L1

L2

X

$$ϕ$$

$$θ$$

In order to solve for the acceleration of the piston, shown in Figure X above, we need to determine the acceleration of X ($\ddot{X}$). In order to find this, we need to first form a closed vector loop which can be seen below in Equation 1:

$$L\_{1}\left(cosθ\vec{i}+sinθ\vec{j}\right)+L\_{2}\left(cosϕ\vec{i}-sinϕ\vec{j}\right)+X\vec{i}=0$$

By taking the derivative of (1) once, we can find the velocity, and twice, we can find the acceleration which can be seen below in Equations 1 and 2, respectively.

$$L\_{1}\left(-\dot{θ}sinθ\vec{i}+\dot{θ}cosθ\vec{j}\right)+L\_{2}\left(-\dot{ϕ}sinϕ\vec{i}-\dot{ϕ}cosϕ\vec{j}\right)+\dot{X}\vec{i}=0$$

$$⇒\dot{X}=L\_{1}\dot{θ}sinθ+L\_{2}\dot{ϕ}sinϕ$$

$$⇒\dot{ϕ}=\frac{L\_{1}\dot{θ}cosθ}{L\_{2}cosϕ}$$

$$L\_{1}\left(\left[-\ddot{θ}sinθ-\dot{θ}^{2}cosθ\right]\vec{i}+\left[\ddot{θ}cosθ-\dot{θ}^{2}sinθ\right]\vec{j}\right)+L\_{2}\left(\left[-\ddot{ϕ}sinϕ-\dot{ϕ}^{2}cosϕ\right]\vec{i}+\left[-\ddot{ϕ}cosϕ+\dot{ϕ}^{2}sinϕ\right]\vec{j}\right)+\ddot{X}\vec{i}=0$$

$$⇒\ddot{X}=L\_{1}(\ddot{θ}sinθ+\dot{θ}^{2}cosθ)+L\_{2}(\ddot{ϕ}sinϕ+\dot{ϕ}^{2}cosϕ)$$

$$⇒\ddot{ϕ}=\frac{\ddot{θ}cosθ-\dot{θ}^{2}sinθ+\dot{ϕ}^{2}sinϕ}{cosϕ}$$

When the RPM = 19250, and $θ=ϕ=0 rad$ the given properties are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Engine Speed | Max Acceleration | Max Force | Max Internal Pressure |
| 19,250 RPM | 9843.9g | 21.245kN | 2.8165MPa |