipation ratio, which is the fraction of energy dissipated, defined as

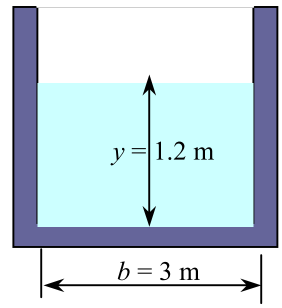
$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/(2g)} = \frac{h_L}{y_1(1 + \mathrm{Fr}_1^2/2)}\,.$$

13-79 Water is flowing in an open channel uniformly. It is to be determined whether the channel slope is mild, critical, or steep for this flow.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

Properties The Manning coefficient for an open channel with finished concrete surfaces is n = 0.012 (Table 13-1).

Analysis The cross-sectional area, perimeter, and hydraulic radius are



$$A_c = yb = (1.2\text{ m})(3\text{ m}) = 3.6\text{ m}^2$$

$$p = b + 2y = 3\text{ m} + 2(1.2\text{ m}) = 5.4\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{3.6\text{ m}^2}{5.4\text{ m}} = 0.6667\text{ m}$$

The flow rate is determined from the Manning equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/\text{s}}{0.012}(3.6\text{ m}^2)(0.6667\text{ m})^{2/3}(0.002)^{1/2} = 10.2\text{ m}^3/\text{s}$$

Noting that the flow is uniform, the specified flow rate is the normal depth and thus $y = y_n$ = 1.2 m. The critical depth for this flow is

$$y_c = \left(\frac{\dot{V}^2}{g b^2}\right)^{1/3} = \left(\frac{(10.2\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(3\text{ m})^2}\right)^{1/3} = 1.06\text{ m}$$

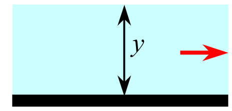
This channel at these flow conditions is classified as **mild** since $y > y_c$, and the flow is subcritical.

Discussion If the flow depth were smaller than 1.06 m, the channel slope would be said to be *steep*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-80 Water is flowing in an wide brick open channel uniformly. The range of flow depth for which the channel can be classified as "steep" is to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

Properties The Manning coefficient for a brick open channel is $n = 0.015$ (Table 13-1).



Analysis The slope of the channel is

$$S_0 = \tan\alpha = \tan 0.4° = 0.006981$$

The hydraulic radius for a wide channel is equal to the flow depth, $R_h = y$. Now assume the flow in the channel to be critical, The channel flow in this case would be critical slope S_c, and the flow depth would be the critical flow depth, which is determined from

$$S_c = \frac{g\,n^2}{a^2 y_c^{1/3}} \quad \rightarrow \quad y_c = \left(\frac{g\,n^2}{a^2 S_c}\right)^3$$

Substituting,

$$y_c = \left(\frac{g\,n^2}{a^2 S_c}\right)^3 = \left(\frac{(9.81\,\text{m/s}^2)(0.015)^2}{(1\ \text{m}^{1/3}\,/\,s)^2(0.006981)}\right)^3 = 0.0316\ \text{m}$$

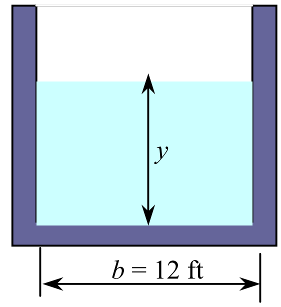
Therefore, this channel can be classified as *steep* for uniform flow depths less than y_c, i.e., $\boldsymbol{y < 0.0316}$ **m**.

Discussion Note that two channels of the same slope can be classified as differently (one mild and the other steep) if they have different roughness and thus different values of n.

13-81E Water is flowing in a rectangular open channel with a specified bottom slope at a specified flow rate. It is to be determined whether the slope of this channel should be classified as mild, critical, or steep. The surface profile is also to be classified for a specified flow depth of 2 m.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

Properties The Manning coefficient of a channel with unfinished concrete surfaces is n = 0.014 (Table 13-1).



