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Classical Thermodynamics 1/e

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FUNDAMENTAL LAWS AND BALANCE EQUATIONS – APPLICATION TO CLOSED STEADY SYSTEMS

Classical thermodynamics is an axiomatic science; that is, starting from a few basic axioms or **laws**, which are assumed to be self evident and always true, the behavior of thermodynamic systems can be predicted. A law is an abstraction of myriads of observations distilled into concise statements that are self-evident and certainly without any contradiction. We have already come across the *zeroth law* of thermodynamics, which establishes temperature as a thermodynamic property, an arbiter of thermal equilibrium between two objects.

In this chapter we introduce fundamental laws of thermodynamics – *conservation of mass*, conservation of energy or the *first law*, and the entropy principle or the *second law*. Applied to a generic unsteady open system, the most general a system can be, these laws are translated to *balance equations* of mass, energy, and entropy equations.

A special class of systems – systems that are *closed* and *steady* – is analyzed in this chapter with the help of the balance equations. Property evaluation, the central topic of the next chapter, is not necessary for such systems. *Heat engines, refrigerators,* and *heat pumps* are shown to be special cases of closed steady systems as far as overall analysis is concerned. Performance related analysis for these devices lead to the concept of *reversibility* and *Carnot efficiency*, and establishment of the absolute temperature scale.

As in chapter 1 we will use the animation module of TEST to illustrate concepts, and, occasionally, TEST daemons to evaluate properties if necessary. For closed steady systems, however, calculations are fairly straightforward as property evaluation can be completely avoided.



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2.1 Balance Equations

Interactions between a system and its surroundings can lead to observable changes in a system. This is illustrated by Anim. 2.A.*openUnsteadySystem*. The global state, represented by the distribution of color inside the system, of a generic, non-uniform, open system continuously changes (indicated by the change of color with time) as mass, heat, and work cross its boundary. Sometimes, even in the absence of any interaction, an isolated system may evolve spontaneously (see Anim. 1.C.*isolatedSystem*). Fortunately, changes in a system, whether spontaneous or driven by interactions, are governed by fundamental laws of thermodynamics, specifically, conservation of mass, conservation of energy or the first law, and the entropy principle or the second law. Before we introduce these laws, let us recapitulate some of the important results from chapter 1 involving interactions and their consequences.

Mass transfer through a port - its rate given by the familiar formula $\dot{m} = \rho AV$ (sec. 1.2.1) - obviously affects the total mass of a system, which may increase, decrease, or remain the same depending on the net rate of transfer. But mass also carries with itself all its attributes – properties such as energy, entropy and, for that matter, all *extensive properties* - that can strongly affect the balance of these attributes in the system at a

given instant. The rate of transport of various properties (KE, J, \dot{S} , etc.), introduced in chapter 1 as part of a flow state, can be expressed through a general *transport equation* (see Anim. 2.A.*transportEqn*), which relates rate of transport \dot{B} of any extensive property (mass, energy, entropy, etc.) with the corresponding specific property b as follows.

$$\dot{B} = \dot{m}b$$
 $\left[\frac{\text{unit of B}}{s} = \frac{\text{kg}}{s}\frac{\text{unit of B}}{\text{kg}}\right]$ (2.1)

Properties of a uniform flow, including the rate of transport of energy and entropy, are described by an extended flow state introduced in sec. 1.3, which builds upon the assumption of local thermodynamic equilibrium or LTE. Beside the transport equation, we will develop many other equations relating the properties of an equilibrium state in chapter 3. The balance equations developed in this chapter will form the foundation of many such relations.

Beside mass transfer, heat and work transfer are the only remaining interactions. The net heat transfer \dot{Q} , expressed by Eq. (1.5), not only affects the energy inventory of



Fig. 2.1 The *generic system* – an open, unsteady, non-uniform system with all possible interactions with its surroundings.

a system, but will be shown to carry entropy with it. In this regard, recall from sec. 1.2.4 that heat transfer through the inlet and exit ports of a system can be neglected and only the transfer across the rest of the boundary needs to be considered.

Work transfer \dot{W} and its different components have been thoroughly discussed in sec. 1.2.5 and expressed by Eq. (1.18) as the sum of two broad categories: flow work \dot{W}_F and external work \dot{W}_{ext} . While the external work consists of easily identifiable boundary, electrical, and shaft work, flow work is somewhat invisible. For a closed system, \dot{W}_F is obviously zero. For an open system, the flow work was added to the rate of transport of stored energy (\dot{E}) in Eq. (1.39), resulting in a new property - the flow energy j. The rate of transport of this flow energy $\dot{J} = \dot{m}j$ bundles flow work with energy transport, eliminating the need for evaluating \dot{W}_F explicitly. For all systems, open or closed, only \dot{W}_{ext} , therefore, needs to be considered.

In what follows, each fundamental thermodynamic law will be expressed as a **balance equation**, a differential equation that keeps inventory of a certain global property – mass for conservation of mass, stored energy for the first law, and entropy for the second law. Each equation will be formulated for the most complex system possible - an unsteady, non-uniform, open system (see Fig. 2.1 or Anim. 2.A.*openUnsteadySystem*), henceforth called the **generic system**, having all possible interactions with its surroundings. Once a governing equation is obtained in its most general form, it can be simplified and customized for any specific system or application. A recapitulation of the image analogy (see sec. 1.3.2) for systems and different types of systems illustrated by Anim. 1.E.*systemsClassified* is recommended before we begin development of the balance equations.

2.1.1 Mass Balance Equation

The **conservation of mass principle** can be stated through the following simple postulate¹.

Mass cannot be created or destroyed.

To translate this fundamental law into a balance equation, we note that the mass of a system can change over time because mass can flow in and out of a system. A balance

¹ A postulate is a statement that is self-evident, does not require a proof, and is accepted by everyone.

equation must take into account all possible ways for these transfer and storage mechanisms.

Consider the system, shown in Fig. 2.2 (and Anim 2.B.*massBalanceEqn*) at two different instants t and $t + \Delta t$, separated by a short period Δt . To simplify the derivation, only a single inlet and a single exit are considered, a limitation that can be readily removed. Also, ignored in these system schematics are the heat and various modes of work transfer, interactions that have no bearing on the mass balance of the system. The open system of interest is enclosed within the red external boundary.

Given that the system is unsteady, let the open system have a mass of m(t) at

time t and $m(t + \Delta t)$ at time $t + \Delta t$. Assuming the interval Δt to be small (we will soon make it vanishingly small), the mass that enters and leaves the system within that period can be expressed as $\dot{m}_i \Delta t$ and $\dot{m}_e \Delta t$ respectively, where \dot{m}_i and \dot{m}_e are the mass flow rates at the inlet and exit at time t. Since there is no other mechanisms through which the mass of the system can be affected, $m(t + \Delta t)$ can be expressed as (see Fig. 2.2 and try different radio buttons in Anim. 2.B.massBalanceEqn)

$$m(t + \Delta t) = m(t) + \dot{m}_i \Delta t - \dot{m}_e \Delta t \qquad [kg]$$
(2.2)

Rearranging and by taking the limit as Δt tends to zero.

$$\lim_{\Delta t \to 0} \frac{m(t + \Delta t) - m(t)}{\Delta t} = \dot{m}_i - \dot{m}_e$$
$$\Rightarrow \frac{dm}{dt} = \dot{m}_i - \dot{m}_e \qquad \left[\frac{\mathrm{kg}}{\mathrm{s}}\right]$$

For multiple inlets and exits, this equation can be generalized by summing over all inlet and exit ports, yielding the **mass balance equation** in its most general form.



The mass balance equation, thus, expresses the rate of increase of mass in an open system as the net rate of transport of mass into the system, which can be visualized through a **flow** diagram such as the one shown in Fig. 2.3, in which the rate term, also known as the **unsteady term**, is represented by the balloon, and the **transport terms** by the big





Fig. 2.2 Schematic used for deriving the mass balance equation.



Fig. 2.3 Flow diagram for the mass balance equation. The red boundary marks the system.

arrows. Although trivial for the mass balance equation, such flow diagrams can be a good visualization tool for the more complex energy and entropy equations.

Just as we used the lake analogy to understand energy, heat, and work in chapter 1 (Sec. 1.2.4), a *checkbook analogy* can be used to remember different terms of a balance equation. In this analogy, the unsteady term can be interpreted as the rate of change of the account balance as a result of deposits (transport at the inlets) and withdrawals (transport at the exit). Since mass cannot be created or destroyed, there is no term in the mass balance equation that mimics interest accrued. Mass balance equation, therefore, resembles a interest free account (see Anim. 2.B.*checkbookAnalogy*).

A balance equation derived for a generic system can be easily customized for specific simpler systems. Consider first a *closed* system. With no possibilities of mass transfer, the transport terms on the RHS drop out. Thus,

$$\frac{dm}{dt} = 0$$
, or $m = \text{constant}$ (2.4)

The mass of a closed system remains constant. Note that for any steady system, the global state remaining frozen in time (Anim. 1.E.*systemsClassified*), the mass of the system remain invariant. But the reverse is not always true - a closed system can be unsteady even though its mass cannot change.

Now consider an open, steady system. By definition of steady state, the time derivative of all extensive properties, including mass m, must be zero. The mass equation simplifies to

$$\frac{d\eta i}{dt}^{0} = \sum_{i} \dot{m}_{i} - \sum_{e} \dot{m}_{e}, \text{ or } \underbrace{\sum_{i} \rho_{i} A_{i} V_{i}}_{\text{What goes in.}} = \underbrace{\sum_{e} \rho_{e} A_{e} V_{e}}_{\text{What comes out.}} \qquad \left[\frac{\text{kg}}{\text{s}}\right] \qquad (2.5)$$

where, $\dot{m} = \rho AV$ (Eq. 1.2) is substituted at all the inlet and exit ports. For an open steady system, the conservation of mass principle reduces to *what goes in is what comes out*. For a single-flow steady system, that is, for systems with only one inlet and one exit, the summation signs can be dropped.

$$\dot{m}_i = \dot{m}_e;$$
 or, $\rho_i A_i V_i = \rho_e A_e V_e;$ or, $\frac{A_i V_i}{v_i} = \frac{A_e V_e}{v_e}$ $\left[\frac{\text{kg}}{\text{s}}\right]$ (2.6)

For incompressible fluids (SL model), density remains constant and the equation further simplifies to $A_i V_i = A_e V_e$, which can be used to understand incompressible flow behavior



Fig. 2.4 A constant-density fluid must accelerate in inverse proportion to flow area in a converging passage.

in a variable area passage (see Fig. 2.4). For example, a steady flow of (constant density) liquid through a converging duct must accelerate and that through a diverging duct must decelerate. However, if the working fluid is a vapor or a gas, change in specific volume introduces complexities that will be fully explored in chapter 15 in connection with high speed flow of gases and vapor through variable area ducts.

The mass equation involves several properties of a flow state including one thermodynamic property, the specific volume v (or its inverse – density ρ). We have to wait until the next chapter before we can explore the local equilibrium that exist at a port and evaluate a flow state manually. In this chapter, the properties at a flow state will be assumed known or will be evaluated using the state daemons.

EXAMPLE 2-1 Application of Mass Balance Equation

Water enters a cylindrical tank with a cross-sectional area of 10 m² through two separate inlets and leaves through a single exit as shown in the accompanying figure. The conditions at the three ports are as follows. State-1: $V_1 = 10 \text{ m/s}$, $A_1 = 50 \text{ cm}^2$; State-2:

 $V_2 = 300 \text{ m/min}, A_2 = 0.011 \text{ m}^2$; State-3: $p_3 = 500 \text{ kPa}, T_3 = 20^{\circ}\text{C}, V_3 = 7 \text{ m/s},$

 $D_3 = 14$ cm. Determine (a) the mass flow rate at the three ports and (b) the rate of increase in height in m/min if the tank has an inner diameter of 5 m. Assume density of water to be constant at 997 kg/m³.

SOLUTION Analyze the mass balance equations for the open system enclosed by the red boundary of Fig. 2.5.

Assumptions The inlet and exit flows are uniform and in LTE (local thermodynamic equilibrium) so that three flow states - state-1, state-2 and state-3 - can be used to describe the flow at the ports.

Analysis The mass flow rates at the three flow states can be calculated from Eq. (1.2).

$$\dot{m}_{1} = \rho_{1}A_{1}V_{1} = (997)\left(\frac{50}{10,000}\right)(10) = 49.85 \frac{\text{kg}}{\text{s}};$$
$$\dot{m}_{2} = \rho_{2}A_{2}V_{2} = (997)(0.011)\left(\frac{300}{60}\right) = 54.84 \frac{\text{kg}}{\text{s}};$$
$$\dot{m}_{3} = \rho_{3}A_{3}V_{3} = (997)\frac{\pi}{4}\left(\frac{14}{100}\right)^{2}(7) = 107.43 \frac{\text{kg}}{\text{s}};$$



The mass equation, Eq. (2.3), produces.

$$\frac{dm}{dt} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 49.85 + 54.84 - 107.43 = -2.748 \frac{\text{kg}}{\text{s}}$$

Since the mass of water in the tank can be expressed as a function of the water level, height h in Fig. 2.5, the unsteady term can be expressed as

$$\frac{dm}{dt} = \frac{d\left(\rho\Psi\right)}{dt} = \frac{d\left(\rho Ah\right)}{dt} = \rho A \frac{dh}{dt}$$

Therefore,

$$\frac{dh}{dt} = \frac{1}{\rho A} \frac{dm}{dt} = \frac{1}{997} \frac{4}{\pi} \left(\frac{1}{5^2}\right) (-2.748) \left(1000 \ \frac{\text{mm}}{\text{m}}\right) \left(60 \ \frac{\text{s}}{\text{min}}\right) = -8.4 \ \frac{\text{mm}}{\text{min}}$$

Discussion The negative sign for the rate of change suggests that the water level is declining in the tank. The mass of water contained in the sections of the pipes that fall within the system boundary can be ignored in this analysis since their amounts remain constant.

