DYNAMIC SCHEMATIZATION OF THE FORCE FUNCTION OF AN INTERNAL-COMBUSTION ENGINE

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Dynamic schematization and the mathematical description of the process of controlling the force function of an internal combusion engine are the most complex and least developed questions in problems involving the dynamics of sets of engines with internal combustion engines. We will understand below by a force function the dependence of the engine torque on the crank angle, i.e., the force function has the meaning of a generalized force.

An analytic description of the force function of an internal combustion engine is given only for the partial dynamics-analysis problem for the residence characteristics in steady-state velocity conditions in existing techniques of dynamically calculating sets of engines [5]. In this case the reliability of results from studying the amplitude and phase spectra of a motor unit is ensured only for engines for which extensive experimental data have been accumulated. We note that the analysis of nonresonance conditions, for which the representation of the force function in the form of a Fourier series is an unnatural and cumbersome formalization of the actual processes, is also of significant interest in actual dynamic calculations of sets of engines with an internal combustion engine.

Dynamic schematization of the control process for the force function of an internal combustion engine does not take into account the pulsed nature of this process in well-known techniques, such pulsing being able to significantly affect the dynamic stability of the automatic speed regulation system under definite conditions. This is particularly substantial for systems with an oscillating-active control object within the effective frequency range [3].

It is necessary to consider interaction between the thermodynamic and mechanical systems in describing the force function of an internal combustion engine. The processes of the filling of the engine cylinders with a gas charge (air or fuel—air mixture) and the compression of this charge, fuel combustion, expansion of the combustion products, and clearing of the cylinders constitute the fundamental functional meaning of the thermodynamic system. The mechanical system of an internal-combustion engine is characterized by structural complexity and a multitude of relations. However, only the geometric characteristics of the crank gears are of importance for a force analysis of the working process of an internal-combustion engine.

The thermodynamic processes in the engine cylinders generate a force function, or torque, when interacting with the crank gears, this torque being discretely distributed along the length of the crank between its crankshafts. To clarify the basic laws of the force characteristics without restricting the generality of the analysis, let us consider an internal-combustion engine with central crank gears (Fig. 1a). The active thermodynamic processes that basically determine the magnitude and nature of the force function of the internal-combustion engine are realized at the stages of gas-charge compression in the cylinders (compression stroke), fuel combustion, and expansion of combustion products (power stroke).

Gas pressure in the engine cylinder varies, in the course of gas-charge compression during the k-th working cycle, in accordance with the equations [4]

$$p_1(\alpha) = p_{\text{ch}} D^{-n}; \quad \alpha \in [a_k, b_k];$$

$$D = 1 - (1-\varepsilon)\left(\sin^2\frac{\alpha}{2} + \lambda^{-1}\sin^2\frac{\beta}{2}\right); \quad \lambda = \frac{R}{L};$$

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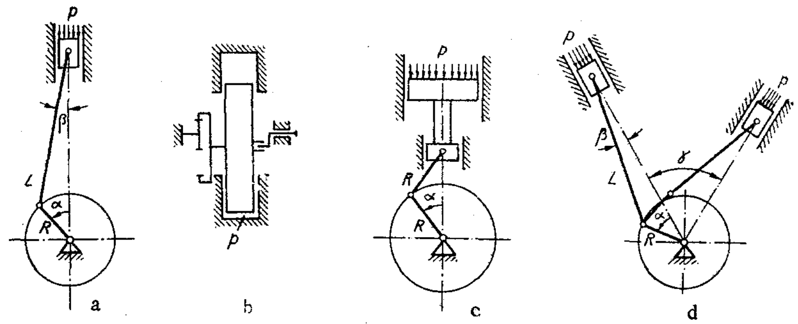

Fig. 1

$$a_k = (2mk - 1)\pi; \quad b_k = 2mk\pi; \quad \beta = \arcsin(\lambda \sin \alpha). \tag{1}$$

Here α is the engine crank angle, counted off from the upper dead point of the piston of the given cylinder, p_{ch} is pressure at the end of the compression stroke, R and L are crankshaft radius and length of the connecting rod of the crank gear, n is the mean value of the compression polytropic exponent, ε and m are the compression and stroke ratio of the internal-combustion engine, m = 1 for two-stroke and m = 2 for four-stroke engines.