Analysis The cross-sectional area, perimeter, and hydraulic radius are

$$A_c = yb = y(12\text{ ft}) = 12y\text{ ft}^2$$

$$p = b + 2y = 12\text{ ft} + 2y = 12 + 2y\text{ ft}$$

$$R_h = \frac{A_c}{p} = \frac{12y\text{ ft}^2}{12 + 2\text{y ft}} = 0.6667\text{ m}$$

Substituting the known quantities into the Manning equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 300\text{ ft}^3\text{/s} = \frac{1.486\text{ ft}^{1/3}\text{ / }s}{0.014}(12y)\left(\frac{12y}{12+2y}\right)^{2/3}(\tan 0.5°)^{1/2}$$

Solving for the flow depth y gives y = 1.95 ft. The critical depth for this flow is

$$y_c = \frac{\dot{V}^2}{gA_c^2} = \frac{(300\text{ ft}^3\text{ / s})^2}{(32.2\text{ ft/s}^2)(12\text{ ft} \times 1.95\text{ ft})^2} = 5.10\text{ ft}$$

This channel at these flow conditions is classified as **steep** since $y < y_c$, and the flow is supercritical. Alternately, we could solve for Froude number and show that Fr > 1 and reach the same conclusion.

The given flow is uniform, and thus $y = y_n$ = 1.95 ft. Therefore, the given value of y = 3 ft during development is between y_c and y_n, and the **flow profile** is **S2** (Table 13-3).

Discussion If the flow depth were larger than 5.19 ft, the channel slope would be said to be *mild*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-82 Water is flowing in a V-shaped open channel with a specified bottom slope at a specified rate. It is to be determined whether the slope of this channel should be classified as mild, critical, or steep.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

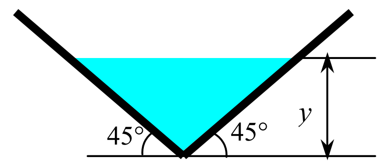
Properties The Manning coefficient for a cast iron channel is $n = 0.013$ (Table 13-1).

Analysis From geometric considerations, the cross-sectional area, perimeter, and hydraulic radius are

$$A_c = y(2y)/2 = y^2$$

$$p = 2\sqrt{y^2 + y^2} = 2\sqrt{2}y$$

$$R_h = \frac{A_c}{p} = \frac{y^2}{2\sqrt{2}\mathrm{y}} = \frac{y}{2\sqrt{2}}$$



Substituting the known quantities into the Manning equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow 3\,\mathrm{m^3/s} = \frac{1\,\mathrm{m^{1/3}/s}}{0.013}(y^2)\left(\frac{\mathrm{y}}{2\sqrt{2}}\right)^{2/3}(0.002)^{1/2}$$

Solving for the flow depth y gives $y = 1.23$ m. The critical depth for this flow is

$$y_c = \frac{\dot{V}^2}{gA_c^2} = \frac{(3\,\mathrm{m^3/s})^2}{(9.81\,\mathrm{m/s^2})(1.23\,\mathrm{m})^2} = 0.61\,\mathrm{m}$$

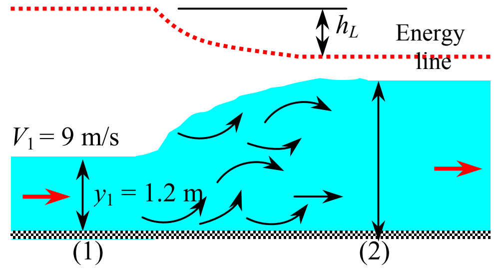
This channel at these flow conditions is classified as **mild** since $y > y_c$, and the flow is subcritical.

Discussion If the flow depth were smaller than 0.61 m, the channel slope would be said to be *steep*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-83 Water at a specified depth and velocity undergoes a hydraulic jump. The depth and Froude number after the jump, the head loss and dissipation ratio, and dissipated mechanical power are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.

Properties The density of water is 1000 kg/m^3.



Analysis (*a*) The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{9\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.2\text{ m})}} = 2.62$$

which is greater than 1. Therefore, the flow is supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(1.2\text{ m})\left(-1+\sqrt{1+8\times 2.62^2}\right) = \mathbf{3.89\text{ m}}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{1.2\text{ m}}{3.89\text{ m}}(9\text{ m/s}) = 2.78\text{ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{2.78\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3.89\text{ m})}} = \mathbf{0.449}$$

(*b*) The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (1.2\text{ m}) - (3.89\text{ m}) + \frac{(9\text{ m/s})^2 - (2.78\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{1.05\text{ m}}$$

The specific energy of water before the jump and the dissipation ratio are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.2\text{ m}) + \frac{(9\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 5.33\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{1.04\text{ m}}{5.33\text{ m}} = \mathbf{0.195}$$

Therefore, 19.5% of the available head (or mechanical energy) of the liquid is wasted (converted to thermal energy) as a result of frictional effects during this hydraulic jump.

