Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)

12-96C The characteristic aspect of Rayleigh flow is its involvement of heat transfer. The main assumptions associated with Rayleigh flow are: the flow is steady, one-dimensional, and frictionless through a constant-area duct, and the fluid is an ideal gas with constant specific heats.

112-97C The points on the Rayleigh line represent the states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations for a given state. Therefore, for a given inlet state, the fluid cannot exist at any downstream state outside the Rayleigh line on a *T-s* diagram.

12-98C In Rayleigh flow, the effect of heat gain is to increase the entropy of the fluid, and the effect of heat loss is to decrease it.

12-99C In Rayleigh flow, the stagnation temperature T_0 always increases with heat transfer to the fluid, but the temperature T decreases with heat transfer in the Mach number range of $0.845 < \text{Ma} < 1$ for air. Therefore, the temperature in this case will decrease.

12-100C Heating the fluid increases the flow velocity in subsonic flow, but decreases the flow velocity in supersonic flow.

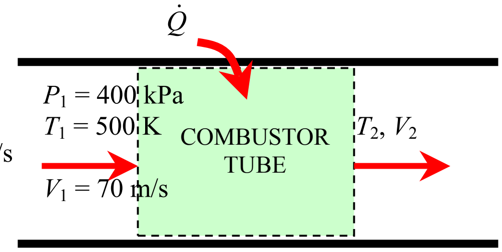
12-101C The flow is choked, and thus the flow at the duct exit will remain sonic.

12-102 Fuel is burned in a tubular combustion chamber with compressed air. For a specified exit Mach number, the exit temperature and the rate of fuel consumption are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Combustion is complete, and it is treated as a heat addition process, with no change in the chemical composition of flow. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet density and mass flow rate of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{400\text{ kPa}}{(0.287\text{ kJ/kgK})(500\text{ K})} = 2.787\text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.787\text{ kg/m}^3)[\pi(0.12\text{ m})^2/4](70\text{ m/s}) = 2.207\text{ kg/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 500\text{ K} + \frac{(70\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 502.4\text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(500\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 448.2\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{70\text{ m/s}}{448.2\text{ m/s}} = 0.1562$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\text{Ma}_1 = 0.1562$: $T_1/T^* = 0.1314$, $T_{01}/T^* = 0.1100$, $V_1/V^* = 0.0566$

$\text{Ma}_2 = 0.8$: $T_2/T^* = 1.0255$, $T_{02}/T^* = 0.9639$, $V_2/V^* = 0.8101$

The exit temperature, stagnation temperature, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0255}{0.1314} = 7.804 \quad\rightarrow\quad T_2 = 7.804T_1 = 7.804(500\text{ K}) = \mathbf{3903\text{ K}}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.1100} = 8.763 \quad\rightarrow\quad T_{02} = 8.763T_{01} = 8.763(502.4\text{ K}) = 4403\text{ K}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.8101}{0.0566} = 14.31 \quad\rightarrow\quad V_2 = 14.31V_1 = 14.31(70\text{ m/s}) = 1002\text{ m/s}$$

Then the mass flow rate of the fuel is determined to be

$$q = c_p(T_{02} - T_{01}) = (1.005\text{ kJ/kg}\cdot\text{K})(4403 - 502.4)\text{ K} = 3920\text{ kJ/kg}$$

$$\dot{Q} = \dot{m}_{\text{air}} q = (2.207\text{ kg/s})(3920\text{ kJ/kg}) = 8650\text{ kW}$$

$$\dot{m}_{\text{fuel}} = \frac{\dot{Q}}{HV} = \frac{8650\text{ kJ/s}}{39{,}000\text{ kJ/kg}} = \mathbf{0.222\text{ kg/s}}$$

Discussion Note that both the temperature and velocity increase during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

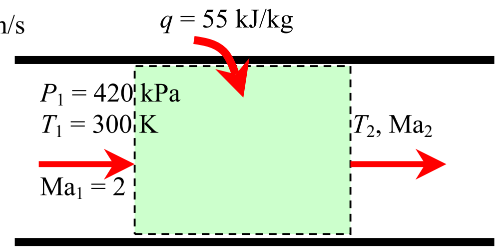
12-103 Fuel is burned in a rectangular duct with compressed air. For specified heat transfer, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 347.2\text{ m/s}$$



$$V_1 = \text{Ma}_1 c_1 = 2(347.2\text{ m/s}) = 694.4\text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300\text{ K} + \frac{(694.4\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 539.9\text{ K}$$

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9\text{ K} + \frac{55\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 594.6\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-15 or from the appropriate relation. At Ma$_1$ = 2 we read $T_{01}/T_0^* = 0.7934$. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9\text{ K}}{0.7934} = 680.5\text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{02}}{T_0^*} = \frac{594.6\text{ K}}{680.5\text{ K}} = 0.8738 \quad \rightarrow \quad \text{Ma}_2 = \mathbf{1.642}$$

Also,

$$\text{Ma}_1 = 2 \quad \rightarrow T_1/T^* = 0.5289$$
$$\text{Ma}_2 = 1.642 \quad \rightarrow T_2/T^* = 0.6812$$

Then the exit temperature becomes

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.6812}{0.5289} = 1.288 \quad \rightarrow \quad T_2 = 1.288T_1 = 1.288(300\text{ K}) = \mathbf{386\text{ K}}$$

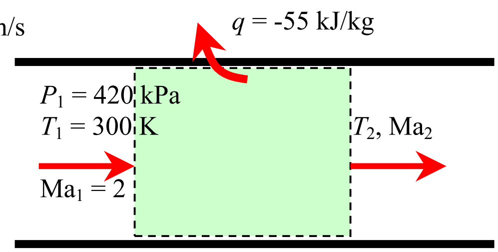
Discussion Note that the temperature increases during this supersonic Rayleigh flow with heating. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-104 Compressed air is cooled as it flows in a rectangular duct. For specified heat rejection, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $C_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are



$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(300 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 347.2 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(347.2 \text{ m/s}) = 694.4 \text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300 \text{ K} + \frac{(694.4 \text{ m/s})^2}{2\times 1.005 \text{ kJ/kg}\cdot\text{K}}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 539.9 \text{ K}$$

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9 \text{ K} + \frac{-55 \text{ kJ/kg}}{1.005 \text{ kJ/kg}\cdot\text{K}} = 485.2 \text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-15 or from the appropriate relation. At $\text{Ma}_1 = 2$ we read $T_{01}/T_0^* = 0.7934$. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9 \text{ K}}{0.7934} = 680.5 \text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{02}}{T_0^*} = \frac{485.2 \text{ K}}{680.5 \text{ K}} = 0.7130 \quad \rightarrow \quad \text{Ma}_2 = \mathbf{2.479}$$

