Chapter 12
COMPRESSIBLE FLOW

Stagnation Properties

12-1C The temperature of the air will rise as it approaches the nozzle because of the stagnation process.

12-2C Stagnation enthalpy combines the ordinary enthalpy and the kinetic energy of a fluid, and offers convenience when analyzing high-speed flows. It differs from the ordinary enthalpy by the kinetic energy term.

12-3C Dynamic temperature is the temperature rise of a fluid during a stagnation process.

12-4C No. Because the velocities encountered in air-conditioning applications are very low, and thus the static and the stagnation temperatures are practically identical.

12-5 The state of air and its velocity are specified. The stagnation temperature and stagnation pressure of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.

Analysis The stagnation temperature of air is determined from

$$T_0 = T + \frac{V^2}{2c_p} = 245.9\ \text{K} + \frac{(470\ \text{m/s})^2}{2 \times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{355.8\ K}$$

Other stagnation properties at the specified state are determined by considering an isentropic process between the specified state and the stagnation state,

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (44\ \text{kPa})\left(\frac{355.8\ \text{K}}{245.9\ \text{K}}\right)^{1.4/(1.4-1)} = \mathbf{160.3\ kPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

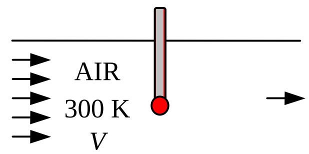
12-6 Air at 300 K is flowing in a duct. The temperature that a stationary probe inserted into the duct will read is to be determined for different air velocities.

Assumptions The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg·K.

Analysis The air which strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature, T_0. It is determined from

$$T_0 = T + \frac{V^2}{2c_p}$$



(*a*) $$T_0 = 300\text{ K} + \frac{(1\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{300.0\ K}$$

(*b*) $$T_0 = 300\text{ K} + \frac{(10\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{300.1\ K}$$

(*c*) $$T_0 = 300\text{ K} + \frac{(100\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{305.0\ K}$$

(*d*) $$T_0 = 300\text{ K} + \frac{(1000\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{797.5\ K}$$

Discussion Note that the stagnation temperature is nearly identical to the thermodynamic temperature at low velocities, but the difference between the two is very significant at high velocities,

12-7 The states of different substances and their velocities are specified. The stagnation temperature and stagnation pressures are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Helium and nitrogen are ideal gases.

Analysis (*a*) Helium can be treated as an ideal gas with c_p = 5.1926 kJ/kg·K and k = 1.667. Then the stagnation temperature and pressure of helium are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(240\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{55.5°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.25\text{ MPa})\left(\frac{328.7\text{ K}}{323.2\text{ K}}\right)^{1.667/(1.667-1)} = \mathbf{0.261\,MPa}$$

(*b*) Nitrogen can be treated as an ideal gas with c_p = 1.039 kJ/kg·K and k =1.400. Then the stagnation temperature and pressure of nitrogen are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(300\text{ m/s})^2}{2\times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{93.3°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.15\text{ MPa})\left(\frac{366.5\text{ K}}{323.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{0.233\ MPa}$$

(*c*) Steam can be treated as an ideal gas with c_p = 1.865 kJ/kg·K and k =1.329. Then the stagnation temperature and pressure of steam are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 350°\text{C} + \frac{(480\text{ m/s})^2}{2\times 1.865\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{411.8°C = 685\ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.1\text{ MPa})\left(\frac{685\text{ K}}{623.2\text{ K}}\right)^{1.329/(1.329-1)} = \mathbf{0.147\ MPa}$$

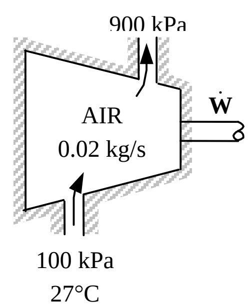
Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

12-8 The inlet stagnation temperature and pressure and the exit stagnation pressure of air flowing through a compressor are specified. The power input to the compressor is to be determined.

Assumptions **1** The compressor is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.

Analysis The exit stagnation temperature of air T_{02} is determined from



$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (300.2\text{ K})\left(\frac{900}{100}\right)^{(1.4-1)/1.4} = 562.4\text{ K}$$

From the energy balance on the compressor,

$$\dot{W}_{\text{in}} = \dot{m}(h_{20} - h_{01})$$

or,

$$\dot{W}_{\text{in}} = \dot{m}c_p(T_{02} - T_{01}) = (0.02\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(562.4 - 300.2)\text{K} = \mathbf{5.27\ kW}$$

Discussion Note that the stagnation properties can be used conveniently in the energy equation.

12-9E Steam flows through a device. The stagnation temperature and pressure of steam and its velocity are specified. The static pressure and temperature of the steam are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Steam is an ideal gas.

Properties Steam can be treated as an ideal gas with c_p = 0.4455 Btu/lbm·R and k =1.329.

Analysis The static temperature and pressure of steam are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 700°\text{F} - \frac{(900\ \text{ft/s})^2}{2 \times 0.4455\ \text{Btu/lbm} \cdot °\text{F}} \left(\frac{1\ \text{Btu/lbm}}{25{,}037\ \text{ft}^2/\text{s}^2} \right) = \mathbf{663.7°F}$$

$$P = P_0 \left(\frac{T}{T_0} \right)^{k/(k-1)} = (120\ \text{psia}) \left(\frac{1123.7\ \text{R}}{1160\ \text{R}} \right)^{1.329/(1.329-1)} = \mathbf{105.5\ psia}$$

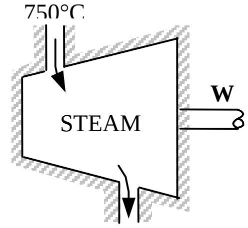
Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

12-10 The inlet stagnation temperature and pressure and the exit stagnation pressure of products of combustion flowing through a gas turbine are specified. The power output of the turbine is to be determined.

Assumptions **1** The expansion process is isentropic. **2** Products of combustion are ideal gases.

Properties The properties of products of combustion are c_p = 1.157 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.33.

Analysis The exit stagnation temperature T_{02} is determined to be



$$T_{02} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{(k-1)/k} = (1023.2\ \text{K}) \left(\frac{0.1}{1} \right)^{(1.33-1)/1.33} = 577.9\ \text{K}$$

Also,

$$c_p = kc_v = k(c_p - R) \longrightarrow c_p = \frac{kR}{k-1} = \frac{1.33(0.287\ \text{kJ/kg} \cdot \text{K})}{1.33-1} = 1.157\ \text{kJ/kg} \cdot \text{K}$$

From the energy balance on the turbine,

$$-w_{\text{out}} = (h_{20} - h_{01})$$

or,

$$w_{\text{out}} = c_p (T_{01} - T_{02}) = (1.157\ \text{kJ/kg} \cdot \text{K})(1023.2 - 577.9)\ \text{K} = \mathbf{515.2\ kJ/kg}$$

Discussion Note that the stagnation properties can be used conveniently in the energy equation.

2-11 Air flows through a device. The stagnation temperature and pressure of air and its velocity are specified. The static pressure and temperature of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at an anticipated average temperature of 600 K are c_p = 1.051 kJ/kg·K and k = 1.376.

