
Normal Shocks in Nozzle Flow

12-68C No, because the flow must be supersonic before a shock wave can occur. The flow in the converging section of a nozzle is always subsonic.

12-69C The Fanno line represents the states which satisfy the conservation of mass and energy equations. The Rayleigh line represents the states which satisfy the conservation of mass and momentum equations. The intersections points of these lines represents the states which satisfy the conservation of mass, energy, and momentum equations.

12-70C No, the second law of thermodynamics requires the flow after the shock to be subsonic..

12-71C (a) decreases, (b) increases, (c) remains the same, (d) increases, and (e) decreases.

12-72C Oblique shocks occur when a gas flowing at supersonic speeds strikes a flat or inclined surface. Normal shock waves are perpendicular to flow whereas inclined shock waves, as the name implies, are typically inclined relative to the flow direction. Also, normal shocks form a straight line whereas oblique shocks can be straight or curved, depending on the surface geometry.

12-73C Yes, the upstream flow have to be supersonic for an oblique shock to occur. No, the flow downstream of an oblique shock can be subsonic, sonic, and even supersonic.

12-74C Yes. Conversely, normal shocks can be thought of as special oblique shocks in which the shock angle is $\beta = \pi/2$, or 90° .

12-75C When the wedge half-angle δ is greater than the maximum deflection angle θ_{\max} , the shock becomes curved and detaches from the nose of the wedge, forming what is called a *detached oblique shock* or a *bow wave*. The numerical value of the shock angle at the nose is be $\beta = 90^\circ$.

12-76C When supersonic flow impinges on a blunt body like the rounded nose of an aircraft, the wedge half-angle δ at the nose is 90° , and an attached oblique shock cannot exist, regardless of Mach number. Therefore, a detached oblique shock must occur in front of *all* such blunt-nosed bodies, whether two-dimensional, axisymmetric, or fully three-dimensional.

12-77C Isentropic relations of ideal gases are *not* applicable for flows across (a) normal shock waves and (b) oblique shock waves, but they *are* applicable for flows across (c) Prandtl-Meyer expansion waves.

12-78 For an ideal gas flowing through a normal shock, a relation for V_2/V_1 in terms of k , Ma_1 , and Ma_2 is to be developed.

Analysis The conservation of mass relation across the shock is $\rho_1 V_1 = \rho_2 V_2$ and it can be expressed as

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{P_1 / RT_1}{P_2 / RT_2} = \left(\frac{P_1}{P_2} \right) \left(\frac{T_2}{T_1} \right)$$

From Eqs. 12-35 and 12-38,

$$\frac{V_2}{V_1} = \left(\frac{1 + kMa_2^2}{1 + kMa_1^2} \right) \left(\frac{1 + Ma_1^2(k-1)/2}{1 + Ma_2^2(k-1)/2} \right)$$

Discussion This is an important relation as it enables us to determine the velocity ratio across a normal shock when the Mach numbers before and after the shock are known.

12-79 Air flowing through a converging-diverging nozzle experiences a normal shock at the exit. The effect of the shock wave on various properties is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K.

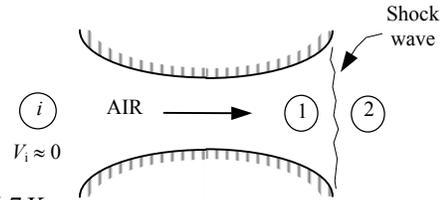
Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Then,

$$P_{01} = P_i = 1 \text{ MPa}$$

$$T_{01} = T_i = 300 \text{ K}$$

Then,

$$T_1 = T_{01} \left(\frac{2}{2 + (k-1)\text{Ma}_1^2} \right) = (300 \text{ K}) \left(\frac{2}{2 + (1.4-1)2^2} \right) = 166.7 \text{ K}$$



and

$$P_1 = P_{01} \left(\frac{T_1}{T_0} \right)^{k/(k-1)} = (1 \text{ MPa}) \left(\frac{166.7}{300} \right)^{1.4/0.4} = 0.1278 \text{ MPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 2.0$ we read

$$\text{Ma}_2 = \mathbf{0.5774}, \quad \frac{P_{02}}{P_{01}} = 0.7209, \quad \frac{P_2}{P_1} = 4.5000, \quad \text{and} \quad \frac{T_2}{T_1} = 1.6875$$

Then the stagnation pressure P_{02} , static pressure P_2 , and static temperature T_2 , are determined to be

$$P_{02} = 0.7209P_{01} = (0.7209)(1.0 \text{ MPa}) = \mathbf{0.721 \text{ MPa}}$$

$$P_2 = 4.5000P_1 = (4.5000)(0.1278 \text{ MPa}) = \mathbf{0.575 \text{ MPa}}$$

$$T_2 = 1.6875T_1 = (1.6875)(166.7 \text{ K}) = \mathbf{281 \text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.5774) \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(281 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{194 \text{ m/s}}$$

Discussion We can also solve this problem using the relations for normal shock functions. The results would be identical.

12-80 Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.

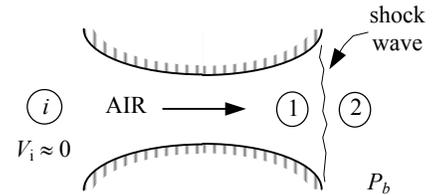
Assumptions 1 Air is an ideal gas. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. 3 The shock wave occurs at the exit plane.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{01} = P_i = 2 \text{ MPa}$$

It is specified that $A/A^* = 3.5$. From Table A-13, Mach number and the pressure ratio which corresponds to this area ratio are the $Ma_1 = 2.80$ and $P_1/P_{01} = 0.0368$. The pressure ratio across the shock for this Ma_1 value is, from Table A-14, $P_2/P_1 = 8.98$. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 8.98P_1 = 8.98 \times 0.0368 P_{01} = 8.98 \times 0.0368 \times (2 \text{ MPa}) = \mathbf{0.661 \text{ MPa}}$$



Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

12-81 Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.

