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Classical Thermodynamics 1/e

Subrata Bhattacharjee •

Prentice Hall
Upper Saddle River, NJ 07458

COMPREHENSIVE ANALYSIS OF STEADY SYSTEMS

Chapter 4

In chapter-2, the fundamental laws of thermodynamics- the conservation of mass principle, the first law, and the second law – were expressed as *balance equations* of mass, energy, and entropy for a generic open unsteady system. Closed steady systems were then analyzed as a special case by simplifying these balance equations. Analysis of any other type of system requires evaluation of states. Having established different material models in chapter 3 for evaluating extended states, we are now in a position to tackle more complex systems.

This chapter is dedicated to the analysis of open steady systems – systems with all types of interactions with the surroundings yet no change in their global states so that total mass, stored energy, or entropy do not change with time. Examples of such systems include pipes, nozzles, diffusers, pumps, compressors, turbines, throttling valves, heat exchangers, mixing chambers, etc., operating at steady state. The objective of this chapter is to gain insight into the steady-state operation of these devices through comprehensive mass, energy, and entropy analysis. The framework developed for the analysis – classify a system through suitable assumptions, customize the balance equations, select an appropriate material model for the working substance, make necessary approximations, obtain a manual solution, use appropriate TEST daemons to verify manual solution, and carry out what-if studies whenever possible– will become a template for system analysis throughout this book.

4.1 Governing Equations and Device Efficiencies

Most engineering devices are designed to operate over a long period of time without much change in the operating conditions. Gas turbines, steam power plants, refrigeration systems, industrial air-conditioning systems, etc., operate sometimes for months before scheduled maintenance shut down. Components such as pipes, turbines, pumps, compressors, nozzles, diffusers, heat exchangers, etc., are open, allowing mass transfer across their boundaries, and are assumed to operate at steady state so that the snapshot taken with a *state camera* (see sec. 1.3.2) does not change with time under the design condition. Open devices, operating at steady states will be referred as **open steady systems** in this book.

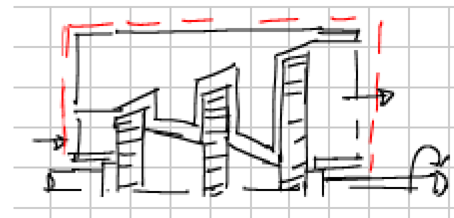


Fig. 4.1 Steam passes through a series of nozzles connected to the casing and blades attached to the rotating shaft in a turbine.

Consider a particular open steady system, a steam turbine illustrated in Anim. 1.C.*turbine* and Fig. 4.1, which produces shaft work at the expense of flow energy as steam passes through an alternating series of stationary and rotating blades. The fixed blades attached to the stationary casing of the turbine create nozzle shaped passages, through which steam expands to a lower pressure and higher velocity (we will learn more about why a nozzle accelerates a flow shortly) before impinging on an array of blades attached to a central shaft. Transfer of momentum from steam to the blades creates a torque on the shaft, making it turn. Steam then enters the next stage and the process is repeated until it leaves the turbine at a temperature and pressure much lower than those at the inlet. The large variation of properties across the device means that the turbine is a non-uniform system. However, properties at a given point do not change with time at steady state – the global state, which is an aggregate of all the local states, remains frozen in time.

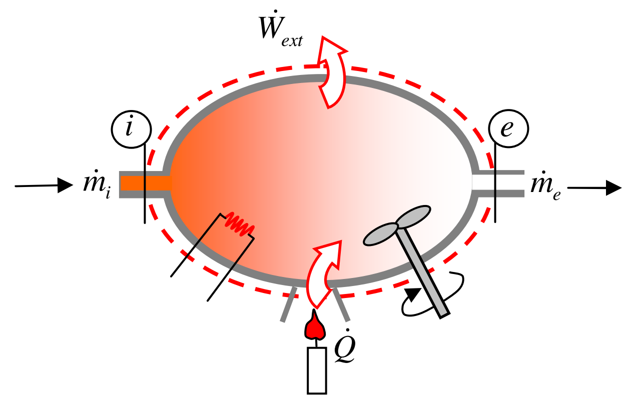


Fig.4.2 The *generic open steady system* – the global state is non-uniform but frozen in time.

This conclusion about the steady turbine holds true for any open steady system, a generic version of which is represented by Fig. 4.2 (or Anim. 4.A.*genericSystem*). The system is non-uniform as indicated by the variation of color, but steady, which is indicated in the animation by the fact that the color pattern does not change with time. The external work transfer in most systems consists primarily of electrical and shaft work and heat transfer occurs mostly with the atmospheric reservoir (TER) unless indicated otherwise.

With the system image remaining frozen, all global properties of the open system, including m, E, and S, must remain constant at steady state. After all, these global properties are sum of the corresponding properties of the local systems comprising the system. Therefore, the unsteady terms in the balance equations – the time derivatives of mass, energy, and entropy – can be set to zero, simplifying the governing balance equations considerably. Furthermore, a large class of devices has only a single inlet and a single exit, that is, a single flow through the system. For such **single-flow devices**, the governing equations are further simplified as the summations of transport terms over multiple ports become unnecessary. The resulting equations are reproduced below from chapter 2 and illustrated in Anim. 4.A.*govEqns*.

Mass: $$\dot{m}_i = \dot{m}_e\text{; or, } \rho_i A_i V_i = \rho_e A_e V_e; \quad \text{or } \frac{A_i V_i}{v_i} = \frac{A_e V_e}{v_e} \tag{4.1}$$

Energy: $$\cancel{\frac{dE}{dt}}^{0} = \dot{m}\left(j_i - j_e\right) + \dot{Q} - \dot{W}_{\text{ext}}; \quad \text{where, } j \equiv h + \text{ke} + \text{pe} \tag{4.2}$$

Entropy: $$\cancel{\frac{dS}{dt}}^{0} = \dot{m}\left(s_i - s_e\right) + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}} \tag{4.3}$$

Different terms of the energy and entropy equations are illustrated in the energy and entropy diagram of Anim. 4.A.*genericSystem*. For many devices, changes in kinetic and potential energies are often negligible, allowing the flow energy j to be replaced by enthalpy h. Another simplification results in the case of adiabatic systems as $\dot{Q}$ is set to zero. Note that by setting the transport terms to zero (no mass transfer), these equations reduce to the set of equations used in the analysis of closed steady systems in sec. 2.2.

4.1.1 TEST and the Open Steady Daemons

The **open steady daemons** build upon the state daemons, introduced in section 1.3.3, and will be extensively used in this chapter to verify manual solutions and pursue what-if studies. Open steady daemons for single flow devices are located in *Daemons*> *Systems*> *Open*> *Steady*> *Generic*> *SingleFlow* page. There, you will see the familiar material models listed just as in state daemon pages. Click on a particular model, say, SL-model, to launch a single-flow daemon where the working fluid is a liquid (or solid – yes, it is possible to have an open system with a solid passing through it).

An open steady daemons looks remarkably similar to the corresponding flow state daemon. Both these daemons have the same default view with identical global control panels and state panels. Two new panels – device and exergy panels that are accessible through the tabs – are added to the open steady daemons. In the state panel, you calculate the inlet and exit states – called the **anchor states** of a device – as completely as possible from the given information.

Now switch to the device panel. In the local control panel, you will see a device identification choice (Device-A, B, etc.), and choices for selecting the anchor states. Under the control panel you will see several device variables – Qdot, Wdot_ext, etc. With the anchor states calculated, all you do in this panel is to select a device identification (A, B, etc.,), choose the inlet and exit states from the calculated state stack, enter the known device variables, and Calculate. The governing equations are solved to produce the unknowns. If an unknown happen to be a state property (j, s, or mdot, for instance), it is posted back to the appropriate state. You can go back to that state (by selecting the state tab and then the state number) and calculate the state completely with the help of the newly evaluated property. Alternatively, the Super-Calculate button does the same by iterating between the device and state panels. It also generates the TEST-codes in the I/O panel that summarizes the solution, and from which the solution can be reproduced at a later time. TEST-codes for all the examples in this chapter can be found in the *TEST.Examples* page.

A discussion of the exergy panel will be postponed until chapter 6. For more details and hands-on examples on the open steady daemons, go through the *Tutorial.Daemons.Chapter04* page or watch the video clips in chapter 4 of the *videoTour* linked from the task bar.

4.1.2 Energetic Efficiency

Efficiency means various things for various systems. However, it can be loosely defined as *the ratio of desired output to required input*. In section 2.2, the energetic efficiency for closed systems was introduced as the ratio of desired energy to the required energy input for a closed steady device. The same definition can be extended for an open steady device. The **energetic efficiency**, also known as the **first law efficiency**, for a steady device, open or closed, is defined as

$$\eta_{\mathrm{I}} \equiv \frac{\text{Desired Energy Output}}{\text{Required Input of Energy}} \qquad (4.4)$$

where η represents efficiency of different types and the subscript I stands for the first law. Specific quantities that appear in the numerator and denominator depend on the purpose of a given device and how each term in its energy balance equation is interpreted physically. With the unsteady term dropping out due to steady-state condition, the energy equation for a generic device, Eq. (2.8), can be interpreted as follows.

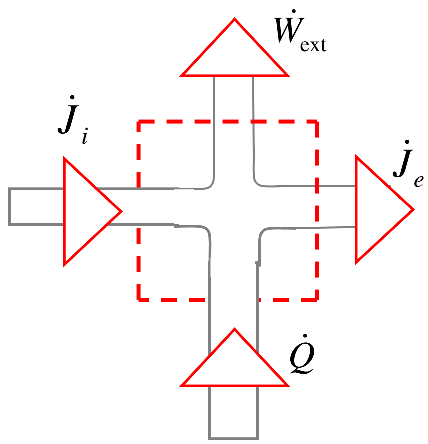


Fig. 4.3 Energy flow diagram for an open steady device shown in Fig. 4.2.

$$0=\underbrace{\sum_i \dot m_i j_i-\sum_e \dot m_e j_e}_{\dot J_{\text{net}}\text{, Net rate of transport of energy into the system.}}+\underbrace{\dot Q}_{\text{Net Rate of heat transfer into the system.}}-\underbrace{\dot W_{\text{ext}}}_{\text{Net Rate of external work transfer out of the system.}}\;;\text{ or, }\;\dot J_{\text{net}}+\dot Q=\dot W_{\text{ext}} \qquad (4.5)$$

For a steady device, the net energy transported by the flow into the device plus the energy transferred by heat must equal the external work delivered by the system. This interpretation is also evident from the energy flow diagram of Fig. 4.3 (also click the Energy button in Anim. 4.A.*genericSystem*). Such diagrams can be helpful to define and visualize the energetic efficiency for a given device. We have already calculated the efficiency of electrical space heater– a closed steady system - to be 100% in Ex. 2-9. Let us perform a similar analysis on an electrically powered water heater (see Anim. 4.A.*waterHeater*) in the following example.

EXAMPLE 4-1 [*mE*[1]] Analysis of a Water Heater

An electric water heater supplies hot water at 150 kPa, 70°C at a flow rate of 10 L/min as shown in the accompanying diagram. The water temperature at the inlet is 15°C. Due to poor insulation, heat is lost at a rate of 2 kW to the surrounding atmosphere at 25°C. Using the SL model for water and neglecting kinetic and potential energy transport, determine (a) the electrical power consumption, (b) the energetic efficiency, and (c) the rate of entropy generation in the heater's universe. Assume no pressure loss in the system.

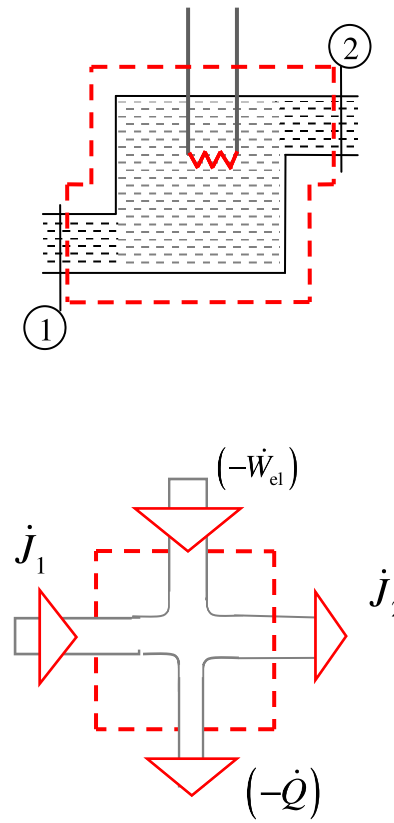


Fig. 4.4 System schematic and energy flow diagram for Ex. 4-1 (visit Anim. 4.A.*waterHeater* for the entropy diagram).

SOLUTION Analyze the heater's universe, enclosed within the red boundary of Fig. 4.4, using the mass, energy, and entropy balance equations.

Assumptions The heater is at steady state. Flow states at the inlet and exit, state 1 and 2, are uniform and at LTE. Since ke and pe are negligible, $j \equiv h+\text{ke}+\text{pe} \cong h$.

Analysis The material properties for water, ρ and c_v, are obtained from Table A-1 or the SL state daemon as 997 kg/m^3 and 4.184 $\text{kJ/kg}\cdot\text{K}$ respectively.

The mass flow rate can be obtained from the volume flow rate by using Eq. (3.4).

$$\dot m=\rho\dot{V\!\!\!\!-}=997\frac{10\times10^{-3}}{60}=0.166\ \frac{\text{kg}}{\text{s}}$$

---

[1] Analysis involving mass, energy, and entropy equations is symbolized by *mEE* in bracket.

The energy equation, Eq. (4.2), can be manipulated to produce the electricity consumption, $\dot{W}_{\text{el}} = \dot{W}_{\text{ext}}$, as follows.

$$
\begin{aligned}
0 &= \dot{m}\left(j_i - j_e\right) + \dot{Q} - \dot{W}_{\text{ext}} \\
\Rightarrow \quad \dot{W}_{\text{ext}} &= \dot{m}\left(j_1 - j_2\right) + \dot{Q} \cong \dot{m}\left(h_1 - h_2\right) + \dot{Q} \\
&= \dot{m}c_v\left(T_1 - T_2\right) + \dot{m}v\cancel{\left(p_2 - p_1\right)}^{0} + \dot{Q} \\
&= (0.166)(4.184)(15 - 70) + (-2) = -40.2 \text{ kW}
\end{aligned}
$$

In this derivation, the formula for enthalpy difference is substituted from Eq. (3.24). The negative sign indicates that external work is delivered to the heater.

The external electrical work clearly is the required input while the energy supplied to the water, $\dot{m}\left(j_2 - j_1\right)$, is the desired output is. The energetic efficiency, therefore, can be expressed as

$$
\eta_I = \frac{\dot{m}\left(j_2 - j_1\right)}{-\dot{W}_{\text{ext}}} = \frac{\dot{Q} - \dot{W}_{\text{ext}}}{-\dot{W}_{\text{ext}}} = \frac{(-2) - (-40.2)}{-(-40.2)} = \frac{38.2}{40.2} = 95.02\ \%
$$

For the system's universe enclosed within the red boundary of Fig. 4.3, the entropy equation, Eq. (4.3), produces

$$
\cancel{\frac{dS}{dt}}^{0} = \dot{m}\left(s_1 - s_2\right) + \frac{\dot{Q}}{T_0} + \dot{S}_{\text{gen,univ}}
$$

$$
\begin{aligned}
\Rightarrow \quad \dot{S}_{\text{gen,univ}} &= \dot{m}\left(s_2 - s_1\right) - \frac{\dot{Q}}{T_0} = \dot{m}c_v \ln\frac{T_2}{T_1} - \frac{\dot{Q}}{T_0} \\
&= (0.166)(4.184)\ln\frac{(273+70)}{(273+15)} - \frac{(-2)}{(273+25)} = 0.128\ \frac{\text{kW}}{\text{K}}
\end{aligned}
$$

TEST Analysis Launch the open steady SL daemon and select Water(L) from the working-fluid menu. Evaluate state-1 from p1, T1, and Voldot1, and state-2 from p2=p1, T2, and mdot2=mdot1. In the device panel, select state-1 and state-2 as the inlet and exit states, enter Qdot as -2 kW, and set T_B as 25 deg-C. Calculate. Wdot_ext and Sdot_gen are evaluated, verifying the manual results. By changing p1 to any other value, and super-calculating, the results can be shown to be independent of the heater pressure.

Discussion Entropy is generated inside the heater as electrical energy is dissipated into heat (electronic friction) and as heat is transferred across a finite temperature difference (thermal friction) inside and in the immediate surroundings of the system. An energetic efficiency of 95% may seem quite satisfactory. However, as we will learn in chapter 6, the accompanying entropy generation is a measure of degradation or wastefulness of useful energy. The same goal of raising the water temperature could be achieved with much less work if a reversible water heater could be invented.

4.1.3 Internally Reversible System

Although entropy generation is pervasive in most systems, idealized systems, where entropy generation can be neglected within the system boundary or even in the entire system's universe, serve as important benchmarks for real systems. Such systems have been introduced in sec. 2.1.3.1 as *internally reversible* and *reversible* systems respectively. Because entropy generation can be looked upon as generalized friction, an internally reversible system often forms an ideal counterpart of an actual device. Analysis of such idealized systems (see Anim. 2.D.*reversibility*), can provide significant insight into understanding the performance limit of many devices.