Also,

$\text{Ma}_1 = 2 \quad \rightarrow T_1/T^* = 0.5289$

$\text{Ma}_2 = 2.479 \quad \rightarrow T_2/T^* = 0.3838$

Then the exit temperature becomes

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.3838}{0.5289} = 0.7257 \quad \rightarrow \quad T_2 = 0.7257T_1 = 0.7257(300 \text{ K}) = \mathbf{218\ K}$$

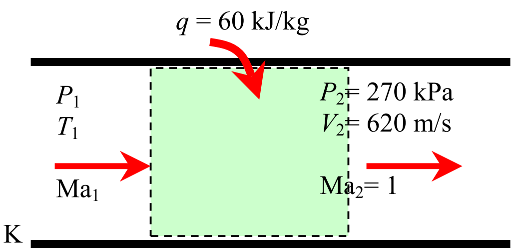
Discussion Note that the temperature decreases and Mach number increases during this supersonic Rayleigh flow with cooling. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-105 Air is heated in a duct during subsonic flow until it is choked. For specified pressure and velocity at the exit, the temperature, pressure, and velocity at the inlet are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Noting that sonic conditions exist at the exit, the exit temperature is



$$c_2 = V_2/\mathrm{Ma}_2 = (620\ \mathrm{m/s})/1 = 620\ \mathrm{m/s}$$

$$c_2 = \sqrt{kRT_2} \rightarrow \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 620\ \mathrm{m/s}$$

It gives $T_2 = 956.7$ K. Then the exit stagnation temperature becomes

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 956.7\ \mathrm{K} + \frac{(620\ \mathrm{m/s})^2}{2\times 1.005\ \mathrm{kJ/kg\cdot K}}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 1148\ \mathrm{K}$$

The inlet stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{01} = T_{02} - \frac{q}{c_p} = 1148\ \mathrm{K} - \frac{60\ \mathrm{kJ/kg}}{1.005\ \mathrm{kJ/kg\cdot K}} = 1088\ \mathrm{K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value in this case is T_{02} since the flow is choked. Therefore, $T_0^* = T_{02} = 1148$ K. Then the stagnation temperature ratio at the inlet, and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{01}}{T_0^*} = \frac{1088\ \mathrm{K}}{1148\ \mathrm{K}} = 0.9478 \quad \rightarrow \quad \mathrm{Ma}_1 = \mathbf{0.7649}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.7649$: $T_1/T^* = 1.017$, $P_1/P^* = 1.319$, $V_1/V^* = 0.7719$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $V_2/V^* = 1$

Then the inlet temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{1.017} \quad \rightarrow \quad T_1 = 1.017T_2 = 1.017(956.7\ \mathrm{K}) = \mathbf{974\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.319} \quad \rightarrow \quad P_1 = 1.319P_2 = 1.319(270\ \mathrm{kPa}) = \mathbf{356\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1}{0.7719} \quad \rightarrow \quad V_1 = 0.7719V_2 = 0.7719(620\ \mathrm{m/s}) = \mathbf{479\ m/s}$$

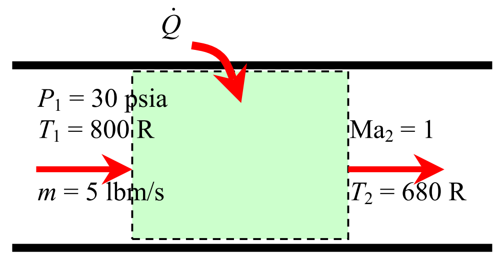
Discussion Note that the temperature and pressure decreases with heating during this subsonic Rayleigh flow while velocity increases. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-106E Air flowing with a subsonic velocity in a round duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the pressure drop are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 0.2400$ Btu/lbm·R, and $R = 0.06855$ Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The inlet density and velocity of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{30 \text{ psia}}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(800 \text{ R})} = 0.1012 \text{ lbm/ft}^3$$

$$V_1 = \frac{\dot{m}_{air}}{\rho_1 A_{c1}} = \frac{5 \text{ lbm/s}}{(0.1012 \text{ lbm/ft}^3)[\pi(4/12 \text{ ft})^2/4]} = 565.9 \text{ ft/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 800 \text{ R} + \frac{(565.9 \text{ ft/s})^2}{2\times 0.2400 \text{ Btu/lbm}\cdot\text{R}}\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) = 826.7 \text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855 \text{ Btu/lbm}\cdot\text{R})(800 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = 1386 \text{ ft/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{565.9 \text{ ft/s}}{1386 \text{ ft/s}} = 0.4082$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.4082$: $\quad T_1/T^* = 0.6310$, $\quad P_1/P^* = 1.946$, $\quad T_{01}/T_0^* = 0.5434$
$\mathrm{Ma}_2 = 1$: $\quad T_2/T^* = 1$, $\quad P_2/P^* = 1$, $\quad T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6310} \quad \rightarrow \quad T_2 = T_1/0.6310 = (800 \text{ R})/0.6310 = 1268 \text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.946} \quad \rightarrow \quad P_2 = P_1/2.272 = (30 \text{ psia})/1.946 = 15.4 \text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.5434} \quad \rightarrow \quad T_{02} = T_{01}/0.1743 = (826.7 \text{ R})/0.5434 = 1521 \text{ R}$$

Then the rate of heat transfer and the pressure drop become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5 \text{ lbm/s})(0.2400 \text{ Btu/lbm}\cdot\text{R})(1521 - 826.7) \text{ R} = \mathbf{834 \text{ Btu/s}}$$

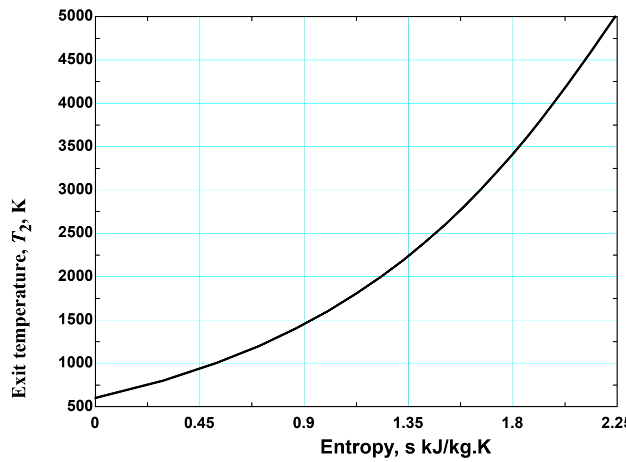
$$\Delta P = P_1 - P_2 = 30 - 15.4 = \mathbf{14.6 \text{ psia}}$$

Discussion Note that the entropy of air increases during this heating process, as expected.

