EXAMPLE - A thin rubber membrane is stretched in such a way that the following strains are measured (plane strain)

$$\begin{pmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y \end{pmatrix} = \begin{bmatrix} 6000 & \frac{3000}{2} \\ \frac{3000}{2} & -5000 \end{bmatrix} \times 10^{-6}$$

How should we orient a rectangular section of the membrane such that the 90° angle of the rectangle remain at 90° after the deformation?

We are looking here for the principal axes since in the principal coordinate system the shear strains are zero hence no change in the angle between the x' & y' principal axes. As an eigenvalue problem

$$\begin{bmatrix} 6-\lambda & 1.5 \\ 1.5 & -5-\lambda \end{bmatrix} \times 10^{-3} = 0$$

$$\lambda^2 - 30 + 5\lambda - 6\lambda - 2.25 = 0$$

$$\lambda^2 - \lambda - 32.25 = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1+129}}{2} = \frac{1 \pm \sqrt{130}}{2} = 6.201, \; -5.201$$

For principal axes

For $\lambda = 6.201$

$$\begin{bmatrix} 6-6.201 & 1.5 \\ 1.5 & -5-6.201 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = 0$$

$$-.201x_1 + 1.5x_2 = 0$$

$$x_1 = 7.4627x_2$$

For unit norm

$$x_1^2 + x_2^2 = 1$$

$$(7.4627x_2)^2 + x_2^2 = 1$$

$$\sin\theta = x_2 = .1328 \qquad x_1 = .99111 = \cos\theta$$

$$\theta = 7.672^\circ$$

Alternatively

$$\tan 2\theta = \frac{2\epsilon_{12}}{\epsilon_1 - \epsilon_2} = \frac{\gamma_{12}}{\epsilon_1 - \epsilon_2} = \frac{3}{6 - -5} = \frac{3}{11}$$

$$2\theta = 15.255^\circ$$

$$\theta = 7.63^\circ$$

Alternately as an eigenvalue problem

$$\det\begin{bmatrix} 1-\lambda & \frac{1}{2} \\ \frac{1}{2} & 4-\lambda \end{bmatrix} \times 10^{-3} = 0$$

$$(1-\lambda)(4-\lambda) - .25 = 0$$

$$4 - 5\lambda + \lambda^2 - .25 = 0$$

$$\lambda^2 - 5\lambda + 3.75 = 0$$

$$\lambda_{1,2} = \frac{5 \pm \sqrt{25-15}}{2} = 4.081,\ .9188$$

For eigenvectors

$$\lambda_2 \quad \begin{pmatrix} 1-.9188 & \frac{1}{2} \\ \frac{1}{2} & 4-.9188 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$.08114\, x_1 + .5\, x_2 = 0$$

$$x_1 = -6.1623\, x_2$$

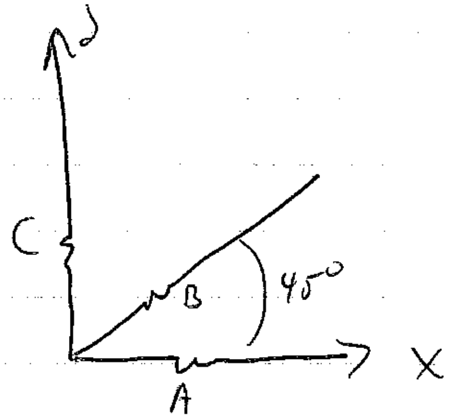
For unit normal

$$x_1^2 + x_2^2 = 1 \qquad x_2^2 + 6.1623 x_2^2 = 1$$

$$\pm \sin\theta = x_2 = .16018 \qquad x_1 = .9871 = \cos\theta$$

$$\theta = -9.212^\circ$$

EXAMPLE: THE STRAIN GAGE ROSETTE BELOW RECORDS THE FOLLOWING STRAINS



$\epsilon_A = .001$
$\epsilon_B = .003$
$\epsilon_C = .004$

a) Determine the plane strain components ϵ_x, ϵ_y and γ_{xy}

b) Determine the principal strains and the orientation of the principal axes

$$\epsilon_A = \epsilon_x \cos^2 0^\circ + \epsilon_y \sin^2 0^\circ + 2\sin 0 \cos 0\, \epsilon_{xy}$$

$$\Rightarrow \quad \epsilon_x = .001$$

$$\epsilon_C = \epsilon_x \cos^2 90^\circ + \epsilon_y \sin^2 90^\circ + 2\sin 90 \cos 90\, \epsilon_{xy}$$

$$\Rightarrow \quad \epsilon_y = .004$$

$$\epsilon_B = \epsilon_{45^\circ} = \epsilon_x \cos^2 45^\circ + \epsilon_y \sin^2 45^\circ + 2\sin 45 \cos 45\, \epsilon_{xy}$$

$$.003 = .001\left(\tfrac{1}{2}\right) + .004\left(\tfrac{1}{2}\right) + \frac{\gamma_{xy}}{2}$$

$$.001 = \gamma_{xy} = 2\epsilon_{xy}$$

b) $\tan 2\theta = \frac{2\epsilon_{xy}}{\epsilon_x - \epsilon_y} = \frac{.001}{.001 - .004}$

$2\theta = -18.435$

$\theta = -9.217$

$\epsilon_x' = (.001)\cos^2\theta + (.004)\sin^2\theta + (.001)\sin\theta\cos\theta$

$= [.974 + .1026 - .1581] \times 10^{-3}$

$= .9185 \times 10^{-3}$

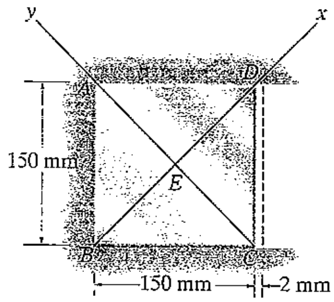
$\epsilon_y' = .001(\sin^2\theta) + .004(\cos^2\theta) - .001\sin\theta\cos\theta$

$= [.0257 + 3.897 + .1581] \times 10^{-3}$

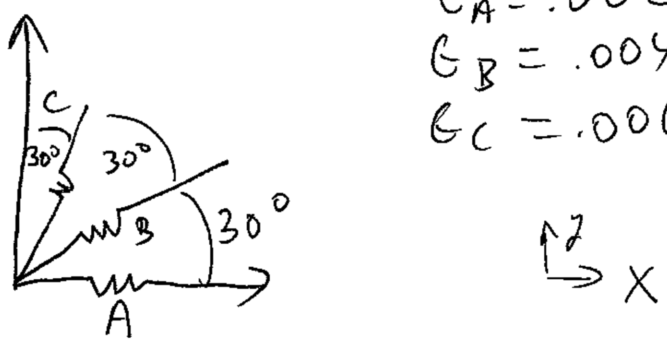
$= 4.0812 \times 10^{-3}$

ME 340 HW - Strain

1. The plate shown in Fig. 2–7*a* is fixed connected along *AB* and held in the horizontal guides at its top and bottom, *AD* and *BC*. If its right side *CD* is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal *AC*, and (b) the shear strain at *E* relative to the *x*, *y* axes.



2. The strain gage rosette shown records the following strains



$\epsilon_A = .002$
$\epsilon_B = .004$
$\epsilon_C = .006$

a) Determine the strain in the xy coordinate system ϵ_x, ϵ_y and γ_{xy}
b) Determine the principal strains
c) Determine the orientation of the principal axes

3. What strains would the following gages record for $\epsilon_x = .004$, $\epsilon_y = .009$, $\gamma_{xy} = .005$