To explore the implication of reversibility in an open steady system, let us do an entropy analysis of a generic internally reversible ($\dot{S}_{\text{gen,int}} = 0$) single-flow system, sketched in Fig. 4.5. The suffix *int.rev.* is added to both heat and external work transfer to remind us that the system is internally reversible. Because there is no internal thermal friction, temperature must vary in a continuous manner from the inlet to the exit. The change in internal boundary temperature requires the entropy transfer by heat to be represented by an integral, and the entropy equation can be manipulated as follows.

$$\cancelto{0}{\frac{dS}{dt}} = \dot{m}\left(s_i - s_e\right) + \int_i^e \frac{\text{đ}\dot{Q}_{\text{int.rev.}}}{T} + \cancelto{0}{\text{đ}\dot{S}_{\text{gen,int}}}$$

$$\Rightarrow \int_i^e \frac{\text{đ}\dot{Q}_{\text{int.rev.}}}{T} - \dot{m}\int_i^e ds = 0; \qquad \Rightarrow \int_i^e \left[\frac{\text{đ}\dot{Q}_{\text{int.rev.}}}{T} - \dot{m}ds\right] = 0 \qquad (4.6)$$

$$\Rightarrow \text{đ}\dot{Q}_{\text{int.rev.}} = \dot{m}Tds$$

For the single-flow system, Eq. (4.6) can be integrated from the inlet to the exit, yielding

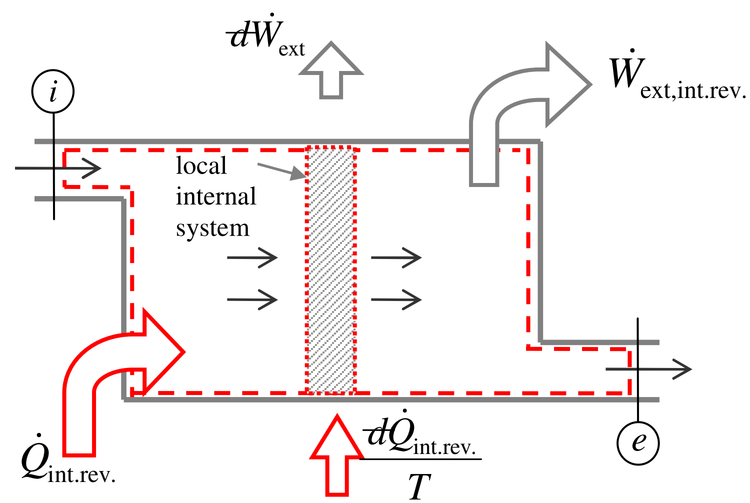


Fig. 4.5 Locations of states 1 and 2 are so close together that the local system (dotted boundary) can be considered uniform.

$$\dot{Q}_{\text{int. rev.}} = \dot{m}\int_i^e T ds \quad [\text{kW}] \tag{4.7}$$

Although derived on purely theoretical ground, this equation has an important implication for any $T-s$ diagram. No matter what is the working fluid, the area under the $T-s$ diagram (see Fig. 4.6) can be interpreted as the heat transfer per unit mass of the working fluid as long as the system can be assumed to be internally reversible. Of course, the same interpretation also applies to reversible systems ($\dot{S}_{\text{gen,univ}} = 0$), which must be internally reversible. Heat transfer to an internally reversible system is called **reversible heat transfer** represented by Eq. (4.7). The significance of reversible heat transfer can be appreciated when we discuss reversible cycles in chapters 7 through 10. For irreversible systems ($\dot{S}_{\text{gen}} > 0$), it can be shown from Eq. (4.6) that the area under the $T-s$ diagram can have contributions from heat transfer as well as internal irreversibilities represented by the internal entropy generation.

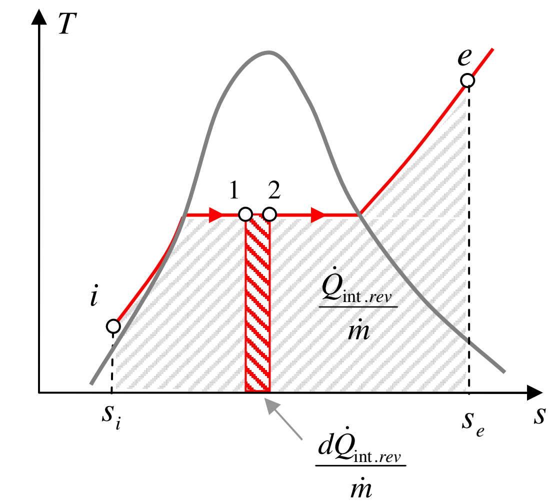


Fig. 4.6 For an internally reversible system, the area under the $T-s$ diagram can be associated with the heat transfer.

The steady flow energy equation, similarly, can be simplified for an *internally reversible* (and, hence, *reversible*) system, producing, a parallel graphical interpretation for the external work transfer.

$$\cancelto{0,\text{ Steady State}}{\frac{dE}{dt}} = \dot{m}(j_i - j_e) + \dot{Q}_{\text{int. rev.}} - \dot{W}_{\text{ext, int. rev.}}$$

$$\Rightarrow \quad \dot{W}_{\text{ext, int. rev.}} = -\dot{m}\int_i^e dj + \dot{m}\int_i^e T ds = \dot{m}\int_i^e (T ds - dj) = \dot{m}\int_i^e \left[-v dp - d(\text{ke}) - d(\text{pe})\right]$$

$$\Rightarrow \quad \dot{W}_{\text{ext, int. rev.}} = -\dot{m}\left[\int_i^e v dp + (\text{ke}_e - \text{ke}_i) + (\text{pe}_e - pe_i)\right] \tag{4.8}$$

When the changes in ke and pe between the inlet and exit are negligible, the equation simplifies to

$$\dot{W}_{\text{ext, int. rev.}} = -\dot{m}\int_i^e v dp \qquad [\text{kW}] \tag{4.9}$$

Notice the misleading resemblance between this equation and the $p d\cancel{V}$ formula, Eq. (1.15), for boundary work involving a closed system. While the external work for an internally reversible system is proportional to the area projected on the p axis (see Fig. 4.7), the $p d\cancel{V}$ work for closed system is proportional to the projected area on the v axis on a $p-v$ diagram. Also note that an internally reversible system allows heat transfer and does not have to be adiabatic.

Equations (4.9) and (4.7) lends important physical meaning to thermodynamic plots. Even without knowing the details of how a device actually produces work or transfers heat, remarkable conclusions can be drawn from the graphical interpretation of these equations. For instance, gas and vapor have much larger v compared to liquids. Other parameters remaining unchanged, the integral in Eq. (4.9) can be expected to be much larger in magnitude for a gas or a vapor compared to a liquid. A compressor that raises the pressure of vapor, therefore, will require much more work than a pump raising the pressure of a liquid by the same amount (compare Figs. 4.7 and 4.8). In the Rankine cycle introduced in Fig. 2.34, the turbine and pump operate with the same pressure differences, yet the turbine produces much more power than what the pump consumes. Once again, Eq. (4.9) is at the root of the correct physical explanation.

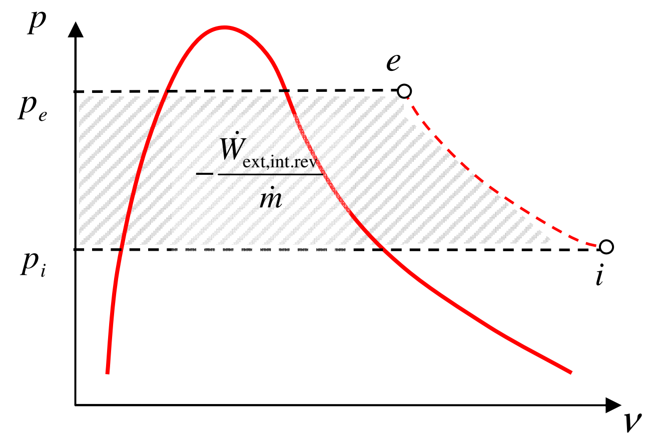


Fig. 4.7 For an internally reversible system, the shaded area is proportional to the magnitude of external work transfer.

4.1.4 Isentropic Efficiency

Systems can be idealized in many different ways. Since friction, quantified by entropy generation, downgrades the performance of most devices, an ideal device can be assumed to be internally reversible, i.e., $\dot{S}_{\text{gen,int}} = 0$. Obviously, devices that rely on friction cannot be internally reversible even at their ideal limit. Many single-flow devices such as pumps, compressors, turbines, nozzles, diffusers, etc., can be regarded as adiabatic (no heat transfer) and operate across fixed inlet and exit pressures under design conditions. We will refer to the inlet state as state-1 and the exit state as state-2. At their ideal limit, these devices can be considered internally reversible. Representing the ideal exit state by state-3, the actual and ideal devices can be compared through an entropy analysis as follows.

$$\text{Actual device: } \cancelto{0,\text{ steady state}}{\frac{dS}{dt}} = \dot{m}(s_1 - s_2) + \frac{\cancelto{0,\text{ adiabatic}}{\dot{Q}}}{T_B} + \dot{S}_{\text{gen,int}} \qquad (4.10)$$

$$\Rightarrow \quad s_2 \geq s_1 \quad \text{since } \dot{S}_{\text{gen,int}} \geq 0$$

$$\text{Ideal device: } \cancelto{0,\text{ steady state}}{\frac{dS}{dt}} = \dot{m}(s_1 - s_3) + \frac{\cancelto{0,\text{ adiabatic}}{\dot{Q}}}{T_B} + \cancelto{0,\text{ internally reversible}}{\dot{S}_{\text{gen,int}}} \qquad (4.11)$$

$$\Rightarrow \quad s_1 = s_3$$

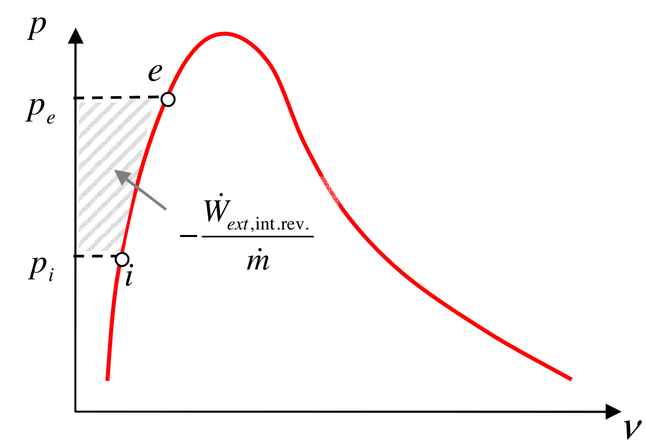


Fig. 4.8 Compared to Fig. 4.7, the work transfer is much less here because the working fluid is a liquid with a much lower specific volume.

The ideal single-flow device, therefore, is simply called an **isentropic** device. Of course, it is mathematically possible for entropy to remain unchanged between the inlet and exit if entropy transfer through heat loss is exactly offset by internal entropy generation; however, such situations are not very likely and an isentropic device implies that it is steady, adiabatic, and internally reversible. On a $T-s$ plot (see Fig. 4.9), such a device is represented by the vertical solid line joining the inlet and ideal exit states. For the corresponding actual device (represented by the dashed lines), entropy generation being non-negative, $s_2 \geq s_1$ and the actual exit state, which is on the same constant pressure line as state-2 in Fig. 4.9, is shifted to the right.

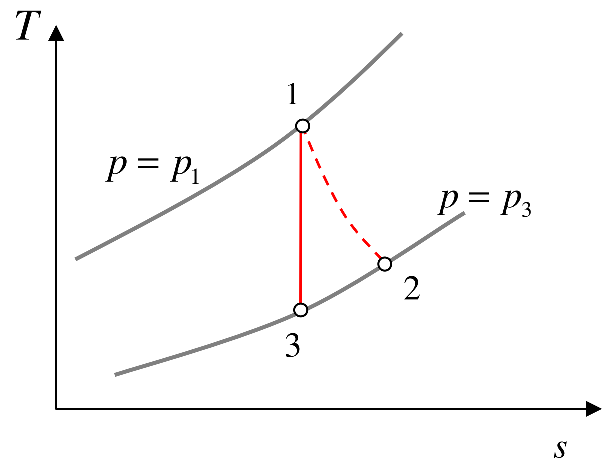


Fig. 4.9 The isentropic and actual exit states share the same exit pressure.

The **isentropic efficiency** compares an energetic term (work output, work input, exit kinetic energy, etc.) from the energy equation of the actual device with the corresponding term of the **isentropic device**.

$$\eta_{\text{device}} = \frac{\text{An Energetic Term of Actual (or Ideal) Device}}{\text{Corresponding Term of Ideal (or Actual) Device}} \qquad (4.12)$$

The exact definition, which is device dependent, ensures that the maximum possible value of the isentropic efficiency does not exceed 100%. As we discuss individual devices in the coming sections, we will introduce these device-specific definitions.

4.2 Comprehensive Analysis

In what follows, we will discuss several open devices, subjecting them to a comprehensive analysis using the mass, energy, and entropy equations. Limiting the discussion of the inner workings to a minimum, we will introduce each device through its functionalities with emphasis on system-surroundings interactions. Suitable assumptions and approximations will be made to simplify the governing equations to a set of equations customized for the particular device.

4.2.1 Pipes, Ducts or Tubes

A **pipe**, **duct**, or a **tube** – to reduce confusion, we will simply use the term a *pipe* - is a hollow cylinder used to conduct a liquid, gas, or finely divided solid particles (see Anim. 4.A.*pipe*). As passive as it may appear, a simple pipe can be turned into interesting devices such as a hair dryer (Anim. 4.A.*hairDryer*), a water heater (Anim. 4.A.*waterHeater*), a boiler tube, etc., depending on heat and work interactions with the surroundings.

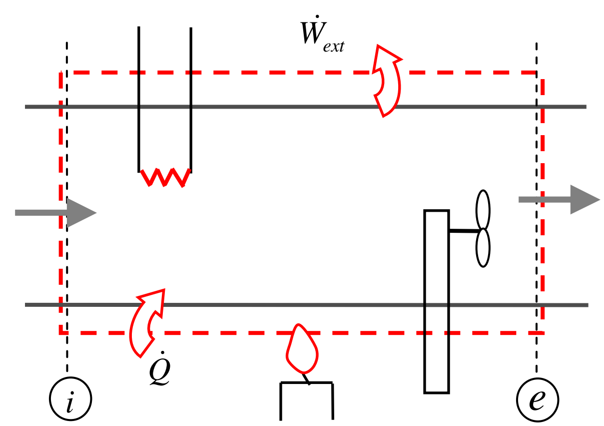


Fig. 4.10 Many useful devices can be represented as a pipe flow with heat and external work transfer.

Consider a steady flow through a constant diameter pipe of a certain length with work and heat interactions as shown in Fig. 4.10. If wall friction in the pipe is not negligible, how will it impact the exit velocity? Before we jump to any conclusion, let us see where the balance equations lead us.

A steady state mass balance between the inlet and exit yields $\dot{m} = A_i V_i / v_i = A_e V_e / v_e$, or $V_i / v_i = V_e / v_e$. Therefore, for an **incompressible** fluid – any fluid that can be treated by the SL model – the mass equation leads to $V_i = V_e$. That is, the velocity must remain constant regardless of friction or orientation. This is a powerful conclusion when you think about liquid water flowing downward through a vertical pipe.

For a (compressible) gas or vapor flow, however, the situation becomes complicated since v is a function of temperature and pressure (for an ideal gas $v = RT/p$). From the free body diagram sketched in Fig. 4.11, it can be deduced that the pressure decreases along the flow to overcome frictional resistance ($p_i A > p_e A$) regardless of fluid density, or heat transfer. Therefore, in the case of isothermal flow ($T =$ constant) or flow with positive heat transfer (T increases), v increases along the flow since $v = RT/p$ for an ideal gas (which is also approximately true for a vapor). An increase in specific volume results in an increase in velocity (recall from mass equation that $V_i / v_i = V_e / v_e$). In fact, advanced gas dynamics analysis can be used to show that the gas velocity can increase up to the speed of sound despite the presence of friction in a heated gas flow.

Did you know?

The longest underwater pipe line, extending more than 500 miles, carries natural gas from Norwegian North Sea gas fields to France.

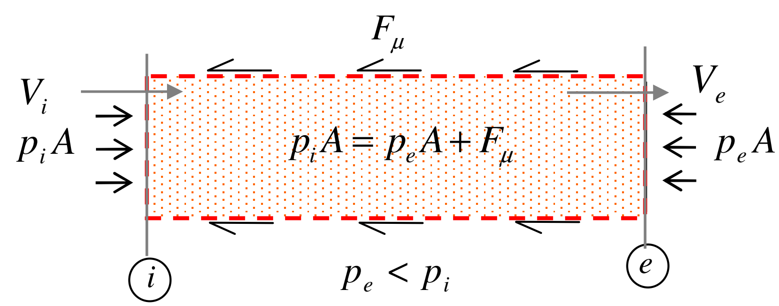


Fig. 4.11 Free body diagram of the fluid in a straight pipe. The force balance is independent of energy or entropy balance.

EXAMPLE 4-2 [*mEE*] Analysis of a Duct Flow.

Helium flows steadily into a long and narrow adiabatic tube of diameter 1 cm with a velocity of 20 m/s at 200 kPa and $30^\circ\mathrm{C}$. At the exit, the pressure drops to 150 kPa to overcome wall friction. Determine the (a) mass flow rate, (b) exit velocity, (c) exit temperature, and (d) the rate of entropy generation in the system's universe. Neglect changes in kinetic and potential energies. ***What-if scenario:*** (e) What would the exit temperature and velocity be if kinetic energy was not neglected?

SOLUTION Analyze the open steady system, the pipe's universe enclosed within the red boundary of Fig. 4.12, using the mass, energy, and entropy balance equations.

Assumptions Steady state, perfect gas behavior for helium, uniform states based on LTE at the inlet and exit, and negligible changes in ke and pe.

Analysis From Table C-1 or any PG daemon, obtain PG model constants $R = 2.078$ kJ/kg·K and $c_p = 5.196$ kJ/kg·K for helium.

The energy equation, Eq. (4.2), for the system shown in Fig. 4.12 can be simplified as follows.

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}\left(j_i - j_e\right) + \dot{Q} - \dot{W}_{\text{ext}} \cong \dot{m}\left(h_i - h_e\right) + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0}$$

$$\Rightarrow \quad h_i - h_e = c_p\left(T_i - T_e\right) = 0; \quad \Rightarrow \; T_e = T_i = 303 \text{ K};$$

Since h is a function of T only for both the PG and IG models, the conclusion that T remains constant in an isenthalpic flow is also valid for any ideal gas.