12-107 Air flowing with a subsonic velocity in a duct. The variation of entropy with temperature is to be investigated as the exit temperature varies from 600 K to 5000 K in increments of 200 K. The results are to be tabulated and plotted.

Analysis We solve this problem using EES making use of Rayleigh functions as follows:

```
k=1.4
cp=1.005
R=0.287

P1=350
T1=600
V1=70
C1=sqrt(k*R*T1*1000)
Ma1=V1/C1

T01=T1*(1+0.5*(k-1)*Ma1^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

F1=1+0.5*(k-1)*Ma1^2
T01Ts=2*(k+1)*Ma1^2*F1/(1+k*Ma1^2)^2
P01Ps=((1+k)/(1+k*Ma1^2))*(2*F1/(k+1))^(k/(k-1))
T1Ts=(Ma1*((1+k)/(1+k*Ma1^2)))^2
P1Ps=(1+k)/(1+k*Ma1^2)
V1Vs=Ma1^2*(1+k)/(1+k*Ma1^2)

F2=1+0.5*(k-1)*Ma2^2
T02Ts=2*(k+1)*Ma2^2*F2/(1+k*Ma2^2)^2
P02Ps=((1+k)/(1+k*Ma2^2))*(2*F2/(k+1))^(k/(k-1))
T2Ts=(Ma2*((1+k)/(1+k*Ma2^2)))^2
P2Ps=(1+k)/(1+k*Ma2^2)
V2Vs=Ma2^2*(1+k)/(1+k*Ma2^2)

T02=T02Ts/T01Ts*T01
P02=P02Ps/P01Ps*P01
T2=T2Ts/T1Ts*T1
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
Delta_s=cp*ln(T2/T1)-R*ln(P2/P1)
```



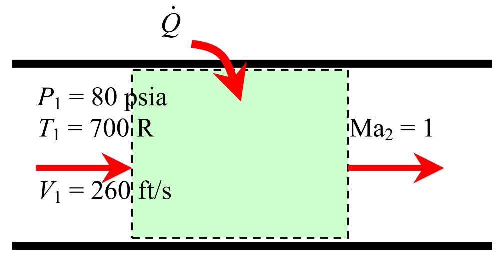
Exit temperature T_2, K	Exit Mach number, Ma_2	Exit entropy relative to inlet, s_2, kJ/kg·K
600	0.143	0.000
800	0.166	0.292
1000	0.188	0.519
1200	0.208	0.705
1400	0.227	0.863
1600	0.245	1.001
1800	0.263	1.123
2000	0.281	1.232
2200	0.299	1.331
2400	0.316	1.423
2600	0.333	1.507
2800	0.351	1.586
3000	0.369	1.660
3200	0.387	1.729
3400	0.406	1.795
3600	0.426	1.858
3800	0.446	1.918
4000	0.467	1.975
4200	0.490	2.031
4400	0.515	2.085
4600	0.541	2.138
4800	0.571	2.190
5000	0.606	2.242

12-108E Air flowing with a subsonic velocity in a square duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the entropy change are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 0.2400$ Btu/lbm·R, and $R = 0.06855$ Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The inlet density and mass flow rate of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{80\text{ psia}}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(700\text{ R})} = 0.3085\text{ lbm/ft}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.3085\text{ lbm/ft}^3)(4\times 4/144\text{ ft}^2)(260\text{ ft/s}) = 8.914\text{ lbm/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 700\text{ R} + \frac{(260\text{ ft/s})^2}{2\times 0.2400\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/lbm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 705.6\text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(700\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)} = 1297\text{ ft/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{260\text{ ft/s}}{1297\text{ ft/s}} = 0.2005$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.2005$: $T_1/T^* = 0.2075$, $P_1/P^* = 2.272$, $T_{01}/T_0^* = 0.1743$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.2075} \quad\rightarrow\quad T_2 = T_1/0.2075 = (700\text{ R})/0.2075 = 3374\text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{2.272} \quad\rightarrow\quad P_2 = P_1/2.272 = (80\text{ psia})/2.272 = 35.2\text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.1743} \quad\rightarrow\quad T_{02} = T_{01}/0.1743 = (705.6\text{ R})/0.1743 = 4048\text{ R}$$

Then the rate of heat transfer and entropy change become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (8.914\text{ lbm/s})(0.2400\text{ Btu/lbm}\cdot\text{R})(4048 - 705.6)\text{ R} = \mathbf{7151\ Btu/s}$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (0.2400\text{ Btu/lbm}\cdot\text{R})\ln\frac{3374\text{ R}}{700\text{ R}} - (0.06855\text{ Btu/lbm}\cdot\text{R})\ln\frac{35.2\text{ psia}}{80\text{ psia}} = \mathbf{0.434\ Btu/lbm\cdot R}$$

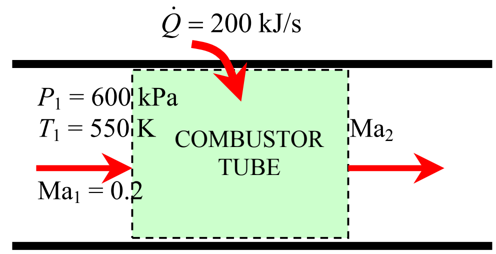
Discussion Note that the entropy of air increases during this heating process, as expected.

12-109 Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are



$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (550\text{ K})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right) = 554.4\text{ K}$$

$$P_{01} = P_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\text{ kPa})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4} = 617.0\text{ kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\text{air}}c_p(T_{02}-T_{01}) \quad\rightarrow\quad 200\text{ kJ/s} = (0.3\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(T_{02}-554.4)\text{ K}$$

It gives

$$T_{02} = 1218\text{ K.}$$

At $Ma_1 = 0.2$ we read from $T_{01}/T_0^* = 0.1736$ (Table A-15). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{554.4\text{ K}}{0.1736} = 3193.5\text{ K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-15)

$$\frac{T_{02}}{T_0^*} = \frac{1218\text{ K}}{3193.5\text{ K}} = 0.3814 \quad\rightarrow\quad \mathrm{Ma}_2 = \mathbf{0.3187}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \quad\rightarrow\quad P_{01}/P_0^* = 1.2346$$
$$\mathrm{Ma}_2 = 0.3187 \quad\rightarrow\quad P_{02}/P_0^* = 1.191$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.191}{1.2346} = 0.9647 \quad\rightarrow\quad P_{02} = 0.9647P_{01} = 0.9647(617\text{ kPa}) = 595.2\text{ kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 595.2 = \mathbf{21.8\ kPa}$$