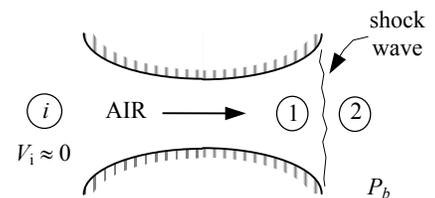
Assumptions 1 Air is an ideal gas. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{0x} = P_i = 2 \text{ MPa}$$

It is specified that $A/A^* = 2$. From Table A-13, the Mach number and the pressure ratio which corresponds to this area ratio are the $Ma_1 = 2.20$ and $P_1/P_{01} = 0.0935$. The pressure ratio across the shock for this M_1 value is, from Table A-14, $P_2/P_1 = 5.48$. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 5.48P_1 = 5.48 \times 0.0935 P_{01} = 5.48 \times 0.0935 \times (2 \text{ MPa}) = \mathbf{1.02 \text{ MPa}}$$



Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

12-82 Air flowing through a nozzle experiences a normal shock. The effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium under the same conditions.

Assumptions 1 Air and helium are ideal gases with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

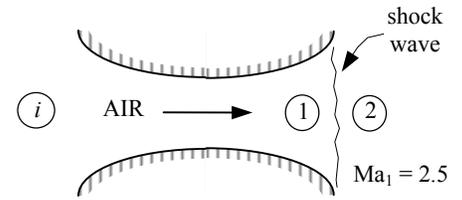
Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K, and the properties of helium are $k = 1.667$ and $R = 2.0769$ kJ/kg·K.

Analysis The air properties upstream the shock are

$$\text{Ma}_1 = 2.5, P_1 = 61.64 \text{ kPa}, \text{ and } T_1 = 262.15 \text{ K}$$

Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions in Table A-14. For $\text{Ma}_1 = 2.5$,

$$\text{Ma}_2 = \mathbf{0.513}, \frac{P_{02}}{P_1} = 8.5262, \frac{P_2}{P_1} = 7.125, \text{ and } \frac{T_2}{T_1} = 2.1375$$



Then the stagnation pressure P_{02} , static pressure P_2 , and static temperature T_2 , are determined to be

$$P_{02} = 8.5261P_1 = (8.5261)(61.64 \text{ kPa}) = \mathbf{526 \text{ kPa}}$$

$$P_2 = 7.125P_1 = (7.125)(61.64 \text{ kPa}) = \mathbf{439 \text{ kPa}}$$

$$T_2 = 2.1375T_1 = (2.1375)(262.15 \text{ K}) = \mathbf{560 \text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.513) \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(560.3 \text{ K}) \left(\frac{1000 \text{ m}^2 / \text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{243 \text{ m/s}}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-14 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k/(k-1) - 1} \right)^{1/2} = \left(\frac{2.5^2 + 2/(1.667-1)}{2 \times 2.5^2 \times 1.667/(1.667-1) - 1} \right)^{1/2} = \mathbf{0.553}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{1 + 1.667 \times 2.5^2}{1 + 1.667 \times 0.553^2} = 7.5632$$

$$\frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2} = \frac{1 + 2.5^2(1.667-1)/2}{1 + 0.553^2(1.667-1)/2} = 2.7989$$

$$\begin{aligned} \frac{P_{02}}{P_1} &= \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} \right) \left(1 + (k-1)\text{Ma}_2^2/2 \right)^{k/(k-1)} \\ &= \left(\frac{1 + 1.667 \times 2.5^2}{1 + 1.667 \times 0.553^2} \right) \left(1 + (1.667-1) \times 0.553^2/2 \right)^{1.667/0.667} = 9.641 \end{aligned}$$

Thus, $P_{02} = 11.546P_1 = (11.546)(61.64 \text{ kPa}) = \mathbf{712 \text{ kPa}}$

$$P_2 = 7.5632P_1 = (7.5632)(61.64 \text{ kPa}) = \mathbf{466 \text{ kPa}}$$

$$T_2 = 2.7989T_1 = (2.7989)(262.15 \text{ K}) = \mathbf{734 \text{ K}}$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.553) \sqrt{(1.667)(2.0769 \text{ kJ/kg} \cdot \text{K})(733.7 \text{ K}) \left(\frac{1000 \text{ m}^2 / \text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{881 \text{ m/s}}$$

12-83 Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions 1 Air and helium are ideal gases with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and the properties of helium are $R = 2.0769 \text{ kJ/kg}\cdot\text{K}$ and $c_p = 5.1926 \text{ kJ/kg}\cdot\text{K}$.

Analysis The entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = (1.005 \text{ kJ/kg}\cdot\text{K}) \ln(2.1375) - (0.287 \text{ kJ/kg}\cdot\text{K}) \ln(7.125) = \mathbf{0.200 \text{ kJ/kg}\cdot\text{K}}$$

For helium, the entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = (5.1926 \text{ kJ/kg}\cdot\text{K}) \ln(2.7989) - (2.0769 \text{ kJ/kg}\cdot\text{K}) \ln(7.5632) = \mathbf{1.14 \text{ kJ/kg}\cdot\text{K}}$$

Discussion Note that shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

12-84E [Also solved by EES on enclosed CD] Air flowing through a nozzle experiences a normal shock. Effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium.

Assumptions 1 Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

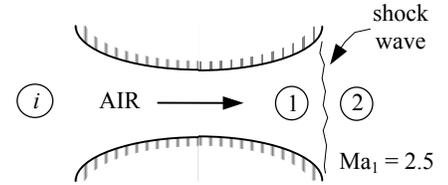
Properties The properties of air are $k = 1.4$ and $R = 0.06855 \text{ Btu/lbm}\cdot\text{R}$, and the properties of helium are $k = 1.667$ and $R = 0.4961 \text{ Btu/lbm}\cdot\text{R}$.