Analysis The static temperature and pressure of air are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 673.2 - \frac{(570\text{ m/s})^2}{2\times 1.051\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{518.6\ K}$$

and

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{518.6\text{ K}}{673.2\text{ K}}\right)^{1.376/(1.376-1)} = \mathbf{0.23\ MPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

Speed of sound and Mach Number

12-12C Sound is an infinitesimally small pressure wave. It is generated by a small disturbance in a medium. It travels by wave propagation. Sound waves cannot travel in a vacuum.

12-13C Yes, it is. Because the amplitude of an ordinary sound wave is very small, and it does not cause any significant change in temperature and pressure.

12-14C The sonic speed in a medium depends on the properties of the medium, and it changes as the properties of the medium change.

12-15C In warm (higher temperature) air since $c = \sqrt{kRT}$

12-16C Helium, since $c = \sqrt{kRT}$ and helium has the highest kR value. It is about 0.40 for air, 0.35 for argon and 3.46 for helium.

12-17C Air at specified conditions will behave like an ideal gas, and the speed of sound in an ideal gas depends on temperature only. Therefore, the speed of sound will be the same in both mediums.

12-18C In general, no. Because the Mach number also depends on the speed of sound in gas, which depends on the temperature of the gas. The Mach number will remain constant if the temperature is maintained constant.

12-19 The Mach number of an aircraft and the speed of sound in air are to be determined at two specified temperatures. √EES

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis From the definitions of the speed of sound and the Mach number,

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(300\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{347\ m/s}$$

and $$\text{Ma} = \frac{V}{c} = \frac{240\ \text{m/s}}{347\ \text{m/s}} = \mathbf{0.692}$$

(b) At 1000 K,

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(1000\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{634\ m/s}$$

and $$\text{Ma} = \frac{V}{c} = \frac{240\ \text{m/s}}{634\ \text{m/s}} = \mathbf{0.379}$$

Discussion Note that a constant Mach number does not necessarily indicate constant speed. The Mach number of a rocket, for example, will be increasing even when it ascends at constant speed. Also, the specific heat ratio k changes with temperature, and the accuracy of the result at 1000 K can be improved by using the k value at that temperature (it would give k = 1.386, c = 619 m/s, and Ma = 0.388).

12-20 Carbon dioxide flows through a nozzle. The inlet temperature and velocity and the exit temperature of CO_2 are specified. The Mach number is to be determined at the inlet and exit of the nozzle. √EES

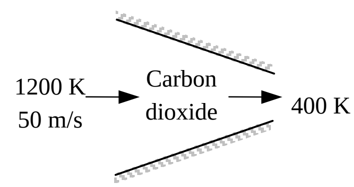
Assumptions **1** CO_2 is an ideal gas with constant specific heats at room temperature. **2** This is a steady-flow process.

Properties The gas constant of carbon dioxide is R = 0.1889 kJ/kg·K. Its constant pressure specific heat and specific heat ratio at room temperature are c_p = 0.8439 kJ/kg·K and k = 1.288.

Analysis (*a*) At the inlet

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.288)(0.1889\ \text{kJ/kg}\cdot\text{K})(1200\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 540.3\ \text{m/s}$$

Thus,



$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{50\ \text{m/s}}{540.3\ \text{m/s}} = \mathbf{0.0925}$$

(*b*) At the exit,

$$c_2 = \sqrt{k_2 R T_2} = \sqrt{(1.288)(0.1889\ \text{kJ/kg}\cdot\text{K})(400\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 312.0\ \text{m/s}$$

The nozzle exit velocity is determined from the steady-flow energy balance relation,

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \quad \rightarrow \quad 0 = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$0 = (0.8439\ \text{kJ/kg}\cdot\text{K})(400 - 1200\ \text{K}) + \frac{V_2^2 - (50\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) \longrightarrow V_2 = 1163\ \text{m/s}$$

Thus,

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{1163\ \text{m/s}}{312\ \text{m/s}} = \mathbf{3.73}$$

Discussion The specific heats and their ratio k change with temperature, and the accuracy of the results can be improved by accounting for this variation. Using EES (or another property database):

At 1200 K: c_p = 1.278 kJ/kg·K, k = 1.173 → c_1 = 516 m/s, V_1 = 50 m/s, Ma_1 = 0.0969

At 400 K: c_p = 0.9383 kJ/kg·K, k = 1.252 → c_2 = 308 m/s, V_2 = 1356 m/s, Ma_2 = 4.41

Therefore, the constant specific heat assumption results in an error of **4.5%** at the inlet and **15.5%** at the exit in the Mach number, which are significant.

12-21 Nitrogen flows through a heat exchanger. The inlet temperature, pressure, and velocity and the exit pressure and velocity are specified. The Mach number is to be determined at the inlet and exit of the heat exchanger. √EES

Assumptions **1** N_2 is an ideal gas. **2** This is a steady-flow process. **3** The potential energy change is negligible.

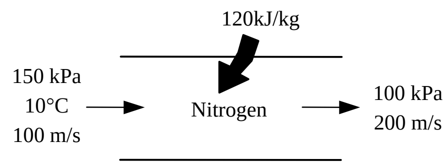
Properties The gas constant of N_2 is R = 0.2968 kJ/kg·K. Its constant pressure specific heat and specific heat ratio at room temperature are c_p = 1.040 kJ/kg·K and k = 1.4.

Analysis

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.400)(0.2968\ \mathrm{kJ/kg\cdot K})(283\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 342.9\ \mathrm{m/s}$$

Thus,

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{100\ \mathrm{m/s}}{342.9\ \mathrm{m/s}} = \mathbf{0.292}$$



From the energy balance on the heat exchanger,

$$q_{\mathrm{in}} = c_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$120\ \mathrm{kJ/kg} = (1.040\ \mathrm{kJ/kg.^\circ C})(T_2 - 10^\circ\mathrm{C}) + \frac{(200\ \mathrm{m/s})^2 - (100\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right)$$

It yields

$$T_2 = 111^\circ\mathrm{C} = 384\ \mathrm{K}$$

$$c_2 = \sqrt{k_2 R T_2} = \sqrt{(1.4)(0.2968\ \mathrm{kJ/kg\cdot K})(384\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 399\ \mathrm{m/s}$$

Thus,

$$\mathrm{Ma}_2 = \frac{V_2}{c_2} = \frac{200\ \mathrm{m/s}}{399\ \mathrm{m/s}} = \mathbf{0.501}$$

Discussion The specific heats and their ratio k change with temperature, and the accuracy of the results can be improved by accounting for this variation. Using EES (or another property database):

At 10°C : c_p = 1.038 kJ/kg·K, k = 1.400 → c_1 = 343 m/s, V_1 = 100 m/s, Ma_1 = 0.292

At 111°C c_p = 1.041 kJ/kg·K, k = 1.399 → c_2 = 399 m/s, V_2 = 200 m/s, Ma_2 = 0.501

Therefore, the constant specific heat assumption results in an error of **4.5%** at the inlet and **15.5%** at the exit in the Mach number, which are almost identical to the values obtained assuming constant specific heats.