(*c*) The mass flow rate of water is

$$\dot{m} = \rho\dot{\boldsymbol{V}} = \rho b y_1 V_1 = (1000\text{ kg/m}^3)(1.2\text{ m})(8\text{ m})(9\text{ m/s}) = 86{,}400\text{ kg/s}$$

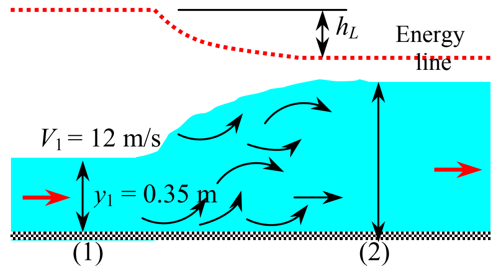
Then the dissipated mechanical power becomes

$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (86{,}400\text{ kg/s})(9.81\text{ m/s}^2)(1.04\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 881{,}000\text{ Nm/s} = \mathbf{881\,kW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 881 kW of power production potential in this case. That is, if the water is routed to a hydraulic turbine instead of being released from the sluice gate, up to 881 kW of power could be produced.

13-84 Water at a specified depth and velocity undergoes a hydraulic jump. The head loss associated with this process is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{12\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.35\text{ m})}} = 6.476$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(0.35\text{ m})\left(-1+\sqrt{1+8\times 6.476^2}\right) = 3.035\text{ m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.35\text{ m}}{3.035\text{ m}}(12\text{ m/s}) = 1.384\text{ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.384\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3.035\text{ m})}} = 0.2536$$

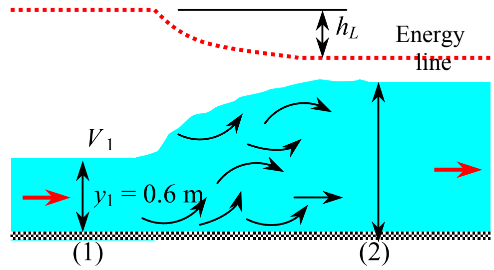
The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.35\text{ m}) - (3.035\text{ m}) + \frac{(12\text{ m/s})^2 - (1.384\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{4.56\text{ m}}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 4.56 m of head in the process.

13-85 The increase in flow depth during a hydraulic jump is given. The velocities and Froude numbers before and after the jump, and the energy dissipation ratio are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the jump is determined from

$$y_2 = 0.5 y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) \rightarrow \quad 3\text{ m} = 0.5\times(0.6\text{ m})\left(-1+\sqrt{1+8\mathrm{Fr}^2}\right)$$

It gives $\mathrm{Fr}_1 = \mathbf{3.873.}$ Then,

$$V_1 = \mathrm{Fr}_1\sqrt{gy_1} = 3.873\sqrt{(9.81\text{ m/s}^2)(0.6\text{ m})} = \mathbf{9.40\text{ m/s}}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.6\text{ m}}{3\text{ m}}(9.40\text{ m/s}) = \mathbf{1.88\text{ m/s}}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.88\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3\text{ m})}} = \mathbf{0.347}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.6\text{ m}) - (3\text{ m}) + \frac{(9.40\text{ m/s})^2 - (1.88\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{1.92\text{ m}}$$

The specific energy of water before the jump and the dissipation ratio are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (0.6\text{ m}) + \frac{(9.40\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 5.10\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{1.92\text{ m}}{5.10\text{ m}} = \mathbf{0.376}$$

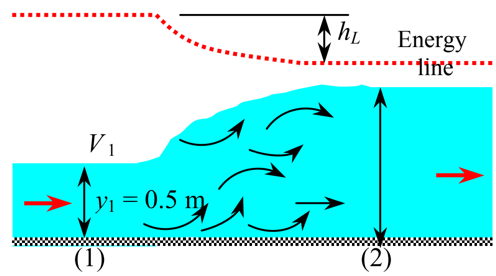
Therefore, 37.6% of the available head (or mechanical energy) of water is wasted (converted to thermal energy) as a result of frictional effects during this hydraulic jump.

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting over one-third of the available head.

13-86 Water flowing in a wide channel at a specified depth and flow rate undergoes a hydraulic jump. The mechanical power wasted during this process is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.

Properties The density of water is 1000 kg/m³.