The mass equation, $\dot{m}_i = \dot{m}_e = \dot{m}$, coupled with the application of IG equation of state, $pv = RT$, produces

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{R T_1} = \frac{\left(7.854\times10^{-5}\right)(20)(200)}{(2.0785)(303)} = 4.99\times10^{-4} \ \frac{\text{kg}}{\text{s}};$$

$$\Rightarrow V_2 = \frac{\dot{m} v_2}{A_2} = \frac{\dot{m} R T_2}{A_2 p_1} = \frac{\left(4.99\times10^{-4}\right)(2.0785)(303)}{\left(7.854\times10^{-5}\right)(150)} = 26.67 \ \frac{\text{m}}{\text{s}};$$

The entropy equation, Eq. (4.3), for the system's universe yields

$$\cancel{\frac{dS}{dt}}^{0,\text{ steady state}} = \dot{m}\left(s_1 - s_2\right) + \frac{\cancel{\dot{Q}}^{0}}{T_0} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}\left(s_2 - s_1\right) = \dot{m}\left(c_p \ln\frac{\cancel{T_2}}{\cancel{T_1}} - R\ln\frac{p_2}{p_1}\right) = 3.0\times10^{-4} \ \frac{\text{kW}}{\text{K}}; \qquad (4.13)$$

Since the system is adiabatic, there is no external entropy generation.

Test Analysis Launch the single-flow PG daemon. Select helium (He) as the working fluid. Calculate the inlet state, state-1, from the known properties. For the exit state (state-2), make Vel2 an unknown, enter p2, and set mdot2 = mdot1, h2 = h1, and A2 = A1. Calculate to find the exit temperature and velocity. In the device panel, select the inlet and exit states, enter Wdot_ext as zero, and Calculate to find Sdot_gen. The manual results are reproduced.

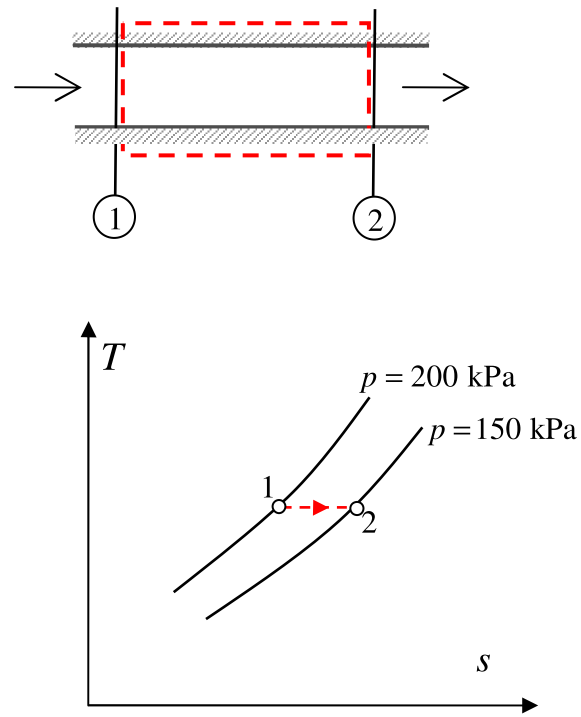


Fig. 4.12 Schematic and $T - s$ diagram for Ex. 4-2.

What-if Scenario To include the effect of change in kinetic energy, use j2 = j1 instead of h2 = h1 in state-2. Super-Calculate to update all calculations. The exit temperature and velocity remain virtually unchanged at 29.97 deg-C and 26.66 m/s respectively.

Discussion Despite the presence of friction, it is the velocity, not temperature, which increases. Although ke seems to have no impact in this problem, $j \cong h$ cannot be automatically assumed. Instead, use TEST to justify the neglect of ke or pe on a case by case basis.

EXAMPLE 4-3 [*mEE*] Analysis of a Hair Dryer.

A hair dryer can be modeled as a steady pipe flow, whereby a small fan pulls the air in and forces it through electrical resistors where it is heated as shown in the accompanying figure. Air enters at the ambient conditions of 100 kPa, $25\,^\circ\mathrm{C}$ at a velocity 10 m/s and leaves with negligible change of pressure at a temperature of $50\,^\circ\mathrm{C}$. The cross-sectional area is 50 cm^2. Heat is lost from the dryer at a rate of 50 W. Determine the (a) exit velocity of the flow, (b) rate of electrical power consumption, and (c) the rate of entropy generation in the system's universe. Use the PG-model for air. ***What-if scenario:*** (d) What would the answers be if the IG model was used?

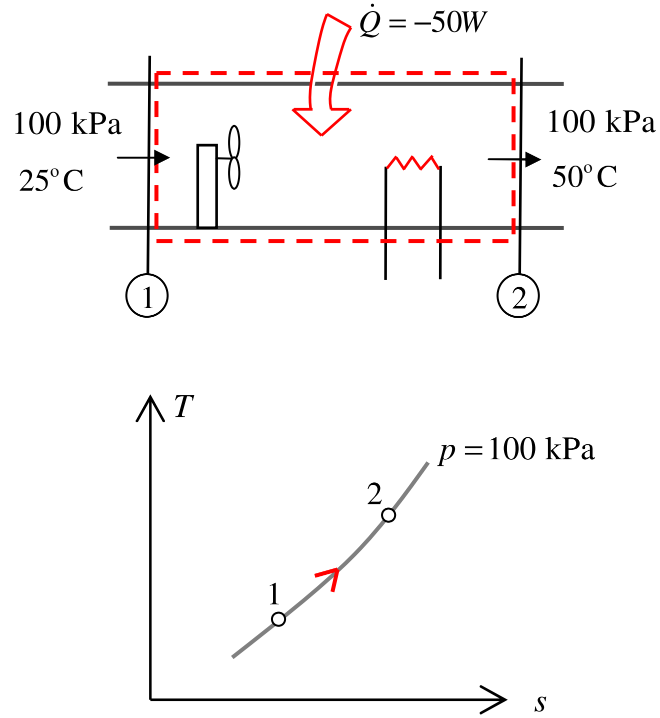


Fig. 4.13 Schematic and $T-s$ diagram for Ex. 4-3.

SOLUTION Analyze the open steady system, the hair dryer's universe enclosed within the red boundary in Fig. 4.13, using the mass, energy, and entropy balance equations.

Assumptions Steady state, perfect gas behavior of air, uniform states at the inlet and exit based on LTE, and negligible changes in ke and pe.

Analysis From Table C-1 or any PG daemon, obtain $R = 0.287$ kJ/kg·K and $c_p = 1.005$ kJ/kg·K for air.

As in the case of the previous example, the mass flow equation, Eq. (4.1), coupled with the ideal gas equation of state produces

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{R T_1} = \frac{(0.005)(10)(100)}{(0.287)(298)} = 0.0585\ \frac{\mathrm{kg}}{\mathrm{s}};$$

$$\Rightarrow V_2 = \frac{\dot{m} v_2}{A_2} = \frac{\dot{m} R T_2}{A_2 p_1} = \frac{(0.0585)(0.287)(323)}{(0.005)(100)} = 10.85\ \frac{\mathrm{m}}{\mathrm{s}};$$

With changes in ke and pe assumed negligible, the energy equation, Eq. (4.2), yields

$$\frac{d\cancel{E}^{0,\text{ steady state}}}{\cancel{dt}} = \dot{m}(j_i - j_e) + \dot{Q} - \dot{W}_{\text{ext}} \cong \dot{m}(h_i - h_e) + \dot{Q} - \dot{W}_{\text{el}}$$

$$\Rightarrow \quad \dot{W}_{\text{el}} = \dot{m}c_p(T_i - T_e) + \dot{Q}$$

$$= (0.0585)(1.005)(25-50) + (-0.05) = -1.52 \text{ kW};$$

The power calculated includes the power consumed by the fan.

The entropy equation, Eq. (4.3), produces the entropy generation rate in the system's universe.

$$\frac{d\cancel{S}^{0,\text{ steady state}}}{\cancel{dt}} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{S}_{\text{gen,univ}} \qquad (4.14)$$

$$\Rightarrow \quad \dot{S}_{\text{gen,univ}} = \dot{m}\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{\cancel{p_2}^{1}}{\cancel{p_1}}\right) + \frac{\dot{Q}}{T_0} = 0.0049 \ \frac{\text{kW}}{\text{K}};$$

Test Analysis Launch the single-flow PG daemon. Select air as the working fluid. Calculate state-1 and 2 as described in the TEST-codes (posted in *TEST.Examples* page). The exit velocity is calculated as part of state-2. In the device panel, select the inlet and exit states, enter Qdot, and Calculate to verify Wdot_ext and Sdot_gen.

What-if Scenario Launch the single-flow IG daemon on a separate browser tab. Copy the TEST-codes generated by the PG solution into the I/O panel, and click the Load button. Results for the IG model can be seen to be almost identical to the PG results because the temperature variation of the gas is not large enough to cause any significant change in the specific heats.

Discussion An isentropic efficiency for a hair dryer does not make any sense because the ideal device cannot operate isentropically (resistance heating inherently generates entropy through electrical and thermal friction). However, an energetic efficiency (see the energy flow diagram of Fig. 4.14) can be defined and evaluated as follows.

$$\eta_{\text{I}} = \frac{\text{Energy Utlized}}{\text{Energy Consumed}} = \frac{\dot{m}(j_2 - j_1)}{(-\dot{W}_{\text{el}})} = \frac{1.47}{1.52} = 96.7\%$$

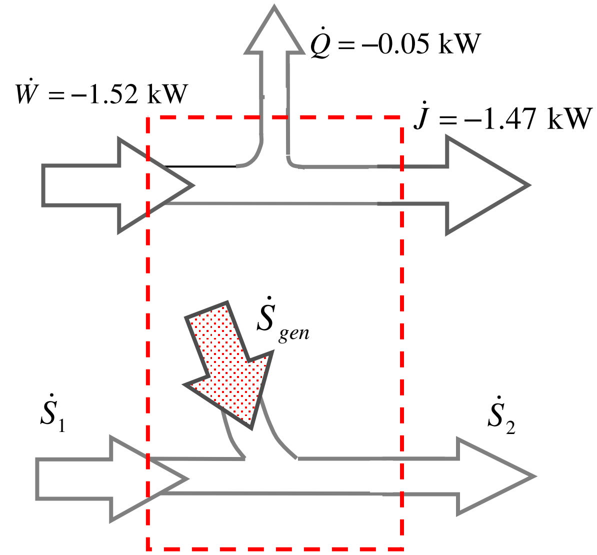


Fig. 4.14 Energy and entropy flow diagram for Ex. 4-3.

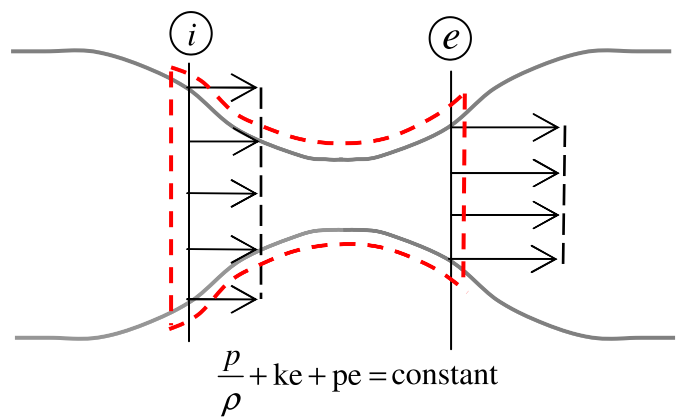


Fig. 4.15 Bernaulli's equation requires a flow to be steady, one-dimensional, incompressible, and internally reversible with no external work transfer.

Bernoulli Equation For internally reversible flow (or reversible), we have already seen how the expressions for external work and heat transfer simplify. It turns out that the inlet and exit conditions can also be related by a simple formula, known in fluid mechanics as the *Bernoulli equation*.

Consider a steady *one-dimensional* flow through a variable diameter pipe (see Fig. 4.15) with no external work transfer. For an incompressible, internally reversible flow, Eq. (4.8) reduces to

$$\cancelto{0}{\dot{W}_{\text{ext, int.rev.}}} = -\dot{m}\left[\int_i^e v dp + (\text{ke}_e - \text{ke}_i) + (\text{pe}_e - pe_i)\right]$$

$$\Rightarrow \quad 0 = \frac{p_e - p_i}{\rho} + (\text{ke}_e - \text{ke}_i) + (\text{pe}_e - pe_i)$$

$$\Rightarrow \quad \frac{p_i}{\rho} + \frac{V_i^2}{2(1000 \text{ J/kJ})} + \frac{gz_i}{(1000 \text{ J/kJ})} = \frac{p_e}{\rho} + \frac{V_e^2}{2(1000 \text{ J/kJ})} + \frac{gz_e}{(1000 \text{ J/kJ})} \quad \left[\frac{\text{kJ}}{\text{kg}}\right] \qquad (4.15)$$

This is the well known Bernoulli equation relating any two flow states in a one-dimensional incompressible flow. Note that the flow does not have to be adiabatic as long as heat transfer, if any, does not introduce any irreversibilities. If the working fluid is a gas or a vapor, the potential energy is often negligible and an increase in velocity (caused by a reduction in flow area since $A_i V_i = A_e V_e$) must accompany a decrease in pressure. This explains the **venturi effect**, where a constriction in the flow area causes a pressure drop to sub-atmospheric level and atmospheric pressure drives the liquid through the straw in Fig. 4.16, causing a spray. Many other applications of Bernoulli's equation can be found in any standard fluid mechanics textbook.

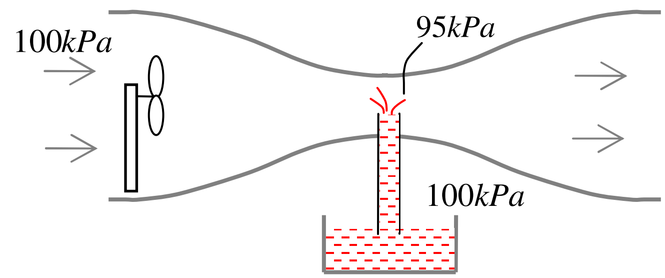


Fig. 4.16 The liquid from the straw sprays into the flowing gas due to venturi effect.

4.2.2 Nozzles and Diffusers

A **nozzle** is a specially designed variable area duct with the purpose of increasing the flow velocity at the expense of a pressure drop (see Fig. 4.17 and Anim. *4.A.nozzle*). Generally a nozzle operates adiabatically between known inlet conditions, p_i and T_i, and a known exit pressure, p_e. With no heat or external work transfer, the single-flow energy equation, Eq. (4.2), assumes a very simple form.

$$0 = \dot{m}j_i - \dot{m}j_e + \cancelto{0}{\dot{Q}} - \cancelto{0}{\dot{W}_{\text{ext}}}; \quad \Rightarrow \quad j_i = j_e \qquad (4.16)$$

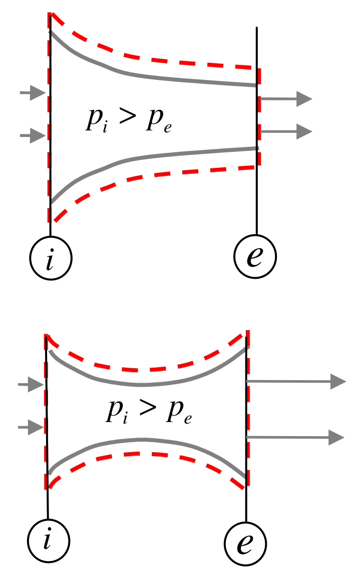


Fig. 4.17 Nozzles are converging or converging/diverging variable area duct.

That is, the specific flow energy remains constant along the flow in an adiabatic nozzle. Neglecting any change in pe and realizing that ke_e is much larger than ke_i (recall the purpose of a nozzle), a simple expression for the exit velocity can be obtained as follows.

$$j_i = j_e; \quad \Rightarrow \quad h_i \cong h_e + \text{ke}_e; \quad \Rightarrow \quad V_e \cong \sqrt{2\left(1000 \text{ J/kJ}\right)\left(h_e - h_i\right)}; \quad (4.17)$$

Clearly, it is the enthalpy difference that drives the exit kinetic energy; however, h_e cannot be independently controlled for a given nozzle.

Most nozzles are contoured in a smooth manner to minimize frictional losses so as to maximize the exit kinetic energy. An ideal nozzle can be expected to satisfy all the conditions – steady, adiabatic, and internally reversible – to be considered *isentropic*. By setting external work and change in potential energy to zero in Eq. (4.8) , we can obtain an expression for the exit kinetic energy of an isentropic nozzle with the subscript es representing the isentropic exit state.

Isentropic nozzle: $$\text{ke}_{es} = h_i - h_{es} = -\int_i^{es} v dp \qquad (4.18)$$

From this equation, it can be seen that dp must be negative, that is, a pressure drop must occur for the exit velocity to be greater than inlet velocity – the higher the pressure drop, the higher is the exit kinetic energy.

We can obtain a simple closed-form expression for isentropic exit velocity if the working fluid can be modeled by the SL or the PG model. For a liquid nozzle, application of SL model (constant v) in Eq. (4.18) results in.

$$\text{ke}_{es} = -\int_i^e v dp = \frac{p_i - p_e}{\rho}; \quad \Rightarrow \quad V_{es} = \sqrt{\frac{2\left(1000 \text{ J/kJ}\right)\left(p_i - p_e\right)}{\rho}} \quad \left[\frac{\text{m}}{\text{s}}\right]; (4.19)$$

The exit velocity, therefore, is driven by the square root of the pressure difference between the inlet and exit. For a perfect gas, the isentropic relation, Eq. (3.64), can be applied, producing

$$\text{ke}_{es} = h_i - h_{es} = c_p\left(T_i - T_{es}\right) = c_p T_i \left[1 - \left(\frac{p_e}{p_i}\right)^{(k-1)/k}\right]; \qquad (4.20)$$

The exit velocity in a gas nozzle is driven by the pressure ratio rather than the pressure difference. The temperature must drop along with pressure along the flow following the

Did you know?

The exhaust velocity of a chemical rocket is about 4 km/s.