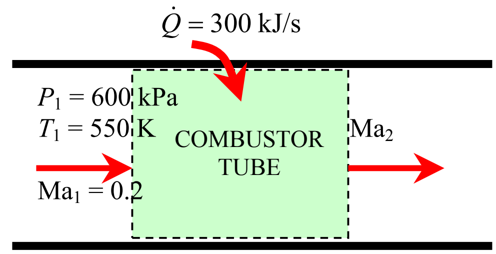
Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-110 Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are



$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (550\text{ K})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right) = 554.4\text{ K}$$

$$P_{01} = P_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\text{ kPa})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4} = 617.0\text{ kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\text{air}}c_p(T_{02}-T_{01}) \quad\rightarrow\quad 300\text{ kJ/s} = (0.3\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(T_{02}-554.4)\text{ K}$$

It gives

$$T_{02} = 1549\text{ K}.$$

At $Ma_1 = 0.2$ we read from $T_{01}/T_0^* = 0.1736$ (Table A-15). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{554.4\text{ K}}{0.1736} = 3193.5\text{ K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-15)

$$\frac{T_{02}}{T_0^*} = \frac{1549\text{ K}}{3193.5\text{ K}} = 0.4850 \quad\rightarrow\quad \mathrm{Ma}_2 = \mathbf{0.3753}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \quad\rightarrow\quad P_{01}/P_0^* = 1.2346$$
$$\mathrm{Ma}_2 = 0.3753 \quad\rightarrow\quad P_{02}/P_0^* = 1.167$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.167}{1.2346} = 0.9452 \quad\rightarrow\quad P_{02} = 0.9452P_{01} = 0.9452(617\text{ kPa}) = 583.3\text{ kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 583.3 = \mathbf{33.7\ kPa}$$

Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

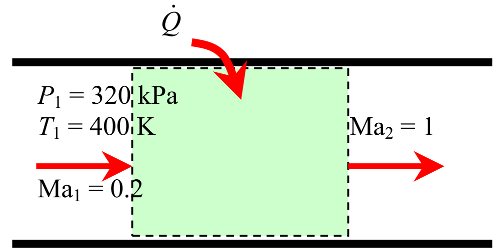
12-111 Argon flowing at subsonic velocity in a constant-diameter duct is accelerated by heating. The highest rate of heat transfer without reducing the mass flow rate is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.

Properties We take the properties of argon to be k = 1.667, c_p =0.5203 kJ/kg·K, and R = 0.2081 kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\text{Ma}_2 = V_2/c_2 = 1$. The inlet stagnation temperature is

$$T_{01} = T_1\left(1+\frac{k-1}{2}\text{Ma}_1^2\right) = (400\text{ K})\left(1+\frac{1.667-1}{2}0.2^2\right) = 405.3\text{ K}$$



The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $\text{Ma}_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\text{Ma}_1^2[2+(k-1)\text{Ma}_1^2]}{(1+k\text{Ma}_1^2)^2} = \frac{(1.667+1)0.2^2[2+(1.667-1)0.2^2]}{(1+1.667\times 0.2^2)^2} = 0.1900$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.1900} \quad \rightarrow \quad T_{02} = T_{01}/0.1900 = (405.3\text{ K})/0.1900 = 2133\text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (0.8\text{ kg/s})(0.5203\text{ kJ/kg}\cdot\text{K})(2133-400)\text{ K} = \mathbf{721\ kW}$$

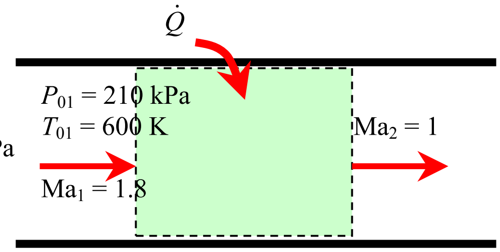
Discussion It can also be shown that T_2 = 1600 K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-15 since they are based on k = 1.4.

12-112 Air flowing at a supersonic velocity in a duct is decelerated by heating. The highest temperature air can be heated by heat addition and the rate of heat transfer are to be determined.

Assumptions **1**The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\mathrm{Ma}_2 = V_2/c_2 = 1$. Knowing stagnation properties, the static properties are determined to be



$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (600\ \mathrm{K})\left(1+\frac{1.4\text{-}1}{2}1.8^2\right)^{-1} = 364.1\ \mathrm{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (210\ \mathrm{kPa})\left(1+\frac{1.4\text{-}1}{2}1.8^2\right)^{-1.4/0.4} = 36.55\ \mathrm{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{36.55\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kgK})(364.1\ \mathrm{K})} = 0.3498\ \mathrm{kg/m^3}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(364.1\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 382.5\ \mathrm{m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.8(382.5\ \mathrm{m/s}) = 688.5\ \mathrm{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.3498\ \mathrm{kg/m^3})[\pi(0.06\ \mathrm{m})^2/4](688.5\ \mathrm{m/s}) = 0.6809\ \mathrm{kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 1.8$: $T_1/T^* = 0.6089$, $T_{01}/T_0^* = 0.8363$
$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6089} \quad \rightarrow \quad T_2 = T_1/0.6089 = (364.1\ \mathrm{K})/0.6089 = \mathbf{598\ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.8363} \quad \rightarrow \quad T_{02} = T_{01}/0.8363 = (600\ \mathrm{K})/0.8363 = \mathbf{717.4\ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p (T_{02} - T_{01}) = (0.6809\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg\cdot K})(717.4 - 600)\ \mathrm{K} = \mathbf{80.3\ kW}$$

Discussion Note that this is the highest temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature will cause the mass flow rate to decrease. Also, once the sonic conditions are reached, the thermodynamic temperature can be increased further by cooling the fluid and reducing the velocity (see the *T-s* diagram for Rayleigh flow).

Adiabatic Duct Flow with Friction (Fanno Flow)

12-113C The characteristic aspect of Fanno flow is its consideration of friction. The main assumptions associated with Rayleigh flow are: the flow is steady, one-dimensional, and adiabatic through a constant-area duct, and the fluid is an ideal gas with constant specific heats.

12-114C The points on the Fanno line represent the states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations for a given inlet state. Therefore, for a given initial state, the fluid cannot exist at any downstream state outside the Fanno line on a *T-s* diagram.