2.1.2 Energy Balance Equation

The **conservation of energy principle**, also known as the **first law** of thermodynamics, can be stated through the following postulates.

i) The specific internal energy u is a thermodynamic property.

ii) The stored energy E = U + KE + PE of a system cannot be created or destroyed, only transported by mass, and transferred across the system boundary through heat and work. The indestructibility of energy is also known as the energy principle.

The first postulate establishes the specific internal energy u as one of the descriptors of thermodynamic equilibrium. As a corollary of the second postulate, *energy of an isolated system must remain constant* because energy cannot be transported or transferred across the boundary nor can it be created or destroyed within the system. Note that there is no problem in the redistribution of energy in its different modes within an isolated system, as illustrated in Anim. 1-C-*isolatedSystem*. By providing a complete account of the interactions that lead to the change in E, the first law allows us to create an inventory of energy in the form of an *energy balance equation*.







Once again, we represent the generic system of Fig. 2.1 by a considerably simplified sketch in Fig. 2.6 at two different instants t and $t + \Delta t$ (see Anim. 2.C.*energyBalanceEqn*) with the open system of interest being enclosed within the red boundary. As before, only a single inlet and a single exit are considered. Likewise, heat transfer is assumed to take place from a single heating source - a formal name for an idealized heating source is a **thermal energy reservoir** or **TER** - and external work is represented by shaft work.

Within the interval Δt the amount of energy transported by mass at the inlet is $\dot{J}_i \Delta t = \dot{m}_i j_i \Delta t$, which includes both transport of stored energy and flow work (see Anim. 2.C.*energyTransport*). Similarly, the energy transported at the exit is $\dot{J}_e \Delta t = \dot{m}_i j_e \Delta t$. With the flow work already accounted for in the energy transport terms, energy transfer out of the system through work is given by $\dot{W}_{ext}\Delta t$, where \dot{W}_{ext} consists of shaft, electricity, and boundary work (see Anim. 2.C.*workTransfer*). The remaining mechanism, heat transfer, adds $\dot{Q}\Delta t$ amount of energy to the system in time Δt , where \dot{Q} is the net rate of heat transfer (see Anim. 2.C.*heatTransfer*). Taking into account the sign convention of work and heat transfer, $E(t + \Delta t)$ can be related to E(t) through (see Anim.

2.C.energyBalanceEqn)

$$E(t + \Delta t) = E(t) + \dot{J}_{i}\Delta t - \dot{J}_{e}\Delta t + \dot{Q}\Delta t - \dot{W}_{ext}\Delta t \qquad [kJ] \qquad (2.7)$$

Dividing by Δt , rearranging, and taking the limit as Δt tends to zero, we obtain.

$$\lim_{\Delta t \to 0} \frac{E(t + \Delta t) - E(t)}{\Delta t} = \dot{J}_i - \dot{J}_e + \dot{Q} - \dot{W}_{ext}$$
$$\Rightarrow \frac{dE}{dt} = \dot{m}_i \dot{j}_i - \dot{m}_e \dot{j}_e + \dot{Q} - \dot{W}_{ext} \qquad \left[\frac{kJ}{s} = kW\right]$$

For multiple inlets and exits, this equation can be generalized by summing, yielding the **energy balance equation** in its most general form.

$$\frac{dE}{dt}_{\text{Rate of increase} \text{ of stored energy} \text{ of the open system.}}} = \sum_{i} \dot{m}_{i} j_{i} - \sum_{e} \dot{m}_{e} j_{e} + \frac{\dot{Q}}{e_{\text{Energy transported}}} - \underbrace{\dot{W}_{ext}}_{\text{work transfer into} \text{ by mass flow out.}}} \begin{bmatrix} kW \end{bmatrix}$$
(2.8)
$$\frac{dE}{dt}_{\text{energy transported} \text{ by mass flow in.}} = h + ke + pe = h + \frac{V^{2}}{2(1000 \text{ J/kJ})} + \frac{gz}{(1000 \text{ J/kJ})}, \text{ and } \dot{W}_{ext} = \dot{W}_{B} + \dot{W}_{O}$$



Fig. 2.7 Flow diagram of the energy balance equation for a generic system.

2-10

Physical meaning of different terms of the energy equations can be explored in Anim. 2.C.*energyBalanceTerms* by clicking each term. As in the mass balance equation, the LHS, the time rate of stored energy, is the *unsteady* term. On the RHS, the first two terms are *transport* terms, energy transported by mass at the inlets and exits. The last two terms, energy transfer across the boundary through heat and external work, are called *boundary transfer* terms. The heat transfer \dot{Q} is the net rate of heat addition due to all possible mechanisms while the external work consists of boundary, shaft, and electrical work, $\dot{W}_{ext} = \dot{W}_{sh} + \dot{W}_{el} + \dot{W}_{B}$ (see Eq. 1.18). Observe that the energy equation preserves the sign convention for heat and work transfer.

In words, the energy equation can be stated as follows: the rate of increase of stored energy of a generic system is equal to the net rate of transport of energy by the flows and the net rate of energy transfer across the boundary through heat and external work. Representing the net rate of energy transport by \dot{J}_{net} , this can be expressed in a succinct form:

$$\underbrace{\frac{dE}{dt}}_{\text{Unsteady term.}} = \underbrace{\dot{J}_{\text{net}}}_{\text{Net transport.}} + \underbrace{\dot{Q} - \dot{W}_{\text{ext}}}_{\text{Net boundary transfer.}}; \text{ where, } \dot{J}_{\text{net}} = \sum_{i} \dot{J}_{i} - \sum_{i} \dot{J}_{e}$$
(2.9)

Like the mass flow diagram, an **energy flow diagram** can be constructed as shown in Fig. 2.7 to visually represent different terms of the energy equation. In our *checkbook analogy*, the unsteady term is analogous to the rate of change of the account balance (the balloon in Fig. 2.7), and the transport terms are counterparts of cash deposits and cash withdrawals as before. The new boundary transfer terms – heat and external work - can be represented by new means of money transfer, say, electronic transfer (for heat transfer) and payments through checks (for external work transfer). As with the mass balance equation, the account is interest free since creation or destruction of energy is forbidden by the *first law*.

Derived for an open system, the energy equation, Eq. (2.8), can be readily adopted for a closed system by dropping the transport terms.

$$\frac{dE}{dt} = \dot{\mathcal{Y}}_{\text{net}}^{0} + \dot{Q} - \dot{W}_{\text{ext}} = \dot{Q} - \dot{W}_{\text{ext}}, \qquad (2.10)$$

Because there is no possibility of any flow work for a closed system,

 $\dot{W} = \dot{W}_{F}^{0} + \dot{W}_{ext} = \dot{W}_{ext}$. Often the symbols \dot{W} and \dot{W}_{ext} are interchangeably used in the energy equation for a closed system.



Fig. 2.8 Flow diagram of the energy balance equation for a single-flow steady system.

Many open devices operate at steady state. The stored energy, like any other extensive property, remains constant as the global state of a steady system does not change with time. For **single-flow** devices - devices with a single inlet and a single exit – the steady-state energy equation simplifies as

$$\frac{dE}{dt}^{0} = \dot{m}_{i}j_{i} - \dot{m}_{e}j_{e} + \dot{Q} - \dot{W}_{ext}; \implies 0 = \dot{m}(j_{i} - j_{e}) + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \underbrace{\dot{m}(h_{i} + ke_{i} + pe_{i}) + \dot{Q}}_{\text{Energy In}} = \underbrace{\dot{m}(h_{e} + ke_{e} + pe_{e}) - \dot{W}_{ext}}_{\text{Energy Out}} \qquad (2.11)$$

$$\Rightarrow \underbrace{\dot{m}(h_{e} + ke_{e} + pe_{e})}_{\text{Energy transported out at the exit.}} = \underbrace{\dot{m}(h_{i} + ke_{i} + pe_{i})}_{\text{Energy transported in at the inlet.}} + \underbrace{\dot{Q}}_{\text{Heat added}} - \underbrace{\dot{W}_{ext}}_{\text{External work supplied.}}$$

in which $\dot{m}_i = \dot{m}_e = \dot{m}$ from the mass equation, Eq. (2.6), has been substituted As shown by the energy flow diagram of Fig. 2.8, this simplified energy equation can be interpreted as a balance between incoming energy through mass and heat and outgoing energy through mass and external work. A further simplification results when the kinetic and potential energy changes between the inlet and exit are not significant. The flow energy can be replaced by enthalpy, a thermodynamic property, which is relatively easier to evaluate.

There are a few other customized forms of the energy balance equation for different types of systems that will be derived later. The examples below are intended to highlight different terms of the energy balance equation. Comprehensive analysis of systems will be undertaken only after entropy equation is developed in the next section.

EXAMPLE 2-2 Energy Analysis: Unsteady Closed System

A cup of coffee is heated in a microwave oven. If heat is added at a constant rate of 200 W and the cup contains 0.3 kg of coffee, determine the rate of increase of the specific internal energy of the coffee. Neglect evaporation.

SOLUTION Simplify the energy equation for the closed, unsteady system and evaluate the unsteady term.

Assumptions The entire amount of heat (in the form of microwave radiation) goes into coffee and the cup itself does not participate.

Analysis With no mass transfer, no external work transfer, and no changes in KE and PE, the energy equation, Eq. (2.8), reduces to





Fig. 2.9 System schematic and the energy diagram for Ex. 2-2.

$$\frac{d \not E^{U}}{dt} = \dot{y}_{net}^{0} + \dot{Q} - \dot{y}_{ext}^{0}; \implies \frac{d(mu)}{dt} = \dot{Q}; \implies \frac{du}{dt} = \frac{\dot{Q}}{m}$$
$$\implies \frac{du}{dt} = \frac{(200 \text{ W})}{(0.3 \text{ kg})(1000 \text{ W/kW})} = 0.667 \frac{\text{kW}}{\text{kg}}$$

Discussion The SL model, to be developed in the next chapter, expresses u as a function of T for an incompressible solid or a liquid. Once such relations are established, the rate of change of temperature, which is a much more interesting quantity, can then be calculated from this result. Note that instead of heat transfer if an electrical heater with $\dot{W}_{el} = -200$ W was used to warm up the coffee, a similar analysis would lead to identical result.

EXAMPLE 2-3 Energy Analysis: Closed Steady System.

A thin wall separates two large chambers, the left chamber containing boiling water at 200 °C and the right chamber containing a boiling refrigerant at 0 °C. The heat transfer rate from the left chamber to the wall is measured at 2 kW. Assuming the wall to be at steady state, determine the rate of heat transfer from the wall to the right chamber.

SOLUTION Analyze the energy equation for the closed system, the wall, where heat transfer with the two chambers are the only interactions between the system and its surroundings.

Assumptions Since the wall remains at steady state, all global properties, including the stored energy E, remain constant.

Analysis Heat transfer at the two surfaces are represented by absolute values \dot{Q}_1 and \dot{Q}_2 with their directions represented by arrows in Fig. 2.10. The net rate of heat addition, therefore, is $\dot{Q} = \dot{Q}_1 - \dot{Q}_2$. Substituting this in the energy equation, Eq. (2.8), and simplifying, we obtain

$$\frac{dE}{dt}^{0, \text{ steady state}} = \dot{y}_{\text{net}}^{0} + \dot{Q}_1 - \dot{Q}_2 - \dot{W}_{\text{ext}}^{0};$$
$$\Rightarrow \dot{Q}_2 = \dot{Q}_1 = 2 \text{ kW};$$

Discussion For the stored energy in the wall to remain constant, the energy equation establishes that heat must leave the wall at the same rate as it enters.



Fig. 2.10 System schematic and energy flow diagram for Ex. 2-3. \dot{Q}_1 and \dot{Q}_2 are absolute values, not algebraic quantities.

EXAMPLE 2-4 Energy Analysis: Open Steady System

A steam turbine with a single inlet and exit produces 1.132 MW of shaft power while losing heat to the ambient atmosphere at a rate of 10 kW. The following data are supplied at the inlet and exit states. State-1: $v_1 = 0.1512 \text{ m}^3/\text{kg}$, $h_1 = 3247.6 \text{ kJ/kg}$, $V_1 = 25 \text{ m/s}$, $A_1 = 121 \text{ cm}^2$, $z_1 = 6 \text{ m}$; State-2: $v_2 = 1.694 \text{ m}^3/\text{kg}$, $h_2 = 2675.5 \text{ kJ/kg}$, $V_2 = 50 \text{ m/s}$, $A_2 = 680 \text{ cm}^2$, $z_2 = 3 \text{ m}$. Determine each term of the (a) mass and (b) energy balance equations.

SOLUTION Evaluate each term of the RHS of both the mass and energy equations for this work producing open system. Using the balance equations, obtain the unsteady terms.

Assumptions The flow is uniform and in LTE at the inlet and exit. We cannot assume that the turbine is operating at steady state.

Analysis Evaluate the mass flow rate and flow energy at the inlet and exit.

State-1:
$$\ker_{1} = \frac{V_{1}^{2}}{2(1000 \text{ J/kJ})} = 0.313 \frac{\text{kJ}}{\text{kg}}; \text{ pe}_{1} = \frac{gz_{1}}{(1000 \text{ J/kJ})} = 0.06 \frac{\text{kJ}}{\text{kg}};$$

 $j_{1} = h_{1} + \ker_{1} + \text{pe}_{1} = 3247.9 \text{ kJ/kg}; \quad \dot{m}_{1} = \frac{A_{1}V_{1}}{v_{1}} = 2.0 \text{ kg/s};$
State-2: $\ker_{2} = \frac{V_{2}^{2}}{2(1000 \text{ J/kJ})} = 1.25 \frac{\text{kJ}}{\text{kg}}; \quad \text{pe}_{2} = \frac{gz_{1}}{(1000 \text{ J/kJ})} = 0.03 \frac{\text{kJ}}{\text{kg}};$
 $j_{2} = h_{2} + \ker_{2} + \text{pe}_{2} = 2676.7 \text{ kJ/kg}; \quad \dot{m}_{2} = \frac{A_{2}V_{2}}{v_{2}} = 2.0 \text{ kg/s};$

The unknown unsteady term of the mass equation can now be evaluated.

$$\frac{dm}{dt} = \underbrace{\dot{m}_1}_{2 \text{ kg/s}} - \underbrace{\dot{m}_2}_{2 \text{ kg/s}} = 0$$

Similarly, the unsteady terms of the energy balance equation are evaluated as follows.



