Position:

$$L_1(\cos\theta\vec{\boldsymbol{\imath}} + \sin\theta\vec{\boldsymbol{\jmath}}) + L_2(\cos\phi\vec{\boldsymbol{\imath}} - \sin\phi\vec{\boldsymbol{\jmath}}) + X\vec{\imath} = 0$$

Differentiate:

$$L_1\left(-\dot{\theta}\sin\theta\vec{\boldsymbol{\imath}} + \dot{\theta}\cos\theta\vec{\boldsymbol{\jmath}}\right) + L_2\left(-\dot{\phi}\sin\phi\vec{\boldsymbol{\imath}} - \dot{\phi}\cos\phi\vec{\boldsymbol{\jmath}}\right) + \dot{X}\vec{\imath} = 0$$

Which Implies:

$$\dot{X} = L_1\dot{\theta}\sin\theta + L_2\dot{\phi}\sin\phi$$

$$\dot{\phi} = \frac{L_1\dot{\theta}\cos\theta}{L_2\cos\phi}$$

Differentiating Again:

$$L_1\left(\left[-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta\right]\vec{\imath} + \left[\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right]\vec{\jmath}\right) + L_2\left(\left[-\ddot{\phi}\sin\phi - \dot{\phi}^2\cos\phi\right]\vec{\imath} + \left[-\ddot{\phi}\cos\phi + \dot{\phi}^2\sin\phi\right]\vec{\jmath}\right)$$
$$+ \ddot{X}\vec{\imath} = 0$$

Which Implies:

$$\ddot{X} = L_1(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) + L_2(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi)$$

$$\ddot{\phi} = \frac{\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta + \dot{\phi}^2\sin\phi}{\cos\phi}$$