12-115C In Fanno flow, the effect of friction is always to increase the entropy of the fluid. Therefore Fanno flow always proceeds in the direction of increasing entropy.

12-116C During subsonic Fanno flow, the stagnation temperature T_0 remains constant, stagnation pressure P_0 decreases, and entropy s increases.

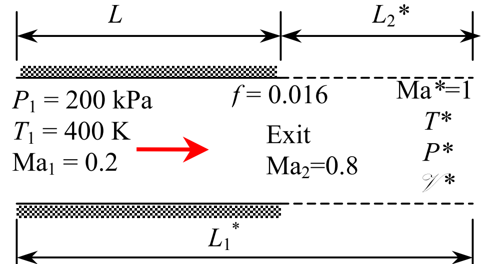
12-117C During supersonic Fanno flow, the stagnation temperature T_0 remains constant, stagnation pressure P_0 decreases, and entropy s increases.

12-118C Friction increases the flow velocity in subsonic Fanno flow, but decreases it in supersonic flow.

12-119C The flow is choked, and thus the flow at the duct exit will remain sonic. The mass flow rate will decrease as a result of extending the duct length.

12-120C The flow at the duct exit will remain sonic. The mass flow rate will remain constant since upstream conditions will not be affected by the added duct length.

12-121 Subsonic airflow in a constant cross-sectional area adiabatic duct is considered. For a specified exit Mach number, the duct length, temperature, pressure, and velocity at the duct exit are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.016.

Analysis The inlet velocity is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(400\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 400.9\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.2(400.9\text{ m/s}) = 80.2\text{ m/s}$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$\text{Ma}_1 = 0.2$: $(fL^*/D_h)_1 = 14.5333$ $T_1/T^* = 1.1905$, $P_1/P^* = 5.4554$, $V_1/V^* = 0.2182$

$\text{Ma}_2 = 0.8$: $(fL^*/D_h)_2 = 0.0723$ $T_2/T^* = 1.0638$, $P_2/P^* = 1.2893$, $V_2/V^* = 0.8251$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0638}{1.1905} = 0.8936 \quad \rightarrow \quad T_2 = 0.8936T_1 = 0.8936(400\text{ K}) = \mathbf{357\text{ K}}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.2893}{5.4554} = 0.2363 \quad \rightarrow \quad P_2 = 0.2363P_1 = 0.2363(200\text{ kPa}) = \mathbf{47.3\text{ kPa}}$$

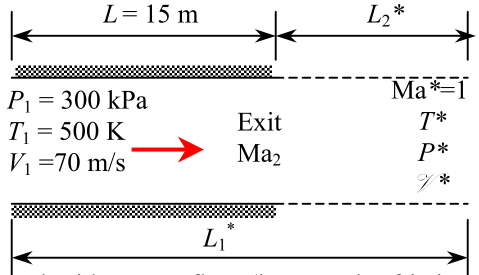
$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.8251}{0.2182} = 3.7814 \quad \rightarrow \quad V_2 = 3.7814V_1 = 3.7814(80.2\text{ m/s}) = \mathbf{303\text{ m/s}}$$

Finally, the actual duct length is determined to be

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (14.5333 - 0.0723)\frac{0.05\text{ m}}{0.016} = \mathbf{45.2\text{ m}}$$

Discussion Note that it takes a duct length of 45.2 m for the Mach number to increase from 0.2 to 0.8. The Mach number rises at a much higher rate as sonic conditions are approached. The maximum (or sonic) duct lengths at the inlet and exit states in this case are L_1^* = 45.4 m and L_2^* = 0.2 m. Therefore, the flow would reach sonic conditions if a 0.2-m long section were added to the existing duct.

12-122 Air enters a constant-area adiabatic duct of given length at a specified state. The exit Mach number, exit velocity, and the mass flow rate are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.023.

Analysis The first thing we need to know is whether the flow is choked at the exit or not. Therefore, we first determine the inlet Mach number and the corresponding value of the function fL^*/D_h ,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(500\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 448.2\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{70\text{ m/s}}{448.2\text{ m/s}} = 0.1562$$

Corresponding to this Mach number we calculate (or read) from Table A-16), $(fL^*/D_h)_1$ = 25.540. Also, using the actual duct length L, we have

$$\frac{fL}{D_h} = \frac{(0.023)(15\text{ m})}{0.04\text{ m}} = 8.625 \quad < \quad 25.540$$

Therefore, flow is *not* choked and exit Mach number is less than 1. Noting that $L = L_1^* - L_2^*$, the function fL^*/D_h at the exit state is calculated from

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL}{D_h} = 25.540 - 8.625 = 16.915$$

The Mach number corresponding to this value of fL^*/D is obtained from Table A-16 to be

$\text{Ma}_2 = \mathbf{0.187}$

which is the Mach number at the duct exit.

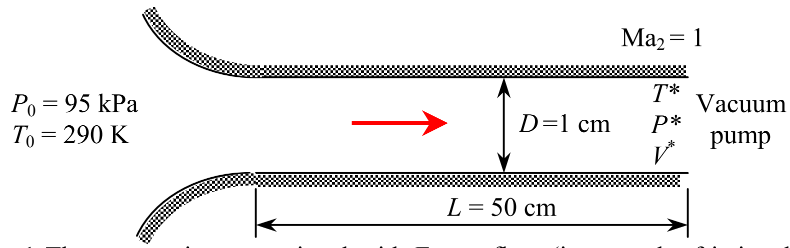
The mass flow rate of air is determined from the inlet conditions to be

$$\rho_1 = \frac{P_1}{RT_1} = \frac{300\text{ kPa}}{(0.287\text{ kJ/kgK})(500\text{ K})}\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = 2.091\text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.091\text{ kg/m}^3)[\pi(0.04\text{ m})^2/4](70\text{ m/s}) = \mathbf{0.184\ kg/s}$$

Discussion It can be shown that L_2^* = 29.4 m, indicating that it takes a duct length of 15 m for the Mach number to increase from 0.156 to 0.187, but only 29.4 m to increase from 0.187 to 1. Therefore, the Mach number rises at a much higher rate as sonic conditions are approached.

12-123 Air enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.018.