Fig. 2.11 Schematic of the turbine and the energy flow diagram for Ex. 2-4.

$$\frac{dE}{dt} = \underbrace{\dot{m}_1 \dot{j}_1}_{j_1 = 6496 \text{ kW}} - \underbrace{\dot{m}_2 \dot{j}_2}_{j_2 = 5354 \text{ kW}} + \underbrace{\dot{Q}}_{-10 \text{ kW}} - \underbrace{\dot{W}_{\text{ext}}}_{1132 \text{ kW}} = 0$$

Discussion The net rate of transport of energy into the system is $\dot{J}_{net} = \dot{J}_i - \dot{J}_e = 6496$ -5354 kW = 1142 kW, out of which 10 kW is lost as heat to the atmosphere (notice the negative sign), and the rest is delivered as shaft work. Note that, even though mass and stored energy remain constant, we cannot declare the system to be steady. The reverse, however, is always true.

Going back to Ex. 1-16, \dot{J}_{net} for the single-flow system was calculated as -20.85 kW. If the device operates at steady state in an adiabatic manner, the energy equation yields $\dot{W}_{ext} = \dot{J}_{net} = -20.85$ kW, the negative value indicating that external work is transferred to the system – a pump in this case - to increase the pressure of the flow.

EXAMPLE 2-5 Energy Analysis for Open Unsteady System

Steam from a supply line enters a rigid tank through a valve as shown in Fig. 2.12. During the filling process, heat is lost to the surroundings at a rate of 1 kW and a paddle-wheel stirs the steam inside the tank at 500 rpm with a torque of 0.01 kN \cdot m to overcome viscous friction. The inlet conditions on the supply side of the valve are: $\dot{m}_1 = 0.11425$ kg/s and $h_1 = 3069.26$ kJ/kg. If the mass and specific internal energy of the steam in the tank at a given instant are 2 kg and 2510 kJ/kg respectively, determine the rate of change of (a) the total internal energy U and (b) specific internal energy u of the system. Neglect ke and pe at the inlet.

SOLUTION Perform a mass and energy balance on the tank and valve as a system enclosed within the red boundary of Fig. 2.12.

Assumptions Uniform flow based on LTE with negligible ke and pe at the port.

Analysis Energy transport at the inlet and the work transfer by shaft can be calculated as follows.

$$\dot{J}_1 = \dot{m}_1 \dot{J}_1 \cong \dot{m}_1 h_1 = (0.11425)(3069.3) = 350.7 \text{ kW};$$

$$\dot{W}_{\rm sh} = -2\pi \frac{N}{60}T = -2\pi \frac{500}{60}(0.01) = -0.523 \text{ kW};$$



Fig. 2.12 Schematic for Ex. 2-5.

The negative sign in the work transfer is necessary to indicate that work is transferred into the system. Likewise, the heat transfer is also negative, $\dot{Q} = -1$ kW, since heat is transferred out of the system. The energy equation, Eq. (2.8), yields,

$$\frac{dE}{dt} \approx \frac{dU}{dt} = \dot{J}_i - \dot{J}_e^0 + \dot{Q} - \dot{W}_{ext} = 350.7 + (-1) - (-0.523) = 350.2 \text{ kW}$$

Since the mass of the system does not remain constant, differentiating the LHS by parts, we get

$$\frac{dU}{dt} = \frac{d(mu)}{dt} = m\frac{du}{dt} + u\frac{dm}{dt} = m\frac{du}{dt} + u\dot{m}_i$$

where, the mass balance equation is used to substitute \dot{m}_i for dm/dt in the last term. Simplifying,

 $\frac{du}{dt} = \frac{1}{m} \left[\frac{dU}{dt} - u\dot{m}_i \right] = \frac{1}{2} \left[350.2 - (2510)(0.11425) \right] = 31.7 \text{ kW}$

Discussion An increase in u usually accompanies an increase in temperature for gases, in which case the temperature in the tank increases much above that in the supply line.

2.1.3 Entropy Balance Equation

The second law of thermodynamics can be stated through the following postulates.

i) Entropy S of a system quantifies its molecular disorder and can be related to the number of ways in which the system's internal energy can be distributed among its molecules; the specific entropy s is a thermodynamic property which is transported by mass like any other specific property.

ii) Entropy is transferred across a boundary by heat at a rate \dot{Q}/T_B , where \dot{Q} is the rate of heat transfer and T_B is the boundary temperature. Work, on the contrary, does not transfer any entropy.

iii) Entropy cannot be destroyed. It can be generated spontaneously until a system comes to equilibrium. That is, $\dot{S}_{gen} \ge 0$, where \dot{S}_{gen} represents the rate of entropy generation within a system. This is also known as the **entropy principle**.



Fig. 2.13 Transport of molecular disorder or entropy by mass.





In chapter 1, we have already discussed entropy from the microscopic viewpoint as a measure of molecular disorder in a system. The first postulate formally establishes entropy as a macroscopic property that helps describe an equilibrium. Because molecules have discrete energy levels, a given amount of energy can be distributed among the molecules of a system in a huge number of possible ways. The more the choices, the more is the disorder (see Anim. 1.F.*entropy*) or entropy. Boltzman defined entropy from a microscopic standpoint as $S = k \ln \Omega$, where k is known as Boltzman constant and Ω is the number of **microstates** - the total number of ways in which the energy can be rearranged in the system without violating the first law. In our macroscopic treatment of thermodynamics, entropy is a macroscopic property just like internal energy and can be related to other properties of an equilibrium state without any reference to Boltzman equation.

Like energy, entropy can be transported by mass; the rate of transport of entropy can be deduced from the generic transport equation, Eq. (2.1), as $\dot{S} = \dot{m}s$ (see Fig. 2.13) and Anim. 2.D.entropyTransport). Additionally, the second postulate provides a fundamental means of entropy transfer by heat (see Fig. 2.14). Heat and work are both energy in transit, but they have a fundamental difference - work does not carry entropy, but heat does according to the second postulate. This can be understood from the fact that work transfer involves only organized or coherent motion - displacement of a boundary, rotation of a shaft, directed movement of electrons, all involve organized motion - and, therefore, have no direct impact on the degree of disorder of a system. Heat, on the other hand, is transferred when molecules of a warmer layer interacts with adjacent molecules of a cooler layer (conduction and convection heat transfer) or photons are transferred (radiation heat transfer). Since energy of molecules (and photons) is quantized, there is a huge number of possible interactions that can lead to a given amount of heat transfer across a plane. Energy transferred by heat, therefore, inherently carries entropy in the direction of heat transfer. Furthermore, if the interface (boundary) temperature is lower, energy levels are more densely packed and the same amount of heat can be transferred with more combinations of molecular interactions, that is, more entropy transfer. While energy transferred by work can be stored in an organized manner in a system (say, in system's kinetic or potential energy as shown in Fig. 2.15) without affecting the system entropy, energy transferred by heat fundamentally involves transfer of entropy (see Anim. 2.D.entropyTransfer) through random molecular interactions and the system entropy must increase as a result. The electrical work transfer in Fig. 2.16 does not carry any entropy into the resistance heater; however, when heat leaves the resistance heater, it must be carrying entropy with it. How does the heater replenish its entropy to maintain



Fig. 2.15 Mechanical work transfer to a rigid body affects all its molecules in an organized manner.



Fig. 2.16 Entropy is generated in the electric heater through *electronic friction* and carried by heat.

steady state? That is where the third postulate comes in –Joule heating is one of several distinguished mechanisms that generate entropy.

The third postulate states that all systems have a natural tendency to generate entropy. Even an isolated system is capable of transforming as its entropy can increase spontaneously until it reaches equilibrium, whereupon no further changes take place. A simple example of such spontaneous changes is illustrated in Fig. 2.17 where two different gases, initially separated, are allowed to mix inside an isolated system. Another example is shown in Fig. 2.18 where two bodies, initially at two different temperatures, are placed in thermal contact inside an isolated changer. In both cases, entropy is generated during spontaneous mixing process and spontaneous equalization of temperature. In each case, entropy is generated as a certain gradient – concentration gradient in the case of mixing and temperature gradient in the case of heat flow - is destroyed or diminished. Friction, which tries to dissipate a velocity gradient, is another common mechanism for entropy generation.

Because entropy cannot be destroyed, the generated entropy is a permanent signature, a second-law footprint if you will, of a natural process. Once entropy is generated, there is no going back – entropy generation, therefore, is irreversible and a fundamental marker of the passage of time. Systems or processes that involve entropy generation are called **irreversible**.

In chapter 6, we will establish a one-to-one connection between entropy generation and destruction of exergy, the useful part of the stored energy. It will be shown that whenever entropy is generated in a system, it is accompanied by destruction of a part of the stored exergy in the system. If electric energy can be stored in a battery in a reversible manner, that is, without any entropy generation, the entire useful energy (electricity in this case) can be extracted back at a later time. However, because of nature's tendency to generate entropy, the stored exergy will decrease over time. The energy stored in a battery can be preserved by isolating it, but its exergy will spontaneously decrease due to relentless generation of entropy. As another example, useful energy can also be stored in the kinetic energy of an isolated flywheel. However, friction will ensure that the stored exergy will decay over time even though the stored energy will remain unchanged. Entropy generation, thus, can be linked to the spontaneous degradation of the quality of stored energy in a system.

Friction being one of the easily identifiable causes of entropy generation, we will refer to the mechanisms of entropy generation, which will be explored throughout this book, as **generalized friction** with the rate \dot{S}_{gen} (pronounced S-dot-gen) quantifying its



Fig. 2.17 Entropy is generated as the concentration gradient is destroyed spontaneously (through *mixing friction*) in this isolated system.



Fig. 2.18 Entropy is generated as the temperature gradient is destroyed (through *thermal friction*) spontaneously in this isolated system.

severity. Just as friction always opposes motion and at best can be zero for a perfectly smooth interface, \dot{S}_{gen} is never negative and at best zero for idealized systems. A few of the entropy generating mechanisms – thermal friction, mixing friction, electronic friction, etc. - are illustrated in animations starting with 2.D.*sGenFriction*.

To express the second law of thermodynamics in the format of a balance equation for entropy, let us use the simplified system of Fig. 2.6 (also see Anim. 2.D.entropyBalanceEqn) used for the derivation of the energy equation. Within the interval Δt , the amount of entropy transported by mass at the inlet and exit are given by $\dot{S}_i \Delta t = \dot{m}_i s_i \Delta t$ and $\dot{S}_e \Delta t = \dot{m}_e s_e \Delta t$ respectively. By the second postulate, during the same interval, heat carries $\dot{Q}\Delta t/T_B$ amount of entropy across the boundary provided the boundary temperature is uniform at T_B . By the third postulate, the entropy that is spontaneously generated is given by $\dot{S}_{gen}\Delta t$, where $\dot{S}_{gen} \ge 0$. Combining all participating mechanisms, $S(t + \Delta t)$ can be related to S(t) as follows.

$$S(t + \Delta t) = S(t) + \dot{S}_{i}\Delta t - \dot{S}_{e}\Delta t + \frac{\dot{Q}\Delta t}{T_{B}} + \dot{S}_{gen}\Delta t \qquad \left[\frac{kJ}{K}\right]$$
(2.12)

Rearranging and taking limit,

$$\lim_{\Delta t \to 0} \frac{S(t + \Delta t) - S(t)}{\Delta t} = \dot{S}_i - \dot{S}_e + \frac{\dot{Q}}{T_B} + \dot{S}_{gen}$$
$$\Rightarrow \frac{dS}{dt} = \dot{m}_i s_i - \dot{m}_e s_e + \frac{\dot{Q}}{T_B} + \dot{S}_{gen} \qquad \left[\frac{kJ}{s} = kW\right]$$

For multiple inlets and exits, this equation leads to the **entropy balance equation** in its most general form and the meaning of each term (see Anim. 2.D.*entropyBalanceTerms*.) explained as follows.

$$\frac{dS}{dt}_{\text{at e of increase}} = \underbrace{\sum_{i} \dot{m}_{i} s_{i}}_{\text{an open system.}} - \underbrace{\sum_{i} \dot{m}_{i} s_{i}}_{\text{Entropy}} - \underbrace{\sum_{e} \dot{m}_{e} s_{e}}_{\text{Entropy}} + \underbrace{\frac{\dot{Q}}{T_{B}}}_{\text{Entropy}} + \underbrace{\frac{\dot{S}_{\text{gen}}}{Rate of}}_{\text{generation of}} \begin{bmatrix} \frac{kW}{K} \end{bmatrix}$$
(2.13)
where, by second law, $\dot{S}_{\text{gen}} \ge 0$

As with the mass and energy equations, the LHS is the *unsteady* term - the rate of change of total entropy in an open unsteady system. The RHS consists of contribution from *transport* of entropy by mass, *boundary transfer* of entropy through heat, and spontaneous *generation* of entropy as dictated by the second law.

In words, the entropy equation states that the rate of increase of entropy of a generic system is equal to the sum of the net transport of entropy by the flows, transfer of entropy by heat transfer, and spontaneous generation of entropy. Representing the net rate of entropy transport by \dot{S}_{net} , Eq. (2.13) can be written in a shorter form:

$$\frac{dS}{dt}_{\text{termsport}} = \underbrace{\dot{S}_{\text{net}}}_{\text{transport}} + \underbrace{\frac{Q}{T_B}}_{\text{Boundary}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Term}}; \text{ where, } \dot{S}_{\text{net}} = \sum_i \dot{S}_i - \sum_i \dot{S}_e \quad (2.14)$$

Like the energy flow diagram, an **entropy flow diagram** can be useful in graphically representing the entropy equation. The only new type of term in this equation is the generation term \dot{S}_{gen} , which is represented by the dotted arrow in Fig. 2.19. In our *checkbook analogy*, the generation term can be compared with the interest accrued in an account. Just as interest rate cannot be negative, a negative \dot{S}_{gen} is forbidden by the second law.