The moment of resistance of the gas forces of a single cylinder during the compression stroke is determined from the equation

$$M_1(\alpha) = V_{cy}\Gamma(\alpha)p_1(\alpha); \quad \alpha \in [a_k, b_k], \tag{2}$$

where $V_{cy} = 2RF$ is the working volume of the cylinder, F is piston area, and $\Gamma(\alpha) = 0.5(\sin\alpha + \cos\alpha \tan\beta)$.

Let us call the expression

$$K(\alpha) = \frac{M_1(\alpha)}{p_{ch}V_{cy}} = \frac{\Gamma(\alpha)}{D^n(\alpha)} \text{ when } \alpha \in [a_k, b_k] \tag{3}$$

the dimensionless compression force function of an internal-combustion engine, while

$$K(\alpha) = 0 \text{ when } \alpha \in [1 + 4(k-1)\pi, \ (4k-1)\pi] \tag{4}$$

for four-stroke engines.

The odd function $K(\alpha)$ describes the effect of a conditional incomplete working process realized on the basis of an ideal, reversible thermodynamic cycle, that is, gas-charge compression during the compression stroke; gas expansion in the piston power stroke in the absence of fuel losses and combustion. The laws of the fuel combustion process in the cylinders of an internal-combustion engine are schematized by a conditional power cycle with two-phase combustion [4]. The first phase is realized by placing the piston near the upper dead point and is depicted on the V vs s diagram as an isochoric process at the end of which gas pressure reaches the value

$$p_z = \lambda_z p_{ch} \tag{5}$$

(λ_z is the degree to which gas pressure increases during fuel combustion).

The second phase of fuel combustion is described by an isobaric process with constant pressure p_z and a variation of the current volume of the working fluid V within the limits

$$1 \leqslant v \leqslant \rho, \tag{6}$$

where $v = V/V_{ch}$, V_{ch} is the volume of the combustion chamber, and ρ is the degree of preliminary expansion of combustion products.

Condition (6) is equivalent during the k-th power cycle to the inequality

$$c_k \leqslant \alpha \leqslant c_k + \alpha_z. \tag{7}$$

Here

$$c_k = 2(k-1)m\pi; \quad \alpha_z \simeq 2\sqrt{\frac{\rho - 1}{(\varepsilon - 1)(\lambda + 1)}}.$$

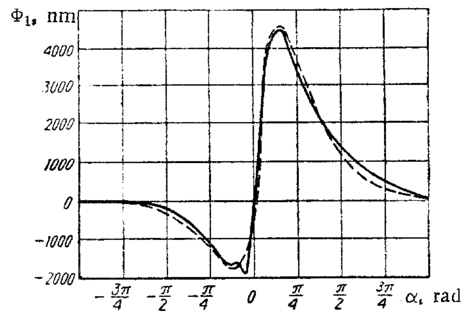

Fig. 2

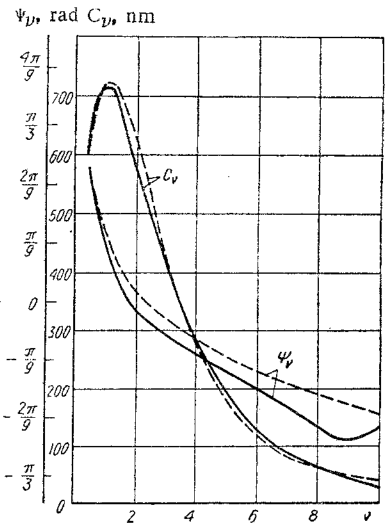

Fig. 3

Gas pressure in a cylinder of an internal-combustion engine is characterized by the dependence

$$p_\rho = \lambda_z \rho^q p_{ch} D^{-q} \text{ when } \alpha \in (c_k + \alpha_z,\ c_k + \pi] \tag{8}$$

(q is the mean value of the expansion of the polytropic exponent) at the end of the fuel combustion process.

The indicated torque of the internal-combustion engine due to gas forces of a single cylinder completing effective positive work as well as the compensating thermodynamic and mechanical losses of the k-th power cycle has the form

$$M_2(\alpha) = V_{cy} \Gamma(\alpha) [p_2(\alpha) - p_1(\alpha)] \text{ when } \alpha \in [c_k, c_k + \pi]. \tag{9}$$

Here

$$p_2(\alpha) = \begin{cases} p_z \text{ when } \alpha \in [c_k, c_k + \alpha_z]; \\ p_\rho \text{ when } \alpha > c_k + \alpha_z. \end{cases}$$

To transform the expression for $M_2(\alpha)$ to a more convenient form, we will use the well-known dependence of the mean indicated pressure of the power cycle of an internal-combustion engine [4];

$$p_i = \frac{\lambda_z p_{ch}}{\varepsilon - 1}\left[\rho - 1 + \frac{\rho}{q-1}(1 - \delta^{1-q}) - \frac{\lambda_z^{-1}}{n-1}(1 - \varepsilon^{1-n})\right], \tag{10}$$

where $\delta = \varepsilon/\rho$.