Analysis Average velocities before and after the jump are

$$V_1 = \frac{70\ \text{m}^3/\text{s}}{(10\ \text{m})(0.5\ \text{m})} = 14\ \text{m/s}$$

$$V_2 = \frac{70\ \text{m}^3/\text{s}}{(10\ \text{m})(4\ \text{m})} = 1.75\ \text{m/s}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.5\ \text{m}) - (4\ \text{m}) + \frac{(14\ \text{m/s})^2 - (1.75\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 6.33\ \text{m}$$

The mass flow rate of water is

$$\dot{m} = \rho \dot{V} = (1000\ \text{kg/m}^3)(70\ \text{m}^3/\text{s}) = 70{,}000\ \text{kg/s}$$

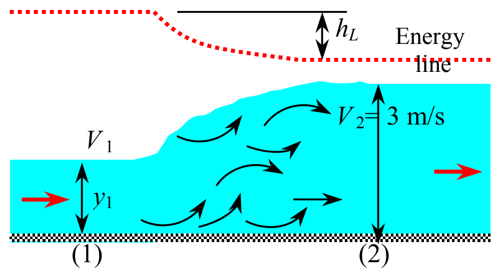
Then the dissipated mechanical power becomes

$$\dot{E}_{\text{dissipated}} = \dot{m} g h_L = (70{,}000\ \text{kg/s})(9.81\ \text{m/s}^2)(6.33\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4350\ \text{kNm/s} = \mathbf{4.35\ MW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 4.35 MW of power production potential in this case.

13-87 The flow depth and average velocity of water after a hydraulic jump are measured. The flow depth and velocity before the jump as well as the fraction of mechanical energy dissipated are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number after the hydraulic jump is

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{3\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2\ \mathrm{m})}} = 0.6773$$

It can be shown that the subscripts in the relation $y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$ are interchangeable. Thus,

$$y_1 = 0.5y_2\left(-1+\sqrt{1+8\mathrm{Fr}_2^2}\right) = 0.5(2\ \mathrm{m})\left(-1+\sqrt{1+8\times 0.6773^2}\right) = 1.16\ \ \mathrm{m}$$

$$V_1 = \frac{y_2}{y_1}V_2 = \frac{2\ \mathrm{m}}{1.161\ \mathrm{m}}(3\ \mathrm{m/s}) = 5.17\ \mathrm{m/s}$$

The Froude number before the jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5.17\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(1.16\ \mathrm{m})}} = 1.53$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (1.16\ \mathrm{m}) - (2\ \mathrm{m}) + \frac{(5.17\ \mathrm{m/s})^2 - (3\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 0.0636\ \mathrm{m}$$

The specific energy of water before the jump and the dissipation ratio is

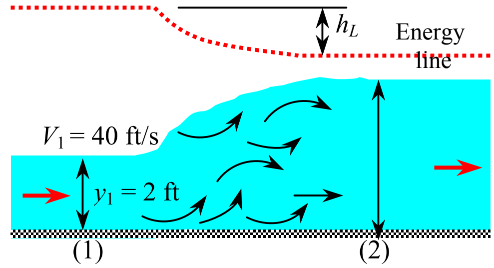
$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.16\ \mathrm{m}) + \frac{(5.17\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 2.52\ \mathrm{m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{0.0636\ \mathrm{m}}{2.52\ \mathrm{m}} = \mathbf{0.025}$$

Discussion Note that this is a "mild" hydraulic jump, and only 2.5% of the available energy is wasted.

13-88E Water at a specified depth and velocity undergoes a hydraulic jump, and dissipates a known fraction of its energy. The flow depth, velocity, and Froude number after the jump and the head loss associated with the jump are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{40\ \text{ft/s}}{\sqrt{(32.2\ \text{m/s}^2)(2\ \text{ft})}} = 4.984$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(2\ \text{ft})\left(-1+\sqrt{1+8\times 4.984^2}\right) = \mathbf{13.1\ ft}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{2\ \text{ft}}{13.1\ \text{ft}}(40\ \text{ft/s}) = \mathbf{6.09\ ft/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{6.091\ \text{ft/s}}{\sqrt{(32.2\ \text{ft/s}^2)(13.13\ \text{m})}} = \mathbf{0.296}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (2\ \text{ft}) - (13.1\ \text{ft}) + \frac{(40\ \text{ft/s})^2 - (6.09\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \mathbf{13.2\ ft}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 13.2 ft of head in the process.