12-22 The speed of sound in refrigerant-134a at a specified state is to be determined. √EES

Assumptions R-134a is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of R-134a is R = 0.08149 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.108.

Analysis From the ideal-gas speed of sound relation,

$$c=\sqrt{kRT}=\sqrt{(1.108)(0.08149\ \text{kJ/kg}\cdot\text{K})(60+273\,\text{K})\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right)}=\mathbf{173\ m/s}$$

Discussion Note that the speed of sound is independent of pressure for ideal gases.

12-23 The Mach number of a passenger plane for specified limiting operating conditions is to be determined. √EES

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis From the speed of sound relation

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(-60+273\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 293\ \text{m/s}$$

Thus, the Mach number corresponding to the maximum cruising speed of the plane is

$$\text{Ma} = \frac{V_{\max}}{c} = \frac{(945/3.6)\ \text{m/s}}{293\ \text{m/s}} = \mathbf{0.897}$$

Discussion Note that this is a subsonic flight since Ma < 1. Also, using a k value at -60°C would give practically the same result.

12-24E Steam flows through a device at a specified state and velocity. The Mach number of steam is to be determined assuming ideal gas behavior. √EES

Assumptions Steam is an ideal gas with constant specific heats.

Properties The gas constant of steam is R = 0.1102 Btu/lbm·R. Its specific heat ratio is given to be k = 1.3.

Analysis From the ideal-gas speed of sound relation,

$$c = \sqrt{kRT} = \sqrt{(1.3)(0.1102\ \text{Btu/lbm}\cdot\text{R})(1160\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/lbm}}\right)} = 2040\ \text{ft/s}$$

Thus,

$$\text{Ma} = \frac{V}{c} = \frac{900\ \text{ft/s}}{2040\ \text{ft/s}} = \mathbf{0.441}$$

Discussion Using property data from steam tables and not assuming ideal gas behavior, it can be shown that the Mach number in steam at the specified state is 0.446, which is sufficiently close to the ideal-gas value of 0.441. Therefore, the ideal gas approximation is a reasonable one in this case.

12-25E Problem 12-24e is reconsidered. The variation of Mach number with temperature as the temperature changes between 350° and 700°F is to be investigated, and the results are to be plotted.

Analysis Using EES, this problem can be solved as follows:

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T=Temperature+460
R=0.1102
V=900
k=1.3
c=SQRT(k*R*T*25037)
Ma=V/c
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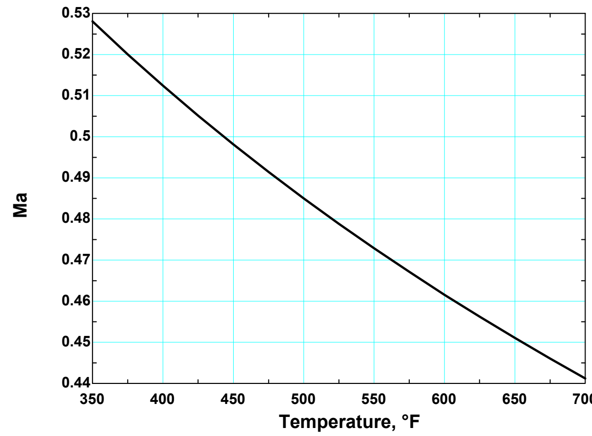
Temperature, T, °F	Mach number Ma
350	0.528
375	0.520
400	0.512
425	0.505
450	0.498
475	0.491
500	0.485
525	0.479
550	0.473
575	0.467
600	0.462
625	0.456
650	0.451
675	0.446
700	0.441



Discussion Note that for a specified flow speed, the Mach number decreases with increasing temperature, as expected.

12-26 The expression for the speed of sound for an ideal gas is to be obtained using the isentropic process equation and the definition of the speed of sound.

Analysis The isentropic relation $Pv^k = A$ where A is a constant can also be expressed as

$$P = A\left(\frac{1}{v}\right)^k = A\rho^k$$

Substituting it into the relation for the speed of sound,

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = \left(\frac{\partial (A\rho)^k}{\partial \rho}\right)_s = kA\rho^{k-1} = k(A\rho^k)/\rho = k(P/\rho) = kRT$$

since for an ideal gas $P = \rho RT$ or $RT = P/\rho$. Therefore,

$$c = \sqrt{kRT}$$

which is the desired relation.

12-27 The inlet state and the exit pressure of air are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.287 kJ/kg·K and k = 1.4. The specific heat ratio k varies with temperature, but in our case this change is very small and can be disregarded.

Analysis The final temperature of air is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (333.2\text{ K})\left(\frac{0.4\text{ MPa}}{1.5\text{ MPa}}\right)^{(1.4-1)/1.4} = 228.4\text{ K}$$

Treating k as a constant, the ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1RT_1}}{\sqrt{k_2RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{333.2}}{\sqrt{228.4}} = \mathbf{1.21}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

12-28 The inlet state and the exit pressure of helium are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Helium is an ideal gas with constant specific heats at room temperature.

Properties The properties of helium are R = 2.0769 kJ/kg·K and k = 1.667.

Analysis The final temperature of helium is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (333.2\text{ K})\left(\frac{0.4}{1.5}\right)^{(1.667-1)/1.667} = 196.3\text{ K}$$

The ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1RT_1}}{\sqrt{k_2RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{333.2}}{\sqrt{196.3}} = \mathbf{1.30}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

12-29E The inlet state and the exit pressure of air are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.06855 Btu/lbm·R and k = 1.4. The specific heat ratio k varies with temperature, but in our case this change is very small and can be disregarded.

Analysis The final temperature of air is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (659.7\text{ R})\left(\frac{60}{170}\right)^{(1.4-1)/1.4} = 489.9\text{ R}$$

Treating k as a constant, the ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1RT_1}}{\sqrt{k_2RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{659.7}}{\sqrt{489.9}} = \mathbf{1.16}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

One Dimensional Isentropic Flow

12-30C (*a*) The exit velocity remain constant at sonic speed, (*b*) the mass flow rate through the nozzle decreases because of the reduced flow area.

12-31C (*a*) The velocity will decrease, (*b*), (*c*), (*d*) the temperature, the pressure, and the density of the fluid will increase.

12-32C (*a*) The velocity will increase, (*b*), (*c*), (*d*) the temperature, the pressure, and the density of the fluid will decrease.

12-33C (*a*) The velocity will increase, (*b*), (*c*), (*d*) the temperature, the pressure, and the density of the fluid will decrease.

12-34C (*a*) The velocity will decrease, (*b*), (*c*), (*d*) the temperature, the pressure and the density of the fluid will increase.

12-35C They will be identical.

12-36C No, it is not possible.

12-37 Air enters a converging-diverging nozzle at specified conditions. The lowest pressure that can be obtained at the throat of the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air at room temperature is $k = 1.4$.

Analysis The lowest pressure that can be obtained at the throat is the critical pressure P^*, which is determined from

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1.2\text{ MPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{0.634\ MPa}$$

Discussion This is the pressure that occurs at the throat when the flow past the throat is supersonic.