Analysis The mass flow rate will be maximum when the flow is choked, and thus the exit Mach number is Ma_2 = 1. In that case we have

$$\frac{fL_1^*}{D} = \frac{fL_1}{D} = \frac{(0.018)(0.50\text{ m})}{0.01\text{ m}} = 0.9$$

The Mach number corresponding to this value of fL^*/D at the tube inlet is obtained from Table A-16 to be Ma_1 = **0.5225**. Noting that the flow in the nozzle section is isentropic, the thermodynamic temperature, pressure, and density at the tube inlet become

$$T_1 = T_{01}\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right)^{-1} = (290\text{ K})\left(1 + \frac{1.4\text{-}1}{2}(0.5225)^2\right)^{-1} = 275.0\text{ K}$$

$$P_1 = P_{01}\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right)^{-k/(k-1)} = (95\text{ kPa})\left(1 + \frac{1.4\text{-}1}{2}(0.5225)^2\right)^{-1.4/0.4} = 78.87\text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{78.87\text{ kPa}}{(0.287\text{ kJ/kgK})(275.0\text{ K})} = 0.9993\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

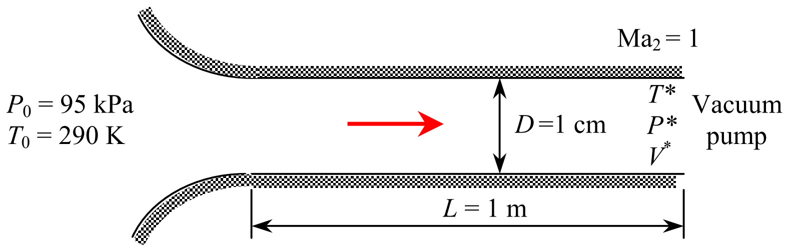
$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(275\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 332.4\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.5225(332.4\text{ m/s}) = 173.7\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.9993\text{ kg/m}^3)[\pi(0.01\text{ m})^2/4](173.7\text{ m/s}) = \mathbf{0.0136\ kg/s}$$

Discussion This is the maximum mass flow rate through the tube for the specified stagnation conditions at the inlet. The flow rate will remain at this level even if the vacuum pump drops the pressure even further.

12-124 Air enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.025.

Analysis The mass flow rate will be maximum when the flow is choked, and thus the exit Mach number is Ma_2 = 1. In that case we have

$$\frac{fL_1^*}{D} = \frac{fL_1}{D} = \frac{(0.025)(1\,\text{m})}{0.01\,\text{m}} = 2.5$$

The Mach number corresponding to this value of fL^*/D at the tube inlet is obtained from Table A-16 to be Ma_1 = 0.3899. Noting that the flow in the nozzle section is isentropic, the thermodynamic temperature, pressure, and density at the tube inlet become

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\text{Ma}_1^2\right)^{-1} = (290\,\text{K})\left(1+\frac{1.4\text{-}1}{2}(0.3899)^2\right)^{-1} = 281.4\,\text{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\text{Ma}_1^2\right)^{-k/(k-1)} = (95\,\text{kPa})\left(1+\frac{1.4\text{-}1}{2}(0.3899)^2\right)^{-1.4/0.4} = 85.54\,\text{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{85.54\,\text{kPa}}{(0.287\,\text{kJ/kgK})(281.4\,\text{K})} = 1.059\,\text{kg/m}^3$$

Then the inlet velocity and the mass flow rate become

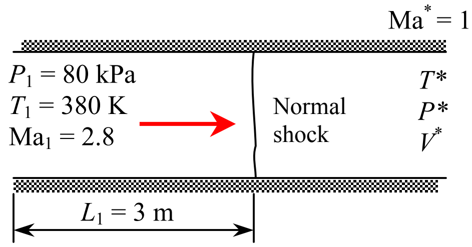
$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\,\text{kJ/kg}\cdot\text{K})(281.4\,\text{K})\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right)} = 336.3\,\text{m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.3899(336.3\,\text{m/s}) = 131.1\,\text{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.059\,\text{kg/m}^3)[\pi(0.01\,\text{m})^2/4](131.1\,\text{m/s}) = \mathbf{0.0109\ kg/s}$$

Discussion This is the maximum mass flow rate through the tube for the specified stagnation conditions at the inlet. The flow rate will remain at this level even if the vacuum pump drops the pressure even further.

12-125 Air enters a constant-area adiabatic duct at a specified state, and undergoes a normal shock at a specified location. The exit velocity, temperature, and pressure are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K. The friction factor is given to be $f = 0.007$.

Analysis The Fanno flow functions corresponding to the inlet Mach number of 2.8 are, from Table A-16,

$\mathrm{Ma}_1 = 2.8$: $(fL^*/D_h)_1 = 0.4898$ $T_1/T^* = 0.4673$, $P_1/P^* = 0.2441$

First we check to make sure that the flow everywhere upstream the shock is supersonic. The required duct length from the inlet L_1^* for the flow to reach sonic conditions is

$$L_1^* = 0.4898\frac{D}{f} = 0.4898\frac{0.05\text{ m}}{0.007} = 3.50\text{ m}$$

which is greater than the actual length 3 m. Therefore, the flow is indeed supersonic when the normal shock occurs at the indicated location. Also, using the actual duct length L_1, we have

$$\frac{fL_1}{D_h} = \frac{(0.007)(3\text{ m})}{0.05\text{ m}} = 0.4200$$

Noting that $L_1 = L_1^* - L_2^*$, the function fL^*/D_h at the exit state and the corresponding Mach number are

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL_1}{D_h} = 0.4898 - 0.4200 = 0.0698 \quad \rightarrow \quad \mathrm{Ma}_2 = 1.315$$

From Table A-16, at $\mathrm{Ma}_2 = 1.315$: $T_2/T^* = 0.8918$, $P_2/P^* = 0.7183$,

Then the temperature, pressure, and velocity before the shock are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.8918}{0.4673} = 1.9084 \quad \rightarrow \quad T_2 = 1.9084T_1 = 1.9084(380\text{ K}) = 725.2\text{ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.7183}{0.2441} = 2.9426 \quad \rightarrow \quad P_2 = 2.9426P_1 = 2.9426(80\text{ kPa}) = 235.4\text{ kPa}$$

The normal shock functions corresponding to a Mach number of 1.315 are, from Table A-14,

$\mathrm{Ma}_2 = 1.315$: $\mathrm{Ma}_3 = 0.7786$, $T_3/T_2 = 1.2001$, $P_3/P_2 = 1.8495$

Then the temperature and pressure after the shock become

$$T_3 = 1.2001T_2 = 1.2001(725.2\text{ K}) = 870.3\text{ K}$$
$$P_3 = 1.8495P_2 = 1.8495(235.4\text{ kPa}) = 435.4\text{ kPa}$$

Sonic conditions exist at the duct exit, and the flow downstream the shock is still Fanno flow. From Table A-16,