Analysis The air properties upstream the shock are

$$\text{Ma}_1 = 2.5, P_1 = 10 \text{ psia}, \text{ and } T_1 = 440.5 \text{ R}$$

Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 2.5$,

$$\text{Ma}_2 = \mathbf{0.513}, \frac{P_{02}}{P_1} = 8.5262, \frac{P_2}{P_1} = 7.125, \text{ and } \frac{T_2}{T_1} = 2.1375$$



Then the stagnation pressure P_{02} , static pressure P_2 , and static temperature T_2 , are determined to be

$$P_{02} = 8.5262P_1 = (8.5262)(10 \text{ psia}) = \mathbf{85.3 \text{ psia}}$$

$$P_2 = 7.125P_1 = (7.125)(10 \text{ psia}) = \mathbf{71.3 \text{ psia}}$$

$$T_2 = 2.1375T_1 = (2.1375)(440.5 \text{ R}) = \mathbf{942 \text{ R}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.513)\sqrt{(1.4)(0.06855 \text{ Btu/lbm}\cdot\text{R})(941.6 \text{ R})\left(\frac{25,037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = \mathbf{772 \text{ ft/s}}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-14 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2k/(k-1) - 1}\right)^{1/2} = \left(\frac{2.5^2 + 2/(1.667-1)}{2 \times 2.5^2 \times 1.667/(1.667-1) - 1}\right)^{1/2} = \mathbf{0.553}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{1 + 1.667 \times 2.5^2}{1 + 1.667 \times 0.553^2} = 7.5632$$

$$\frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2} = \frac{1 + 2.5^2(1.667-1)/2}{1 + 0.553^2(1.667-1)/2} = 2.7989$$

$$\begin{aligned} \frac{P_{02}}{P_1} &= \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2}\right) \left(1 + (k-1)\text{Ma}_2^2/2\right)^{k/(k-1)} \\ &= \left(\frac{1 + 1.667 \times 2.5^2}{1 + 1.667 \times 0.553^2}\right) \left(1 + (1.667-1) \times 0.553^2/2\right)^{1.667/0.667} = 9.641 \end{aligned}$$

Thus, $P_{02} = 11.546P_1 = (11.546)(10 \text{ psia}) = \mathbf{115 \text{ psia}}$

$$P_2 = 7.5632P_1 = (7.5632)(10 \text{ psia}) = \mathbf{75.6 \text{ psia}}$$

$$T_2 = 2.7989T_1 = (2.7989)(440.5 \text{ R}) = \mathbf{1233 \text{ R}}$$

$$V_2 = \text{Ma}_2c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.553)\sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(1232.9 \text{ R})\left(\frac{25,037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = \mathbf{2794 \text{ ft/s}}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

12-85E Reconsider Prob. 12-84E. Using EES (or other) software, study the effects of both air and helium flowing steadily in a nozzle when there is a normal shock at a Mach number in the range $2 < M_x < 3.5$. In addition to the required information, calculate the entropy change of the air and helium across the normal shock. Tabulate the results in a parametric table.

" Also calculate the entropy change of the air and helium across the normal shock."
 "Results are given in the Parametric Table for $2 < M_x < 3.5$ "

```

Procedure NormalShock(M_x,k:M_y,PyOPx, TyOTx,RhoyORhox, PoyOPox, PoyOPx)
  If M_x < 1 Then
    M_y = -1000;PyOPx=-1000;TyOTx=-1000;RhoyORhox=-
  1000
    PoyOPox=-1000;PoyOPx=-1000
  else
    M_y=sqrt( (M_x^2+2/(k-1)) / (2*M_x^2*k/(k-1)-1) )
    PyOPx=(1+k*M_x^2)/(1+k*M_y^2)
    TyOTx=( 1+M_x^2*(k-1)/2 )/(1+M_y^2*(k-1)/2 )
    RhoyORhox=PyOPx/TyOTx
    PoyOPox=M_x/M_y*( (1+M_y^2*(k-1)/2) / (1+M_x^2*(k-1)/2)
)^(k+1)/(2*(k-1)))
    PoyOPx=(1+k*M_x^2)*(1+M_y^2*(k-1)/2)^(k/(k-
1))/(1+k*M_y^2)
  Endif
End
  
```

```

Function ExitPress(P_back,P_crit)
If P_back>=P_crit then ExitPress:=P_back "Unchoked Flow Condition"
If P_back<P_crit then ExitPress:=P_crit "Choked Flow Condition"
End
  
```

```

Procedure GetProp(Gas$:Cp,k,R) "Cp and k data are from Text Table A.2E"
  M=MOLARMASS(Gas$) "Molar mass of Gas$"
  R= 1545/M "Particular gas constant for Gas$, ft-lbf/lbm-R"
  "k = Ratio of Cp to Cv"
  "Cp = Specific heat at constant pressure"
  if Gas$='Air' then
    Cp=0.24"Btu/lbm-R"; k=1.4
  endif
  if Gas$='CO2' then
    Cp=0.203"Btu/lbm_R"; k=1.289
  endif
  if Gas$='Helium' then
    Cp=1.25"Btu/lbm-R"; k=1.667
  endif
End
  
```

"Variable Definitions:"
 "M = flow Mach Number"
 "P_ratio = P/P_o for compressible, isentropic flow"
 "T_ratio = T/T_o for compressible, isentropic flow"
 "Rho_ratio= Rho/Rho_o for compressible, isentropic flow"
 "A_ratio=A/A* for compressible, isentropic flow"
 "Fluid properties before the shock are denoted with a subscript x"
 "Fluid properties after the shock are denoted with a subscript y"
 "M_y = Mach Number down stream of normal shock"

"PyOverPx= P_y/P_x Pressue ratio across normal shock"
 "TyOverTx =T_y/T_x Temperature ratio across normal shock"
 "Rho_yOverRho_x=Rho_y/Rho_x Density ratio across normal shock"
 "PoyOverPox = P_{oy}/P_{ox} Stagantion pressure ratio across normal shock"
 "PoyOverPx = P_{oy}/P_x Stagnation pressure after normal shock ratioed to pressure before shock"

"Input Data"

{P_x = 10 "psia"} "Values of P_x, T_x, and M_x are set in the Parametric Table"

{T_x = 440.5 "R"}

{M_x = 2.5}

Gas\$='Air' "This program has been written for the gases Air, CO2, and Helium"

Call GetProp(Gas\$:Cp,k,R)

Call NormalShock(M_x,k:M_y,PyOverPx, TyOverTx,Rho_yOverRho_x, PoyOverPox, PoyOverPx)

P_{oy}_air=P_x*PoyOverPx "Stagnation pressure after the shock"

P_y_air=P_x*PyOverPx "Pressure after the shock"

T_y_air=T_x*TyOverTx "Temperature after the shock"

M_y_air=M_y "Mach number after the shock"

"The velocity after the shock can be found from the product of the Mach number and speed of sound after the shock."