12-38 Helium enters a converging-diverging nozzle at specified conditions. The lowest temperature and pressure that can be obtained at the throat of the nozzle are to be determined.

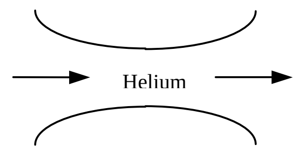
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are k = 1.667 and cBB$_p$ = 5.1926 kJ/kg·K.

Analysis The lowest temperature and pressure that can be obtained at the throat are the critical temperature T* and critical pressure P*. First we determine the stagnation temperature T_0 and stagnation pressure P_0,

$$T_0 = T + \frac{V^2}{2c_p} = 800\text{ K} + \frac{(100\text{ m/s})^2}{2 \times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 801\text{ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.7\text{ MPa})\left(\frac{801\text{ K}}{800\text{ K}}\right)^{1.667/(1.667-1)} = 0.702\text{ MPa}$$



Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (801\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{601\ K}$$

and

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.702\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.342\ MPa}$$

Discussion These are the temperature and pressure that will occur at the throat when the flow past the throat is supersonic.

12-39 The critical temperature, pressure, and density of air and helium are to be determined at specified conditions.

Assumptions Air and Helium are ideal gases with constant specific heats at room temperature.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, k = 1.4, and c_p = 1.005 kJ/kg·K. The properties of helium at room temperature are R = 2.0769 kJ/kg·K, k = 1.667, and c_p = 5.1926 kJ/kg·K.

Analysis (*a*) Before we calculate the critical temperature T^*, pressure P^*, and density ρ^*, we need to determine the stagnation temperature T_0, pressure P_0, and density ρ_0.

$$T_0 = 100°\text{C} + \frac{V^2}{2c_p} = 100 + \frac{(250\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot°\text{C}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 131.1°\text{C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{404.3\ \text{K}}{373.2\ \text{K}}\right)^{1.4/(1.4-1)} = 264.7\ \text{kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{264.7\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(404.3\ \text{K})} = 2.281\ \text{kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (404.3\ \text{K})\left(\frac{2}{1.4+1}\right) = \mathbf{337\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (264.7\ \text{kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{140\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (2.281\ \text{kg/m}^3)\left(\frac{2}{1.4+1}\right)^{1/(1.4-1)} = \mathbf{1.45\ kg/m^3}$$

(*b*) For helium, $T_0 = T + \frac{V^2}{2c_p} = 40 + \frac{(300\ \text{m/s})^2}{2\times 5.1926\ \text{kJ/kg}\cdot°\text{C}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 48.7°\text{C}$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{321.9\ \text{K}}{313.2\ \text{K}}\right)^{1.667/(1.667-1)} = 214.2\ \text{kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{214.2\ \text{kPa}}{(2.0769\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(321.9\ \text{K})} = 0.320\ \text{kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (321.9\ \text{K})\left(\frac{2}{1.667+1}\right) = \mathbf{241\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{97.4\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (0.320\ \text{kg/m}^3)\left(\frac{2}{1.667+1}\right)^{1/(1.667-1)} = \mathbf{0.208\ kg/m^3}$$

Discussion These are the temperature, pressure, and density values that will occur at the throat when the flow past the throat is supersonic.

12-40 Quiescent carbon dioxide at a given state is accelerated isentropically to a specified Mach number. The temperature and pressure of the carbon dioxide after acceleration are to be determined.

Assumptions Carbon dioxide is an ideal gas with constant specific heats at room temperature.

Properties The specific heat ratio of the carbon dioxide at room temperature is $k = 1.288$.

Analysis The inlet temperature and pressure in this case is equivalent to the stagnation temperature and pressure since the inlet velocity of the carbon dioxide is said to be negligible. That is, $T_0 = T_i = 400$ K and $P_0 = P_i = 800$ kPa. Then,

$$T = T_0\left(\frac{2}{2+(k-1)\mathrm{Ma}^2}\right) = (400\ \mathrm{K})\left(\frac{2}{2+(1.288-1)(0.6)^2}\right) = \mathbf{380\ K}$$

and

$$P = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (800\ \mathrm{kPa})\left(\frac{380\ \mathrm{K}}{400\ \mathrm{K}}\right)^{1.288/(1.288-1)} = \mathbf{636\ kPa}$$

Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

12-41 Air flows through a duct. The state of the air and its Mach number are specified. The velocity and the stagnation pressure, temperature, and density of the air are to be determined. √EES

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air at room temperature are $R = 0.287$ kPa.m^3/kg.K and $k = 1.4$.

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(373.2\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 387.2\ \mathrm{m/s}$$

Thus,

$$V = \mathrm{Ma}\times c = (0.8)(387.2\ \mathrm{m/s}) = \mathbf{310\ m/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{200\ \mathrm{kPa}}{(0.287\ \mathrm{kPa\cdot m^3/kg\cdot K})(373.2\ \mathrm{K})} = 1.867\ \mathrm{kg/m^3}$$

Then the stagnation properties are determined from

$$T_0 = T\left(1+\frac{(k-1)\mathrm{Ma}^2}{2}\right) = (373.2\ \mathrm{K})\left(1+\frac{(1.4-1)(0.8)^2}{2}\right) = \mathbf{421\ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \mathrm{kPa})\left(\frac{421.0\ \mathrm{K}}{373.2\ \mathrm{K}}\right)^{1.4/(1.4-1)} = \mathbf{305\ kPa}$$

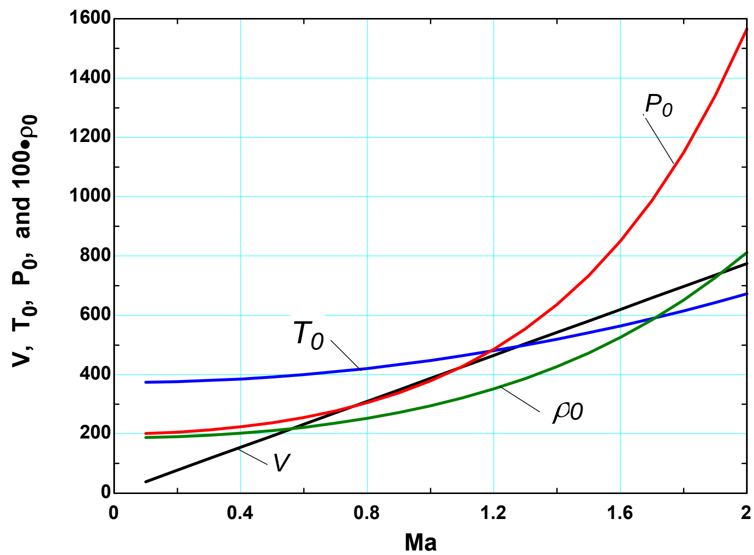
$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (1.867\ \mathrm{kg/m^3})\left(\frac{421.0\ \mathrm{K}}{373.2\ \mathrm{K}}\right)^{1/(1.4-1)} = \mathbf{2.52\ kg/m^3}$$

Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

12-42 Problem 12-41 is reconsidered. The effect of Mach number on the velocity and stagnation properties as the Ma is varied from 0.1 to 2 are to be investigated, and the results are to be plotted.