The entropy equation can be simplified in the same way as the energy or mass equation. For a closed system, for instance, the transport terms drop out, resulting in

$$\frac{dS}{dt} = \dot{S}_{net}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen} = \frac{\dot{Q}}{T_B} + \dot{S}_{gen} , \qquad (2.15)$$

Furthermore, if the system is isolated, there is no heat transfer either. While the energy of an isolated system remains constant, the entropy may increase since $\dot{S}_{gen} \ge 0$. Attributed to Clausius, this is one of several proclamations of the second law.

Most open devices generally operate at steady state. For a single-flow device the steady-state entropy equation can be further simplified as

$$\frac{dS'^{0}}{dt} = \dot{m}_{i}s_{i} - \dot{m}_{e}s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{gen}; \implies 0 = \dot{m}(s_{i} - s_{e}) + \frac{\dot{Q}}{T_{B}} + \dot{S}_{gen};, \qquad (2.16)$$

$$\Rightarrow \underbrace{\dot{m}s_{e}}_{\text{Entropy Out}} = \underbrace{\dot{m}s_{i} + \dot{Q}/T_{B}}_{\text{Entropy In}} + \underbrace{\dot{S}_{gen}}_{\text{Entropy Generated}}$$



Fig. 2.19 Flow diagram for the entropy balance equation.



Fig. 2.20 Entropy is generated internally and externally for this turbine.

From this simplified form, it can be easily established that the entropy at the inlet must be equal to that at the exit for an adiabatic, steady, single-flow device with no internal (generalized) friction. – an important conclusion that will be fully explored in chapter 4.

2.1.3.1 Entropy and Reversibility

The transformation of a system from one equilibrium state to another is called a **process**. Reversing a process and restoring the system and its surroundings to their original conditions have been a long standing goal for engineers (think how nice it would be if all polluting processes could be easily reversed). Most practical processes, however involve some types of *generalized friction*, which leaves an indelible footprint in the form of entropy generation within and around the system. Since entropy cannot be destroyed, such processes cannot be completely reversed. They are called **irreversible processes**. The degree of **irreversibility** of a process is directly related to the amount (or rate) of entropy generation - more the entropy generation more the irreversibility of the process . Likewise, a system with a positive \dot{S}_{gen} is called an **irreversible system** at a given instant.

In most practical devices, entropy is generated not only inside the system but also in the immediate surroundings. This is particularly so for devices that exchanges heat with the surroundings. For instance, in the turbine of Fig. 2.20, entropy is not only generated inside the turbine but also in the immediate vicinity due to **thermal friction** (see Anim. 1.D.*sGen-thermalFriction*) -heat loss from the turbine into the atmosphere over a finite temperature difference. The entropy generation rate \dot{S}_{gen} , therefore, depends on the precisely how the boundary is drawn – the external red boundary in Fig. 2.20 captures not only all the entropy generated inside the system but also that generated in the immediate surroundings. For a given system, entropy generated within the system is called **internal irreversibility** and the generation outside is called **external irreversibility**. An extended enclosure such as the red boundary of Fig. 2.20 captures all the entropy generated internal, and the extended system is known as the **system's universe**. Entropy generated internally and externally are additive and their sum constitutes the total entropy generation rate in the system universe.

$$\dot{S}_{\text{gen,univ}} = \dot{S}_{\text{gen,int}} + \dot{S}_{\text{gen,ext}} \qquad \left[\frac{kW}{K}\right]$$
 (2.17)

It may appear that calculating $\dot{S}_{\text{gen,univ}}$ is a daunting task, given the subjective nature of immediate surroundings. Quite to the contrary, it is actually easier to calculate $\dot{S}_{\text{gen,univ}}$



Fig. 2.21 A reversible system must be internally and externally reversible.

than $\dot{S}_{\text{gen,int}}$ in most cases. Consider, for instance, the turbine's universe enclosed within the red external boundary of Fig. 2.20. Since the boundary passes through the surrounding air, T_B in the entropy balance equation can be replaced by the constant T_0 , the atmospheric temperature. On the other hand, calculation of $\dot{S}_{\text{gen,int}}$ would require a complete knowledge of the distribution of T_B over the internal boundary.

Remember that the second law does not mandate generation of entropy; it only stipulates that entropy cannot be destroyed. It is therefore possible for a particular system (during a process or steady-state operation) not to generate entropy inside, outside, or in the entire system's universe. The system is said to be **internally reversible** when $\dot{S}_{\text{gen,int}} = 0$, **externally reversible** when $\dot{S}_{\text{gen,ext}} = 0$, and **reversible** when $\dot{S}_{\text{gen,univ}} = 0$ (see Fig. 2.21 and Anim. 2.D.*reversibility*). Obviously, a reversible system must be both internally and externally reversible since $\dot{S}_{\text{gen,univ}} = 0$ implies $\dot{S}_{\text{gen,int}} = 0$ and $\dot{S}_{\text{gen,ext}} = 0$ from Eq. (2.17). With no *thermal friction* in the immediate surroundings, adiabatic systems are externally reversible and $\dot{S}_{\text{gen,univ}} = \dot{S}_{\text{gen,int}}$ for such systems. Although most practical systems are highly unlikely to be reversible, reversibility will be shown to play a very important role in thermodynamic analysis by helping us define ideal system performance.

For many practical systems with variable boundary temperature, the boundary can be divided into k segments, each at a uniform temperature T_k , and the entropy equation can be modified as follows

$$\frac{dS}{dt} = \sum_{i} \dot{m}_{i} s_{i} - \sum_{e} \dot{m}_{e} s_{e} + \sum_{k} \frac{\dot{Q}_{k}}{T_{k}} + \dot{S}_{gen} \quad \left[\frac{kW}{K}\right]$$
(2.18)

Whether \dot{S}_{gen} in this equation represents $\dot{S}_{gen,int}$ or $\dot{S}_{gen,univ}$ depends on what boundary, internal or external, is selected. For example, suppose we are interested in evaluating $\dot{S}_{gen,univ}$ for the closed steady system in Fig. 2.22 exchanging heat with three different reservoirs - \dot{Q}_1 and \dot{Q}_2 coming from sources at T_1 and T_2 and \dot{Q}_0 rejected to the ambient atmosphere at T_0 . Noting that the heat transfer symbols represents absolute values with their direction indicated by the arrows in Fig. 2.22, Eq. (2.18) can be simplified producing



Fig. 2.22 An entropy analysis is much simpler for the system's universe enclosed within the red external boundary.

$$\frac{dS^{\prime 0}}{dt} = \sum_{i} \dot{p}_{i}^{\prime 0} s_{i} - \sum_{e} \dot{p}_{e}^{\prime 0} s_{e} + \frac{\dot{Q}_{1}}{T_{1}} + \frac{\dot{Q}_{2}}{T_{2}} - \frac{\dot{Q}_{0}}{T_{3}} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{0}}{T_{3}} - \frac{\dot{Q}_{1}}{T_{1}} - \frac{\dot{Q}_{2}}{T_{2}}$$

Notice that a similar analysis to evaluate $\dot{S}_{\text{gen,int}}$ would be much more difficult as temperature along the internal boundary most likely varies in a complicated manner. The entropy generation in the entire system's universe is due to *thermal friction* in this case.

In the following example we analyze closed and open steady systems to explore viscous and thermal friction as mechanisms of entropy generation. Besides these, electrical resistance heating (see Anim. 2.D.electronicFriction), free expansion, mixing, phase transformation, and chemical reactions are other prominent mechanisms of entropy generation.

EXAMPLE 2-6 Entropy Generation through Heat Transfer (Thermal Friction)

A thin wall separates two large chambers, the left one containing boiling water at $200 \,^{\circ}$ C and the right one containing a boiling refrigerant at $0 \,^{\circ}$ C. The heat transfer through the wall at steady state is measured at 2 kW. Assuming each wall surface to be at the respective chamber temperature, determine the rate of entropy transfer (a) from the left chamber to the wall and (b) from the wall to the right chamber. (c) Explain the difference in entropy flow from left to right.

SOLUTION Analyze the entropy equation for the wall, a closed steady system, where heat transfer at the two faces are the only interactions.

Assumptions Since the wall remains at steady state, all global properties, including the total entropy S of the system enclosed within the red boundary of Fig. 2.33, remain constant.

Analysis We have already established in Ex. 2-3 that the heat flow from the left chamber into the wall must be equal in magnitude to the heat flow from the wall into the right chamber. Using the same symbols used previously, the absolute values of entropy transported by heat at the two faces can be obtained from the second law.

Left face:
$$\frac{\dot{Q}_1}{T_1} = \frac{20}{(273 + 200)} = 0.0423 \frac{\text{kW}}{\text{K}};$$





Fig. 2.23 System schematics and entropy flow diagram for Ex. 2-6 (also see Anim. 2-D.*sGen-thermalFriction*.

Right face:
$$\frac{\dot{Q}_2}{T_2} = \frac{20}{273} = 0.0733 \frac{\text{kW}}{\text{K}};$$

The direction of the entropy transfer, obviously, coincides with the heat flow direction as shown in Fig. 2.23.

At steady state, the total entropy of the wall remaining constant, the unsteady term of the entropy balance equation must be zero. Therefore, Eq. (2.18) simplifies to

$$\frac{dS'}{dt}^{0, \text{ steady state}} = \sum_{i} \dot{p}_{i}^{0} s_{i} - \sum_{e} \dot{p}_{e}^{0} s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{gen};$$

$$\Rightarrow 0 = \frac{\dot{Q}}{T_{1}} - \frac{\dot{Q}}{T_{2}} + \dot{S}_{gen}; \Rightarrow \dot{S}_{gen} = 0.0732 - 0.0423 = 0.0309 \frac{\text{kW}}{\text{K}};$$

The generation of entropy in the wall explains the difference in the entropy transfer at the two faces of the wall.

Discussion In this example, we have used the entropy balance equation to uncover one of the fundamental mechanisms of entropy generation - heat transfer across a finite temperature difference or, in simpler terms, thermal friction. For a given rate of heat transfer, \dot{S}_{gen} depends on the severity of temperature difference and is independent of the thickness or any other properties of the wall. Entropy generation through heat transfer is illustrated in Anim. 2.D.*sGen-ThermalFriction*. It is the thermal counterpart of friction, the most prominent mechanism of entropy generation, and will be referred in this textbook as thermal friction.

Imagine what would happen if the natural direction of heat transfer was from the right chamber to the left, against the temperature gradient. An entropy analysis would then lead to a negative value of \dot{S}_{gen} , which is expressly forbidden by the second law. Given our daily life experience, the natural direction of heat transfer seems trivial, but without the help of the second law, this simple fact is impossible to establish. In fact, one of the original pronouncements of the second law goes as follows:

It is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler body to a hotter body.

This is known as the **Clausius statement**, deduced here as an exercise of the entropy balance equation.

Did you know?

Clausius statement and Kelvin-Planck statements are the original pronouncements of the Second Law, which preceded development of entropy as a property. If the temperature difference between the two chambers is reduced, \dot{S}_{gen} decreases and approach zero as $T_1 \rightarrow T_2$. In the limit of infinitesimal ΔT , heat transfer is said to be **reversible** since no permanent signature results the form of \dot{S}_{gen} . However, temperature difference being the driving force of heat transfer, you may wonder if significant heat transfer is possible with an infinitesimal ΔT . But recall from Sec. 1.2.5 that Q not only depends on ΔT , but also on A, and the duration t. Therefore, by providing a large area of contact and sufficient duration, finite amount of reversible heat transfer is possible, however impractical that may be.

EXAMPLE 2-7 Entropy Generation through Friction and Heat Transfer

A fan recirculates air in a closed chamber while consuming 1 kW of electric power. The chamber temperature increases and reaches a steady value of $70 \,^{\circ}$ C at which point the electrical work is balanced by an equivalent amount of heat transfer out of the chamber into the surroundings at $25 \,^{\circ}$ C. Determine the entropy generation rate (a) inside the chamber and (b) in the chamber's universe. (c) Identify the mechanisms of entropy generation.

SOLUTION Analyze the energy and entropy equation for the internal system and the system's universe.

Assumptions The chamber wall is not considered as part of the system

Analysis At steady state, all global properties, including the total stored energy E and entropy S, remain constant. The energy equation applied to the system or its universe (either boundary of Fig. 2.24) produces

$$\frac{dE}{dt}^{0, \text{ steady state}} = \sum_{i} \dot{p}_{i}^{0} j_{i} - \sum_{e} \dot{p}_{e}^{0} j_{e} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \dot{W}_{\text{ext}} = \dot{W}_{\text{el}} = -1 \text{ kW};$$

For the two boundaries shown in Fig. 2.24, the entropy equation produces.





System:
$$\frac{dS'}{dt}^{(0, \text{ steady state})} = \sum_{i} \dot{p}_{i}^{(0)} s_{i} - \sum_{e} \dot{p}_{e}^{(0)} s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,int}};$$

 $\Rightarrow \dot{S}_{\text{gen,int}} = -\frac{\dot{Q}}{T_{B}} = -\frac{(-1)}{273 + 70} = 0.0029 \frac{\text{kW}}{\text{K}};$
Universe: $\frac{dS'}{dt}^{(0, \text{ steady state})} = \sum_{i} \dot{p}_{i}^{(0)} s_{i} - \sum_{e} \dot{p}_{e}^{(0)} s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,univ}};$
 $\Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_{B}} = -\frac{(-1)}{273 + 25} = 0.0034 \frac{\text{kW}}{\text{K}};$

Discussion While $\dot{S}_{\text{gen,int}}$ can be attributed primarily to the dissipation of kinetic energy of the air into internal energy through *viscous friction*, the difference $\dot{S}_{\text{gen,univ}} - \dot{S}_{\text{gen,int}}$ is due to *thermal friction*. Note that the solution would remain identical if the fan was driven by a shaft instead of the electric motor. Although work transfer, in this problem, ends up generating entropy, it is not always the case. For instance, if work transfer is used to raise a weight, impart rotational kinetic energy into a flywheel, or charge a battery, entropy generation could be reduced or even eliminated. It should be stressed that unlike heat, work does not transfer entropy with it.