C_y_air = sqrt(k*R"ft-lbf/lbm_R"*T_y_air"R"*32.2 "lbm-ft/lbf-s^2")

V_y_air=M_y_air*C_y_air

DELTA_s_air=entropy(air,T=T_y_air, P=P_y_air) -entropy(air,T=T_x,P=P_x)

Gas2\$='Helium' "Gas2\$ can be either Helium or CO2"

Call GetProp(Gas2\$:Cp_2,k_2,R_2)

Call NormalShock(M_x,k_2:M_{y2},PyOverPx2, TyOverTx2,Rho_yOverRho_{x2}, PoyOverPox2, PoyOverPx2)

P_{oy}_he=P_x*PoyOverPx2 "Stagnation pressure after the shock"

P_y_he=P_x*PyOverPx2 "Pressure after the shock"

T_y_he=T_x*TyOverTx2 "Temperature after the shock"

M_y_he=M_{y2} "Mach number after the shock"

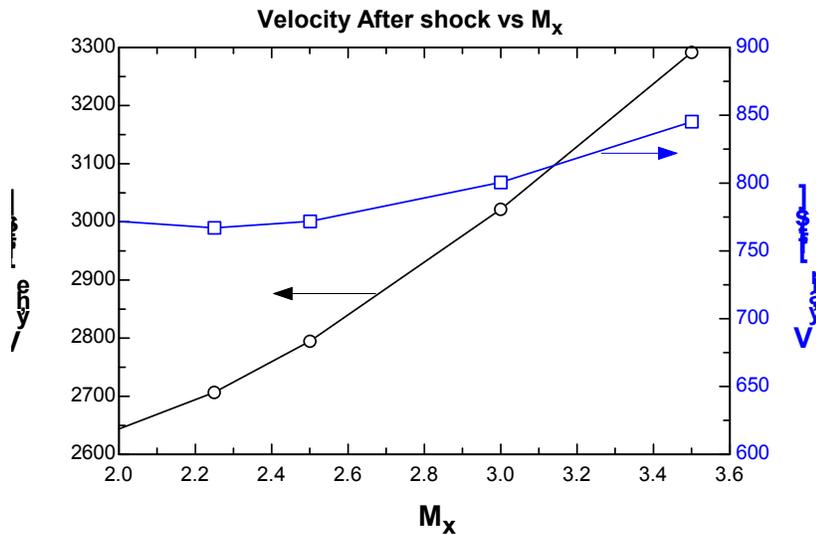
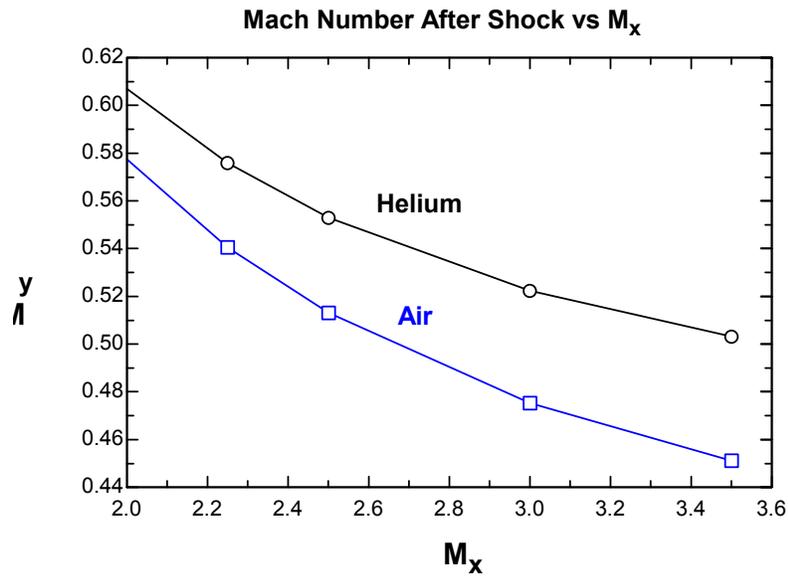
"The velocity after the shock can be found from the product of the Mach number and speed of sound after the shock."

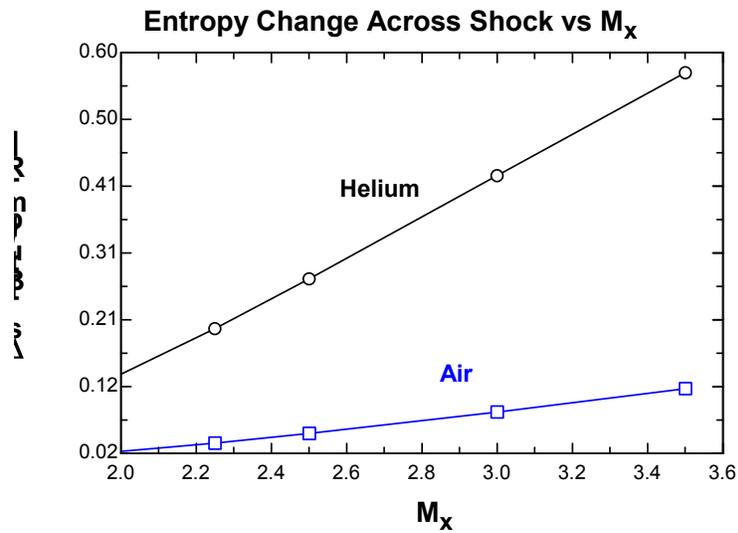
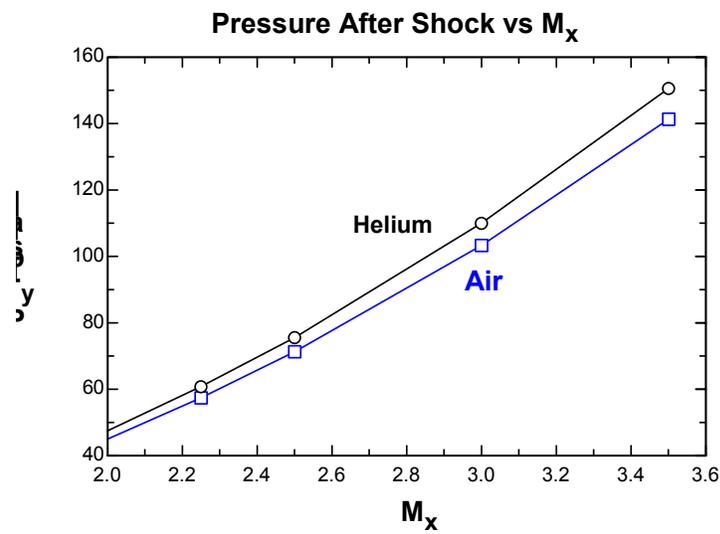
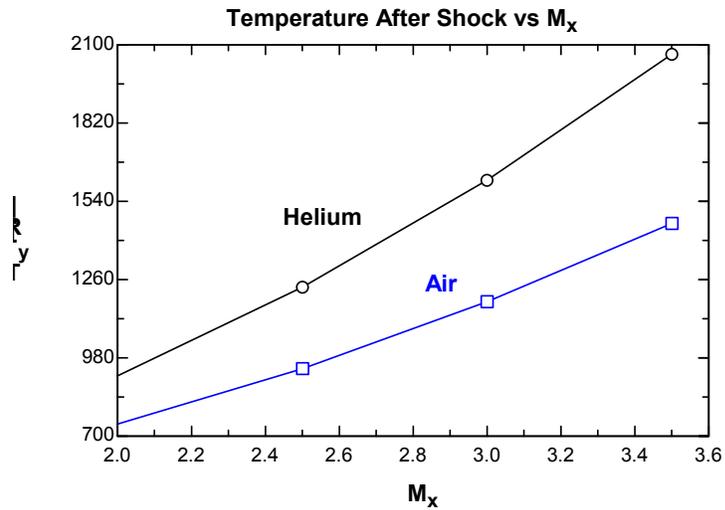
C_y_he = sqrt(k_2*R_2"ft-lbf/lbm_R"*T_y_he"R"*32.2 "lbm-ft/lbf-s^2")