Analysis Using EES, the problems is solved as follows:

```
P=200
T=100+273.15
R=0.287
k=1.4
c=SQRT(k*R*T*1000)
Ma=V/c
rho=P/(R*T)

"Stagnation properties"
T0=T*(1+(k-1)*Ma^2/2)
P0=P*(T0/T)^(k/(k-1))
rho0=rho*(T0/T)^(1/(k-1))
```



Mach num. Ma	Velocity, V, m/s	Stag. Temp, T_0, K	Stag. Press, P_0, kPa	Stag. Density, ρ_0, kg/m^3
0.1	38.7	373.9	201.4	1.877
0.2	77.4	376.1	205.7	1.905
0.3	116.2	379.9	212.9	1.953
0.4	154.9	385.1	223.3	2.021
0.5	193.6	391.8	237.2	2.110
0.6	232.3	400.0	255.1	2.222
0.7	271.0	409.7	277.4	2.359
0.8	309.8	420.9	304.9	2.524
0.9	348.5	433.6	338.3	2.718
1.0	387.2	447.8	378.6	2.946
1.1	425.9	463.5	427.0	3.210
1.2	464.7	480.6	485.0	3.516
1.3	503.4	499.3	554.1	3.867
1.4	542.1	519.4	636.5	4.269
1.5	580.8	541.1	734.2	4.728
1.6	619.5	564.2	850.1	5.250
1.7	658.3	588.8	987.2	5.842
1.8	697.0	615.0	1149.2	6.511
1.9	735.7	642.6	1340.1	7.267
2.0	774.4	671.7	1564.9	8.118

Discussion Note that as Mach number increases, so does the flow velocity and stagnation temperature, pressure, and density.

12-43E Air flows through a duct at a specified state and Mach number. The velocity and the stagnation pressure, temperature, and density of the air are to be determined. √EES

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.06855 Btu/lbm.R = 0.3704 psia·ft^3/lbm.R and k = 1.4.

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.06855\ \text{Btu/1bm}\cdot\text{R})(671.7\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/1bm}}\right)} = 1270.4\ \text{ft/s}$$

Thus,

$$V = \text{Ma}\times c = (0.8)(1270.4\ \text{ft/s}) = \mathbf{1016\ \ ft/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{30\ \text{psia}}{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(671.7\ \text{R})} = 0.1206\ \text{1bm/ft}^3$$

Then the stagnation properties are determined from

$$T_0 = T\left(1+\frac{(k-1)\text{Ma}^2}{2}\right) = (671.7\ \text{R})\left(1+\frac{(1.4\text{-}1)(0.8)^2}{2}\right) = \mathbf{758\ \ R}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (30\ \text{psia})\left(\frac{757.7\ \text{R}}{671.7\ \text{R}}\right)^{1.4/(1.4-1)} = \mathbf{45.7\ \ psia}$$

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (0.1206\ \text{1bm/ft}^3)\left(\frac{757.7\ \text{R}}{671.7\ \text{R}}\right)^{1/(1.4-1)} = \mathbf{0.163\ \ 1bm/ft^3}$$

Discussion Note that the temperature, pressure, and density of a gas increases during a stagnation process.

12-44 An aircraft is designed to cruise at a given Mach number, elevation, and the atmospheric temperature. The stagnation temperature on the leading edge of the wing is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.287 kPa.m^3/kg.K, c_p = 1.005 kJ/kg·K, and k = 1.4 .

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(236.15\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 308.0\ \text{m/s}$$

Thus,

$$V = \text{Ma}\times c = (1.4)(308.0\ \text{m/s}) = 431.2\ \text{m/s}$$

Then,

$$T_0 = T + \frac{V^2}{2c_p} = 236.15 + \frac{(431.2\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{329\ \ K}$$

Discussion Note that the temperature of a gas increases during a stagnation process as the kinetic energy is converted to enthalpy.

Isentropic Flow Through Nozzles

12-45C (*a*) The exit velocity will reach the sonic speed, (*b*) the exit pressure will equal the critical pressure, and (*c*) the mass flow rate will reach the maximum value.

12-46C (*a*) None, (*b*) None, and (*c*) None.

12-47C They will be the same.

12-48C Maximum flow rate through a nozzle is achieved when Ma = 1 at the exit of a subsonic nozzle. For all other Ma values the mass flow rate decreases. Therefore, the mass flow rate would decrease if hypersonic velocities were achieved at the throat of a converging nozzle.

12-49C Ma* is the local velocity non-dimensionalized with respect to the sonic speed at the throat, whereas Ma is the local velocity non-dimensionalized with respect to the local sonic speed.

12-50C The fluid would accelerate even further instead of decelerating.

12-51C The fluid would decelerate instead of accelerating.

12-52C (*a*) The velocity will decrease, (*b*) the pressure will increase, and (*c*) the mass flow rate will remain the same.

12-53C No. If the velocity at the throat is subsonic, the diverging section will act like a diffuser and decelerate the flow.

12-54 It is to be explained why the maximum flow rate per unit area for a given ideal gas depends only on $P_0/\sqrt{T_0}$. Also for an ideal gas, a relation is to be obtained for the constant a in $\dot{m}_{max}/A^* = a\left(P_0/\sqrt{T_0}\right)$.

Properties The properties of the ideal gas considered are R = 0.287 kPa.m^3/kg·K and k = 1.4.

Analysis The maximum flow rate is given by

$$\dot{m}_{\max} = A^* P_0 \sqrt{k/RT_0}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

or

$$\dot{m}_{\max}/A^* = \left(P_0/\sqrt{T_0}\right)\sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

For a given gas, k and R are fixed, and thus the mass flow rate depends on the parameter $P_0/\sqrt{T_0}$.

$\dot{m}_{\max}/A^*$ can be expressed as a $\dot{m}_{\max}/A^* = a\left(P_0/\sqrt{T_0}\right)$ where

$$a = \sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} = \sqrt{\frac{1.4}{(0.287\text{ kJ/kg.K})\left(\dfrac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}}\left(\frac{2}{1.4+1}\right)^{2.4/0.8} = 0.0404\text{ (m/s)}\sqrt{\text{K}}$$

Discussion Note that when sonic conditions exist at a throat of known cross-sectional area, the mass flow rate is fixed by the stagnation conditions.

12-55 For an ideal gas, an expression is to be obtained for the ratio of the speed of sound where Ma = 1 to the speed of sound based on the stagnation temperature, c*/c_0.