Setting $\rho = 1$ and $q = n$ when $p_i = 0$ in Eq. (10) from self-evident physical concepts, we determine the indicated torque of an internal-combustion engine due to the gas forces of a single cylinder based on Eqs. (5)-(10),

$$M_2(\alpha) = \frac{\partial M_2}{\partial p_2}\frac{\partial p_2}{\partial \lambda_z}\frac{\partial \lambda_z}{\partial p_i} p_i = p_i V_{cy} \frac{G\Gamma(\alpha)}{\sigma^q}. \tag{11}$$

Here

$$\sigma = \begin{cases} 1 & \text{when } \alpha \in [c_k, c_k + \alpha_z]; \\ D\rho^{-1} & \text{when } \alpha \in (c_k + \alpha_z, c_k + \pi]; \end{cases} \qquad G = \frac{(\varepsilon - 1)(q-1)}{(\rho - 1)(q-1) + \rho(1 - \delta^{1-q})}.$$

We will call the expression

$$S(\alpha) = \begin{cases} \dfrac{M_2(\alpha)}{p_i V_{cy}} = \dfrac{G\Gamma(\alpha)}{\sigma^q} \text{ when } \alpha \in [c_k, c_k + \pi]; \\ 0 \qquad \text{when } \alpha \in (c_k + \pi, b_k) \end{cases} \tag{12}$$

the dimensionless indicated force function of an internal-combustion engine.

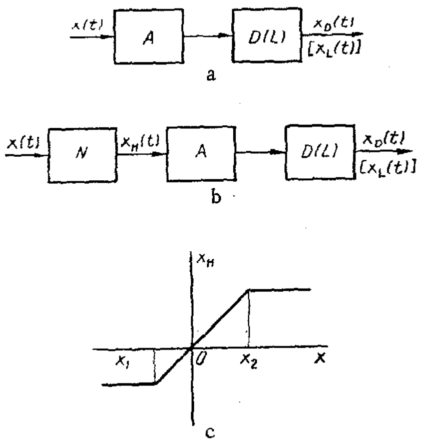

Fig. 4

The force effect of the active indicated power cycle due to fuel combustion followed by expansion of combustion products and conversion of thermal energy into mechanical energy is described based on the function $S(\alpha)$.

Numerical analysis of the functions $K(\alpha)$ and $S(\alpha)$ in the space of the parameters ε, λ, n, q, and ρ for different classes of internal-combustion engines (diesels, carburetor, and gas engines) demonstrated that these functions are not very sensitive to variations in the parameters λ, n, q, and ρ. The engine compression ratio ε exerts the determining influence on the quantitative characteristics of these functions. Our results reasonably agree with experiments that have demonstrated that the compression ratio of the y coordinate of the curves (2) and (9) are connected, correspondingly, with the values p_{ch} and p_i by nearly linear dependences [5] for internal combustion engines of the same type.

Studies have shown that, based on the above, the compression and indicated dimensionless force functions of an internal-combustion engine are approximated to within a high degree of accuracy by the equations

$$K(\alpha) = \varkappa_k [\alpha_x \exp(-\zeta_k \alpha_x) - \alpha_y \exp(-\zeta_k \alpha_y)];$$
$$S(\alpha) = \varkappa_s \alpha_x \exp(-\zeta_s \alpha_x), \tag{13}$$

where

$$\alpha_x = \alpha - 2m\pi E(x); \quad \alpha_y = 2m\pi - \alpha_x; \quad x = \frac{\alpha}{2m\pi};$$

and E(x) is the integral part of x.