V_y_he=M_y_he*C_y_he

DELTA_s_he=entropy(helium,T=T_y_he, P=P_y_he) -entropy(helium,T=T_x,P=P_x)

$V_{y,he}$ [ft/s]	$V_{y,air}$ [ft/s]	$T_{y,he}$ [R]	$T_{y,air}$ [R]	T_x [R]	$P_{y,he}$ [psia]	$P_{y,air}$ [psia]	P_x [psia]	$P_{oy,he}$ [psia]	$P_{oy,air}$ [psia]	$M_{y,he}$	$M_{y,air}$	M_x	ΔS_{he} [Btu/lbm-R]	ΔS_{air} [Btu/lbm-R]
2644	771.9	915.6	743.3	440.5	47.5	45	10	63.46	56.4	0.607	0.5774	2	0.1345	0.0228
2707	767.1	1066	837.6	440.5	60.79	57.4	10	79.01	70.02	0.5759	0.5406	2.25	0.2011	0.0351
2795	771.9	1233	941.6	440.5	75.63	71.25	10	96.41	85.26	0.553	0.513	2.5	0.2728	0.04899
3022	800.4	1616	1180	440.5	110	103.3	10	136.7	120.6	0.5223	0.4752	3	0.4223	0.08
3292	845.4	2066	1460	440.5	150.6	141.3	10	184.5	162.4	0.5032	0.4512	3.5	0.5711	0.1136





12-86 Air flowing through a nozzle experiences a normal shock. Various properties are to be calculated before and after the shock.

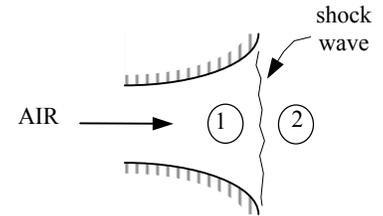
Assumptions 1 Air is an ideal gas with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are $k = 1.4$, $R = 0.287 \text{ kJ/kg}\cdot\text{K}$, and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$.

Analysis The stagnation temperature and pressure before the shock are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 217 + \frac{(680 \text{ m/s})^2}{2(1.005 \text{ kJ/kg}\cdot\text{K}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)} = 447.0 \text{ K}$$

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1} \right)^{k/(k-1)} = (22.6 \text{ kPa}) \left(\frac{447.0 \text{ K}}{217 \text{ K}} \right)^{1.4/(1.4-1)} = 283.6 \text{ kPa}$$



The velocity and the Mach number before the shock are determined from

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(217.0 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{295.3 \text{ m/s}}$$

and

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{680 \text{ m/s}}{295.3 \text{ m/s}} = \mathbf{2.30}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 2.30$ we read

$$\text{Ma}_2 = \mathbf{0.5344}, \quad \frac{P_{02}}{P_1} = 7.2937, \quad \frac{P_2}{P_1} = 6.005, \quad \text{and} \quad \frac{T_2}{T_1} = 1.9468$$

Then the stagnation pressure P_{02} , static pressure P_2 , and static temperature T_2 , are determined to be

$$P_{02} = 7.2937P_1 = (7.2937)(22.6 \text{ kPa}) = \mathbf{165 \text{ kPa}}$$

$$P_2 = 6.005P_1 = (6.005)(22.6 \text{ kPa}) = \mathbf{136 \text{ kPa}}$$

$$T_2 = 1.9468T_1 = (1.9468)(217 \text{ K}) = \mathbf{423 \text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5344)\sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(422.5 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{220 \text{ m/s}}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

12-87 Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions 1 Air is an ideal gas with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$.

Analysis The entropy change across the shock is determined to be

$$\begin{aligned} s_2 - s_1 &= c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\ &= (1.005 \text{ kJ/kg}\cdot\text{K}) \ln(1.9468) - (0.287 \text{ kJ/kg}\cdot\text{K}) \ln(6.005) \\ &= \mathbf{0.155 \text{ kJ/kg}\cdot\text{K}} \end{aligned}$$

Discussion Note that shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

12-88 The entropy change of air across the shock for upstream Mach numbers between 0.5 and 1.5 is to be determined and plotted.