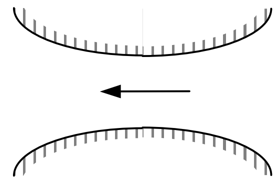
Analysis For an ideal gas the speed of sound is expressed as $c = \sqrt{kRT}$. Thus,

$$\frac{c^*}{c_0} = \frac{\sqrt{kRT^*}}{\sqrt{kRT_0}} = \left(\frac{T^*}{T_0}\right)^{1/2} = \left(\frac{2}{k+1}\right)^{1/2}$$

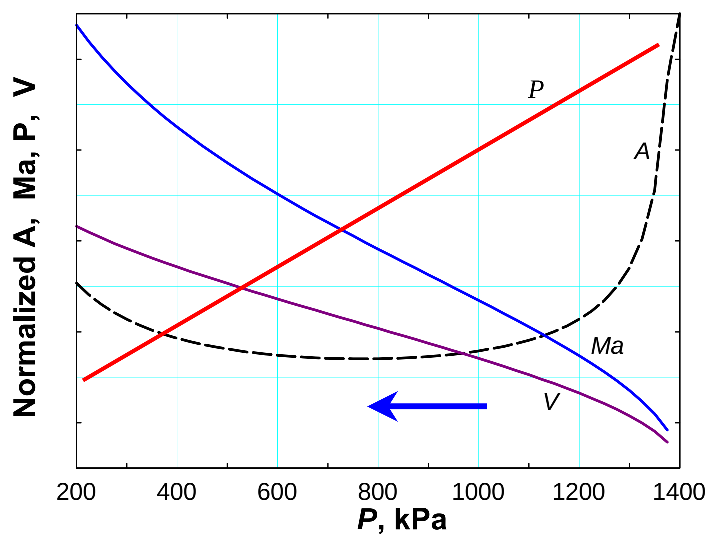
Discussion Note that a speed of sound changes the flow as the temperature changes.

12-56 For subsonic flow at the inlet, the variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.

Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure drops from a stagnation value of 1400 kPa to 200 kPa. Note that the curve for *A* represents the shape of the nozzle, with horizontal axis serving as the centerline.

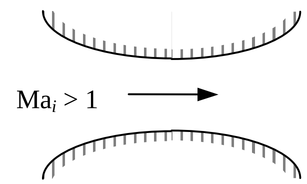



```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```

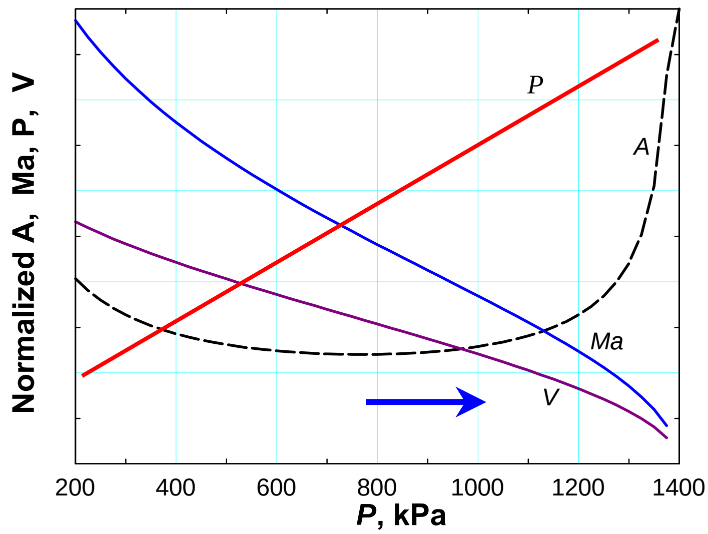


12-57 For supersonic flow at the inlet, the variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.

Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure rises from 200 kPa at a very high velocity to the stagnation value of 1400 kPa. Note that the curve for *A* represents the shape of the nozzle, with horizontal axis serving as the centerline.




```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```



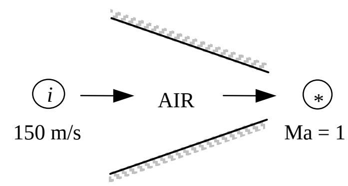
Discussion Note that this problem is identical to the proceeding one, except the flow direction is reversed.

12-58 Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4 and c_p = 1.005 kJ/kg·K.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.



$$T_0 = T_i + \frac{V_i^2}{2c_p} = 350\text{ K} + \frac{(150\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 361.2\text{ K}$$

and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (0.2\text{ MPa})\left(\frac{361.2\text{ K}}{350\text{ K}}\right)^{1.4/(1.4-1)} = 0.223\text{ MPa}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma = 1, we read T/T_0 = 0.8333, P/P_0 = 0.5283. Thus,

$T = 0.8333T_0 = 0.8333(361.2\text{ K}) = \mathbf{301\ K}$

and

$P = 0.5283P_0 = 0.5283(0.223\text{ MPa}) = \mathbf{0.118\ MPa}$

Also,

$$c_i = \sqrt{kRT}_i = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(350\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 375\text{ m/s}$$

and

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{150\text{ m/s}}{375\text{ m/s}} = 0.40$$

From Table A-13 at this Mach number we read A_i/A^* = 1.5901. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{1.5901} = \mathbf{0.629}$$

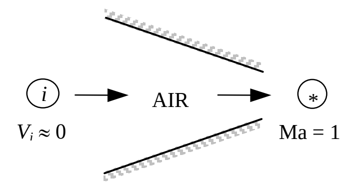
Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-59 Air enters a nozzle at specified temperature and pressure with low velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is k = 1.4.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.



$$T_0 = T_\mathrm{i} = 350\ \mathrm{K}$$

$$P_0 = P_\mathrm{i} = 0.2\ \mathrm{MPa}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma =1,

we read T/T_0 =0.8333, P/P_0 = 0.5283. Thus,

$$T = 0.8333T_0 = 0.8333(350\ \mathrm{K}) = \mathbf{292\ K}$$

and

$$P = 0.5283P_0 = 0.5283(0.2\ \mathrm{MPa}) = \mathbf{0.106\ MPa}$$

The Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. From Table A-13 at this Mach number we read $A_\mathrm{i}/A^* = \infty$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

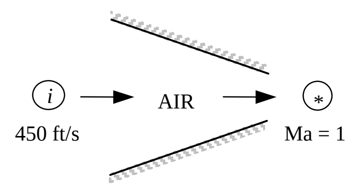
12-60E Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4 and c_p = 0.240 Btu/lbm·R (Table A-2Ea).

Analysis The properties of the fluid at the location where Ma =1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T + \frac{V_i^2}{2c_p} = 630\text{ R} + \frac{(450\text{ ft/s})^2}{2\times 0.240\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/1bm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 646.9\text{ R}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (30\text{ psia})\left(\frac{646.9\text{ K}}{630\text{ K}}\right)^{1.4/(1.4-1)} = 32.9\text{ psia}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283. Thus,

T = 0.8333T_0 = 0.8333(646.9 R) = **539 R**

and

P = 0.5283P_0 = 0.5283(32.9 psia) = **17.4 psia**

Also,

$$c_i = \sqrt{kRT}_i = \sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(630\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1230\text{ ft/s}$$

and

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{450\text{ ft/s}}{1230\text{ ft/s}} = 0.3657$$

From Table A-13 at this Mach number we read A_i/A^* = 1.7426. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{1.7426} = \mathbf{0.574}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-61 Air enters a converging-diverging nozzle at a specified pressure. The back pressure that will result in a specified exit Mach number is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

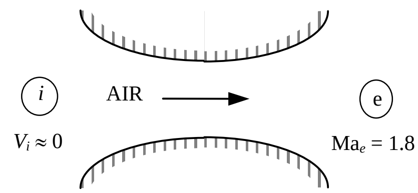
Properties The specific heat ratio of air is k = 1.4.