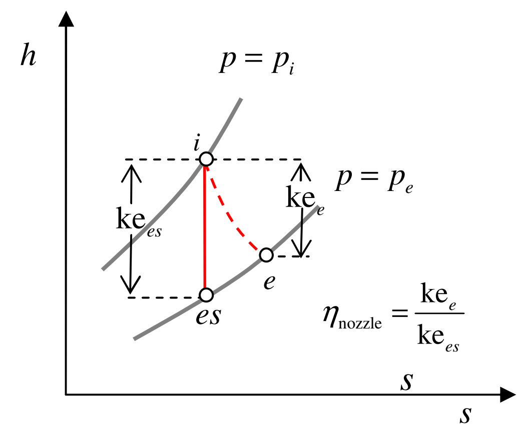


Fig. 4.18 Graphical interpretation of nozzle efficiency (see Anim. 4.A.*nozzleEff*.

isentropic relation (Eq. 3.64). For a given pressure ratio, a nozzle with a higher inlet temperature will produce greater exit velocity, which is also evident from Eq. (4.18), predicting a higher exit kinetic energy for a working fluid with higher average specific volume. Even if the working fluid is an ideal gas or a vapor, these conclusions apply in a qualitative sense. In a liquid-fueled rocket engine, hydrogen and oxygen must be ignited to create a high exit velocity (and, hence, thrust). A cold flow, operating under the same pressure difference, will produce a much smaller exit velocity (and thrust) due to smaller value of average specific volume.

Using the isentropic nozzle as a benchmark, the performance of an actual nozzle can be measured by its isentropic efficiency, defined as (see Anim. *4.A.isentropicNozzle*)

$$\eta_{\text{nozzle}} \equiv \frac{\dot{\mathrm{KE}}_e}{\dot{\mathrm{KE}}_{es}} = \frac{\dot{m}(\mathrm{ke}_e)}{\dot{m}(\mathrm{ke}_{es})} = \frac{j_i - h_e}{j_i - h_{es}} \cong \frac{h_i - h_e}{h_i - h_{es}} \qquad (4.21)$$

Sketched in Fig. 4.18, a $h-s$ diagram, which is quite similar to the $T-s$ diagram for the IG or PG model, is more convenient for nozzle flow since the difference in enthalpies can be directly interpreted as the kinetic energy. For an actual adiabatic nozzle, $s_e \geq s_i$ as established by Eq. (4.10) , and the exit kinetic energy can be seen to be reduced in Fig. 4.18.

We will revisit nozzles in chapter 15, and deduce the shapes of isentropic nozzles through advanced analysis. At this point it is sufficient to mention that a **subsonic** nozzle, which has an exit velocity $V_e \leq a_e$, where a_e is the speed of sound at the cross section with the smallest area known as the throat of the nozzle, has a converging contour while a **supersonic** nozzle with a $V_e > a_e$ has a converging-diverging contour as shown in Fig. 4.17. Well contoured nozzles have typical efficiencies of 90% or more.

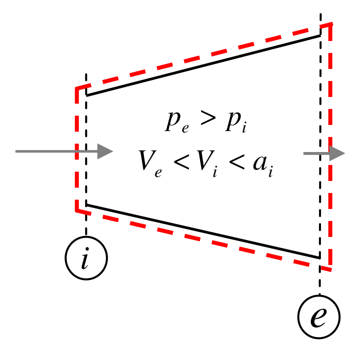


Fig. 4.19 A diffuser is an exact opposite of a nozzle. It increases pressure at the expense of kinetic energy.

A **diffuser** works in an exact opposite manner to a nozzle (see Fig. 4.19 and Anim. 4.A.*diffuserEff*); its purpose is to increase the pressure of a flow at the expense of its kinetic energy. As a result, the kinetic energy at the inlet of a diffuser is much greater than the exit kinetic energy, which can be neglected. Also, the exit pressure is not fixed and depends on the irreversibilities present in the diffuser. As in a nozzle, these irreversibilities are quantified through the **diffuser efficiency**. To define the isentropic efficiency, let us neglect the exit kinetic energy (a more refined analysis will be carried out in chapter 15). With no heat or external work transfer, the single-flow energy equation, Eq. (4.2), for the actual and corresponding isentropic diffuser can be written as

$$\text{Actual:} \qquad j_i = j_e; \quad \Rightarrow \quad h_e \cong h_i + \mathrm{ke}_i; \qquad (4.22)$$

$$\text{Isentropic:}\quad j_i = j_{es}; \quad \Rightarrow \quad h_{es} \cong h_i + \text{ke}_i = h_e; \qquad (4.23)$$

Note that the exit enthalpies of the actual and isentropic diffusers are equal, which is indicated by the horizontal dotted line in Fig. 4.20 connecting states *e* and *es*. However, the isentropic exit state is not used in defining the diffuser efficiency. Instead, a different exit state, state-es' in Fig. 4.20, which has the same pressure as the actual exit pressure, $p_{es'} = p_e$, and is isentropic to state-*i*, $s_{es'} = s_i$, with negligible kinetic energy, $\text{ke}_{es'} \cong 0$, is used as the target exit state for a given diffuser. Everything else at the inlet remaining unchanged, the reduced inlet kinetic energy, $\text{ke}_{is'}$, required to reach this target exit state, state-es', can be obtained from the energy equation.

$$j_{is'} = j_{es'}; \quad \Rightarrow \quad h_i + \text{ke}_{is'} = h_{e,s'} + \text{ke}_{es'}; \quad \Rightarrow \quad \text{ke}_{is'} \cong h_{es'} - h_i; \qquad (4.24)$$

The isentropic efficiency is then defined (see Anim. 4.A.*diffuserEff*) as the ratio of $\text{ke}_{is'}$ and ke_i, both of which produce the same exit pressure p_e.

$$\eta_{\text{diffuser}} \equiv \frac{\text{ke}_{is'}}{\text{ke}_i} = \frac{j_{es'} - h_i}{j_e - h_i} \cong \frac{h_{es'} - h_i}{h_e - h_i} \qquad (4.25)$$

Thus, a 90% efficiency means that with 90% of the inlet kinetic energy, the actual exit pressure can be produced by an isentropic diffuser. As the diffuser efficiency decreases, state-es' in Fig. 4.20 moves down along the constant entropy line and state-*e* shifts to the right (states *i* and *es* remain invariant for a given inlet condition); however, both states still belong to the same constant pressure line at the lowered exit pressure.

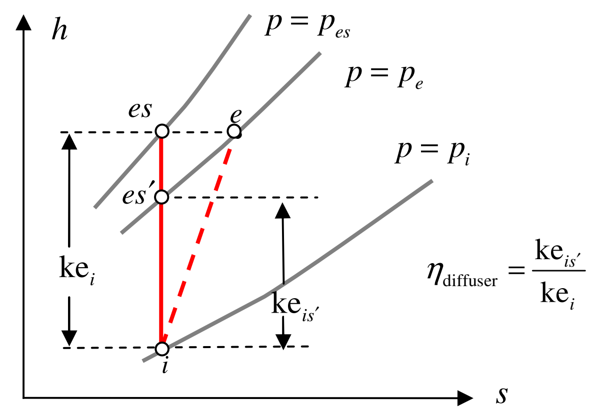


Fig. 4.20 Isentropic efficiency for a diffuser (see Anim. 4.A.*diffuserEff*.

EXAMPLE 4-4 [*mE*] Analysis of a Nozzle

Air at 0.15 MPa, $30\,^\circ\text{C}$ enters an insulated nozzle with a velocity of 10 m/s and leaves at a pressure of 0.1 MPa and a velocity of 200 m/s. (a) Determine the exit temperature of air. Assume air to behave as a perfect gas. (b) Is the nozzle isentropic?

SOLUTION Analyze the open steady system, the nozzle enclosed within the red boundary in Fig. 4.21, using the mass and energy balance equations.

Assumptions Steady state, PG model for air with $R = 0.287$ kJ/kg·K and $c_p = 1.005$ kJ/kg·K (Table C.1), uniform states at the inlet and exit based on LTE, and negligible pe's.

Analysis

Using the PG model to evaluate enthalpy difference, the energy equation for the nozzle, Eq. (4.17), can be simplified to produce the exit temperature.

$$j_i = j_e;$$
$$\Rightarrow \quad h_i + \mathrm{ke}_i = h_e + \mathrm{ke}_e; \quad \Rightarrow \quad h_i - h_e = \mathrm{ke}_e - \mathrm{ke}_i;$$
$$\Rightarrow \quad c_p(T_i - T_e) = \mathrm{ke}_e - \mathrm{ke}_i; \quad \Rightarrow \quad T_e = T_i - \frac{\mathrm{ke}_e - \mathrm{ke}_i}{c_p}$$
$$\Rightarrow \quad T_e = 30 - \frac{200^2 - 10^2}{(2000)(1.005)} = 10.1\ ^\circ\mathrm{C};$$

The change in entropy between the inlet and exit can be evaluated from Eq. (3.63).

$$\Delta s = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$
$$= (1.005)\ln\frac{273+10.1}{273+30} - (0.287)\ln\frac{0.1}{0.15} = 0.041\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};$$

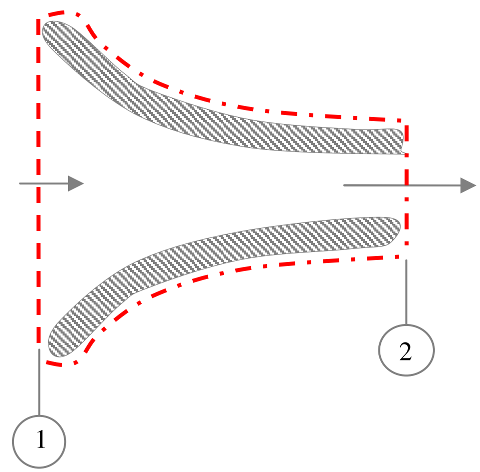
The only way entropy can increase in an adiabatic, steady nozzle, is through entropy generation; therefore, the nozzle is clearly not isentropic.

Test Analysis Launch the single-flow PG daemon, and select air as the working fluid. Calculate state-1 from p1, T1, and Vel1, and state-2 from p2, Vel2, and j2 = j1. T2 and s2 are evaluated as part of the exit state.

Discussion Using the daemon, you can determine the shape of the isentropic nozzle relative to a given inlet area (assume it to be 1 m^2). Since pressure continuously decreases along the nozzle, an intermediate location between the inlet and exit can be identified by a pressure between 0.15 MPa and 0.1 MPa. To find an intermediate state, say, state-3, pick a p3, and enter p3, mdot3 = mdot1, j3 = j1, and s3 = s1. Calculate to find A3. Repeat with different values of p3 until you can deduce the shape of the nozzle (converging or converging/diverging).

EXAMPLE 4-5 [*mEE*] Analysis of a Nozzle

Steam at 0.5 MPa, 250 °C enters an adiabatic nozzle with a velocity of 30 m/s and leaves at 0.3 MPa, 200 °C. If the mass flow rate is 5 kg/s, determine the (a) inlet and exit areas, (b) exit velocity, (c) isentropic exit velocity, and (d) isentropic efficiency.



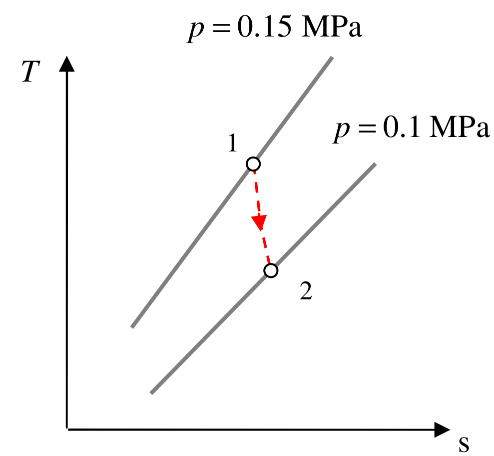


Fig. 4.21 Schematic and $T-s$ diagram for the nozzle in Ex. 4-4.

SOLUTION Analyze the open steady system, the nozzle enclosed within the red boundary in Fig. 4.22, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for steam, uniform states based on LTE at the inlet and exit, negligible inlet ke, and negligible pe's.

Analysis Use TEST or the manual approach to determine the anchor states – state-1 for the inlet, state-2 for the actual exit, and state-3 for the isentropic exit (see Fig. 4.22).

State-1 (given p_1, T_1, V_1 and $\dot{m}$)

$$\Rightarrow v_1 = 0.4744\ \frac{\text{m}^3}{\text{kg}};\quad h_1 = 2960.6\ \frac{\text{kJ}}{\text{kg}};\quad s_1 = 7.271\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$j_1 = h_1 + \text{ke}_1 = 2960.6 + \frac{30^2}{2000} = 2961.1\frac{\text{kJ}}{\text{kg}};\ A_1 = \frac{\dot{m}}{(V_1/v_1)} = 791\ \text{cm}^2$$

State-2 (given p_2, T_2, and $\dot{m}$)

$$\Rightarrow v_2 = 0.7163\ \frac{\text{m}^3}{\text{kg}};\quad h_2 = 2865\ \frac{\text{kJ}}{\text{kg}};$$

The energy equation for the nozzle, Eq. (4.17), produces

$$V_2 = \sqrt{2000(\text{ke}_2)} = \sqrt{2000(j_1 - h_2)} = \sqrt{2000(96.1)} = 438\ \frac{\text{m}}{\text{s}};$$

Therefore, $A_2 = \dfrac{\dot{m}}{(V_2/v_2)} = 82\ \text{cm}^2$

For the isentropic exit state, state-3, the exit pressure and entropy are known. Also the energy equation for the adiabatic nozzle produces $j_3 = j_1$.

State-3 (given $p_3 = p_2$, $s_3 = s_2$, and $j_3 = j_1$)
Once again, applying Eq. (4.17)

$$V_3 = \sqrt{2000(\text{ke}_3)} = \sqrt{2000(j_1 - h_3)} = \sqrt{2000(114.1)} = 478\ \frac{\text{m}}{\text{s}}$$

The isentropic efficiency can now be calculated from Eq. (4.21).

$$\eta_{\text{nozzle}} = \frac{\text{ke}_e}{\text{ke}_{es}} = \frac{\text{ke}_2}{\text{ke}_3} = \frac{438^2}{478^2} = 84.0\%$$

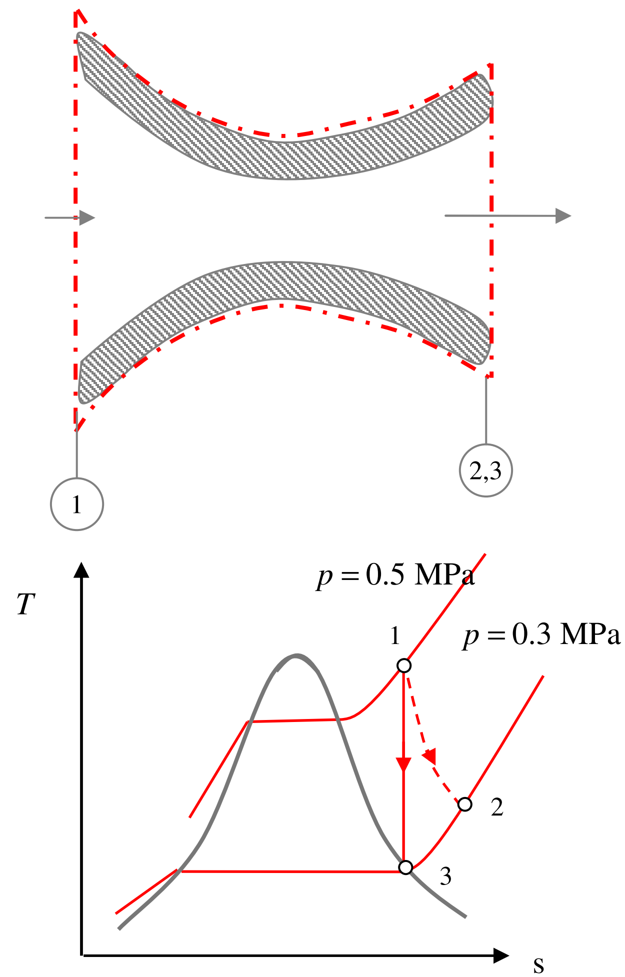


Fig. 4.22 Schematic and $T-s$ diagram for the nozzle in Ex. 4-5.

Test Analysis Launch the single-flow PC daemon and select H2O as the working fluid. Calculate states 1, 2, and 3 as described in the TEST-codes (posted in *TEST.Examples*). To calculate the isentropic efficiency, ke3 and ke2 must be manually calculated in the I/O panel by entering expressions '= Vel3^2/2000' and '= Vel2^2/2000' respectively.

Discussion From the Test analysis, compare the exit areas of the actual and isentropic nozzles, A2 = 82 cm2 and A3 = 74 cm2. A nozzle contour that does not follow the isentropic shape can cause eddies and vortices in an otherwise streamlined flow, resulting in entropy generation (internal irreversibilities) associated with viscous friction.

EXAMPLE 4-6 [*mEE*] Analysis of a Diffuser

Helium at 100 kPa, 300 K enters a 90% efficient adiabatic diffuser with a velocity of 200 m/s. If the exit velocity is negligible, determine the exit (a) temperature, and (c) pressure.

SOLUTION Analyze the open steady system, the diffuser enclosed within the red boundary in Fig. 4.23, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PG model for helium, uniform states based on LTE at the inlet and exit, and negligible pe's.

Analysis From Table C-1, or any PG daemon obtain $R = 2.0785$ kJ/kg·K, $c_p = 5.1926$ kJ/kg·K, and $k = 1.667$ for helium.

The energy equation, Eq.(4.22), coupled with the PG model, applied between the actual inlet and exit states, state-1 and state-2 in Fig. 4.23, yields the exit temperature as follows.

$$j_2 = j_1; \quad \Rightarrow h_2 + \cancel{\mathrm{ke}_2}^{0} \cong h_1 + \mathrm{ke}_1; \quad \Rightarrow c_p\left(T_2 - T_1\right) = \mathrm{ke}_1$$

Therefore, $T_2 = T_1 + \dfrac{V_1^2}{2000c_p} = 300 + \dfrac{200^2}{(2000)(5.1926)} = 303.8\text{ K};$

The exit pressure p_2 is still unknown. However, the maximum possible exit pressure can be found by evaluating the isentropic exit state, state-3, with $V_3 = 0$. The energy equation with $j_3 = j_1$, and $V_3 = 0$ produces $T_3 = T_2$. The isentropic relation, Eq. (3.64), for the PG model, produces p_3.