Assumptions 1 Air is an ideal gas. 2 Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

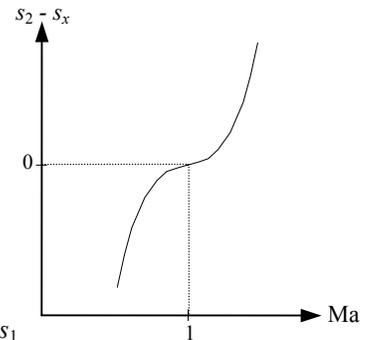
Properties The properties of air are $k = 1.4$, $R = 0.287 \text{ kJ/kg}\cdot\text{K}$, and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$.

Analysis The entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

where

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k / (k-1) - 1} \right)^{1/2}, \quad \frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2}, \quad \text{and} \quad \frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2}$$



The results of the calculations can be tabulated as

Ma_1	Ma_2	T_2/T_1	P_2/P_1	$s_2 - s_1$
0.5	2.6458	0.1250	0.4375	-1.853
0.6	1.8778	0.2533	0.6287	-1.247
0.7	1.5031	0.4050	0.7563	-0.828
0.8	1.2731	0.5800	0.8519	-0.501
0.9	1.1154	0.7783	0.9305	-0.231
1.0	1.0000	1.0000	1.0000	0.0
1.1	0.9118	1.0649	1.2450	0.0003
1.2	0.8422	1.1280	1.5133	0.0021
1.3	0.7860	1.1909	1.8050	0.0061
1.4	0.7397	1.2547	2.1200	0.0124
1.5	0.7011	1.3202	2.4583	0.0210

Discussion The total entropy change is negative for upstream Mach numbers Ma_1 less than unity. Therefore, normal shocks cannot occur when $\text{Ma}_1 < 1$.

12-89 Supersonic airflow approaches the nose of a two-dimensional wedge and undergoes a straight oblique shock. For a specified Mach number, the minimum shock angle and the maximum deflection angle are to be determined.

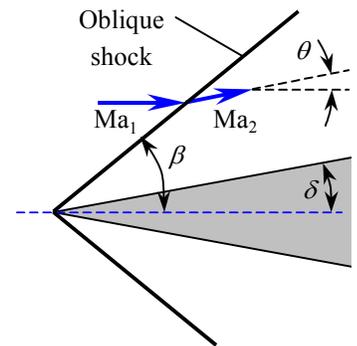
Assumptions Air is an ideal gas with a constant specific heat ratio of $k = 1.4$ (so that Fig. 12-41 is applicable).

Analysis For $Ma = 5$, we read from Fig. 12-41

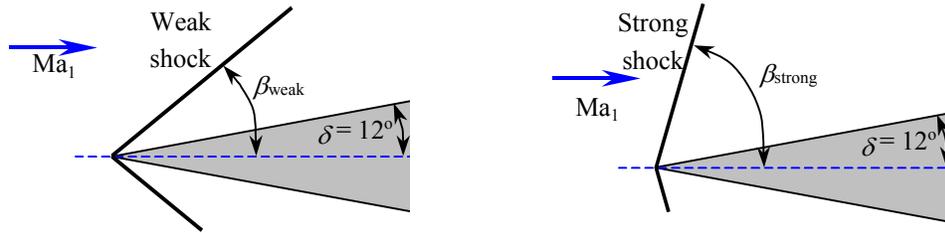
Minimum shock (or wave) angle: $\beta_{\min} = 12^\circ$

Maximum deflection (or turning) angle: $\theta_{\max} = 41.5^\circ$

Discussion Note that the minimum shock angle decreases and the maximum deflection angle increases with increasing Mach number Ma_1 .



12-90 Air flowing at a specified supersonic Mach number impinges on a two-dimensional wedge. The shock angle, Mach number, and pressure downstream of the weak and strong oblique shock formed by a wedge are to be determined.



Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 12^\circ$. Then the two values of oblique shock angle β are determined from

$$\tan \theta = \frac{2(\text{Ma}_1^2 \sin^2 \beta - 1) / \tan \beta}{\text{Ma}_1^2 (k + \cos 2\beta) + 2} \rightarrow \tan 12^\circ = \frac{2(3.4^2 \sin^2 \beta - 1) / \tan \beta}{3.4^2 (1.4 + \cos 2\beta) + 2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 26.75^\circ$ and $\beta_{\text{strong}} = 86.11^\circ$. Then the upstream “normal” Mach number $\text{Ma}_{1,n}$ becomes

Weak shock: $\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 3.4 \sin 26.75^\circ = 1.531$

Strong shock: $\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 3.4 \sin 86.11^\circ = 3.392$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

Weak shock: $\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.531)^2 + 2}{2(1.4)(1.531)^2 - 1.4 + 1}} = 0.6905$

Strong shock: $\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(3.392)^2 + 2}{2(1.4)(3.392)^2 - 1.4 + 1}} = 0.4555$

The downstream pressure for each case is determined to be

Weak shock: $P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (60 \text{ kPa}) \frac{2(1.4)(1.531)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{154 \text{ kPa}}$

Strong shock: $P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (60 \text{ kPa}) \frac{2(1.4)(3.392)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{796 \text{ kPa}}$

The downstream Mach number is determined to be

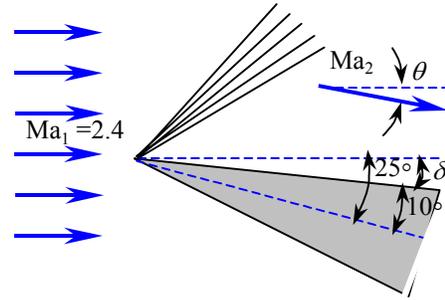
Weak shock: $\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.6905}{\sin(26.75^\circ - 12^\circ)} = \mathbf{2.71}$

Strong shock: $\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.4555}{\sin(86.11^\circ - 12^\circ)} = \mathbf{0.474}$

Discussion Note that the change in Mach number and pressure across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock cases, $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.

12-91 Air flowing at a specified supersonic Mach number undergoes an expansion turn over a tilted wedge. The Mach number, pressure, and temperature downstream of the sudden expansion above the wedge are to be determined.

Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.



Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta = 25^\circ - 10^\circ = 15^\circ$. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\text{Ma}) = \sqrt{\frac{k+1}{k-1}} \tan^{-1} \left(\sqrt{\frac{k-1}{k+1} (\text{Ma}^2 - 1)} \right) - \tan^{-1} \left(\sqrt{\text{Ma}^2 - 1} \right)$$

Upstream:

$$\nu(\text{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}} \tan^{-1} \left(\sqrt{\frac{1.4-1}{1.4+1} (2.4^2 - 1)} \right) - \tan^{-1} \left(\sqrt{2.4^2 - 1} \right) = 36.75^\circ$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\text{Ma}_2) = \theta + \nu(\text{Ma}_1) = 15^\circ + 36.75^\circ = 51.75^\circ$$

Now Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

$$\text{Downstream: } \nu(\text{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}} \tan^{-1} \left(\sqrt{\frac{1.4-1}{1.4+1} (\text{Ma}_2^2 - 1)} \right) - \tan^{-1} \left(\sqrt{\text{Ma}_2^2 - 1} \right) = 51.75^\circ$$

It gives $\text{Ma}_2 = 3.105$. Then the downstream pressure and temperature are determined from the isentropic flow relations

$$P_2 = \frac{P_2 / P_0}{P_1 / P_0} P_1 = \frac{[1 + \text{Ma}_2^2 (k-1) / 2]^{-k/(k-1)}}{[1 + \text{Ma}_1^2 (k-1) / 2]^{-k/(k-1)}} P_1 = \frac{[1 + 3.105^2 (1.4-1) / 2]^{-1.4/0.4}}{[1 + 2.4^2 (1.4-1) / 2]^{-1.4/0.4}} (70 \text{ kPa}) = \mathbf{23.8 \text{ kPa}}$$

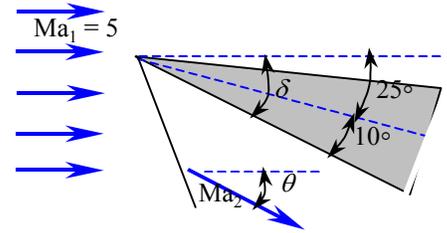
$$T_2 = \frac{T_2 / T_0}{T_1 / T_0} T_1 = \frac{[1 + \text{Ma}_2^2 (k-1) / 2]^{-1}}{[1 + \text{Ma}_1^2 (k-1) / 2]^{-1}} T_1 = \frac{[1 + 3.105^2 (1.4-1) / 2]^{-1}}{[1 + 2.4^2 (1.4-1) / 2]^{-1}} (260 \text{ K}) = \mathbf{191 \text{ K}}$$

Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html.

12-92 Air flowing at a specified supersonic Mach number undergoes a compression turn (an oblique shock) over a tilted wedge. The Mach number, pressure, and temperature downstream of the shock below the wedge are to be determined.

Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.



Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta = 25^\circ + 10^\circ = 35^\circ$. Then the two values of oblique shock angle β are determined from

$$\tan \theta = \frac{2(\text{Ma}_1^2 \sin^2 \beta - 1) / \tan \beta}{\text{Ma}_1^2 (k + \cos 2\beta) + 2} \rightarrow \tan 35^\circ = \frac{2(5^2 \sin^2 \beta - 1) / \tan \beta}{5^2 (1.4 + \cos 2\beta) + 2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 49.86^\circ$ and $\beta_{\text{strong}} = 77.66^\circ$. Then for the case of strong oblique shock, the upstream “normal” Mach number $\text{Ma}_{1,n}$ becomes

$$\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 5 \sin 77.66^\circ = 4.884$$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

$$\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(4.884)^2 + 2}{2(1.4)(4.884)^2 - 1.4 + 1}} = 0.4169$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (70 \text{ kPa}) \frac{2(1.4)(4.884)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{1940 \text{ kPa}}$$

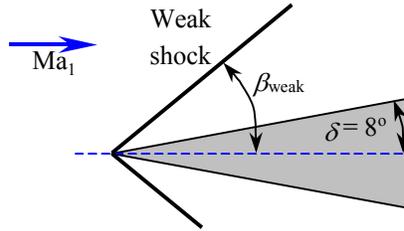
$$T_2 = T_1 \frac{P_2}{P_1} \frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1} \frac{2 + (k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (260 \text{ K}) \frac{1940 \text{ kPa}}{70 \text{ kPa}} \frac{2 + (1.4-1)(4.884)^2}{(1.4+1)(4.884)^2} = \mathbf{1450 \text{ K}}$$

The downstream Mach number is determined to be

$$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.4169}{\sin(77.66^\circ - 35^\circ)} = \mathbf{0.615}$$

Discussion Note that $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ and Ma_2 are subsonic. Also note the huge rise in temperature and pressure across the strong oblique shock, and the challenges they present for spacecraft during reentering the earth’s atmosphere.

12-93E Air flowing at a specified supersonic Mach number is forced to turn upward by a ramp, and weak oblique shock forms. The wave angle, Mach number, pressure, and temperature after the shock are to be determined.



Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the ramp, i.e., $\theta \approx \delta = 8^\circ$. Then the two values of oblique shock angle β are determined from

$$\tan \theta = \frac{2(\text{Ma}_1^2 \sin^2 \beta - 1) / \tan \beta}{\text{Ma}_1^2 (k + \cos 2\beta) + 2} \rightarrow \tan 8^\circ = \frac{2(2^2 \sin^2 \beta - 1) / \tan \beta}{2^2 (1.4 + \cos 2\beta) + 2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 37.21^\circ$ and $\beta_{\text{strong}} = 85.05^\circ$. Then for the case of weak oblique shock, the upstream “normal” Mach number $\text{Ma}_{1,n}$ becomes

$$\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 2 \sin 37.21^\circ = 1.209$$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

$$\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.209)^2 + 2}{2(1.4)(1.209)^2 - 1.4 + 1}} = 0.8363$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (8 \text{ psia}) \frac{2(1.4)(1.209)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{12.3 \text{ psia}}$$

$$T_2 = T_1 \frac{P_2}{P_1} \frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1} \frac{2 + (k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (480 \text{ R}) \frac{12.3 \text{ psia}}{8 \text{ psia}} \frac{2 + (1.4-1)(1.209)^2}{(1.4+1)(1.209)^2} = \mathbf{544 \text{ R}}$$

The downstream Mach number is determined to be

$$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.8363}{\sin(37.21^\circ - 8^\circ)} = \mathbf{1.71}$$

Discussion Note that $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock (it is *subsonic* across the strong oblique shock).