Analysis The stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. It remains constant throughout the nozzle since the flow is isentropic,

$$P_0 = P_i = 0.8 \text{ MPa}$$

From Table A-13 at Ma_e =1.8, we read P_e/P_0 = 0.1740.

Thus, $P = 0.1740P_0 = 0.1740(0.8 \text{ MPa}) = $ **0.139 MPa**



Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-62 Nitrogen enters a converging-diverging nozzle at a given pressure. The critical velocity, pressure, temperature, and density in the nozzle are to be determined.

Assumptions **1** Nitrogen is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

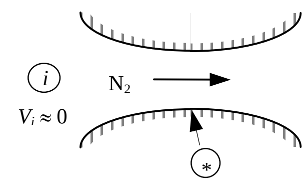
Properties The properties of nitrogen are k = 1.4 and R = 0.2968 kJ/kg·K.

Analysis The stagnation pressure in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle,

$P_0 = P_i$ = 700 kPa

$T_0 = T_i$ = 400 K

$$\rho_0 = \frac{P_0}{RT_0} = \frac{700\text{ kPa}}{(0.2968\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(400\text{ K})} = 5.896\text{ kg/m}^3$$



Critical properties are those at a location where the Mach number is Ma = 1. From Table A-13 at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283, and ρ/ρ_0 = 0.6339. Then the critical properties become

$T^* = 0.8333T_0$ = 0.8333(400 K) = **333 K**

$P^* = 0.5283P_0$ = 0.5283(700 kPa) = **370 MPa**

$\rho^* = 0.6339\rho_0$ = 0.6339(5.896 kg/m^3) = **3.74 kg/m^3**

Also,

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.4)(0.2968\text{ kJ/kg}\cdot\text{K})(333\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{372\text{ m/s}}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-63 An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 2.4 is specified. The flow area where Ma = 1.2 is to be determined. √EES

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio is given to be k = 1.4.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where Ma_2 = 1.2 is determined using A/A^* data from Table A-13 to be

$$\text{Ma}_1 = 2.4: \frac{A_1}{A^*} = 2.4031 \longrightarrow A^* = \frac{A_1}{2.4031} = \frac{25\text{ cm}^2}{2.4031} = 10.40\text{ cm}^2$$

$$\text{Ma}_2 = 1.2: \frac{A_2}{A^*} = 1.0304 \longrightarrow A_2 = (1.0304)A^* = (1.0304)(10.40\text{ cm}^2) = \mathbf{10.7\text{ cm}^2}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-64 An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 2.4 is specified. The flow area where Ma = 1.2 is to be determined. √EES

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where $\mathrm{Ma}_2 = 1.2$ is determined using the A/A^* relation,

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left\{\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right\}^{(k+1)/2(k-1)}$$

For $k = 1.33$ and $\mathrm{Ma}_1 = 2.4$:

$$\frac{A_1}{A^*} = \frac{1}{2.4}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}2.4^2\right)\right\}^{2.33/2\times 0.33} = 2.570$$

and, $$A^* = \frac{A_1}{2.570} = \frac{25\ \mathrm{cm}^2}{2.570} = 9.729\ \mathrm{cm}^2$$

For $k = 1.33$ and $\mathrm{Ma}_2 = 1.2$:

$$\frac{A_2}{A^*} = \frac{1}{1.2}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}1.2^2\right)\right\}^{2.33/2\times 0.33} = 1.0316$$

and $$A_2 = (1.0316)A^* = (1.0316)(9.729\ \mathrm{cm}^2) = \mathbf{10.0\ cm^2}$$

Discussion Note that the compressible flow functions in Table A-13 are prepared for $k = 1.4$, and thus they cannot be used to solve this problem.

12-65 [*Also solved by EES on enclosed CD***]** Air enters a converging nozzle at a specified temperature and pressure with low velocity. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure are to be calculated and plotted.

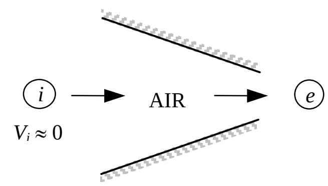
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4, R = 0.287 kJ/kg·K, and c_p = 1.005 kJ/kg·K.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.,

$$P_0 = P_i = 900 \text{ kPa}$$

$$T_0 = T_i = 400 \text{ K}$$



The critical pressure is determined to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (900 \text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 475.5 \text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$P_e = P_b$ for $P_b \geq 475.5$ kPa

$P_e = P^* = 475.5$ kPa for $P_b < 475.5$ kPa (choked flow)

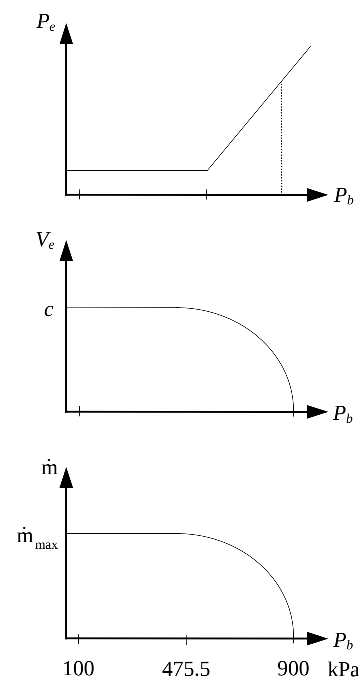
Thus the back pressure will not affect the flow when $100 < P_b < 475.5$ kPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from

Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (400 \text{ K})\left(\frac{P_e}{900}\right)^{0.4/1.4}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005 \text{ kJ/kg}\cdot\text{K})(400 - T_e)\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287 \text{ kPa}\cdot\text{m}^3/kg\cdot K)T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001 \text{ m}^2)$$



The results of the calculations can be tabulated as

P_b, kPa	P_e, kPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	$\dot{m}$, kg / s
900	900	400	0	7.840	0
800	800	386.8	162.9	7.206	1.174
700	700	372.3	236.0	6.551	1.546
600	600	356.2	296.7	5.869	1.741
500	500	338.2	352.4	5.151	1.815
475.5	475.5	333.3	366.2	4.971	1.820
400	475.5	333.3	366.2	4.971	1.820
300	475.5	333.3	366.2	4.971	1.820
200	475.5	333.3	366.2	4.971	1.820
100	475.5	333.3	366.2	4.971	1.820