The coefficients $\varkappa_k$, ζ_k, and $\varkappa_s$, ζ_s found based on minimizing the standard deviation of the approximating functions on the segment $[0, 2\pi]$ are linear dependences on ε for internal-combustion engines with central crank gears and for rotary-piston [2] and rod-free [1] internal-combustion engines:

$$\varkappa_k = d_k + e_k\varepsilon; \qquad \zeta_k = f_k + g_k\varepsilon;$$
$$\varkappa_s = d_s + e_s\varepsilon; \qquad \zeta_s = f_s + g_s\varepsilon. \tag{14}$$

The coefficients d_k, e_k, f_k, and g_k, and d_s, e_s, f_s, and g_s have the values

$$d_k = 0.99; \quad e_k = 0.008; \quad f_k = 1.42; \quad g_k = 0.083;$$
$$d_s = 1.62; \quad e_s = 0.28; \quad f_s = 1.43; \quad g_s = 0.062$$

for internal combustion engines with central crank gears (Fig. 1a),

$$d_k = 0.37; \quad e_k = 0.0055; \quad f_k = 0.78; \quad g_k = 0.055;$$
$$d_s = 0.57; \quad e_s = 0.086; \quad f_s = 0.77; \quad g_s = 0.043$$

for rotary piston engines (in describing the force function of a single chamber, Fig. 1b), and

$$d_k = 0.79; \quad e_k = 0.0067; \quad f_k = 1.25; \quad g_k = 0.074;$$
$$d_s = 1.31; \quad e_s = 0.2; \quad f_s = 1.16; \quad g_s = 0.065$$

for rod-free engines (in describing the first function of a single cylinder, Fig. 1c).

The dependences $\varkappa_k$, ζ_k, and $\varkappa_s$, ζ_s are as follows for the force function of the lateral cylinder of an internal-combustion engine with "pull-type" crank gears:

$$\varkappa_k = 1.12 - 0.05\gamma - (0.0027 + 0.0019\gamma)\varepsilon;$$
$$\zeta_k = 1.38 + 0.092\varepsilon; \quad \varkappa_s = 1.9 - 0.12\gamma + 0.31\varepsilon;$$
$$\zeta_s = 1.55 - 0.027\gamma + (0.062 + 0.005\gamma)\varepsilon,$$

where γ is the camber angle in radians for a block of pull-type cylinders (Fig. 1d).

The value of m in Eqs. (13) must be set equal to three in analyzing rotary-piston internal-combustion engines.

The force function $\Phi_1(\alpha)$ that can be generated by the power process in a single cylinder of an internal-combustion engine can be represented in the form (Fig. 2)

$$\Phi_1(\alpha) = V_{cy}[p_{ch}K(\alpha) + p_i S(\alpha)], \tag{15}$$

where p_{ch} and p_i are, in the general case, functions of the high-velocity and load conditions of the engine.

It can be easily proved that force functions of the type of Eq. (15) correspond to the general formalism of Lagrange mechanics as forces having the potential Π

$$\Pi = V_{cy}[p_{ch}(G_{k,x} - G_{k,y}) + p_i G_{s,x}]. \tag{16}$$

Here

$$G_{i,j} = \frac{\varkappa_i}{\zeta_i}\left(\alpha_j + \frac{1}{\zeta_i}\right)\exp(-\zeta_i\alpha_j) \qquad (i = k, s;\ j = x, y).$$

We may describe, using Eq. (15), the general force function of any in-line engine having z cylinders (chambers) as a set of z force functions (15) acting on the corresponding crankshaft throw. Each of the constituent functions is shifted in argument in accordance with the distribution diagram of ignitions for cylinders (chambers) of the engine. The sum of the z_1 force functions of the form (15), shifted relative to each other in accordance with the ignition order in the z_1 cylinders acting on the crank, act on each of its crankshaft throws in the case of a multi-in-line internal combustion engine.

A Fourier trigonometric series corresponding to the force function (15) of a single cylinder is the most effective way of calculating the form of the representation of the force function in estimating the resonance characteristics of a set of engines with an internal-combustion engine. The constituent amplitude C and phase Ψ spectra of this series can be represented, based on Eq. (15), in the form (Fig. 3)

$$C_\nu = \sqrt{A_\nu^2 + B_\nu^2};\quad \psi_\nu = \operatorname{arc\,tg}\frac{A_\nu}{B_\nu} \tag{17}$$

$$(\nu = 1, 2, \ldots).$$

Here

$$A_\nu = \frac{p\,V_{cy}}{m\pi}\frac{\varkappa_s(\zeta_s^2 - \nu_1^2)}{(\zeta_s^2 + \nu_1^2)^2};\qquad \nu_1 = \frac{\nu}{m};$$

$$B_\nu = \frac{4p_{ch}V_{cy}\nu_1}{m\pi}\left[\frac{\varkappa_k\zeta_k}{(\zeta_k^2 + \nu_1^2)^2} + \frac{p_i}{2p_{ch}}\frac{\varkappa_s\zeta_s}{(\zeta_s^2 + \nu_1^2)^2}\right];$$

ν_1 is the ratio of the frequency of the ν-th harmonic component of the Fourier series to the rate of rotation of the engine.