12-94 Air flowing at a specified supersonic Mach number undergoes an expansion turn. The Mach number, pressure, and temperature downstream of the sudden expansion along a wall are to be determined.

Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15^\circ$. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\text{Ma}) = \sqrt{\frac{k+1}{k-1}} \tan^{-1} \left(\sqrt{\frac{k-1}{k+1} (\text{Ma}^2 - 1)} \right) - \tan^{-1} \left(\sqrt{\text{Ma}^2 - 1} \right)$$

Upstream:

$$\nu(\text{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}} \tan^{-1} \left(\sqrt{\frac{1.4-1}{1.4+1} (3.6^2 - 1)} \right) - \tan^{-1} \left(\sqrt{3.6^2 - 1} \right) = 60.09^\circ$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\text{Ma}_2) = \theta + \nu(\text{Ma}_1) = 15^\circ + 60.09^\circ = 75.09^\circ$$

Now Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

$$\text{Downstream: } \nu(\text{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}} \tan^{-1} \left(\sqrt{\frac{1.4-1}{1.4+1} \text{Ma}_2^2 - 1} \right) - \tan^{-1} \left(\sqrt{\text{Ma}_2^2 - 1} \right) = 75.09^\circ$$

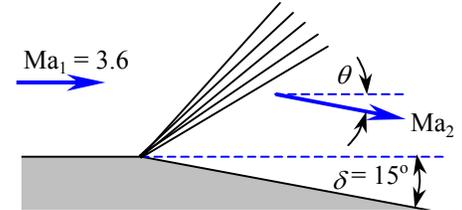
It gives $\text{Ma}_2 = 4.81$. Then the downstream pressure and temperature are determined from the isentropic flow relations

$$P_2 = \frac{P_2 / P_0}{P_1 / P_0} P_1 = \frac{[1 + \text{Ma}_2^2 (k-1) / 2]^{-k/(k-1)}}{[1 + \text{Ma}_1^2 (k-1) / 2]^{-k/(k-1)}} P_1 = \frac{[1 + 4.81^2 (1.4-1) / 2]^{-1.4/0.4}}{[1 + 3.6^2 (1.4-1) / 2]^{-1.4/0.4}} (40 \text{ kPa}) = \mathbf{8.31 \text{ kPa}}$$

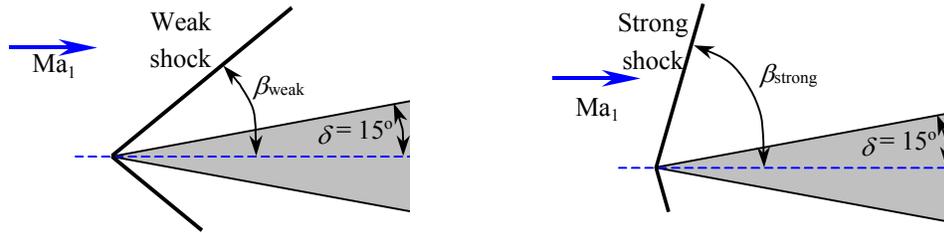
$$T_2 = \frac{T_2 / T_0}{T_1 / T_0} T_1 = \frac{[1 + \text{Ma}_2^2 (k-1) / 2]^{-1}}{[1 + \text{Ma}_1^2 (k-1) / 2]^{-1}} T_1 = \frac{[1 + 4.81^2 (1.4-1) / 2]^{-1}}{[1 + 3.6^2 (1.4-1) / 2]^{-1}} (280 \text{ K}) = \mathbf{179 \text{ K}}$$

Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html.



12-95E Air flowing at a specified supersonic Mach number is forced to undergo a compression turn (an oblique shock). The Mach number, pressure, and temperature downstream of the oblique shock are to be determined.



Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin. 3 Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15^\circ$. Then the two values of oblique shock angle β are determined from

$$\tan \theta = \frac{2(\text{Ma}_1^2 \sin^2 \beta - 1) / \tan \beta}{\text{Ma}_1^2 (k + \cos 2\beta) + 2} \rightarrow \tan 15^\circ = \frac{2(2^2 \sin^2 \beta - 1) / \tan \beta}{2^2 (1.4 + \cos 2\beta) + 2}$$

which is implicit in β . Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 45.34^\circ$ and $\beta_{\text{strong}} = 79.83^\circ$. Then the upstream “normal” Mach number $\text{Ma}_{1,n}$ becomes

Weak shock: $\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 2 \sin 45.34^\circ = 1.423$

Strong shock: $\text{Ma}_{1,n} = \text{Ma}_1 \sin \beta = 2 \sin 79.83^\circ = 1.969$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

Weak shock: $\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.423)^2 + 2}{2(1.4)(1.423)^2 - 1.4 + 1}} = 0.7304$

Strong shock: $\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.969)^2 + 2}{2(1.4)(1.969)^2 - 1.4 + 1}} = 0.5828$

The downstream pressure and temperature for each case are determined to be

Weak shock: $P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (6 \text{ psia}) \frac{2(1.4)(1.423)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{13.2 \text{ psia}}$

$$T_2 = T_1 \frac{P_2}{P_1} \frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1} \frac{2 + (k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (480 \text{ R}) \frac{13.2 \text{ psia}}{6 \text{ psia}} \frac{2 + (1.4-1)(1.423)^2}{(1.4+1)(1.423)^2} = \mathbf{609 \text{ R}}$$

Strong shock: $P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k + 1} = (6 \text{ psia}) \frac{2(1.4)(1.969)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{26.1 \text{ psia}}$

$$T_2 = T_1 \frac{P_2}{P_1} \frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1} \frac{2 + (k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (480 \text{ R}) \frac{26.1 \text{ psia}}{6 \text{ psia}} \frac{2 + (1.4-1)(1.969)^2}{(1.4+1)(1.969)^2} = \mathbf{798 \text{ R}}$$

The downstream Mach number is determined to be

Weak shock: $\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.7304}{\sin(45.34^\circ - 15^\circ)} = \mathbf{1.45}$

Strong shock: $\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.5828}{\sin(79.83^\circ - 15^\circ)} = \mathbf{0.644}$

Discussion Note that the change in Mach number, pressure, temperature across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock

cases, $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.