$\mathrm{Ma}_3 = 0.7786$: $T_3/T^* = 1.0702$, $P_3/P^* = 1.3286$
$\mathrm{Ma}_4 = 1$: $T_4/T^* = 1$, $P_4/P^* = 1$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_4}{T_3}=\frac{T_4/T^*}{T_3/T^*}=\frac{1}{1.0702} \quad \rightarrow \quad T_4=T_3/1.0702=(870.3\text{ K})/1.0702=\mathbf{813\ K}$$

$$\frac{P_4}{P_3}=\frac{P_4/P^*}{P_3/P^*}=\frac{1}{1.3286} \quad \rightarrow \quad P_4=P_3/1.3286=(435.4\text{ kPa})/1.3286=\mathbf{328\ kPa}$$

$$V_4=\text{Ma}_4c_4=(1)\sqrt{kRT_4}=\sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(813\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}=\mathbf{572\ m/s}$$

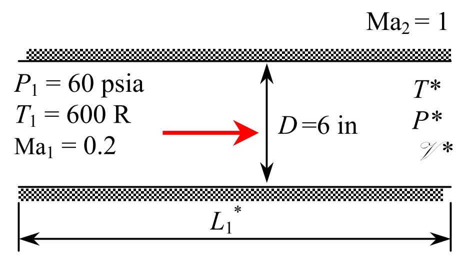
Discussion It can be shown that L_3^* = 0.67 m, and thus the total length of this duct is 3.67 m. If the duct is extended, the normal shock will move further upstream, and eventually to the inlet of the duct.

12-126E Helium enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of helium to be k = 1.667, c_p = 1.2403 Btu/lbm·R, and R = 0.4961 Btu/lbm·R. The friction factor is given to be f = 0.025.

Analysis The Fanno flow function fL^*/D corresponding to the inlet Mach number of 0.2 is (Table A-16):



$$\frac{fL_1^*}{D} = 14.5333$$

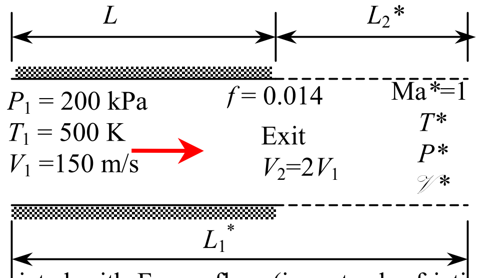
Noting that * denotes sonic conditions, which exist at the exit state, the duct length is determined to be

$$L_1^* = 14.5333D/f = 14.5333(6/12\ \text{ft})/0.025 = \mathbf{291\ ft}$$

Thus, for the given friction factor, the duct length must be 291 ft for the Mach number to reach Ma = 1 at the duct exit.

Discussion This problem can also be solved using appropriate relations instead of tabulated values for the Fanno functions.

12-127 Subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The duct length from the inlet where the inlet velocity doubles and the pressure drop in that section are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K. The average friction factor is given to be $f = 0.014$.

Analysis The inlet Mach number is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(500\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 448.2\ \text{m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{150\ \text{m/s}}{448.2\ \text{m/s}} = 0.3347$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$\text{Ma}_1 = 0.3347$: $(fL^*/D_h)_1 = 3.924$ $P_1/P^* = 3.2373$, $V_1/V^* = 0.3626$

Therefore, $V_1 = 0.3626V^*$. Then the Fanno function V_2/V^* becomes

$$\frac{V_2}{V^*} = \frac{2V_1}{V^*} = \frac{2\times 0.3626V^*}{V^*} = 0.7252$$

The corresponding Mach number and Fanno flow functions are, from Table A-16,

$\text{Ma}_2 = 0.693$, $(fL^*/D_h)_1 = 0.2220$, and $P_2/P^* = 1.5099$.

Then the duct length where the velocity doubles, the exit pressure, and the pressure drop become

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (3.924 - 0.2220)\frac{0.20\ \text{m}}{0.014} = \mathbf{52.9\ m}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.5099}{3.2373} = 0.4664 \quad \rightarrow \quad P_2 = 0.4664P_1 = 0.4664(200\ \text{kPa}) = 93.3\ \text{kPa}$$

$$\Delta P = P_1 - P_2 = 200 - 93.3 = \mathbf{106.7\ kPa}$$

Discussion Note that it takes a duct length of 52.9 m for the velocity to double, and the Mach number to increase from 0.3347 to 0.693. The maximum (or sonic) duct lengths at the inlet and exit states in this case are $L_1^* = 56.1$ m and $L_2^* = 3.2$ m. Therefore, the flow would reach sonic conditions if there is an additional 3.2 m of duct length.

12-128E Air enters a constant-area adiabatic duct of given length at a specified state. The velocity, temperature, and pressure at the duct exit are to be determined.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

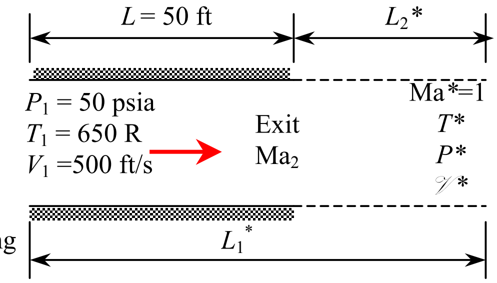
Properties We take the properties of helium to be k = 1.4, c_p = 0.2400 Btu/lbm·R, and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R. The friction factor is given to be f = 0.025.

Analysis The first thing we need to know is whether the flow is choked at the exit or not. Therefore, we first determine the inlet Mach number and the corresponding value of the function fL^*/D_h ,

$$T_1 = T_{01} - \frac{V_1^2}{2c_p} = 650\text{ R} - \frac{(500\text{ ft/s})^2}{2\times 0.2400\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/lbm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 629.2\text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(629.2\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1230\text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{500\text{ m/s}}{1230\text{ ft/s}} = 0.4066$$



Corresponding to this Mach number we calculate (or read) from Table A-16), $(fL^*/D_h)_1$ = 2.1911. Also, using the actual duct length L, we have

$$\frac{fL}{D_h} = \frac{(0.02)(50\text{ ft})}{6/12\text{ ft}} = 2 \;<\; 2.1911$$

Therefore, the flow is *not* choked and exit Mach number is less than 1. Noting that $L = L_1^* - L_2^*$, the function fL^*/D_h at the exit state is calculated from

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL}{D_h} = 2.1911 - 2 = 0.1911$$

The Mach number corresponding to this value of fL^*/D is obtained from Table A-16 to be Ma$_2$ = 0.7091. The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