The solid curves in Figs. 3 and 2 correspond to experiment and the broken curves, to the calculation.

It is useful to write the force function of the power process in a single engine cylinder in the form

$$\Phi_1(\alpha, y) = \Phi_{1.0}(\alpha) + \Phi_{1,p}(\alpha, y) \tag{18}$$

in dynamically analyzing the automatic control system for the rate of rotation of an internal-combustion engine. Here $\Phi_{1,0}(\alpha)$ is a regular perturbing function, or the unregulated part of the force function determined from Eq. (15); $\Phi_{1,p}(\alpha, y)$ is the regulating pulse, or the controlled component of the force function,

$$\Phi_{1,p}(\alpha, y) = \frac{2m\pi M_e}{z y_{pump}} H(\alpha) S(\alpha); \tag{19}$$

M_e is the mean effective torque of the internal-combustion engine in terms of external characteristics for the given velocity condition, y_{pump} is the displacement of the fuel apparatus feeder (fuel pump measuring rod, throttle) as the load conditions vary from idle to maximal, $H(\alpha) = y(\alpha_k)$ is the fuel delivery function for diesel engines [3], $H(\alpha) = \alpha_f^{-1}\int_{\alpha_k}^{\alpha_k+\alpha_f} y(\alpha)d\alpha$ is for carburetor and gas engines, $\alpha_k = c_k - \alpha_0$, α_0 is the fuel–air mixture injection advance angle in carburetor and gas internal-combustion engines ($\alpha_0 \simeq 2\pi$ for the latter), α_f is the phase fuel–air mixture delivery period ($\alpha_0 \simeq \pi$), and y is the current value of the displacement of the feeder relative to the equilibrium position (input signal).

The elastic properties of the crankshaft are not taken into account in our analysis of the dynamic properties of the automatic control system for the rate of rotation of an internal-combustion engine. In this case, the general force function of the engine is the result of summing the force functions of the cylinders with sequenced period $T_S = 2\pi m/z\Omega$ (Ω is the mean angular rate of the engine under these conditions). Thus, according to Eq. (19), the control process for the regulated component of the general force function of an internal combustion engine can be considered as a pulsed process with retardation and amplitude modulation [3]. The laws of this control process can be justifiably described by a linear, continuously operating model [3].

Such a continuous linear model (Fig. 4a) consists of two sequentially connected lengths of a directed action: link A of pure retardation by the magnitude $\tau_0 = \alpha_0/\Omega$, and static link D (for diesel engines) or L (for carburetor and gas engines). The input signal is a dimensionless shift $x = y/y_l$ of the feeder of the internal-combustion engine fuel apparatus and is transformed into an output signal, namely, the regulating force effect on the crankshaft. The amplitude $R(\omega)$ and $\psi(\omega)$ frequency characteristics of the links D and L have the form

$$R_D(\omega) = \frac{M_e}{1+\varphi^2}; \quad \psi_D(\omega) = -\operatorname{arc\,tg}\frac{2\varphi}{1-\varphi^2};$$

$$R_L(\omega) = \frac{|\sin\theta|}{\theta} R_D(\omega); \quad \psi_L(\omega) = \theta - \psi_D(\omega) \quad \left(\varphi = \frac{\omega}{\Omega\zeta_s}; \quad \theta = \frac{\pi\omega}{2\Omega}\right).$$

It is necessary to take into account great displacements of the fuel apparatus feeder in solving some problems in control. Here it is necessary to take into account the nonlinear dependence of the delivery function on displacement $y(\alpha_k)$ or the integral $\int_{\alpha_k}^{\alpha_k+\alpha_f} y(\alpha)d\alpha$. This is taken into account by introducing a noninertialess link N (Fig. 4b) at the input to the linear model at the level of our schematization of the control process for the force function of an internal-combustion engine. The piecewise-linear characteristics of the link N consists of a linear segment with angular coefficient 1 and two saturation zones corresponding to the values x_1 and x_2 of the displacement x at which fuel delivery ceases and is cut off (Fig. 4c).

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