EXAMPLE 2-8 Entropy Analysis of an Open Steady System

The following additional data are supplied for the steam turbine of Ex. 2-4. State-1: $s_1 = 7.127 \text{ kJ/}(\text{kg} \cdot \text{K})$; State-2: $s_2 = 7.359 \text{ kJ/}(\text{kg} \cdot \text{K})$. Also, the temperature of the turbine surface remains constant at 150 °C while the surrounding atmosphere is at 25 °C. Assuming steady state operation, determine the entropy generation rate (a) inside the turbine and (b) in the turbine's universe.

SOLUTION Analyze entropy balance equation for the system and its universe.

Assumptions Same as in Ex. 2-4.

Analysis Simplifying the entropy balance equation, Eq. (2.13), for the internal system (see Fig. 2.25) and substituting the given properties, we obtain





Fig. 2.25 System schematics and entropy flow diagram for Ex. 2-8.

$$\frac{dS'}{dt}^{0,\text{steady state}} = \sum_{i} \dot{m}_{i} s_{i} - \sum_{e} \dot{m}_{e} s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,int}} \left[\frac{\text{kW}}{\text{K}}\right]$$

$$\Rightarrow \dot{S}_{\text{gen,int}} = -\dot{m} s_{1} + \dot{m} s_{2} - \frac{\dot{Q}}{T_{B}}$$

$$= -(2)(7.127) + (2)(7.359) - \frac{(-10)}{(273+150)}$$

$$= 0.488 \text{ kW/K}$$

For the system's universe, shown by the red boundary in Fig. 2.25, $T_{B} = T_{0}$. Therefore,

$$\frac{dS'^{0}}{dt} = \sum_{i} \dot{m}_{i} s_{i} - \sum_{e} \dot{m}_{e} s_{e} + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,univ}} \left[\frac{\text{kW}}{\text{K}}\right]$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \underbrace{\dot{m} s_{1}}_{\dot{S}_{1}=14.25 \text{ kW/K}} - \underbrace{\dot{m} s_{2}}_{\dot{S}_{2}=14.72 \text{ kW/K}} + \underbrace{\frac{\dot{Q}}{T_{0}}}_{-0.034 \text{ kW/K}} = 0.498 \text{ kW/K}$$

Discussion Entropy transport rates \dot{S}_1 and \dot{S}_2 or the energy transfers \dot{Q} and \dot{W}_{sh} for the turbine are independent of the boundary - internal or external - selected for the analysis. The only difference is in the calculation of entropy generation rate, which also disappears if the system is adiabatic. That is why the internal and external boundaries are used interchangeably in many analyses.

2.2 Closed Steady Systems

A large class of important systems – closed steady systems- can be analyzed without having to evaluate states, which entails an elaborate understanding of equilibrium, material models, and tables and charts. In what follows, we undertake a comprehensive analysis of closed steady systems with the help of the mass, energy, and entropy equations.

A closed system has no mass interactions with its surroundings, and a steady system does not change its global state over time. When both these conditions are satisfied, we have a closed steady system. Notice how the generic open unsteady system of Anim. 2.A.*genericSystem* reduces to a generic closed steady system in Anim.

2.E.*genericSystem*. At steady state, the color composition of the image, that is, the picture does not change with time despite the presence of work and heat interactions; however, the spatial variation remains, indicating the possibility of the system to be non-uniform.

Although most of the well-known thermodynamic devices - turbines, pumps, compressors, etc. – are open systems, we can identify plenty of closed steady systems if we look around. This textbook is a trivial but legitimate closed (no mass transfer) steady (the global state does not change with time) system. Non-trivial examples include a light bulb, a gear box, an electric heater, an electrical adapter, or even a refrigerator (watch animations starting with 2.E.*lightBulb*) – the global states of these closed systems remain unchanged over time.

The mass m of a closed system obviously remains constant, and so do all the global properties of a steady system, including E and S. With the unsteady and transport terms set to zero, the mass, energy and entropy equations reduce to:

 $Mass \qquad m = constant \qquad [kg] \tag{2.19}$

 $0 = \dot{O} - \dot{W} = \dot{O} - (\dot{W}_{r} + \dot{W}_{r} + \dot{W}_{r})$ [kW]

Energy

Entropy
$$0 = \frac{\dot{Q}}{T_{B}} + \dot{S}_{gen} \left[\frac{kW}{K}\right]$$
 (2.21)

(2.20)

Even a cursory look at the energy and entropy equations reveals that these equations do not involve any state properties, thereby, simplifying the analysis considerably. We have already analyzed a few closed steady systems in Examples 2-3, 2-6, and 2-7. Before we analyze a few more systems, let us define a commonly used performance parameter – the energetic or first-law efficiency.

Efficiency is one of the frequently used terms in engineering. We will come across many different types of efficiencies – energetic or first-law efficiency, thermal efficiency, isentropic efficiency, exergetic or second-law efficiency, combustion efficiency, and many more. Broadly speaking, efficiency is used to compare the performance of a device with that of an ideal device, or to compare a desired output with the required input.

The **energetic efficiency**, also known as the **first-law efficiency** (see Fig. 2.26), is defined as

$$\eta_{\rm I} \equiv \frac{\text{Desired Energy Output}}{\text{Required Energy Input}}$$
(2.22)



Fig. 2.26 An energetic efficiency of a system is defined as the ratio of the desired energy term to the required energy transfer.

where, the symbol η (eta) is reserved for efficiency of any kind and the subscript I symbolizes the first law. Device specific in nature, the numerator and denominator in this definition usually appear as individual terms of the energy balance equation for the system. Often, an energy flow diagram can be helpful in formulating the definition of the energetic efficiency.

EXAMPLE 2-9 Analysis of an Electric Heater

An electric heater, operating steadily, maintains a chamber at 30° C while consuming a current of 1 A from a 120-V source. If the temperature of the heater surface is 250° C, determine (a) the rate of heat transfer, (b) the energetic efficiency of the heater, (c) the rate of internal entropy generation, and (d) the rate of entropy generation in the heater's universe.

SOLUTION Analyze the energy and entropy balance equations for the closed steady system enclosed within the black (internal) and red (external) boundaries in Fig. 2.27.

Assumptions The heater and its universe are at steady state.

Analysis The electrical power consumption with appropriate sign is

$$\dot{W}_{el} = -\frac{VI}{(1000 \text{ J/kJ})} = -\frac{(120)(10)}{(1000 \text{ J/kJ})} = -1.2 \text{ kW}$$

The energy equation, Eq. (2.20), reduces into identical form for either the system or its universe (red boundary) sketched in Fig. 2.27.

$$\frac{dE}{dt}^{0} = \dot{Q} - \dot{W} = \dot{Q} - \dot{W}_{el}; \quad \Rightarrow \quad \dot{Q} = \dot{W}_{el} = -1.2 \text{ kW}$$

The desired energy output clearly is the heat produced by the heater while the required input is the electrical power consumed.

$$\eta_I = \frac{\dot{Q}}{\dot{W}_{\rm el}} = \frac{-1.2}{-1.2} = 100 \%$$

With the internal system boundary at 250 °C, an entropy balance for the internal system (enclosed by the black boundary in Fig. 2.27) produces



Fig. 2.27 System schematics and entropy flow diagram for Ex. 2-9 (see Anim. 2.D.*sGenelectronicFriction*).

$$\frac{dS^{\prime 0}}{dt} = \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,int}}; \implies \dot{S}_{\text{gen,int}} = -\frac{-1.2}{(273 + 250)} = 0.00229 \frac{\text{kW}}{\text{K}}$$

For the system's universe (enclosed by the red boundary of Fig. 4.20), which is also at steady state

$$\frac{dS'^0}{dt} = \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}$$
$$\Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B} = -\frac{(-1.2)}{(273+30)} = 0.00396 \frac{\text{kW}}{\text{K}}$$

Discussion The difference between $\dot{S}_{gen,univ}$ and $\dot{S}_{gen,int}$ is $\dot{S}_{gen,ext}$, which is the external entropy generation in the immediate surroundings of the heater. $\dot{S}_{gen,int}$ is caused by the dissipation of electrical work into internal energy or what can be called *electronic friction* while $\dot{S}_{gen,ext}$ must be due to *thermal friction*, a mechanism investigated in Ex. 2-7. The latter should not be confused with entropy transfer through heat.

EXAMPLE 2-10 Shaft Work Transfer in a Gear Box

A car engine delivers 150 kW of shaft power steadily at 4000 RPM into the gearbox with a gear ratio of 2 as shown in the accompanying figure. The surface of the gearbox is 10° C warmer than the atmosphere, which is at 25° C. If the heat lost to the surroundings through convection is 3 kW, determine the (a) power and torque delivered by the output shaft, (b) energetic efficiency, (c) entropy generation rate in the gearbox, and (d) entropy generation rate in the system's universe.

SOLUTION Analyze the energy and entropy balance equations for the closed steady systems enclosed within the black and red boundaries of Fig. 2.28.

Assumptions The system and its universe are at steady state.

Analysis The energy equation, Eq.(3.42), for either boundary in Fig. 2.28 can be written as

$$\frac{dE^{\prime 0}}{dt} = \dot{Q} - \dot{W} = \left(-\dot{Q}_{\rm conv}\right) - \left(\dot{W}_{\rm sh,out} - \dot{W}_{\rm sh,in}\right); \text{ or, } \dot{W}_{\rm sh,in} = \dot{W}_{\rm sh,out} + \dot{Q}_{\rm conv}$$





Fig. 2.28 System schematic, energy, and entropy flow diagrams for Ex.2-10.

where \dot{Q}_{conv} , $\dot{W}_{sh,out}$, and $\dot{W}_{sh,in}$ are absolute values of the respective energy transfer rates. Solving for $\dot{W}_{sh,out}$ from the given quantities,

$$\dot{W}_{\rm sh,out} = \dot{W}_{\rm sh,in} - \dot{Q}_{\rm conv} = 150 - 3 = 147 \text{ kW}$$

The torque can be calculated from Eq. (1.11) given the RPM of the output shaft as 4000/2 = 2000 RPM.

$$T = \frac{60}{2\pi N} \dot{W}_{\rm sh} = \frac{60}{2\pi (2000)} (147) = 0.702 \text{ kN} \cdot \text{m}$$

The energetic efficiency can be defined as the ratio of the shaft power delivered by the system to the required input, the power delivered to the gearbox.

$$\eta_I = \frac{\dot{W}_{\rm sh,out}}{\dot{W}_{\rm sh,in}} = \frac{147}{150} = 98 \%$$

An entropy balance for the internal system (black boundary) produces

$$\frac{dS'^{0}}{dt} = \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,int}} = \frac{-\dot{Q}_{\text{conv}}}{T_{B}} + \dot{S}_{\text{gen}}$$
$$\Rightarrow \quad \dot{S}_{\text{gen,int}} = \frac{\dot{Q}_{\text{conv}}}{T_{B}} = \frac{3}{(273+35)} = 0.0097 \frac{\text{kW}}{\text{K}}$$

The entropy balance equation over the system's universe (enclosed by the red boundary) yields

$$\frac{dS'^{0}}{dt} = \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,univ}} = \frac{-\dot{Q}_{\text{conv}}}{T_{0}} + \dot{S}_{\text{gen,univ}}$$
$$\Rightarrow \quad \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{conv}}}{T_{0}} = \frac{3}{(273+25)} = 0.01003 \frac{\text{kW}}{\text{K}}$$

Discussion $\dot{S}_{\text{gen,univ}}$ can be seen to be only marginally greater than \dot{S}_{gen} , indicating that the bulk of the entropy generation takes place due to friction inside the gearbox with the rest contributed by thermal friction in the immediate vicinity of the system.

2.2.1 Cycles – A Special Case of Closed Steady Systems

It is quite easy to construct a device that converts work into heat. In fact, the electric heater of Ex. 3-9 converts electric work into heat with the perfect energetic efficiency of 100%. As yet another example, consider the paddle-wheel device of Fig. 2.29. As the paddle stirs the fluid, viscous friction dissipates the kinetic energy into internal energy; the temperature rises, and heat starts conducting across the wall into the outside atmosphere. Eventually, a steady state is reached and the simplified governing equations can be used to evaluate the energetic efficiency and the entropy generated in the system's universe.

$$\dot{W}_{\rm sh,in} = \dot{Q}_{\rm out}$$
; therefore, $\eta_I \equiv \frac{\dot{Q}_{\rm out}}{\dot{W}_{\rm sh,in}} = 100\%$; and, $\dot{S}_{\rm gen,univ} = \frac{\dot{Q}_{\rm out}}{T_0}$; (2.23)

where, $\dot{W}_{\rm sh,in}$ and $\dot{Q}_{\rm out}$ represent absolute values with their subscripts indicating the direction of energy transfer. The energetic efficiency of the device is 100% to satisfy the conservation of energy. Also, $\dot{Q}_{\rm out}$ and T_0 being positive, $\dot{S}_{\rm gen,univ} > 0$, satisfying the second law.

While conversion of useful work into heat is a relatively simple to accomplish (rubbing hands against each other, for instance), the reverse is not so. A device that can steadily convert heat to useful work is called a **heat engine** (see Anim. 2.F.*heatEngine*). Building heat engines has been such a historical obsession for engineers that the word *engin*eer owes its origin to early heat engines.