Ma$_1$ = 0.4066: T_1/T^* = 1.1616, P_1/P^* = 2.6504, V_1/V^* = 0.4383
Ma$_2$ = 0.7091: T_2/T^* = 1.0903, P_2/P^* = 1.4726, V_2/V^* = 0.7404

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0903}{1.1616} = 0.9386 \qquad \rightarrow \quad T_2 = 0.9386T_1 = 0.9386(629.2\text{ R}) = \mathbf{591\,R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.4726}{2.6504} = 0.5556 \qquad \rightarrow \quad P_2 = 0.5556P_1 = 0.5556(50\text{ psia}) = \mathbf{27.8\,psia}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.7404}{0.4383} = 1.6893 \qquad \rightarrow \quad V_2 = 1.6893V_1 = 1.6893(500\text{ ft/s}) = \mathbf{845\,ft/s}$$

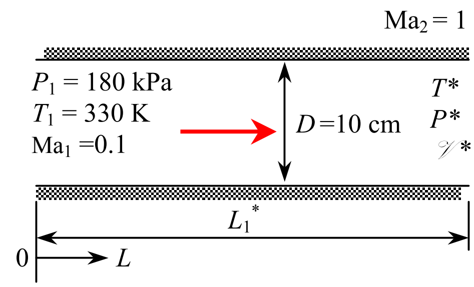
Discussion It can be shown that L_2^* = 4.8 ft, indicating that it takes a duct length of 50 ft for the Mach number to increase from 0.4066 to 0.7091, but only 4.8 ft to increase from 0.7091 to 1. Therefore, the Mach number rises at a much higher rate as sonic conditions are approached.

12-129 Choked subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The variation of duct length with Mach number is to be investigated, and the results are to be plotted.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.02.

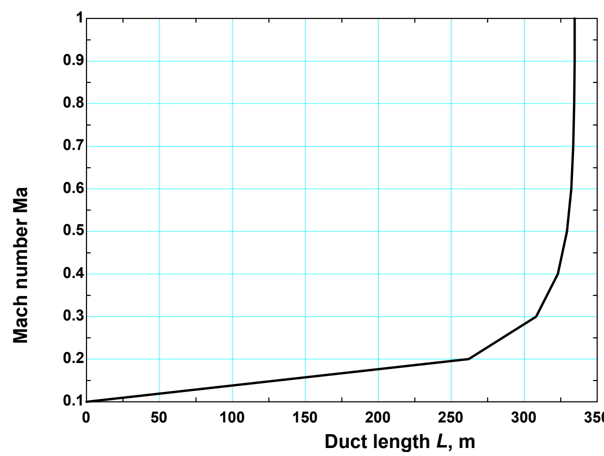
Analysis The flow is choked, and thus Ma_2 =1. Corresponding to the inlet Mach number of Ma_1 = 0.1 we have, from Table A-16, fL^*/D_h = 66.922, Therefore, the original duct length is



$$L_1^* = 66.922\frac{D}{f} = 66.922\frac{0.10\text{ m}}{0.02} = 335\text{ m}$$

Repeating the calculations for different Ma_2 as it varies from 0.1 to 1 results in the following table for the location on the duct from the inlet:

Mach number, Ma	Duct length L, m
0.10	0
0.20	262
0.30	308
0.40	323
0.50	329
0.60	332
0.70	334
0.80	334
0.90	335
1.00	335



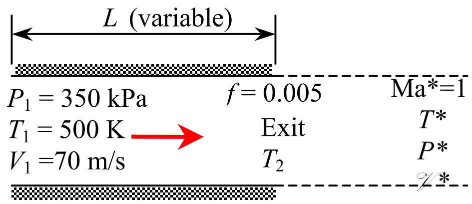
EES program:

```
k=1.4
cp=1.005
R=0.287

P1=180
T1=330
Ma1=0.1
"Ma2=1"
f=0.02
D=0.1

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1
```

```
P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that the Mach number increases very mildly at the beginning, and then rapidly near the duct outlet. It takes 262 m of duct length for Mach number to increase from 0.1 to 0.2, but only 1 m to increase from 0.7 to 1.

12-130 The flow of argon gas in a constant cross-sectional area adiabatic duct is considered. The variation of entropy change with exit temperature is to be investigated, and the calculated results are to be plotted on a *T-s* diagram.

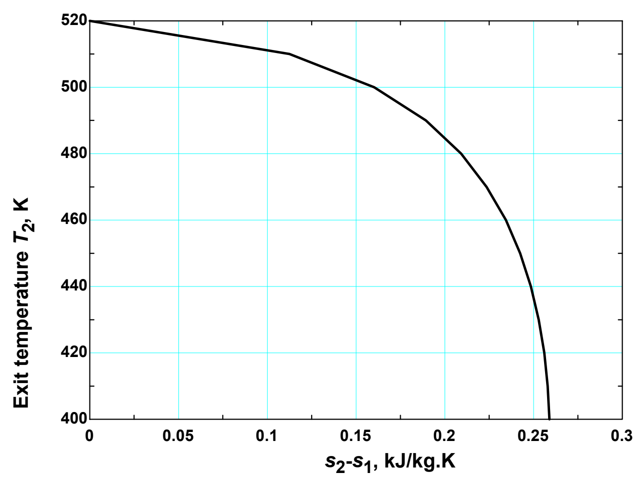


Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties The properties of argon are given to be k = 1.667, c_p = 0.5203 kJ/kg·K, and R = 0.2081 kJ/kg·K. The average friction factor is given to be f = 0.005.

Analysis Using EES, we determine the entropy change and tabulate and plot the results as follows:

Exit temp. T_2, K	Mach umber Ma_2	Entropy change Δs, kg/kg·K
520	0.165	0.000
510	0.294	0.112
500	0.385	0.160
490	0.461	0.189
480	0.528	0.209
470	0.591	0.224
460	0.649	0.234
450	0.706	0.242
440	0.760	0.248
430	0.813	0.253
420	0.865	0.256
410	0.916	0.258
400	0.967	0.259



Discussion Note that entropy increases with increasing duct length and Mach number (and thus decreasing temperature). It reached a maximum value of 0.259 kJ/kg.K when the Mach number reaches Ma_2 =1 and thus the flow is choked.

EES Program:

```
k=1.667
cp=0.5203
R=0.2081

P1=350
T1=520
V1=70
"T2=400"
f=0.005
D=0.08

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
T2=T2Ts/T1Ts*T1
V2=V2Vs/V1Vs*V1
Del_s=cp*ln(T2/T1)-R*ln(P2/P1)
```