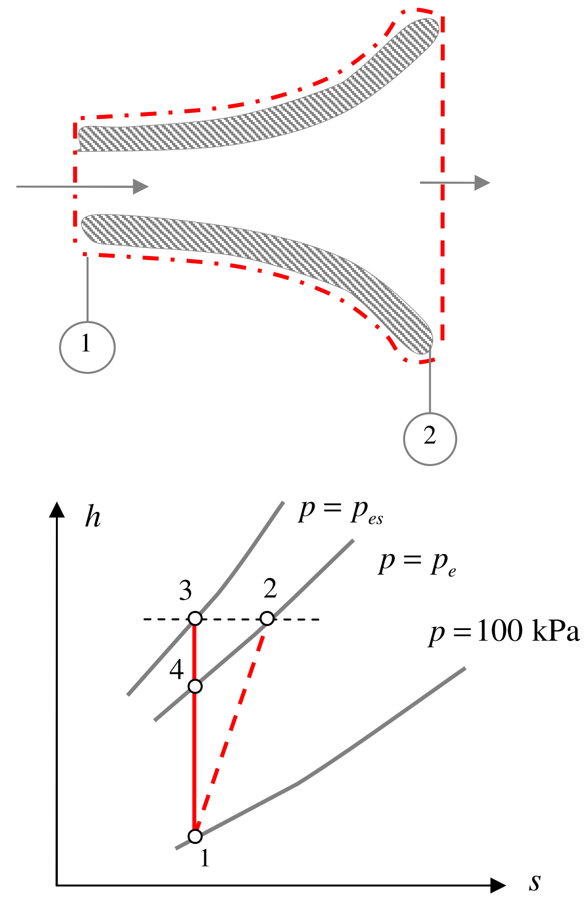


Fig. 4.23 System schematic and the $T - s$ diagram for Ex. 4-6.

$$\frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{(k-1)/k} \quad \Rightarrow p_3 = p_1\left(\frac{T_3}{T_1}\right)^{k/(k-1)} = 103.2 \text{ kPa};$$

Using the definition of the diffuser efficiency, Eq. (4.25), we can relate the actual exit state, state-2, with the isentropic exit state, state-4, at the actual exit pressure (see Fig. 4.23) and obtain $p_2 = p_4$.

$$\eta_{\text{diffuser}} \cong \frac{h_4 - h_1}{h_3 - h_1}; \quad \Rightarrow (T_4 - T_1) = (T_3 - T_1)\eta_{\text{diffuser}};$$

$$\Rightarrow \quad T_1\left[\left(\frac{p_4}{p_1}\right)^{(k-1)/k} - 1\right] = \eta_{\text{diffuser}} T_1\left[\left(\frac{p_3}{p_1}\right)^{(k-1)/k} - 1\right];$$

Except for p_4, all other variables in this equation are known. Solving, we obtain $p_4 = p_2 = 102.9$ kPa.

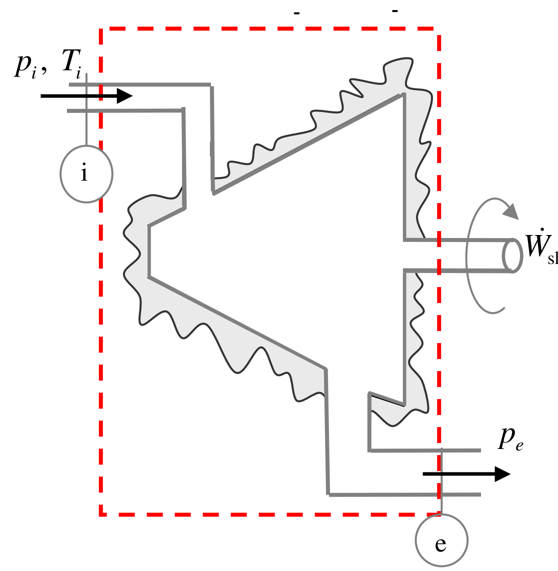
Test Analysis Launch the single-flow PG daemon, and select helium as the working fluid. Calculate states 1, 2, 3 and 4 as described on the TEST-codes (posted in *TEST.Examples*). State-2, being the actual exit state, contains the answers.

Discussion The temperature increases in a diffuser and decreases in a nozzle when the working fluid is a gas or a vapor. For a liquid, however, entropy being a function of temperature only (SL model), isentropic flow also implies isothermal flow.

4.2.3 Turbines

A **turbine** is a rotary device which delivers shaft work $\dot{W}_{\text{sh}}$ at the expense of flow energy of the working fluid (see Anim. 4.A.*turbine*). The inlet conditions, p_i and T_i, and the exit pressure p_e are generally known for a given turbine (see Fig. 4.24). To simplify analysis, changes in ke and pe are often neglected.

Although an actual turbine is not reversible, Eq. (4.9) can be used as a guide to explore its idealized behavior. According to this equation, the work produced is proportional to the mass flow rate $\dot{m}$ and the integral of vdp - the shaded area in Fig. 4.25. Clearly, the output can be increased if the average specific volume of the working fluid can be increased. For a given mass flow rate and pressure difference, a vapor turbine, therefore, will produce more work than a hydraulic turbine with liquid water as



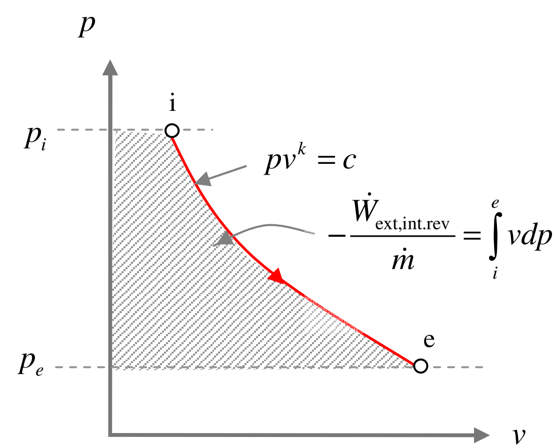


Fig. 4.24 Work output of an internally reversible turbine is proportional to the shaded area.

the working fluid. For a gas or vapor turbine, an increase in the inlet temperature would increase the average specific volume and, hence, the output.

There are three main categories of turbines based on the working fluid – steam (vapor) turbine, gas turbines, and hydraulic (water) turbine. In a steam turbine, steam exits at a sub-atmospheric pressure to a device called condenser (discussed in sec. 4.2.7). In a gas turbine, the exhaust pressure is atmospheric since the gas is normally expelled to the atmosphere. Heat transfer from turbine is undesirable and is usually minimized using good insulation.

Typical inlet, exit, and the isentropic exit states of a steam turbine are shown in the $T-s$ diagram of Fig. 4.25. The governing equations, Eqs. (4.2) and (4.3), for an actual and the corresponding isentropic turbine can be simplified as follows

$$\text{Actual:} \quad \dot{W}_{\text{ext}} = \dot{W}_{\text{sh}} = \dot{m}\left(j_i - j_e\right) \cong \dot{m}\left(h_i - h_e\right); \quad s_e = s_i + \underbrace{\dot{S}_{\text{gen}}/\dot{m}}_{\geq 0} \qquad (4.26)$$

$$\text{Isentropic:} \quad \dot{W}_{\text{sh},s} = \dot{m}\left(j_i - j_{es}\right) \cong \dot{m}\left(h_i - h_{es}\right); \qquad s_{es} = s_i \qquad (4.27)$$

Using these results, the isentropic **turbine efficiency**, which compares the performance of the actual turbine with the corresponding isentropic turbine, can be established as (see Anim. 4.A.*turbineEff*)

$$\eta_{\text{turbine}} \equiv \frac{\dot{W}_{\text{sh}}}{\dot{W}_{\text{sh},s}} = \frac{j_i - j_e}{j_i - j_{es}} \cong \frac{h_i - h_e}{h_i - h_{es}} \qquad (4.28)$$

The isentropic state can be evaluated from a known $p_{es} = p_e$ and $s_{es} = s_i$. Knowledge of the isentropic efficiency relates h_e to a known h_{es}, and the actual exit state can be evaluated from p_e and h_e.

For a gas or vapor turbine, a drop in pressure also accompanies a drop in temperature and increase in specific volume (use isentropic PG relation as a guide). With no significant change in velocity, the exit area must be greater than the inlet area of a vapor or gas turbine. In a steam turbine, the vapor quality is kept sufficiently high, 85% or higher, at the turbine exit to avoid turbine damage.

The equations presented above are applicable to both vapor and gas turbines. For a hydraulic turbine shown in Fig. 4.26 the inlet and exit pressures and velocities are not known. However, the analysis can be simplified by treating the surfaces on the two sides of a dam (see Fig. 4.27) as the inlet and exit states. Equations (4.26) and (4.28) still apply

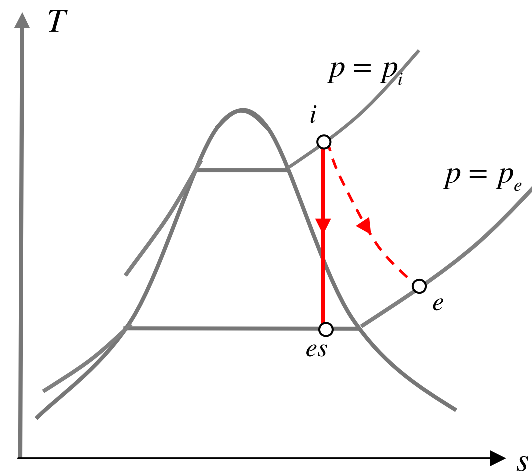


Fig. 4.25 $T-s$ diagram for an actual and isentropic steam turbine.

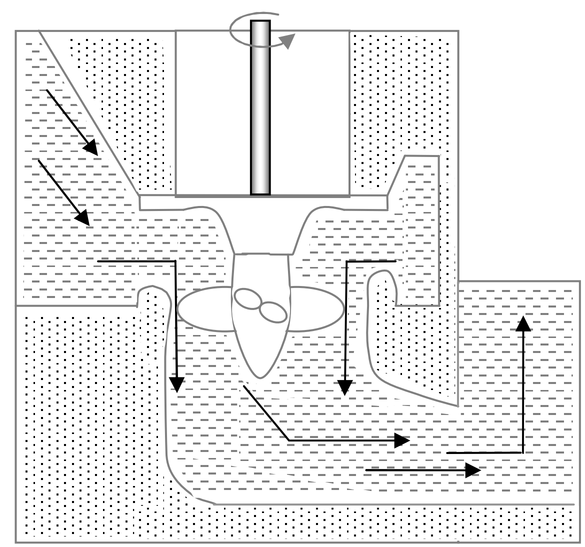


Fig. 4.26 A hydraulic turbine.

as long as we do not assume that $j \cong h$. While ke's can be neglected due to large surface areas, the change in pe then becomes the driving force and must be included in the energy equation. Since $p_i = p_e = p_0$ (see Fig. 4.26), application of SL model produces,

$$j_i - j_e = (h_i - h_e) + (\text{pe}_i - \text{pe}_e) = (u_i - u_e) + \frac{(p_i - p_e)}{\rho} + (\text{pe}_i - \text{pe}_e)$$

$$\Rightarrow \quad j_i - j_e = c_v (T_i - T_e) + g(z_i - z_e)/(1000 \text{ J/kJ}) \quad [\text{kJ/kg}] \qquad (4.29)$$

The output of an actual hydraulic turbine, $\dot{W}_{\text{sh}} = \dot{m}(j_i - j_e)$, therefore, depends on the **available head** $z_i - z_e$ and the temperature change brought about by friction. For an isentropic turbine, $s_i = s_e$ implies $T_i = T_e$ for the SL model. Therefore, the power of an isentropic hydraulic turbine reduces to the simple expression $\dot{W}_{\text{sh},s} = \dot{m}g(z_i - z_e)/1000$, which can be directly deduced from Eq. (4.8). Since the power output depends mainly on the product of the mass flow rate and the available head, it is possible to produce considerable amount of power even with a small head if the available mass flow rate is very large.

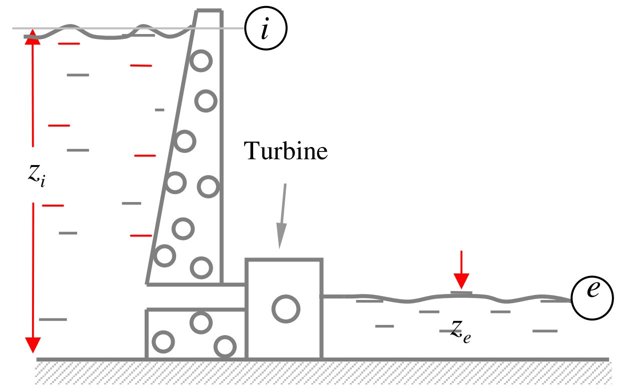


Fig. 4.27 The driving force for a hydraulic turbine is the difference in the available head $z_i - z_e$.

EXAMPLE 4-7 [*mEE*] Analysis of a Vapor Turbine

A steam turbine operates steadily with a mass flow rate of 5 kg/s. The steam enters the turbine at 500 kPa, 400 $^\circ$C and leaves at a pressure of 7.5 kPa and a quality of 0.95. Neglecting the changes in ke and pe as well as any heat loss from the turbine to the surroundings, determine (a) the power developed by the turbine, (b) the isentropic efficiency, and (c) the entropy generation rate. ***What-if scenario:*** (d) What would the answers be if the steam left the turbine as saturated vapor?

SOLUTION Analyze the open steady system, the turbine enclosed within the red boundary in Fig. 4.28, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for steam, uniform states based on LTE at the inlet and exit, and negligible changes in ke and pe.

Analysis Representing the inlet, exit, and the isentropic exit states by states 1, 2 and 3 respectively, use TEST or the manual approach to obtain the following state properties.

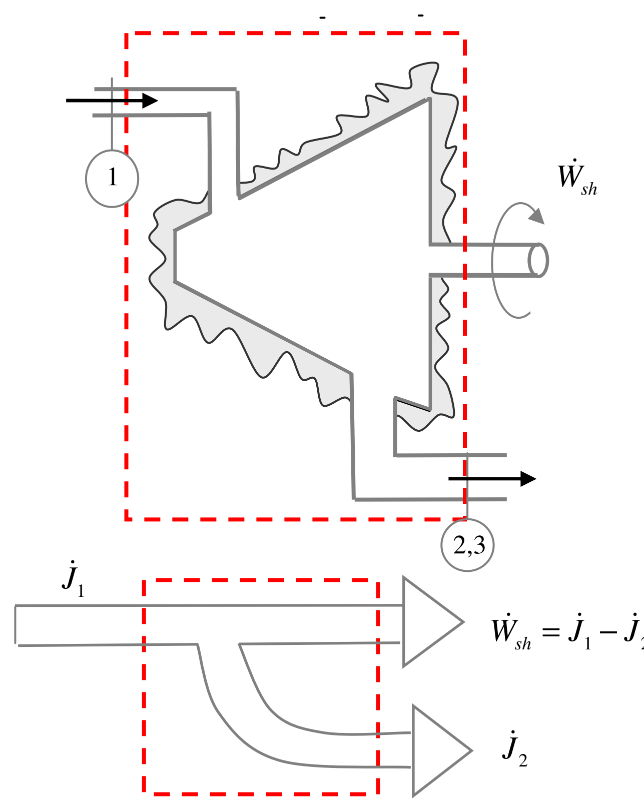


Fig. 4.28 System schematic and the energy flow diagram for Ex. 4-7.

State-1 (given p_1, T_1): $j_1 \cong h_1 = 3271.8\ \frac{\text{kJ}}{\text{kg}}$; $s_1 = 7.794\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$;

State-2 (given p_2, x_2): $j_2 \cong h_2 = 2454.5\ \frac{\text{kJ}}{\text{kg}}$; $s_2 = 7.868\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$;

State-3 (given $p_3 = p_2$, $s_3 = s_2$): $j_3 \cong h_3 = 2429.0\ \frac{\text{kJ}}{\text{kg}}$;

The energy equation, Eq. (4.26), for the actual and ideal turbines yields

Actual: $\dot{W}_{\text{sh}} = \dot{m}(j_1 - j_2) = (5)(3271.8 - 2454.5) = 4086.5\text{ kW}$;

Isentropic: $\dot{W}_{\text{sh},s} = \dot{m}(j_1 - j_3) = (5)(3271.8 - 2431.1) = 4203.5\text{ kW}$;

The isentropic efficiency now can be evaluated as

$$\eta_{\text{turbine}} \equiv \frac{\dot{W}_{\text{sh}}}{\dot{W}_{\text{sh},s}} = \frac{4086.5}{4203.5} = 97.2\%$$

The entropy balance equation, Eq. (4.3), applied on the turbine's universe produces

$$\cancel{\frac{dS}{dt}}^{0,\text{ steady state}} = \dot{m}(s_1 - s_2) + \frac{\cancel{\dot{Q}}^{0}}{T_0} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) = (5)(7.868 - 7.794) = 0.370\ \frac{\text{kW}}{\text{K}};$$

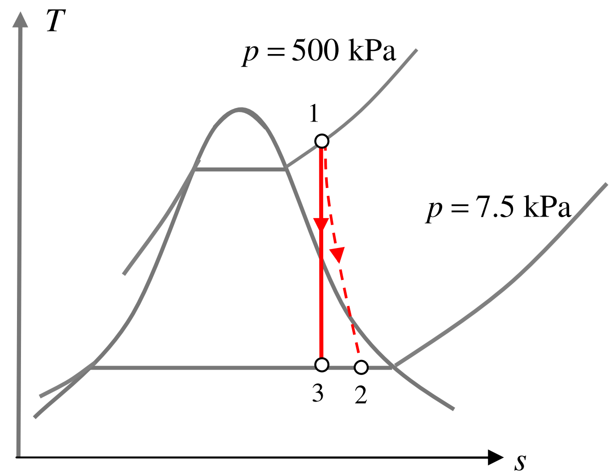


Fig. 4.29 $T - s$ diagram for Ex 4-7.

Test Analysis Launch the single-flow PC daemon and select H2O as the working fluid. Evaluate the three states as described in the TEST-codes (posted in *TEST.Examples*). Draw the $T-s$ diagram (see Fig. 4.29) using the plot menu. In the device panel, set up device-A as the actual turbine with state-1 and state-2 as the anchor states. Enter Qdot = 0 and Calculate. The shaft work and entropy generation rate are calculated. Similarly, set up device-B as the isentropic turbine. Obtain the isentropic efficiency by finding the ratio of Wdot_ext from the two devices.