At first thought, building a heat engine does not appear to be a difficult task. Useful work can be obtained by heating a gas in a piston-cylinder assembly as shown in Fig. 2.30 (also see Anim. 1.D.*intEnergy*); however, such a conversion is a one time event and is not sustainable on a steady basis. Historically, many conceptual designs have been proposed for a heat engine. In Fig. 2.31, work is produced at a steady rate of \dot{W}_{out} without any other interactions with the surroundings. With $\dot{Q} = 0$, the steady-state energy equation, Eq. (2.20), produces $\dot{W}_{ext} = \dot{W}_{out} = 0$. Any steady work output from this engine is, therefore, a clear violation of the first law. Such a fictitious engine is called the perpetual machine of the first kind or **PMM1**. The second device, shown in Fig. 2.32, produces work at a steady rate of \dot{W}_{out} as heat \dot{Q}_H is added from a reservoir at T_H . An energy analysis quickly yields $\dot{W}_{out} = \dot{Q}_H$, establishing an energetic efficiency of 100%. Clearly, the first law is no longer violated by this improved design. However, the entropy









Fig. 2.30 Although this device produces electricity from heat, it is not a heat engine because the process cannot be indefinitely sustained.

balance equation, Eq. (2.21), applied to the engine's universe produces, $\dot{S}_{\text{gen,univ}} = -\frac{Q_{\text{H}}}{T_{H}}$, a

negative but impossible value given $\dot{Q}_{\rm H}$ and $T_{\rm H}$ are both positive quantities. To emphasize the violation of the second law by this type of fictitious engines, they arecalled the perpetual machines of the second kind or **PMM2**.

Kelvin-Planck Statement The realization that PMM2 is forbidden by fundamental laws came before the development of the second law. In fact, the Kelvin-Planck statement of the second law asserts what we have just established from the entropy equation.

It is impossible to build a heat engine that exchanges heat with a single reservoir.

A reservoir or, more precisely, a **thermal energy reservoir** (TER) has been already introduced as a large reservoir of energy, a heating source or sink, whose temperature does not change regardless of the amount of heat transfer. The atmosphere, for instance, is a TER at T_0 .

The Clausius statement, derived in the previous section, and the Kelvin- Planck statement above are the pioneering pronouncements of the second-law of thermodynamics, which, historically, led to the discovery of entropy as a property. In our postulative approach, these statements are derived from the entropy balance equation. Towards the end of chapter 5, we will reconcile these two approaches by reconstructing the arguments that led to the concept of entropy.

2.2.1.1 Heat Engine

If the second perpetual motion machine discussed above is modified to reject heat \dot{Q}_c to a second TER (see Fig. 2.33) at T_c , the energy and entropy balance equations, Eqs. (2.20) and (2.21), can both be satisfied.

$$\dot{Q}_{H} - \dot{Q}_{C} = \dot{W}_{\text{net}};$$
 Therefore, $\dot{W}_{\text{net}} > 0$ if $\dot{Q}_{H} > \dot{Q}_{C};$ (2.24)
 $\dot{Q}_{H} = \dot{Q}_{C};$ (2.24)

$$\dot{S}_{\text{gen,univ}} = \frac{Q_C}{T_C} - \frac{Q_H}{T_H};$$
 Therefore, $\dot{S}_{\text{gen,univ}} \ge 0$ if $\dot{Q}_C \ge \dot{Q}_H \frac{T_C}{T_H};$ (2.25)

Here, all the heat and work transfer symbols represent absolute values, and appropriate signs are added while they are used in the energy or entropy equation. The analysis above establishes that positive work output by the engine is possible provided some of the heat received from the hot reservoir is rejected to a cold one. With the contradictions of the







Fig. 2.32 Perpetual machine of the second kind violates second law of thermodynamics.

perpetual machines removed, the construct of Fig. 2.33 forms a fundamental framework for a practical heat engine. The net work output of such a concept **heat engine** has to be less than the heat input, resulting in an energetic efficiency, $\dot{W}_{net} / \dot{Q}_H$, of less than 100%. The cold reservoir is generally is the atmosphere and the rejected heat \dot{Q}_C is completely dissipated into internal energy and wasted (hence, the name **waste heat** used in industries), leaving \dot{W}_{net} as the only desirable output and \dot{Q}_H as the required input.

Chapters 7 through 10 are exclusively devoted to analysis of heat engines and refrigeration cycles. For an overall analysis, however, a system executing a cycle can be treated as a closed steady system, allowing simplified analysis. Consider the schematic of a Rankine cycle, the analytical model of a steam power plant, shown in Fig. 2.34 and Anim. 2.F. openCycle, where several open devices are connected in series to form a closed loop. External work is supplied to the **pump** at a rate \dot{W}_{in} to raise the pressure of water so as to force it through the entire cycle. In the **boiler**, heat is added at a rate \dot{Q}_{μ} from an external high-temperature TER, say, a coal or gas fired furnace, to transform water into vapor. The turbine extracts a significant fraction of the flow energy (exergy, to be precise) of the high-pressure, high-temperature vapor as useful shaft work \dot{W}_{out} . The low-pressure (sub-atmospheric) vapor, still hotter than atmospheric air, is condensed to liquid state by a heat exchanger called the **condenser**, where heat is rejected at a rate \dot{Q}_c to a colder TER, usually the atmosphere or water of a nearby reservoir. The condensed water returns to the pump, completing the cycle. Power producing cycles of this type, where open steady devices are connected back to back to form a loop, are called open **power cycles** and exclusively covered in chapters 8 (Gas Power) and 9 (Vapor Power).

Observe that there is only heat and work transfer – and no mass transfer - across the red boundary of the system drawn around the Rankine cycle of Fig. 2.34 (and Anim. 2.F.*openCycle*). The resulting closed system internalizes all the open devices, leaving only heat and external work interactions with its surroundings. As complex as the interior of the heat engine may be, it is rather easy to decide if it is at steady state or not. The image of the global system, taken with the *state camera*, will be quite colorful since the state varies from point to point within a device and from device to device in an overall fashion. However, if the engine operates at steady state, any two snapshots taken at two different *macroscopic instants* will be identical. Temperature and pressure sensors connected at various locations along the path of steam will show stationary readings, when fluctuations within a macroscopic instant are averaged out. The Rankine engine,







Fig. 2.34 The Rankine cycle consists of open steady devices connected in a loop.

therefore, qualifies as a closed steady device as described by the abstract diagram of Fig. 2.33 or Anim. 2.F.*openCycle*.

Heat engines commonly found in automobiles are called reciprocating engines because of the reciprocating motion of the piston. There are two types of reciprocating engines - the spark-ignition or SI engines used in gasoline-powered vehicles are modeled by the air standard Otto cycle, and the compression-ignition or CI engines used in diesel powered vehicles are modeled by the air standard Diesel cycle. To briefly describe one of these cycles, say, the Otto cycle, (see Anims. 1.B.closedPowerCycle and 2.F.closedCycle), consider the four processes executed by a fixed mass of air trapped in a piston-cylinder device of Fig. 2.35. In the compression process, the piston is pushed from the bottom location, called the **bottom dead center** or **BDC**, to the top location, the **top dead center** or **TDC**, requiring W_{in} amount of boundary work. At the end of the compression process, heat Q_{H} is transferred to the air almost instantly at constant volume - this is to simulate the rapid combustion process that takes place in an actual spark ignition engine. The high pressure and high temperature air expands, pushing the piston back to the BDC and transferring W_{out} amount of boundary work into the crank shaft. Finally, waste heat in the amount Q_c is rejected at constant volume (to simulate the actual expulsion followed by fresh intake) until the air comes to the original state, completing the cycle. The atmospheric work (work done by or against the atmospheric pressure) during expansion and compression are equal and opposite in sign leaving the net work delivered to the shaft unaffected, i.e., $W_{\text{cycle}} = W_{\text{out}} - W_{\text{in}}$. Such cycles executed by a closed system through a sequence of processes forming a loop are known as reciprocating or closed cycles, discussed exclusively in chapter 7.

During a cycle, the closed system consisting of the trapped gas cannot be considered steady, its state changing in a cyclic nature as the piston goes through the reciprocating motion. However, suppose the shortest time scale we are interested in is long enough for a large number of cycles to have finished execution. With the state camera exposed to multiple cycles in this macroscopic instant, the piston will appear as a blur between the TDC and BDC (see Fig. 2.36 or click the steady state button in Anim. 2.F.*closedCycle*), and the intermittent transfers of work and heat will appear continuous. If *n* is the fixed number of cycles executed every second, the cyclic quantities can be converted into rate basis through $\dot{W}_{net} = nW_{cycle}$, $\dot{Q}_{H} = nQ_{H}$, and $\dot{Q}_{C} = nQ_{C}$. The simplified sketch of Fig. 2.33 or Anim. 2.F.*heatEngine*, once again, can be used to conceptually represent a heat engine, this time implemented by a closed cycle.



Fig. 2.35 The Otto cycle consists of a series of processes forming a loop.



Fig. 2.36 Averaged over many cycles, even a reciprocating system can be regarded as closed and steady.

In terms of the absolute symbols used in Fig. 2.33 for the energy transfers, the energy balance equation for the closed steady engine, regardless of how it is implemented (Rankine or Otto cycle), can be written as

Energy:
$$0 = \underbrace{\left(\dot{Q}_{H} - \dot{Q}_{C}\right)}_{\dot{Q}_{\text{net}}} - \dot{W}_{\text{net}}; \quad \Rightarrow \quad \dot{Q}_{\text{net}} = \dot{W}_{\text{net}}$$
(2.26)

The energetic efficiency of a heat engine is known as the **thermal efficiency** and is defined as follows (see Anim. 2.F.effAndCOP).

$$\eta_{\rm I} = \eta_{\rm th} \equiv \frac{\text{Desired Energy Output}}{\text{Required Energy Input}} = \frac{\dot{W}_{\rm net}}{\dot{Q}_{\rm H}} = \frac{\dot{Q}_{\rm H} - \dot{Q}_{\rm C}}{\dot{Q}_{\rm H}} = 1 - \frac{\dot{Q}_{\rm C}}{\dot{Q}_{\rm H}}$$
(2.27)

Operating expenses of a heat engine primarily depends on \dot{Q}_{H} supplied from the burning of fossil fuels or other alternative sources. One of the goals in the design of an ideal heat engine, therefore, is to maximize η_{th} , which also means minimization of the ratio of waste heat to the heat supplied, \dot{Q}_{C}/\dot{Q}_{H} .

EXAMPLE 2-11 Energy Analysis of Heat Engine

The fuel efficiency of a compact car is rated at 40 miles/gallon while operating at a steady speed of 70 mph with the engine producing a power of 40 hp to overcome aerodynamic drag and other resistances. If the fuel has a heating value of 44 MJ/kg and a density of 700 kg/m³, (a) determine the thermal efficiency of the engine. (b) *What-if scenario:* How would a 5% increase in the thermal efficiency affect the fuel mileage?

SOLUTION Determine the rate of heat release \dot{Q}_{H} and obtain the efficiency from Eq. (2.27).

Assumptions The engine runs at steady state. Entire heating value of fuel is converted to heat.

Analysis Use the unit converter daemon to express all quantities into SI units. The volume flow rate of fuel to the engine is

 $\dot{V}_F = \frac{\text{mph}}{\text{mpg}} = \frac{70}{40} = 1.75 \frac{\text{gal}}{\text{h}} = 1.84 \times 10^{-6} \frac{\text{m}^3}{\text{s}}$

The fuel consumption rate in kg/s, therefore, can now be obtained.

Did you know?

>Why are there two M's in MMBTU (a million BTU)? The two M stand for a million in Roman numerals.

>What is a Therm? A Therm is equivalent to the heating value of 100 cubic ft of natural gas and is equal to 105.5 MJ.

>What is a Quad and a Q?

They are measures of large amount of energy. A Quad is 1.055×10^{12} MJ or 10^{15} Btu. A Q is 1000 Quad/s. or 33000 GW.

$$\dot{m}_F = \rho_F \dot{V}_F = \left(700 \ \frac{\text{kg}}{\text{m}^3}\right) \left(1.84 \times 10^{-6} \ \frac{\text{m}^3}{\text{s}}\right) = 1.288 \times 10^{-3} \ \frac{\text{kg}}{\text{s}}$$

The net power output of the engine is $\dot{W}_{net} = 40$ hp = 29.83 kW and the heat added due to combustion of fuel is $\dot{Q}_{H} = \dot{m}_{F}$ (HV). Therefore, the thermal efficiency can be obtained from Eq. (2.27).

$$\eta_{\rm th} = \frac{\dot{W}_{\rm net}}{\dot{Q}_{\rm H}} = \frac{\dot{W}_{\rm net}}{\dot{m}_{\rm F} (\rm HV)} = \frac{29.83}{(1.288 \times 10^{-3})(44000)} = 52.63\%$$

What-If-Scenario Everything else remaining unchanged, mpg can be related to thermal efficiency as follows.

$$mpg = \frac{mph}{\dot{V_F}} = \rho_F \frac{mph}{\dot{m}_F} = \rho_F (HV) \frac{mph}{\dot{Q}_H} = \eta_{th} \rho_F (HV) \frac{mph}{\dot{W}_{net}}$$
(2.28)

A 5% increase in the thermal efficiency, therefore, will increase the fuel mileage by 5%. From the above equation, it may appear that an increase in the speed of a car may increase its fuel mileage. This is not true because an increase in speed also increases the power production \dot{W}_{net} by the engine in a disproportionate manner (see Ex. 1-4), and the fuel mileage actually decreases as a result, especially at high velocity.

Discussion The entire heating value is generally not completely utilized due to incomplete combustion and other reasons that will be discussed in chapter 13. An energetic efficiency, called combustion efficiency, is used to obtain a more realistic value of heat release, $\dot{Q}_{H} = \eta_{\text{combustion}} \dot{m}_{F} (\text{HV})$.

2.2.1.2 Refrigerator and Heat Pump

Being an idealized model of a steam power plant, Rankine cycle is a reversible cycle. Conceptually, the Rankine cycle can therefore be run backwards, causing heat to flow from the cold reservoir to the working fluid of the cycle and then from the working fluid to the hot reservoir. Such a cycle is emulated by a **refrigerator** to keep a refrigerated space colder than the environment, and by a **heat pump** to keep a heated space warmer than the environment.

A typical vapor compression refrigeration cycle behind a household refrigerator (Anim. 2.F.*refrigerator*) is sketched in Fig. 2.39 (and Anim. 2.F.*refriCycle*). The working

Did you know?