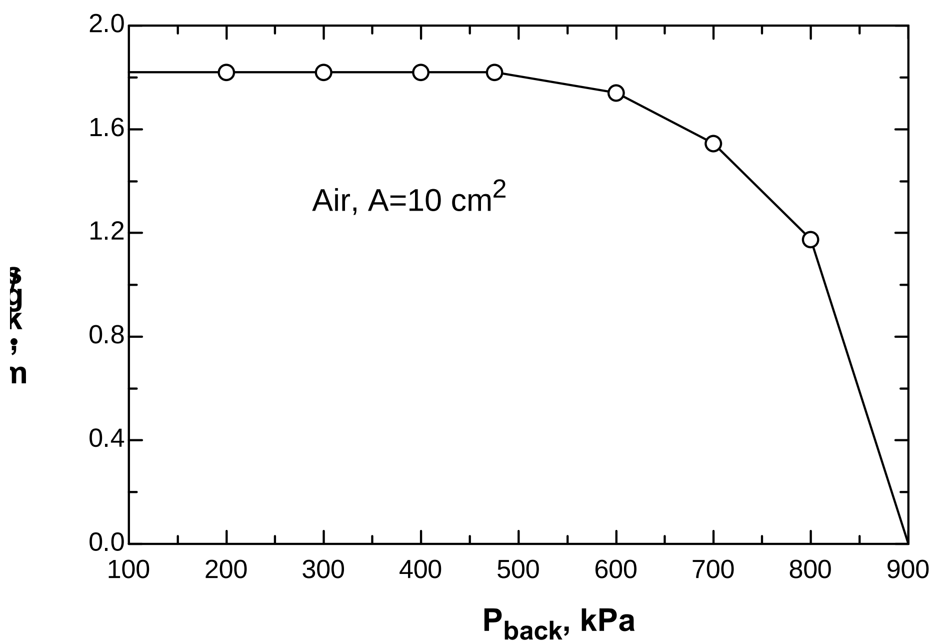
12-66 Reconsider Prob. 16-65. Using EES (or other) software, solve the problem for the inlet conditions of 1 MPa and 1000 K.

```
Air at 900 kPa, 400 K enters a converging nozzle with a negligible velocity.
The throat area of the nozzle is 10 cm2.  Assuming isentropic flow, calculate
and plot the exit pressure, the exit velocity, and the mass flow rate versus the
back pressure Pb for 0.9>= Pb >=0.1 MPa."

Procedure ExitPress(P_back,P_crit : P_exit, Condition$)
If  (P_back>=P_crit)  then
        P_exit:=P_back                  "Unchoked Flow Condition"
        Condition$:='unchoked'
else
        P_exit:=P_crit                  "Choked Flow Condition"
        Condition$:='choked'
Endif
End

"Input data from Diagram Window"
{Gas$='Air'
A_cm2=10                        "Throat area, cm2"
P_inlet = 900"kPa"
T_inlet= 400"K"}
{P_back =475.5  "kPa"}

A_exit = A_cm2*Convert(cm^2,m^2)
C_p=specheat(Gas$,T=T_inlet)
C_p-C_v=R
k=C_p/C_v
M=MOLARMASS(Gas$)               "Molar mass of  Gas$"
R= 8.314/M                      "Gas constant for Gas$"

"Since the inlet velocity is negligible, the stagnation temperature = T_inlet;
and, since the nozzle is isentropic, the stagnation pressure = P_inlet."

P_o=P_inlet                     "Stagnation pressure"
T_o=T_inlet                     "Stagnation temperature"
P_crit /P_o=(2/(k+1))^(k/(k-1))         "Critical pressure from Eq. 16-22"
Call ExitPress(P_back,P_crit : P_exit, Condition$)

T_exit /T_o=(P_exit/P_o)^((k-1)/k)      "Exit temperature for isentopic flow, K"

V_exit ^2/2=C_p*(T_o-T_exit)*1000       "Exit velocity, m/s"

Rho_exit=P_exit/(R*T_exit)              "Exit density, kg/m3"

m_dot=Rho_exit*V_exit*A_exit            "Nozzle mass flow rate, kg/s"

"If you wish to redo the plots, hide the diagram window and remove the { } from
the first 4 variables just under the procedure.  Next set the desired range of
back pressure in the parametric table.  Finally, solve the table (F3).  "
```

SOLUTION

A_cm2=10 [cm^2]
A_exit=0.001 [m^2]
Condition$='choked'
C_p=1.14 [kJ/kg-K]
C_v=0.8532 [kJ/kg-K]
Gas$='Air'
k=1.336
M=28.97 [kg/kmol]
m_dot=1.258 [kg/s]
P_back=300 [kPA]
P_crit=539.2 [kPA]
P_exit=539.2 [kPA]
P_inlet=1000 [kPA]
P_o=1000 [kPA]
R=0.287 [kJ/kg-K]
Rho_exit=2.195 [m^3/kg]
T_exit=856 [K]
T_inlet=1000 [K]
T_o=1000 [K]
V_exit=573 [m/s]

m [kg/s]	P_{exit} [kPa]	T_{exit} [K]	V_{exit} [m/s]	ρ_{exit} [kg/m^3]	P_{back} [kPa]
1.819	475.5	333.3	366.1	4.97	100
1.819	475.5	333.3	366.1	4.97	200
1.819	475.5	333.3	366.1	4.97	300
1.819	475.5	333.3	366.1	4.97	400
1.819	475.5	333.3	366	4.97	475.5
1.74	600	356.2	296.6	5.868	600
1.546	700	372.3	236	6.551	700
1.176	800	386.8	163.1	7.207	800
0	900	400	0	7.839	900



12-67E Air enters a converging-diverging nozzle at a specified temperature and pressure with low velocity. The pressure, temperature, velocity, and mass flow rate are to be calculated in the specified test section.

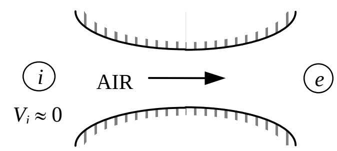
Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4 and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$P_0 = P_i = 150 \text{ psia}$$

$$T_0 = T_i = 100°\text{F} = 560 \text{ R}$$



Then,

$$T_e = T_0\left(\frac{2}{2+(k-1)\text{Ma}^2}\right) = (560\text{ R})\left(\frac{2}{2+(1.4\text{-}1)2^2}\right) = \mathbf{311\,R}$$

and

$$P_e = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (150\text{ psia})\left(\frac{311}{560}\right)^{1.4/0.4} = \mathbf{19.1\ psia}$$

Also,

$$\rho_e = \frac{P_e}{RT_e} = \frac{19.1\text{ psia}}{(0.3704\text{ psia.ft}^3/1\text{bm}\cdot\text{R})(311\text{ R})} = 0.166\,1\text{bm/ft}^3$$

The nozzle exit velocity can be determined from V_e = Ma$_e c_e$, where c_e is the speed of sound at the exit conditions,

$$V_e = \text{Ma}_e c_e = \text{Ma}_e\sqrt{kRT_e} = (2)\sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(311\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = \mathbf{1729\ \ ft/s}$$

Finally,

$$\dot{m} = \rho_e A_e V_e = (0.\,166\text{ 1bm/ft}^3)(5\text{ ft}^2)(1729\text{ ft/s}) = \mathbf{1435\ 1bm/s}$$

Discussion Air must be very dry in this application because the exit temperature of air is extremely low, and any moisture in the air will turn to ice particles.