What-if Scenario Change x2 to 1 in state-2, and Super-Calculate. Power output in the two device panels are updated to 3485.1 kW and 4203.4 kW respectively, resulting in a reduced isentropic efficiency of 82.9%. The entropy generation rate increases significantly to 2.291 kW/K.

Discussion An energetic efficiency for the turbine can be defined as $\dot{W}_{\text{sh}}/(\dot{J}_1-\dot{J}_2)$, which must be 100% since the turbine is adiabatic (see the energy diagram sketched in Fig. 4.28). The isentropic efficiency, clearly, is a better measure of the turbine performance, telling us how much potential output is 'lost' to internal irreversibilities.

EXAMPLE 4-8 [*mEE*] Analysis of a Hydraulic Turbine

Water enters the intake pipe of a hydraulic turbine at 110 kPa, $15\,^{\circ}\text{C}$, 1 m/s, and exits at a point 150 m below the intake at 100 kPa, 10 m/s. If the turbine output is 700 kW for a flow rate of 1000 kg/s, determine (a) the exit temperature and (b) the isentropic efficiency.

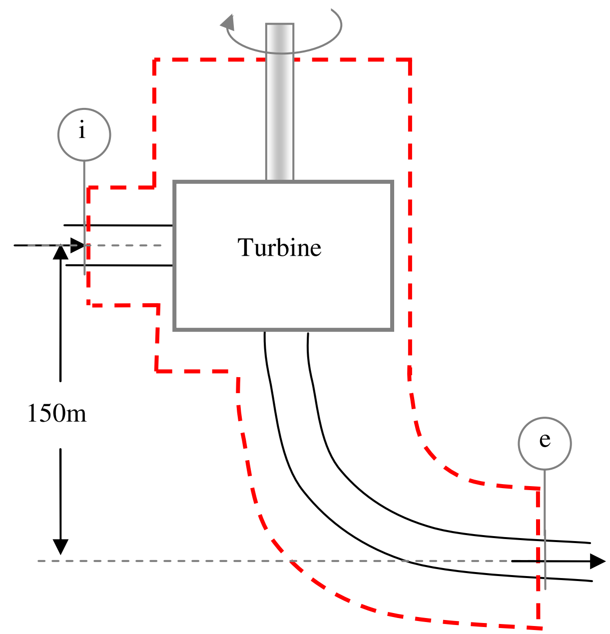


Fig. 4.30 Schematic for Ex. 4-8.

SOLUTION Analyze the open steady system, the turbine enclosed within the red boundary in Fig. 4.30, using the mass, energy, and entropy balance equations.

Assumptions Steady state, SL model for water, uniform states based on LTE at the inlet and exit. Negligible heat transfer.

Analysis From Table A-2 or any SL daemon obtain ρ = 997 kg/m^3 and c_v = 4.187 kJ/kg$\cdot$K for liquid water.

The energy equation, Eq. (4.2), coupled with the SL model yields

$$\dot{W}_{\text{sh}} = \dot{m}(j_i - j_e) = \dot{m}(h_i - h_e) + \dot{m}(\Delta \text{pe} + \Delta ke)$$

$$= \dot{m}(u_i - u_e) + \frac{\dot{m}(p_i - p_e)}{\rho} + \dot{m}(\text{pe}_i - \text{pe}_e) + \dot{m}(\text{ke}_i - \text{ke}_e)$$

$$= \dot{m}c_v(T_i - T_e) + \dot{m}\left[\frac{50}{997} + \frac{9.81\{0-(-100)\}}{1000} + \frac{1-10^2}{2000}\right]$$

$$\Rightarrow T_e = T_i - \frac{\dot{W}_{\text{sh}}}{\dot{m}c_v} + \frac{0.9816}{c_v} = 15.08\ ^{\circ}\text{C};$$

The isentropic power can be obtained by substituting $T_i = T_e$ in the expression for turbine output.

$$\dot{W}_{\text{sh},\,s} = \frac{\dot{m}(p_i - p_e)}{\rho} + \dot{m}(\text{pe}_i - \text{pe}_e) + \dot{m}(\text{ke}_i - \text{ke}_e) = 981.6\ \text{kW}$$

The isentropic efficiency, therefore, can be obtained as

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{sh,actual}}}{\dot{W}_{\text{sh,isentropic}}} = \frac{700}{981.6} = 71.3\%$$

Test Analysis Launch the single-flow SL daemon. Select water as the working fluid. Calculate states 1, 2 and 3 and analyze devices A and B as described on the TEST-codes (see *TEST.Examples*).

Discussion Although the temperature increase may appear insignificant, it accounts for a significant (about30%) loss of turbine power due to internal irreversibilities.

4.2.4 Compressors, Fans and Pumps

Compressors, **fans**, **blowers** and **pumps** raise the pressure of a fluid at the expense of useful work, which is usually delivered through a shaft. The distinction among these devices stems from the fluid they handle – a vapor or a gas for compressors, fans, and blowers, and a liquid for pumps. While a fan or blower increases the pressure of a gas just enough to create a desired mass flow, a compressor is capable of delivering a gas at a very high pressure. From our discussion in sec. 4.1.3, we would expect a compressor to require more power (compare Figs. 4.7 and 4.8) than a pump for a given mass flow rate and rise in pressure. As in a turbine, the inlet conditions, p_i and T_i, and the exit pressure p_e are generally given. Also, changes in ke and pe are quite small compared to the change in flow energy j, and are generally neglected.

Different types of compressors and pumps are classified in Fig. 4.30 and illustrated in Anim. 4.A.*compressors*. The most common type is the **rotary** type, in which the rotating blades (called impeller) impart kinetic energy to the incoming working fluid. The high speed fluid passes through a diffuser section where it slows down, resulting in an increase in pressure. In an **axial** flow device, the direction of flow is parallel to the rotating shaft while in a **centrifugal** device the fluid enters axially through the central hub and exits in the radial direction. In a **reciprocating** device, the inlet valve opens during the intake stroke at the end of which a fixed volume of fluid is trapped. During the compression stroke both valves are closed and the exit valve opens after the fluid reaches the desired pressure or volume. A reciprocating device is unsteady during a given cycle; however, when averaged over the time resolution of interest, the cyclic fluctuations disappear and the device can be treated as an open steady system (read the

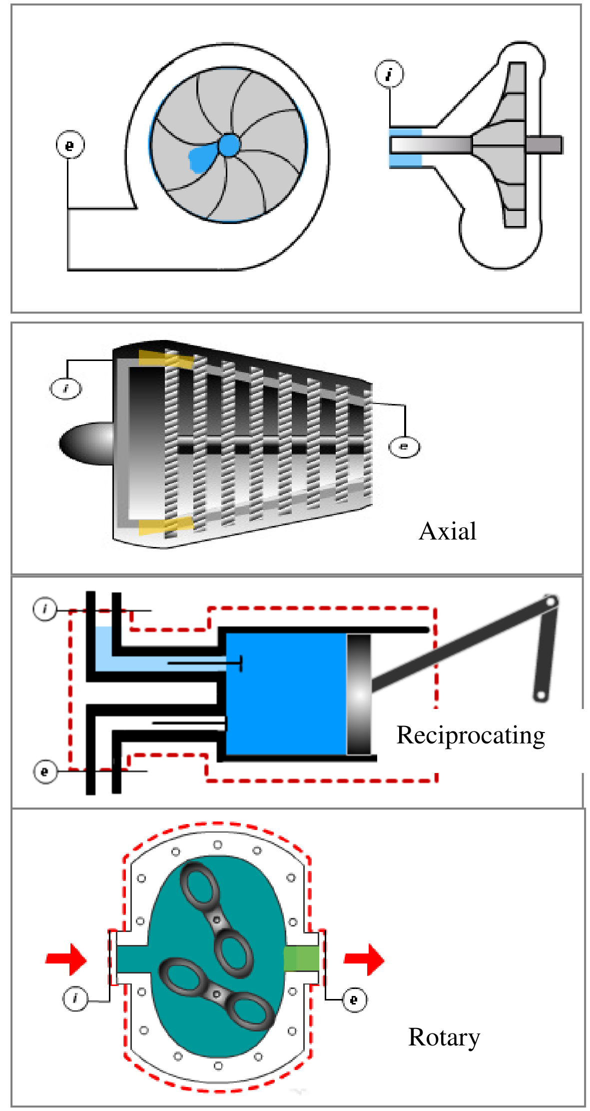


Fig. 4.30 Screen shots from Anim. 4.A.*compressors*.

discussion in sec. 2.2.1.1 on how the reciprocating heat engines can be treated as closed steady systems).

Given that the work requirement for compression is proportional the integral of vdp, it is desirable to have small value of specific volume to minimize the work required for compression. Since temperature increases during isentropic compression, heat rejection is the simplest way to cool the gas and reduce its specific volume. However, due to high volume flow rates in axial compressors, sufficient time is not available for any significant heat rejection. One solution to that is to use multistage compression with **intercooling**, where the hot gas at the end of the first stage of compression is cooled, ideally, back to the original inlet temperature without any loss of pressure. Reciprocating compressors, on the other hand, can be equipped with fins or water jackets to enhance heat rejection as more time is available during the compression process. By default, however, we will assume all compressors and pumps to be adiabatic unless otherwise mentioned.

The governing balance equation for an adiabatic compressor look almost identical to those derived for a turbine, except the work carries a negative sign.

$$\text{Actual:} \quad \dot{W}_{\text{ext}} = \dot{W}_{\text{sh}} = \dot{m}\left(j_i - j_e\right) \cong \dot{m}\left(h_i - h_e\right); \quad s_e = s_i + \underbrace{\dot{S}_{\text{gen}} / \dot{m}}_{\geq 0} \qquad (4.30)$$

$$\text{Isentropic:} \quad \dot{W}_{\text{ext},\,s} = \dot{W}_{\text{sh},\,s} = \dot{m}\left(j_i - j_{es}\right) \cong \dot{m}\left(h_i - h_{es}\right); \qquad s_{es} = s_i \qquad (4.31)$$

The isentropic compressor efficiency, therefore, can be defined as (see Anim. 4.A.*compressorEff*)

$$\eta_{\text{compressor/pump}} \equiv \frac{\dot{W}_{\text{sh,s}}}{\dot{W}_{\text{sh}}} = \frac{j_i - j_{es}}{j_i - j_e} \cong \frac{h_i - h_{es}}{h_i - h_e} \qquad (4.32)$$

Compare this expression with the corresponding definition for a turbine. The numerator and denominator are switched for a work consuming device to ensure that the efficiency is equal or less than 100%.

The expression for pumping power can be simplified by substituting Eq. (4.29) for the SL model into Eq. (4.30). Furthermore, for an isentropic pump $s_i = s_e$ implies $T_i = T_e$ for a liquid. Neglecting the changes in ke and pe across the pump, the pumping power can be expressed as (see Anim. 4.A.*pumpingSystem*)

$$\text{Actual:} \quad \dot{W}_{\text{pump}} = \dot{m}\left(j_i - j_e\right) \cong \dot{m}c_v\left(T_i - T_e\right) + \left(p_i - p_e\right)/\rho \qquad (4.33)$$

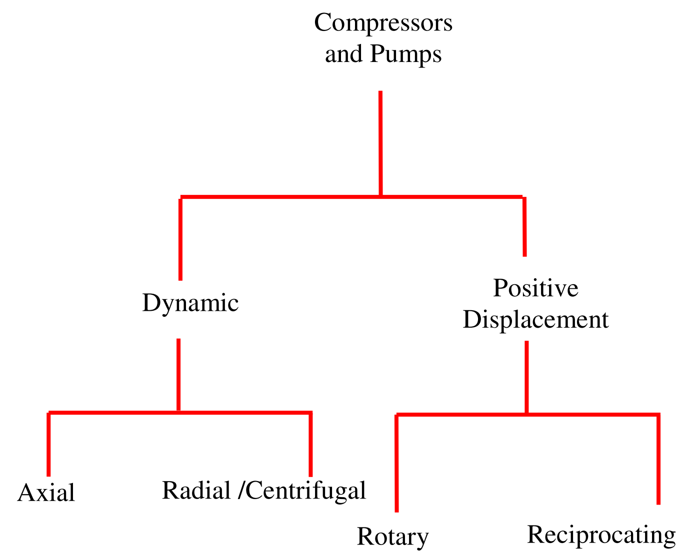


Fig. 4.31 Different categories of pumps and compressors.

Isentropic: $\dot{W}_{\text{pump},\,s} = \dot{m}\left(j_i - j_{e,s}\right) \cong \dot{m}\left(p_i - p_e\right)/\rho;$ (4.34)

The temperature rise in an actual pump, therefore, is a sign of irreversibilities in the pump and raises its power requirement.

EXAMPLE 4-9 [*mE*] Analysis of a Pump

In Ex. 3-5 determine the power consumption per unit mass of water by the pump by (a) including and (b) neglecting the effect of kinetic energy.

SOLUTION Analyze the open steady system, the pumping system enclosed within the red boundary in Fig. 4.31, using the mass and energy balance equations.

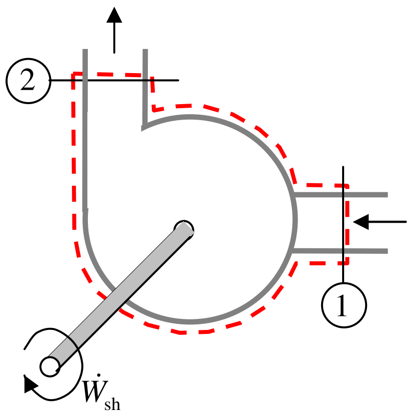


Fig. 4.31 System schematic for Ex. 4-9.

Assumptions Steady state, SL model for water, uniform states based on LTE at the inlet and exit. Isentropic condition implies that the system is adiabatic.

Analysis Having evaluated the difference in enthalpy and flow energy in Ex. 3-5, we employ the energy equation for the pump, Eq. (4.33), to produce the pumping power per unit mass.

$$\frac{\dot{W}_{\text{sh},\,s}}{\dot{m}} = \left(j_1 - j_2\right) = -\Delta j = 1.01\ \frac{\text{kJ}}{\text{kg}};$$

If the change in ke is neglected,

$$\frac{\dot{W}_{\text{sh},\,s}}{\dot{m}} = -\Delta j = -\Delta h - \Delta\text{pe} - \cancel{\Delta\text{ke}}^{\,0} = -\Delta h - \Delta\text{pe}$$

$$= -0.9 - \frac{5}{1000}(9.81) = -0.95\ \frac{\text{kJ}}{\text{kg}};$$

TEST Analysis Launch the single-flow SL daemon, and select water. Evaluate the inlet and exit states, states 1 and 2 (see TEST-codes) using mdot1 = 1 kg/s as the basis. In the device panel, select the anchor states, enter Qdot = 0, and Calculate. The pumping power for the isentropic pump is verified. To find out the effect of neglecting kinetic energy, set Vel2 = Vel1 and Super-Calculate.

Discussion Note that the analysis is completely independent of what type of pump is used as long as the inlet and exit conditions are the same.

EXAMPLE 4-10 [*mEE*] Analysis of Compressors

Air is compressed from an inlet condition of 100 kPa, 300 K to an exit pressure of 1000 kPa by an internally reversible compressor. Determine the compressor power per unit mass flow rate if the device is (a) isentropic, (b) polytropic with $n = 1.3$, or (c) isothermal. (d) Assuming compression to be polytropic with an exponent of 1.3 and the intercooler to be ideal, determine the ideal intermediate pressure if two stage compression when intercooling is used.

SOLUTION Analyze the open steady systems – alternative compression devices - using the mass, energy, and entropy balance equations.

Assumptions Steady state, PG model for air, uniform states based on LTE at the inlet and exit.

Analysis Obtain $R = 0.287$ and $k = 1.4$ for air from Table C-1 or any PG daemon. Use the isentropic relation, Eq. (3.64), to simplify the energy equation, Eq. (4.30), for the isentropic compressor with the inlet and exit represented by state-1 and state-2 respectively.

$$\dot{W}_{\text{sh,1-2}} = \dot{m}(j_1 - j_2) \cong \dot{m}(h_1 - h_2) = \dot{m}c_p(T_1 - T_2)$$
$$= \dot{m}c_pT_1\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = \frac{\dot{m}kRT_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = -280.5 \text{ kW};$$

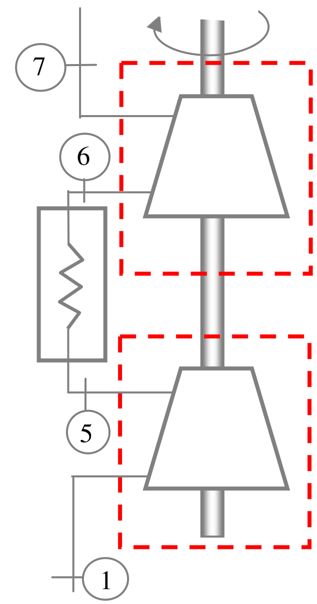


Fig. 4.32 Compressors with intercooler in option (d) in Ex. 4-10.

For the polytropic compressor, let state-3 represent the exit state ($p_3 = p_2$). Recall from sec. 3.61.2 that a polytropic relation has the same form as the corresponding isentropic relation k replaced by n. Therefore,

$$\dot{W}_{\text{sh,1-3}} = \frac{\dot{m}nRT_1}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(n-1)/n}\right] = -261.6 \text{ kW};$$

For the isothermal reversible compressor, operating between state-1 and state-4 ($p_4 = p_2$), the energy and entropy equations produce

$$\dot{W}_{\text{sh,1-4}} = \dot{m}(j_1 - j_4) + \dot{Q}_{1-4} \cong \dot{m}(h_1 - h_4) + \dot{Q}_{1-4}$$
$$= \dot{m}c_p\cancelto{0}{(T_1 - T_4)} + \dot{Q}_{1-4} = \dot{Q}_{1-4}$$

$\dot{Q}_{1-4}$ can be evaluated from the entropy equation as follows.

$$0 = \dot{m}(s_1 - s_4) + \frac{\dot{Q}_{1-4}}{T_1} + \cancel{\dot{S}_{gen}}^{\,0}$$

Therefore, $\dot{Q}_{1-4} = \dot{m}T_1\left(c_p \ln \cancel{\frac{T_4}{T_1}}^{\,1} - R\ln\frac{p_4}{p_1}\right) = \dot{m}RT\ln\frac{p_2}{p_1}$

$\Rightarrow \dot{W}_{\text{sh},1\text{-}4} = \dot{Q}_{1-4} = -198.3\text{ kW};$

The same result, of course, could be obtained by applying Eq. (4.9).