A window air conditioner with a rating of 5000 Btu/hr with an EER of 10 means that $\dot{Q}_c = 1.47$ kW and COP=2.93 so that $\dot{W}_{\rm net} = 0.5$ kW.

substance, which has a freezing temperature well below the freezing point of water, is called a **refrigerant**. In the **evaporator**, the temperature of the refrigerant is designed to be slightly below that of the refrigerated space, causing it to absorb \dot{Q}_C , which is called the cooling load of a refrigerator. Change of phase (liquid to vapor) enables the refrigerant to remain at a constant temperature until it turns completely into vapor at the evaporator exit. An adiabatic **compressor**, consuming work at a rate of \dot{W}_{net} , raises the pressure of the vapor to force it through the high-pressure condenser. An increase in pressure in the compressor also accompanies an increase in temperature (as in a bicycle pump) above the ambient temperature. Consequently, as the vapor enters the **condenser**, heat is rejected to the atmosphere at a rate \dot{Q}_H . An **expansion valve** throttles the condensate into the evaporator, causing the pressure and temperature to drastically drop (the reason for this will become clear when we study the expansion valve in chapter 4), completing the cycle. This refrigeration cycle can also be used for the opposite purpose - transferring heat from a cold space to a warm space. The resulting device is then called a **heat pump** and \dot{Q}_H is called the desired heating load.

The essence of a refrigeration cycle or heat pump can be represented by the closed steady system of Fig. 2.38 or Anim. 2.F.*effAndCOP* (click on the Refrigerator button) in exactly the same manner as the heat engines were found equivalent to the closed steady system of Fig. 2.33. This closed steady system can represent a refrigerator, which extracts heat at a rate \dot{Q}_c from the refrigerated space in order to keep it at a low temperature T_c against heat leakage or normal operational load. Looked upon as a heat pump, the same closed steady system supplies \dot{Q}_H to the hot reservoir to keep it warm at T_H against any heat loss to a cold surroundings at T_c . The external work necessary to run the compressor is the required input in both configurations.

The energy equation, Eq. (2.26), derived for the heat engine, also applies to the refrigerator and heat pump described by the steady system of Fig. 2.38. The energetic efficiency for these devices can exceed 100%, and that is why it is called the **coefficient of performance** or **COP**.

$$COP_{R} \equiv \frac{Desired Energy Transfer}{Required Input} = \frac{\dot{Q}_{C}}{\dot{W}_{net}} = \frac{\dot{Q}_{C}}{\dot{Q}_{H} - \dot{Q}_{C}} = \frac{1}{\dot{Q}_{H} / \dot{Q}_{C} - 1} \quad (2.29)$$

$$COP_{HP} \equiv \frac{Desired Energy Transfer}{Required Input} = \frac{\dot{Q}_{H}}{\dot{W}_{net}} = \frac{\dot{Q}_{H}}{\dot{Q}_{H} - \dot{Q}_{C}} = \frac{1}{1 - \dot{Q}_{C} / \dot{Q}_{H}} \quad (2.30)$$



Fig. 2.37 Vapor compression cycle used in a refrigerator or heat pump.



Fig. 2.38 Refrigerator or heat pump as a closed steady system.

Manipulating these equations, it can be shown that $\text{COP}_{HP} = \text{COP}_R + 1$. Obviously, COP of a heat pump is always greater than 1. Although COP is a convenient measure of energetic performance of refrigerators and heat pumps, a different measure called the Energy Efficiency Rating or EER is often used in the USA. An **EER** is the number of Btu's removed from a cooled space for every watt-hour of electricity consumed. Since 1 W \cdot h = 3.412 Btu, an EER is related to the COP by EER=3.412 COP.

EXAMPLE 2-12 Energy Analysis of a Refrigerator

Due to ineffective sealing, heat leaks into a kitchen refrigerator at a rate of 5 kW. If the COP of the refrigerator is 3.5, determine (a) the rate of heat rejection by the refrigerator into the kitchen, and (b) the net rate of energy transfer into the kitchen. Treat the kitchen as a closed system.

SOLUTION Perform an energy balance on the refrigerator as a closed steady system, and the kitchen as a closed unsteady system.

Assumptions The kitchen can be considered an adiabatic closed system while the refrigerator runs a closed sub-system that runs at a steady state.

Analysis To keep the refrigerated space at a constant temperature, heat should be removed exactly at the same rate as the leakage rate. Therefore, $\dot{Q}_c = 5$ kW (see Anim. 2.F.*refrigerator*). The net work and the rejected heat can be obtained as follows.

 $\dot{W}_{\text{net}} = \frac{\dot{Q}_C}{\text{COP}_{\text{R}}} = \frac{5}{3.5} = 1.43 \text{ kW}; \quad \dot{Q}_H = \dot{Q}_C + \dot{W}_{\text{net}} = 6.43 \text{ kW};$

As explained by the energy flow diagram of Fig. 2.39, the net rate of heat transfer from the refrigerator to the kitchen is 6.43-5 = 1.43 kW.

Discussion Note that if we take the entire kitchen as the system, the only mode of energy transfer is electrical work. Therefore, regardless of how the electrical power is utilized inside the kitchen, $\dot{W}_{el} = \dot{W}_{net} = 1.43 \text{ kW}$

2.2.1.3 The Carnot Cycle

Because a heat engine must reject some heat in order to satisfy the second law of thermodynamics, it can never achieve a thermal efficiency of 100%. What, then, is the



Fig. 2.39 Schematic for Example 2-12.

maximum limit for η_{th} ? Parallel questions can be raised regarding the COP of a refrigerator or a heat pump.

Purely through deductive reasoning, a young engineer named Sadi Carnot answered those questions in 1811, about 30 years before the first law was formally established, and along the way gave a mathematical representation of the second law. The cycle he proposed, called the **Carnot cycle**, constitutes the most efficient heat engine possible, and when its direction is reversed, yields the highest possible COP for a refrigerator or heat pump.

2.2.1.3.1 Carnot Heat Engine

The Carnot cycle is a completely reversible cycle, executed without any entropy generation, internal or external. It operates between two reservoirs (TER), one at a high temperature T_H and another at a low temperature T_C (usually the temperature of the ambient atmosphere T_0). Since the cycle is reversible, heat transfer cannot take place across any finite temperature difference. T_C and T_H remaining constant, both heat addition and rejection must take place isothermally with only a differential temperature difference driving the heat transfers between the reservoirs and the engine (recall the discussion of reversible heat transfer in Ex. 2-6).

Postponing specific implementations of Carnot cycle to chapter 7 (see Anim. 7.A.*carnotCycle* for a preview), energy and entropy analysis can be performed on the concept Carnot engine sketched in Fig. 2.40, which differs from a generic heat engine (compare Fig. 2.40 with Fig. 2.33) in two respects: the temperatures of heat addition and rejection in a Carnot engine must remain constant, and $\dot{S}_{gen,univ} = 0$ because the Carnot engine is reversible. While the Rankine or Otto cycle is also reversible, recall that heat transfers in those engines do not take place at constant temperatures.

The energy equation for the Carnot engine remains the same as Eq. (2.26), derived for the generic heat engine. However, when $\dot{S}_{gen,univ} = 0$ is substituted in the entropy equation, Eq. (2.25), a simple yet powerful conclusion emerges.

$$0 = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{\text{gen,univ}}^0; \quad \Rightarrow \quad \frac{\dot{Q}_C}{\dot{Q}_H} = \frac{T_C}{T_H}$$
(2.31)

To ensure reversibility, heat transfer in a Carnot cycle must be proportional to the absolute temperature of the reservoir, independent of the details of the cycle. Introducing



Fig. 2.40 Energy and entropy flow in a Carnot heat engine.

this result into the definition of thermal efficiency, Eq. (2.27), results in the following expression for the **Carnot efficiency** (see Anim. 2.F.*carnotEff*)

$$\eta_{\text{Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{H}} = 1 - \frac{\dot{Q}_{C}}{\dot{Q}_{H}} = 1 - \frac{T_{C}}{T_{H}}$$
(2.32)

The Carnot efficiency is a remarkably simple yet elegant result that bypasses all the internal complexities of a cycle and gets to the bottom line in terms of easily measurable quantities, the absolute temperatures of the hot and cold reservoirs. The cold reservoir is usually the atmosphere; therefore, only the highest temperature used in any heat engine needs to be known in order to estimate an upper limit of its thermal efficiency.

The following theorems, proved by Carnot, purely through deductive reasoning from the Kelvin-Planck statement, can be derived as corollaries of Eq. (2.32).

Theorem 1 All Carnot engines operating between the same two temperatures, T_H and T_C , must have the same efficiency.

Equation (2.32), which was derived without any reference to the implementation of the cycle, shows that thermal efficiency of the Carnot engine is a function of the TER temperatures alone (see Fig. 2.41). Two different Carnot engines, implemented completely differently (see Anims. 7.A.*carntoClosedCycle*, 9.A.*carnotCycle*, and Fig. 2.41) but operating between the same pair of reservoirs must produce the same efficiency as dictated by Eq. (2.32).

Theorem 2 *The efficiency of a Reversible engine is higher than any other engine operating between the same two temperatures.*

The efficiency of an irreversible engine operating between two fixed-temperatures can be derived by retaining the entropy generation term in the entropy equation, Eq. (2.25).

$$0 = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{\text{gen,univ}}; \quad \Rightarrow \quad \frac{\dot{Q}_C}{\dot{Q}_H} = \frac{T_C}{T_H} + \dot{S}_{\text{gen,univ}} \frac{T_C}{\dot{Q}_H}$$
(2.33)

The thermal efficiency of an irreversible heat engine, therefore, is given by

$$\eta_{\text{th,irrev}} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{T_C}{T_H} - \dot{S}_{\text{gen,univ}} \frac{T_C}{\dot{Q}_H} = \eta_{\text{Carnot}} - \dot{S}_{\text{gen,univ}} \frac{T_C}{\dot{Q}_H}$$
(2.34)

With $\dot{S}_{\text{gen,univ}}$, \dot{Q}_{H} , and T_{C} , all being positive, $\eta_{\text{Carnot}} \ge \eta_{\text{th,irrev}}$.



Fig. 2.41 All Carnot cycles have the same efficiency regardless of how they are implemented.

Theorem 3 For the same cold reservoir temperature T_c , the Carnot engine that has the larger ΔT has the higher efficiency.

This is obvious from the expression of η_{Carnot} derived in Eq. (2.32). A plot of η_{Carnot} against $\Delta T = T_H - T_C$, where T_C is assumed to be the standard atmospheric temperature T_0 is shown in Fig. 2.42. Notice how rapidly the Carnot efficiency increases with ΔT , especially when the efficiency is below 40%.

2.2.1.3.2 Carnot Refrigerator and Heat Pump

Because Carnot heat engine is a completely reversible cycle, it can be simply reversed to serve as a refrigerator or a heat pump (compare Fig. 2.40 with Fig. 2.43). The entropy equation for this reversed configuration shown in Fig. 2.43 produces the same relation between the heat transfers – proportionality of heat transfer rate with absolute reservoir temperature - as in the case of the Carnot heat engine.

$$0 = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen,univ}}^{0}; \quad \Rightarrow \quad \frac{\dot{Q}_C}{\dot{Q}_H} = \frac{T_C}{T_H}$$
(2.35)

Substituting this result in Eq. (2.29) and (2.30), the COP's of the Carnot refrigerator and heat pump can be expressed as (see Anim. 2.F.*carnotCOP*) follows:

$$COP_{R,Carnot} = \frac{\dot{Q}_{C}}{\dot{W}_{net}} = \frac{\dot{Q}_{C}}{\dot{Q}_{H} - \dot{Q}_{C}} = \frac{1}{\dot{Q}_{H} / \dot{Q}_{C} - 1} = \frac{T_{C}}{T_{H} - T_{C}}$$
(2.36)

$$COP_{HP,Carnot} = \frac{\dot{Q}_{H}}{\dot{W}_{net}} = \frac{\dot{Q}_{H}}{\dot{Q}_{H} - \dot{Q}_{C}} = \frac{T_{H}}{T_{H} - T_{C}} = 1 + COP_{R,Carnot}$$
(2.37)

Once again the striking simplicity of the formulas makes them some of the most elegant results in engineering. The maximum possible COP of a refrigerator or heat pump can be calculated merely from the knowledge of the temperatures of reservoirs. Further simplification can be achieved from the fact that one of the reservoirs – the hot reservoir for a refrigerator and the cold reservoir for a heat pump – is often the atmosphere. To study the sensitivity of the Carnot COP on the temperature difference between the reservoirs, Eq. (2.36) is used with T_H set to the standard atmospheric temperature T_0 . The resulting plot of Fig. 2.44 shows a dramatic increase in COP as T_C approaches T_0 . Setting the temperature of the refrigerated space a few degrees closer to outside temperature, therefore, can result in considerable saving of work due to two reasons –



Fig. 2.42 Carnot efficiency monotonically increases with temperature difference.



Fig. 2.43 Energy and entropy flow in a Carnot refrigeration/heat pump cycle (see Anim. 2.F.*carnotCOP*).

improvement of COP and a reduction in \dot{Q}_c (due to a reduction in the driving force, $T_0 - T_c$, for heat leakage).

Although actual refrigerators and heat pumps do not operate as Carnot cycles, Fig. 2.44 can be used as a qualitative guide for predicting actual COP. As an example, consider a Carnot heat pump designed for $T_H = 20$ °C (temperature inside a house), $T_C = 0$ °C (outside temperature), and $\dot{Q}_H = 5$ kW (heat pumped to the house). The compressor, therefore, must be rated at 0.34 kW since the COP can be calculated from Eq. (2.37) as 14.65. Now suppose the temperature in a particular night drops down to, say, -20 °C so that $T_C = -20$ °C; consequently, the COP decreases to 7.325, almost half its original value. To pump heat at the same rate, the compressor must be able to operate at twice its original power rating. Compounding the problem, the heating load can be expected to go up due to increased heat loss to the colder surroundings. Heat pumps, therefore, are not suitable for extreme climates. It is remarkable that we can reach such a powerful conclusion even though a detailed analysis of a refrigerator or a heat pump will be taken up in chapter 7.