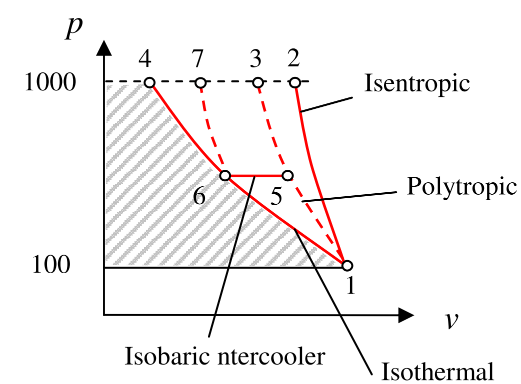
For the two stage compression (see Fig. 4.32), let us represent the intermediate pressure by p_x (see Fig. 4.33). Since the inlet temperature is the same for each stage, the expression for the polytropic power can be applied to each stage, producing

$$\dot{W}_{\text{sh},1\text{-}5\text{-}6\text{-}7} = \dot{W}_{\text{sh},1\text{-}5} + \dot{W}_{\text{sh},6\text{-}7} = \frac{\dot{m}nRT_1}{n-1}\left[1-\left(\frac{p_x}{p_1}\right)^{(n-1)/n} + 1-\left(\frac{p_7}{p_x}\right)^{(n-1)/n}\right]$$

To determine p_x that will minimize the total work, we differentiate the expression with respect to p_x and set it to zero. A little manipulation of the resulting equation produces $p_x = \sqrt{p_1 p_7} = 316.23\text{ kPa}$. Substituting this result in the above expression, we obtain $\dot{W}_{\text{sh},1\text{-}5\text{-}6\text{-}7} = 227.1\text{ kW}$.

Test Analysis Launch the single-flow PG daemon and select air as the working fluid. Calculate states 1-7 as described in the TEST-codes (see *TEST.Examples*). Notice how the polytropic relation is used to enter v3, v5 and v7. For two stage compression with intercooling, p5 is arbitrarily assumed to be 500 kPa initially. Two devices – device-D and E – represent the two stages. The sum of Wdot_ext for the two stages is calculated as 225 kW. Now change p5 to a new value, Super-Calculate and find the net power. Repeat until the optimum pressure is found.

Discussion The isothermal compressor requires the least amount of work for a given compression ratio, which is also evident from the $p-v$ diagram of Fig. 4.33 (minimum shaded area). One way to approach the isothermal limit is to indefinitely increase the number of stages of intercooling. In practice, the number of stages is decided by balancing the additional cost against the marginal power saving resulting from a new stage.



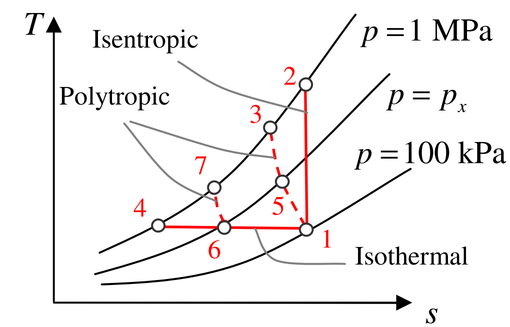


Fig. 4.33 $p-v$ and $T-s$ diagrams for different compressors in Ex. 4-10. Work consumption is the least for an isothermal compressor.

4.2.5 Throttling Valves

A **throttling device** is a special restriction - an **orifice** in a plate, a **porous plug**, a **capillary tube**, or an **adjustable valve** (see Fig. 4.34 and Anim. 4.A.*expansionValve*) – designed to enhance frictional resistance to a flow in order to create a large pressure drop.

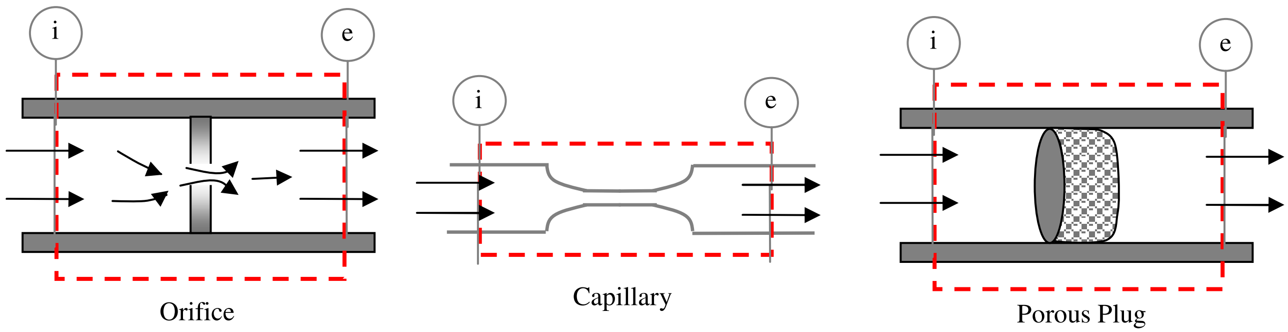
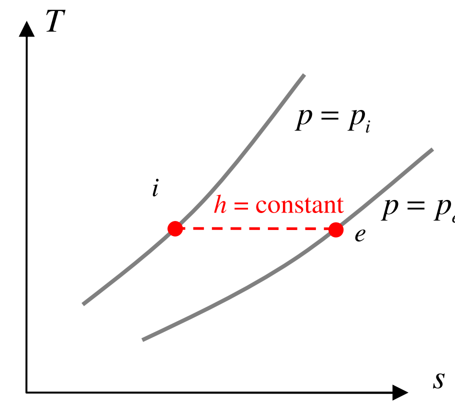


Fig. 4.34 Different types of throttling devices. The expansion valve is illustrated in Anim. 4.A.*throttlingValve*.



IG/PG model

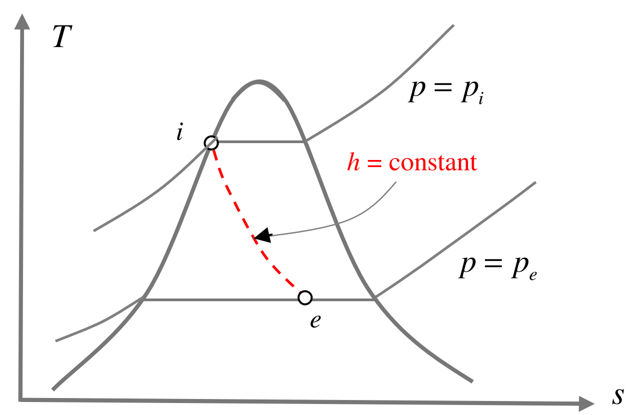


Fig. 4.35 Constant enthalpy lines on the $T-s$ diagram reveals if there is a temperature change in throttling.

Paradoxically, the flow velocity sometimes increases (due to an increase in specific volume), although the changes in ke is not significant. There is no external work transfer, and heat transfer is also negligible (due to insulation and high mass flow rate). For this adiabatic, single-flow, open steady device, Eqs. (4.2) and (4.3) simplify as

$$\text{Energy: } j_i = j_e; \quad \Rightarrow \quad h_i \cong h_e; \tag{4.35}$$

$$\text{Entropy: } s_e = s_i + \underbrace{\dot{S}_{\text{gen}}/\dot{m}}_{\geq 0}; \quad \Rightarrow \quad s_e > s_i; \tag{4.36}$$

Throttling, as can be seen from the energy equation, can be regarded as an *isenthalpic* process. If the working fluid is an ideal (or perfect) gas, enthalpy being a function of temperature only, we cannot expect any temperature change when a gas is throttled. However, for a PC fluid, the conclusion can be drastically different. As can be seen from the $T-s$ diagram (see Fig. 4.35) of a PC fluid, the constant-enthalpy lines inside the saturation dome have negative slopes. To verify this, draw a constant enthalpy line in the plot panel after evaluating any saturated liquid state using a PC state daemon. This means that if a saturated liquid is throttled, the isenthalpic requirement produces a saturated mixture at the exit that must be at a much lower temperature as shown in Fig. 4.35.

Physically what happens is that the liquid finds itself at a pressure much below the saturation pressure as it passes through the restriction and starts boiling in its quest for equilibrium. In the lack of heat transfer from the surroundings, the enthalpy of vaporization is supplied from the working fluid itself which cools down rapidly as a result. The *sensible cooling* produced by throttling is exploited in applications such as refrigeration and air-conditioning. Since frictional pressure drop is at the core of this phenomenon, frictionless or isentropic valve is meaningless.

The rate of change of temperature with respect to pressure for an **isenthalpic process** is called the Joule-Thompson coefficient

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h \quad \left[\frac{\mathrm{K}}{\mathrm{kPa}}\right]; \qquad (4.37)$$

Obviously, μ_J is zero for an ideal or perfect gas. A positive μ_J means that temperature decreases during throttling, while a negative μ_J means just the opposite. This newly defined thermodynamic property will be further explored in chapter 11.

EXAMPLE 4-11 [*mEE*] Analysis of a Throttling Valve

R-134a enters the throttling valve of a refrigeration system at 1.5 MPa and $50^\circ\mathrm{C}$. The valve is set to produce an exit pressure of 150 kPa. Determine (a) the quality and (b) temperature at the exit, and (c) the rate of entropy generation per unit mass of refrigerant. Assume the atmospheric temperature to be $25^\circ\mathrm{C}$. ***What-if scenario:*** (d) What would the answer in part (c) be if the exit pressure was reduced to 100 kPa?

SOLUTION Analyze the open steady system, the valve enclosed within the red boundary in Fig. 4.36, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for R-134a, uniform states based on LTE at the inlet and exit, and negligible changes in ke and pe.

Analysis Use TEST or the manual approach described to evaluate the following states.

State-1 (given p_1 and T_1) $\qquad h_1 = 123.0\ \frac{\mathrm{kJ}}{\mathrm{kg}};\ s_1 = 0.439\ \frac{\mathrm{kJ}}{\mathrm{kg}};$

State-2 (given $h_2 = h_1$ and p_2)

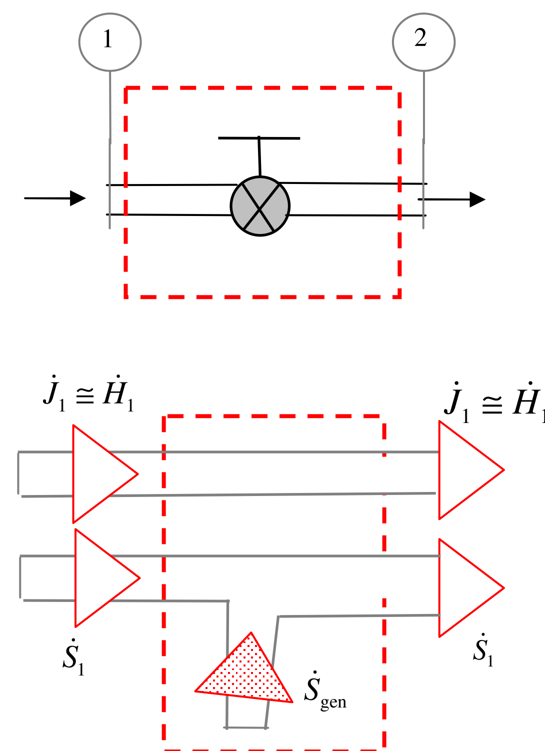


Fig. 4.36 System schematic and energy and entropy flow diagram in Ex 4-11.

$x_2 = 45.01\ \%$; $T_2 = -17.3\ ^0\mathrm{C}$; $s_2 = 0.4856\ \frac{\mathrm{kJ}}{\mathrm{kg}}$;

The entropy equation, Eq. (4.38), produces

$$\dot{S}_{\mathrm{gen,univ}} / \dot{m} = \left(s_e - s_i\right) = 0.0466\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};$$

Since the system is adiabatic, all the entropy generation takes place inside the valve, which can be attributed to viscous friction.

Test Analysis Launch the single-flow PC daemon, and select R-134a. Calculate the inlet and exit states from the given conditions as described in the TEST-codes (see *TEST.Examples*). Analyze the device on the basis of a mass flow rate of 1 kg/s.

What-if Scenario Change p2 to 100 kPa, press the Enter key, and Super-Calculate to update all calculations. In the device panel, find the updated value of Sdot_gen as 0.0615 kJ/kg.K.

Discussion The ratio of specific volume between the exit and inlet can be calculated as about 65.5. The velocity, therefore, can increase significantly unless the exit area is made significantly larger. To evaluate the effect of kinetic energy, assume a mass flow rate, an inlet velocity, and use A2 = A1, mdot2 = mdot1, and j2 = j1 for state-2. In spite of a large change in velocity, the exit temperature can be seen to remain unchanged. Only the quality at the exit and entropy generation rate change slightly.

4.2.6 Heat Exchangers

Heat Exchangers allow two streams of fluid to exchange heat without mixing while minimizing any heat transfer to the surroundings. The fluid that is heated may change phase as in the case of a **boiler** of a steam power plant or an **evaporator** of a refrigerator. On the other hand, the fluid that is cooled may condense as in a **condenser** of a power plant or a refrigerator. The simplest form of heat exchanger is the **tube-and-shell** type sketched in Fig. 4.37 (also see Anim. 4.B.*heatExchanger* to learn about co-flow and counter-flow configurations of heat exchangers). In it, the inner stream flows through the tube and the outer stream flows through the shell, usually, in a counter-flow configuration. Loss of pressure in either stream is considered negligible as are the changes in ke or pe. There is no possibility of any external work, and heat transfer for the overall system is neglected provided there is good insulation. Note that if a single stream is selected as the system, as in the case of a boiler, evaporator, or condenser, the analysis reduces to a pair of pipe flow analysis.

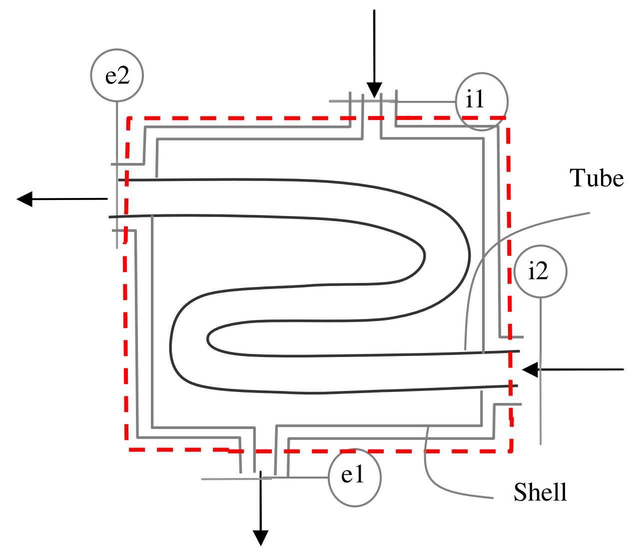


Fig. 4.37 Schematic of a counter-flow tube-and-shell heat exchanger. The multi-flow daemons in TEST allow two inlets and two exits.

4.2.7 TEST and the Multi-Flow Non-Mixing Daemons

In TEST heat exchangers are classified as multi-flow non-mixing device and the relevant daemons are located in the open, steady, generic, multi-flow, non-mixing branch. Since the two streams may carry different fluids, a large number of composite models are offered. In such a model, say PC/IG model, there are two material selectors, one for a phase-change fluid (PC model) and another for a gas (IG model). Also, in the device panel there is provision for two inlets and two exits. When the inlet and exit states are selected, the system schematic dynamically adjusts to the selected configuration. The procedure for evaluating the anchor states and setting up a device remains the same as in the case of a single-flow device. Additional information on this daemon can be found in chapter 4 of *Tutorial.Daemons* page and in the video clips in the *visualTour* linked from the task bar.

EXAMPLE 4-12 [*mEE*] Analysis of a Heat Exchanger

Steam enters the condenser of a steam power plant at 15 kPa and a quality of 90% with a mass flow rate of 25,000 kg/h. It is cooled by circulating water from a nearby lake at $25\,^{\circ}\mathrm{C}$ as shown in the accompanying sketch. If the water temperature is not to rise above $35\,^{\circ}\mathrm{C}$ and the steam is to leave the condenser as saturated liquid, determine (a) the mass flow rate of cooling water, (b) the rate of heat removal from the steam, and (c) the rate of entropy generation in the system's universe. ***What-if scenario:*** (e) What would the mass flow rate be if the exit temperature of cooling water was allowed to be only $5\,^{\circ}\mathrm{C}$ warmer? Assume atmospheric conditions to be 100 kPa and $25\,^{\circ}\mathrm{C}$.

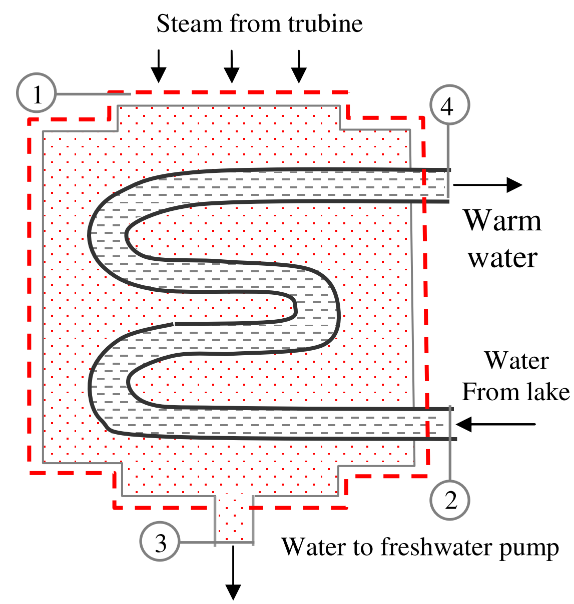


Fig. 4.38 Condenser in Ex. 4-12.