Fig. 2.44 Carnot COP rapidly decreases if the temperature difference becomes more severe.

EXAMPLE 2-13 Carnot Heat Engine

In the engine described in Ex. 2-11, the maximum temperature achieved during the cycle is 1500 K. If the atmospheric temperature is 298 K, evaluate if the fuel mileage of 40 mpg claimed by the manufacturer is reasonable.

SOLUTION Evaluate the Carnot efficiency and compare it with the actual thermal efficiency of the engine.

Analysis The most efficient engine for the car is a Carnot engine running between 1500 K and 298 K. The highest possible efficiency, therefore, is the Carnot efficiency.

$$\eta_{\text{th,Max}} = \eta_{\text{Carnot}} = 1 - \frac{T_c}{T_H} = 1 - \frac{298}{1500} = 80.1\%$$

The actual thermal efficiency, calculated in Ex. 3-11, is only 52.63%, much below the Carnot efficiency. Therefore, the fuel mileage claimed by the manufacturer is well within the theoretical limit.

TEST Solution A closed steady daemon to calculate thermal efficiencies and COPs can be found in the *Daemons*> *Closed*> *Steady* page of TEST. However, given the simplicity of calculations, a TEST verification is probably not necessary.

Discussion Approaching the Carnot efficiency is the ultimate goal of any energy efficient combustion engine. In chapters 7 through 10, we will discuss various approaches to improve efficiencies of actual heat engine and refrigeration cycles. Fuel cells (introduced in chapter 14), however, are not heat engines and, therefore, much of the concepts developed in this section cannot be applied to fuel cells.

EXAMPLE 2-14 Reversible Refrigerator

A kitchen refrigerator maintains the freezer compartment at a temperature -10 °C and the main compartment at 1 °C while the outside temperature is 30 °C. The refrigerator consumes a power of 2 kW while removing heat from the two compartments at a rate of 10 MJ/h and 5 MJ/h respectively (see accompanying figure). Determine (a) the COP of the refrigerator and (b) the rate of entropy generation in the refrigerator's universe. (c) *What-if scenario:* What would the power consumption be if the refrigerator operated in a reversible manner?

SOLUTION Perform an energy and entropy balance on the refrigerator's universe, a closed system enclosed within the red boundary of Fig. 2.45.

Assumptions The refrigerator can be modeled as a closed steady system.

Analysis Referring to the accompanying sketch, the known heat transfer rates are $\dot{Q}_F = 10 \text{ MJ/h} = 2.778 \text{ kW}$ and $\dot{Q}_M = 5 \text{ MJ/h} = 1.389 \text{ kW}$. By definition, the coefficient of performance is

$$COP_{R} = \frac{Desired Energy Transfer}{Required Input} = \frac{\dot{Q}_{F} + \dot{Q}_{M}}{\dot{W}_{net}} = \frac{2.778 + 1.389}{2} = 2.08$$

An energy balance at steady state yields

$$0 = (\dot{Q}_F + \dot{Q}_M - \dot{Q}_0) - (-\dot{W}_{net});$$

$$\Rightarrow \dot{Q}_0 = \dot{Q}_F + \dot{Q}_M - \dot{W}_{net} = 2.778 + 1.389 - (-2) = 6.167 \text{ kW};$$

An entropy balance over the system's universe (red boundary) produces



Fig. 2.45 System schematic and energy flow diagram for Ex. 2-14.

$$0 = \frac{\dot{Q}_F}{T_F} + \frac{\dot{Q}_M}{T_M} - \frac{\dot{Q}_0}{T_0} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_0}{T_0} - \frac{\dot{Q}_F}{T_F} - \frac{\dot{Q}_M}{T_M} = \frac{6.167}{303} - \frac{2.778}{263} - \frac{1.389}{274} = 0.0047 \frac{\text{kW}}{\text{K}}$$

When a reversible refrigerator replaces the actual one, \dot{Q}_0 and \dot{W}_{net} would both change as $\dot{S}_{gen,univ}$ goes to zero. An entropy balance for the reversible system produces

$$0 = \frac{\dot{Q}_F}{T_F} + \frac{\dot{Q}_M}{T_M} - \frac{\dot{Q}_{0,rev}}{T_0} + \dot{S}_{gen,univ}^0$$

$$\Rightarrow \dot{Q}_{0,rev} = T_0 \left(\frac{\dot{Q}_F}{T_F} + \frac{\dot{Q}_M}{T_M}\right) = 303 \left(\frac{2.778}{263} + \frac{1.389}{274}\right) = 4.736 \text{ kW}$$

Substituting this in the energy balance equation,

$$0 = (\dot{Q}_F + \dot{Q}_M - \dot{Q}_{0,\text{rev}}) - (-\dot{W}_{\text{net, rev}});$$

$$\Rightarrow \dot{W}_{\text{net, rev}} = \dot{Q}_{0,\text{rev}} - \dot{Q}_F - \dot{Q}_M = 4.736 - 2.778 - 1.389 = 0.569 \text{ kW};$$

The COP of the reversible refrigerator now can be calculated as

$$\text{COP}_{\text{R, rev}} = \frac{\dot{Q}_F + \dot{Q}_M}{\dot{W}_{\text{net, rev}}} = \frac{2.778 + 1.389}{0.569} = 7.32$$

Discussion Note that using an average cold-space temperature of $T_c = 268.5$ K, results in a Carnot COP of 268.5/(303-268.5) = 7.78, which overestimates the correct answer.

EXAMPLE 2-15 Heat Pump and its Alternatives

A house requires 600 MJ of heat per day to maintain its temperature at 20° C while the outside temperature is 2° C. If the cost of electricity is 10 cents per kW h and the cost of natural gas is 75 cents per Therm, compare the daily operational cost of the following alternative heating systems: (a) Electrical heating, (b) Gas heating system with an energetic efficiency of 90%. (c) A Carnot heat pump.

SOLUTION Perform an energy analysis on each alternative, shown by the subsystems in Fig. 2.46.





Assumptions Each heating subsystems behaves as a closed steady system.

Analysis Use the unit converter daemon to convert different energy units into MJ: 1 kW \cdot h = 3.6 MJ, 1 Therm = 105.5 MJ. Therefore, the cost of electricity and natural gas per unit MJ can be calculated as 2.857 cents and 0.711 cents respectively.

An energy balance for electrical heater as a closed steady system (see Fig. 2.46) produces

$$0 = (-\dot{Q}_{\rm H}) - (-\dot{W}_{\rm el}); \implies \dot{W}_{\rm el} = \dot{Q}_{\rm H}; \implies \frac{W_{\rm el}}{\Delta t} = \frac{Q_{\rm H}}{\Delta t};$$
$$\implies W_{\rm el} = Q_{\rm H} = 600 \text{ MJ}$$

Electrical Heating Cost = $(600)(0.0278) = 16.67 \frac{\$}{day}$

For gas heating, the energetic efficiency relates the heat transferred to the house to the heat released by gas \dot{Q}_{gas} . An energy balance for the furnace yields

$$0 = (\eta_{I, heater} \dot{Q}_{gas} - \dot{Q}_{H}) - \dot{W}_{ext}^{0};$$

$$\Rightarrow Q_{gas} = \dot{Q}_{gas} \Delta t = \frac{\dot{Q}_{H}}{\eta_{I, heater}} \Delta t = \frac{Q_{H}}{\eta_{I, heater}} = \frac{600}{0.9} = 666.7 \text{ MJ}$$

Gas Heating Cost = (666.7)(0.00711) = 4.74 $\frac{\$}{day}$

A Carnot heat pump operating between the temperature inside and outside the house will have a COP of

$$\text{COP}_{\text{HP,Carnot}} = \frac{T_H}{T_H - T_0} = \frac{293}{18} = 16.3$$

Therefore, cost of the work input, supplied by electrical power, can be calculated as

HP Electricity Cost =
$$\left(\frac{600}{16.3}\right)(0.0278) = 1.02 \frac{\$}{\text{day}}$$

Discussion The best alternative, clearly, is the reversible heat pump as far as the operating cost is concerned. However, there are a few caveats. A real heat pump has a much lower COP than its Carnot counterpart. Also, if the outside temperature decreases below 2° C, the COP of the heat pump deteriorates drastically (see Fig. 2.44).

2.2.1.4 The Kelvin Temperature Scale

While discussing temperature as a property in section 1.3.4.5, the Kelvin scale for absolute temperature was asserted as a better alternative to arbitrary Celcius or Fahrenheit markings on a thermometer. Having discussed the Carnot theorems, we are now in a position to understand the arguments that led Kelvin to establish a purely *thermodynamic* temperature scale that bears his name.

When Carnot proposed his theorem (see sec. 2.2.1.3), he did not have the benefit of the balance equations. In fact, the concept of entropy or even heat as a form of energy in transit did not exist. Although by Carnot efficiency, we routinely mean $\eta_{Carnot} = 1 - T_C / T_H$, what Carnot actually proposed as Theorem-1 is

$$\eta_{\text{th,Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{H}} = 1 - \frac{\dot{Q}_{C}}{\dot{Q}_{H}} = 1 - f(T_{C}, T_{H}); \text{ or, } \frac{\dot{Q}_{C}}{\dot{Q}_{H}} = f(T_{C}, T_{H})$$
(2.38)

That is, the thermal efficiency of a reversible engine operating between two constanttemperature reservoirs is a function of the temperatures of the reservoirs alone. He arrived at this conclusion purely through deductive reasoning from Kelvin-Planck statement of the second law.

Kelvin realized that Carnot's conclusion is independent of the unit used for temperature and set out to explore the temperature dependent unknown function f of Eq. (2.38). To follow Kelvin's argument, consider the three Carnot engines - two in series and one in parallel - as shown in Fig. 2.47. Applying Eq. (2.38) to each engine, we obtain

$$\frac{\dot{Q}_{1}}{\dot{Q}_{2}} = f(T_{1}, T_{2}); \ \frac{\dot{Q}_{2}}{\dot{Q}_{3}} = f(T_{2}, T_{3}); \text{ and, } \frac{\dot{Q}_{1}}{\dot{Q}_{3}} = f(T_{1}, T_{3});$$

$$\Rightarrow f(T_{1}, T_{3}) = \frac{\dot{Q}_{1}}{\dot{Q}_{3}} = \frac{\dot{Q}_{1}}{\dot{Q}_{2}} \frac{\dot{Q}_{2}}{\dot{Q}_{3}} = f(T_{1}, T_{2}) f(T_{2}, T_{3})$$
(2.39)

The nature of the function f must be such that T_2 disappears on the right hand side of Eq. (2.39). This is possible only if f has the following form.



Fig. 2.47 Illustration used to develop the absolute temperature scale.

$$f(T_H, T_C) = \frac{g(T_H)}{g(T_C)}$$

$$(2.40)$$

so that
$$f(T_1, T_2) f(T_2, T_3) = \frac{g(T_1)}{g(T_2)} \frac{g(T_2)}{g(T_3)} = f(T_1, T_3)$$

Kelvin proposed the simplest possible form, g(T) = T, for the function g so that Eq. (2.38) reduces to the familiar form

$$\frac{T_H}{T_C} = \left(\frac{\dot{Q}_H}{\dot{Q}_C}\right)_{\text{Carnot Cycle}}$$
(2.41)

Equation (2.41) defines the new temperature scale known as the **Kelvin scale**. By taking the limit of $\dot{Q}_C \rightarrow 0$ or $\dot{Q}_H \rightarrow \infty$ we can see that the lower and upper limit of the Kelvin temperature are zero and infinity respectively – hence, the name **absolute temperature** for this thermodynamic scale of temperature.

2.3 Closure

In this chapter, we have introduced the fundamental laws of thermodynamics, translated them into balance equations, and applied those to the study of closed steady systems. The chapter begins with derivation of with the mass, energy, and entropy balance equations. Flow diagrams for energy and entropy are introduced to visualize different terms of the balance equations for energy and entropy. The first and second laws of thermodynamics are introduced in a postulative manner and the mechanisms of entropy generation are discussed with several examples. Energetic efficiency and reversibility are discussed in a general manner. The remainder of the chapter is devoted to comprehensive analysis of closed steady systems. Heat engines, refrigerators, and heat pumps are studied as special cases of closed steady systems. The thermal efficiency of heat engines and COP of refrigerators and heat pumps are introduced as special energetic efficiencies. Carnot efficiency and Carnot COP's are derived from an energy and entropy analysis of the reversible Carnot cycles. The Kelvin temperature scale, introduced in chapter 1, is finally established from a theoretical standpoint as a closing note.

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- 1. Balance equations without the burden of property evaluation.
- 2. All equations treated in a similar manner with emphasis on similarities.
- 3. Use of specific flow energy in a consistent manner. Wdot_ext in a consistent manner for both closed and open systems.
- 4. Energy and entropy flow diagrams for systems.
- 5. Using the daemons to understand the balance equations.
- 6. Closed steady systems without properties
- 7. Closed cycles treated as a special case of closed steady systems.
- 8. We make a point that a large class of systems can be analyzed without tables and charts many students don't know that.

To do: 1. Rename 2.A.openUnsteady to openUnsteadySystem.

To do: check all references with 1.xx etc. Also all other references to 1.

Pluses: Derivation in the same template. Entropy as the basis. Different statements are derived. Much more indepth discussion of second law. Entropy analysis of closed steady systems showing mechanisms of entropy generation through entropy diagram is unique. Derivation of T-ds equation is much more general with much more rigor. Right in chapter 2 one uses the comprehensive analysis without the clutter of property evaluation. Use of energy and entropy flow diagrams. Analysis of cycles without the clutter of property evaluation early.

Note: For a closed system with dQ, dW = 0; therefore, dU = dQ. TdS equation will produce TdS = dU+TdSgen,int. Therefore dSgen = 0. That means with TB = T you have to heat reversibly. The dSGen in the T-ds derivation, therefore, comes solely from the work lost. dQ cannot cause entropy generation as there is no temperature discontinuity. The T-ds proof has to be slightly rewritten.