SOLUTION Analyze the open steady system, the heat exchanger enclosed within the red boundary in Fig. 4.38, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for water, negligible changes in ke, pe, and pressure in each stream, uniform states based on LTE at the inlets and exits.

Analysis Use TEST or the manual approach to determine the anchor states – state-1 and state-2 for the inlets, and state-3 and state-4 for the exits as shown in the accompanying figure.

State-1 (given p_1, x_1, $\dot{m}_1$)

$$\Rightarrow j_1 \cong h_1 = 2361.7\ \frac{\text{kJ}}{\text{kg}};\quad s_1 = 7.284\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

State-2 (given p_2 and T_2)

$$\Rightarrow j_2 \cong h_2 = 105.0\ \frac{\text{kJ}}{\text{kg}};\quad s_2 = 0.3673\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

State-3 (given $p_3 = p_1$ and $x_3 = 0$, $\dot{m}_3 = \dot{m}_1$)

$$\Rightarrow j_3 \cong h_3 = 225.8\ \frac{\text{kJ}}{\text{kg}};\quad s_3 = 0.7543\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

State-4 (given $p_4 = p_2$ and T_4)

$$\Rightarrow j_4 \cong h_4 = 146.8\ \frac{\text{kJ}}{\text{kg}};\quad s_4 = 0.5052\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

The mass equation for each stream produces $\dot{m}_1 = \dot{m}_3$ and $\dot{m}_2 = \dot{m}_4$. The energy equation for the entire heat exchanger, Eq. (2.8), yields

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 + \dot{m}_2 j_2 - \dot{m}_3 j_3 - \dot{m}_4 j_4; \Rightarrow \dot{m}_1 \left(j_1 - j_3 \right) = \dot{m}_2 \left(j_4 - j_2 \right);$$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 \left(j_1 - j_3 \right) / \left(j_4 - j_2 \right) = 354.9\ \text{kg/s};$$

To obtain the internal heat exchange, analyze one of the streams, say, the circulating water, as a single-flow sub-system. Employing the energy equation, Eq. (4.2), we obtain

$$0 = \dot{m}_2 \left(j_2 - j_4 \right) + \dot{Q};\quad \Rightarrow \dot{Q} = \dot{m}_2 \left(j_4 - j_2 \right) = 14.83\ \text{MW};$$

The entropy equation, Eq. (2.13), applied to the overall adiabatic system produces

$$\cancel{\frac{dS}{dt}}^{0} = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}_1 \left(s_3 - s_1 \right) + \dot{m}_2 \left(s_4 - s_2 \right) = 3.60\ \frac{\text{kW}}{\text{K}};$$

The energy and entropy diagrams of Fig. 4.39 visually represents how the two streams interact to generate entropy in the system's universe. While energy and entropy are transferred from the hotter stream to the cooler stream, entropy generated due to thermal friction boosts the entropy transported by the exit flows.

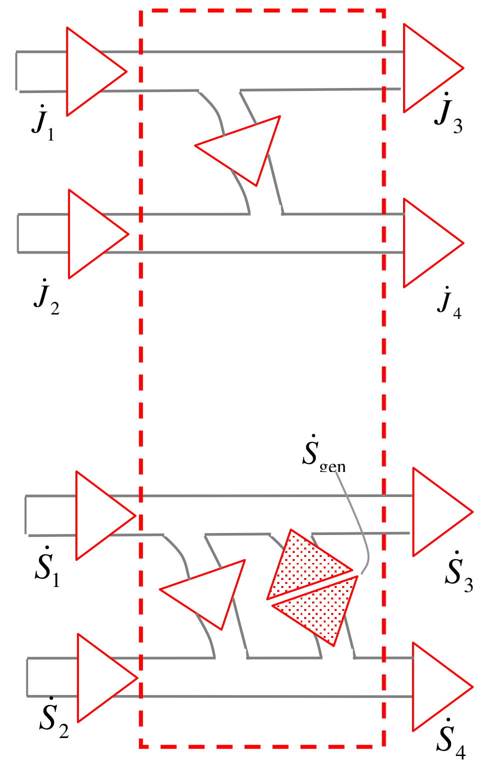


Fig. 4.39 Energy and entropy flow diagrams in in Ex. 4-12.

Test Analysis Launch the non-mixing multi-flow PC daemon (because the fluids are identical no need for the PC/PC composite model), and select H2O as the working fluid. Calculate states 1-4 as described in the TEST-codes (see *TEST.Examples*). In the device panel, set up the overall heat exchanger as device-A, selecting appropriate inlet and exit states, and entering Qdot = Wdot_ex t = 0. Make sure that the non-mixing option is turned on. Super-Calculate to evaluate mdot2 and Sdot_gen. To calculate the heat transfer between the streams, set up device-B with states 2 and 4 as the anchor states. Calculate Qdot after setting Wdot_ext = 0.

What-if Scenario Change T4 to T2 + 15, press the Enter key and Super-Calculate. The new mass flow rate is calculated as mdot2 = 236.7 kg/s.

Discussion The pumping power necessary to circulate the cooling water can be shown (see Eq. (4.33)) to be $\dot{W}_{\text{sh,s}} = \dot{m}\Delta p / \rho$, where Δp is the pressure drop in the pipe. A 5 $^\circ$C increase in the exit temperature of cooling water reduces the mass flow rate by 33.3% and, thereby, the pumping power by at least as many percent. Generally the warmer water is dumped into a lake or a river, and increase in the exit temperature may have adverse environmental consequences on the local echo system.

4.2.8 Mixing Chambers and Separators

A **mixing chamber** is a device where two streams of fluids merge to produce a single stream (see Anim. 4.C.*mixingChamber*). A simple T-elbow can serve as a mixing chamber. The small size of the device ensures that heat transfer can be neglected. Changes in ke and pe are also negligible and there is no external work transfer. For the two flows to enter the chamber, pressures at the two inlets must be equal; otherwise, a back flow will be created through the low-pressure inlet. Frictional pressure loss in the chamber is often neglected resulting in the assumption of a constant pressure throughout the system. When the two mixing streams have different temperatures, the mixing chamber works as a heat exchanger between the two; hence, the name **direct-contact heat exchanger** is sometime used for a mixing chamber. Mixing between two different fluids will be discussed in chapter 11. A **separator** is just the opposite of a mixing chamber, where a flow is bifurcated into two dissimilar streams. Separators can be found in turbines with bleeding (chapter 9) and flash chambers (chapter 10) and their analysis is quite analogous to mixing flow analysis.

4.2.9 TEST and the Multi-Flow Mixing Daemons

A mixing chamber in TEST is classified as a multi-flow, mixing, open steady device. The corresponding daemons are located in open, steady, generic, multi-flow, mixing branch. The state panel of a mixing daemon is identical to that of a single-flow daemon. The device panel allows up to two inlets and two exits. The same daemon can be used as a mixing chamber or a separator. For a mixing chamber, only one exit should be used and the other left at its null-state (that is closed). Likewise, one of the inlet states is not used for a separator. The procedure for evaluating the anchor states and setting up a device remains the same as in the case of a single-flow device, discussed in sec. 4.1.1. Devices with more than two inlets can be creatively handled by splitting a multiple-inlet device into two or more devices, each with two inlets, connected in series.

EXAMPLE 4-13 [*mEE*] Analysis of a Mixing System

Superheated ammonia at 200 kPa, -10°C enters an adiabatic mixing chamber with a flow rate of 2 kg/s where it mixes with a flow of saturated mixture of ammonia at a quality of 20%. The desuperheated ammonia exits as saturated vapor. Assuming pressure to remain constant, and neglecting change in ke and pe, determine (a) the mass flow rate of the saturated mixture and (b) the rate of entropy generation in the chambers universe. ***What-if scenario:*** (c) What would the answer in part (a) be if the superheated ammonia entered the chamber at 0°C ?

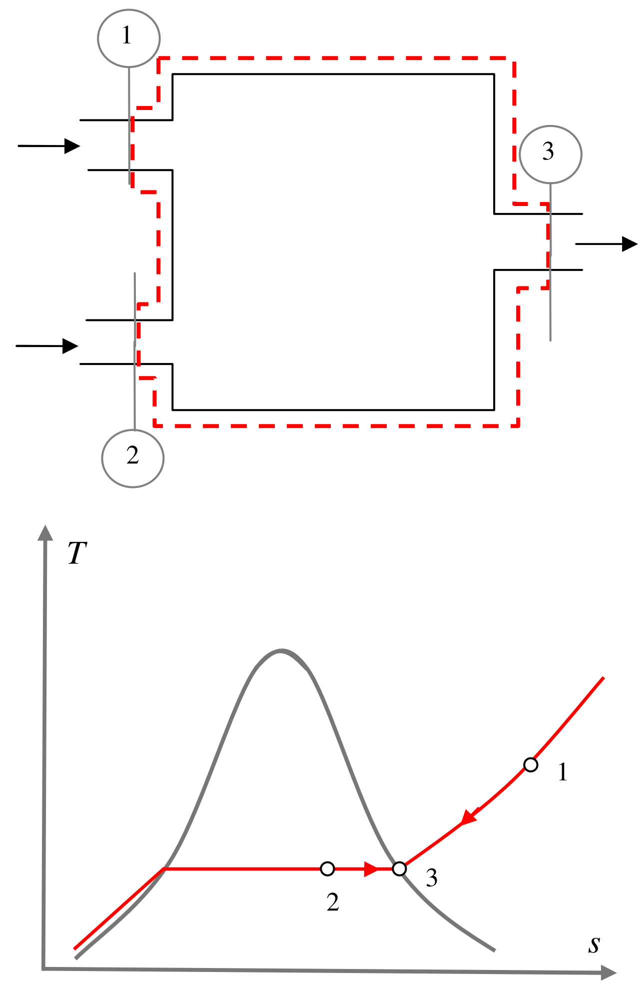


Fig. 4.40 Schematic and $T-s$ diagram for Ex. 4-13.

SOLUTION Analyze the open steady system, the mixing chamber enclosed within the red boundary in Fig. 4.40, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for ammonia, negligible changes in ke and pe, no pressure drop in the chamber so that $p_1 = p_2 = p_3$, uniform states based on LTE at the inlet and exit of each device.

Analysis Use TEST or the manual approach to determine the anchor states.

State-1 (given p_1, T_1, $\dot{m}_1$)

$$\Rightarrow j_1 \cong h_1 = 1440.3 \ \frac{\text{kJ}}{\text{kg}}; \quad s_1 = 5.677 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

State-2 (given $p_2 = p_1$ and x_2)

$$\Rightarrow j_2 \cong h_2 = 359.2 \ \frac{\text{kJ}}{\text{kg}}; \quad s_2 = 1.428 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

State-3 (given $p_3 = p_2$ and $x_3 = 1$)

$$\Rightarrow j_3 \cong h_3 = 1419 \ \frac{\text{kJ}}{\text{kg}}; \quad s_3 = 5.599 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

The steady-state mass equation, Eq. (2.3) simplifies as $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. Substituting it in the energy balance equation, Eq. (2.8), results in

$$0 = \dot{m}_1 j_1 + \dot{m}_2 j_2 - \dot{m}_3 j_3 = \dot{m}_1 j_1 + \dot{m}_2 j_2 - \dot{m}_1 j_3 - \dot{m}_2 j_3$$

$$\Rightarrow \quad \dot{m}_2 = 0.0393 \text{ kg/s};$$

The entropy equation, Eq. (2.13), applied to the overall adiabatic system produces

$$\cancel{\frac{dS}{dt}}^{0} = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}_1 \left(s_3 - s_1\right) + \dot{m}_2 \left(s_3 - s_2\right) = 0.0067 \ \frac{\text{kW}}{\text{K}};$$

Test Analysis Launch the multi-flow mixing PC daemon and select H2O as the working fluid. Set up states 1-3 and the device as described in the TEST-codes (see *TEST.Examples*). In the device panel, select the inlet and exit states, enter Qdot and Wdot_ext as 0 and Super-Calculate to update the entire solution.

What-if Scenario Change T1 to $0\,^\circ\text{C}$, press the Enter key and then Super-Calculate. The new mass flow rate is mdot2 = 0.084 kg/s and the entropy generation rate is 0.0142 kW/K.

Discussion As stated in sec. 2.1.3, mixing is a fundamental entropy generating mechanism. The more the difference between the two stream in terms of temperature or phase composition, the more would be the mixing irreversibilities. This can be easily demonstrated by carrying out multiple what-if studies using the mixing daemon.

EXAMPLE 4-14 [*mEE*] Analysis of a Composite System

Saturated liquid water at 1.0 MPa is throttled in an expansion valve to a pressure of 500 kPa, which then flows into a flash chamber as shown in the accompanying schematic. The saturated liquid exits the chamber at the bottom while the saturated vapor, after exiting near the top, expands in an isentropic turbine to a pressure of 10 kPa. Assuming

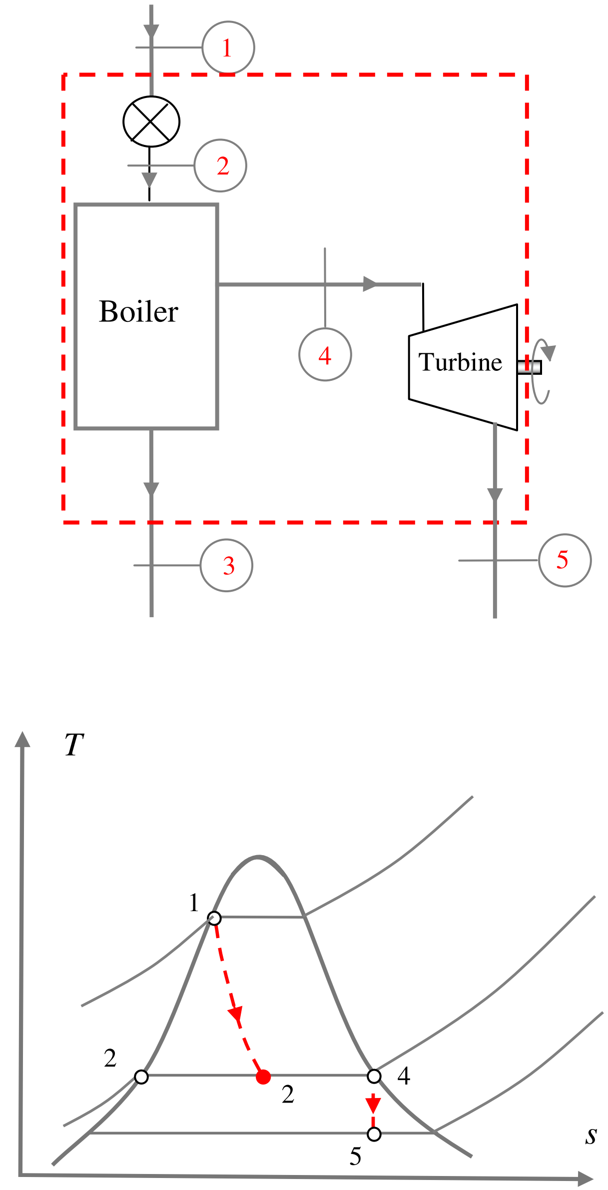


Fig. 4.41 Schematic and $T-s$ diagram for Ex. 4-14.

all components to be adiabatic and a steady flow rate of 10 kg/s at the inlet, determine (a) the quality at the turbine exit, (b) power produced by the turbine in kW, and (c) entropy generated in the system's universe. ***What-if scenario:*** (e) What would the turbine power and exit quality be if saturated liquid entered the system at 1.5 MPa? Assume atmospheric conditions to be 100 kPa and $25\,^{\circ}\mathrm{C}$.

SOLUTION Analyze the open steady system, the composite system enclosed within the red boundary in Fig. 4.41, using the mass, energy, and entropy balance equations.

Assumptions Steady state, PC model for water, negligible changes in ke and pe, uniform states based on LTE at the inlet and exit of each device.

Analysis Use TEST or the manual approach to determine the five principal states shown in the schematic of Fig. 4.41. To evaluate state-2, the energy equation for an adiabatic valve, $j_1 = j_2$ (sec. 4.2.5), is used. The flash chamber separates the mixture into its component phases – saturated vapor and saturated liquid.

State-1 (given p_1, $x_1 = 0$, $\dot{m}_1$)

$$\Rightarrow j_1 \cong h_1 = 762.8\ \frac{\mathrm{kJ}}{\mathrm{kg}};\quad s_1 = 2.139\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};$$

State-2 (given p_2 and $j_2 = j_1$)

$$\Rightarrow j_2 \cong h_2 = 762.8\ \frac{\mathrm{kJ}}{\mathrm{kg}};\quad s_2 = 2.149\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};\quad x_2 = 5.823\ \%;$$

State-3 (given $p_3 = p_2$ and $x_3 = 0$)

$$\Rightarrow j_3 \cong h_3 = 640.0\ \frac{\mathrm{kJ}}{\mathrm{kg}};\quad s_3 = 1.860\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};$$

$$\dot{m}_3 = \dot{m}_2\left(1 - x_2\right) = 9.418\ \frac{\mathrm{kg}}{\mathrm{s}};$$

State-4 (given $p_4 = p_2$ and $x_4 = 1$)

$$\Rightarrow j_4 \cong h_4 = 2748.6\ \frac{\mathrm{kJ}}{\mathrm{kg}};\quad s_4 = 6.822\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}};\ \dot{m}_4 = \dot{m}_2 x_2 = 0.582\ \frac{\mathrm{kg}}{\mathrm{s}};$$

State-5 (given p_5 and $s_5 = s_4$)

$$\Rightarrow j_5 \cong h_5 = 2160.9\ \frac{\mathrm{kJ}}{\mathrm{kg}};\quad x_5 = 82.3\ \%;$$

Energy and entropy balance on the combined system yield

$$0 = \dot{m}_1 j_1 - \dot{m}_3 j_3 - \dot{m}_5 j_5 - \dot{W}_{sh} \quad \Rightarrow \quad \dot{W}_{sh} = 342 \text{ kW};$$

$$0 = \dot{m}_1 s_1 - \dot{m}_3 s_3 - \dot{m}_5 s_5 + \dot{S}_{gen,univ}; \quad \Rightarrow \quad \dot{S}_{gen,univ} = 0.103 \text{ kW/K};$$

Test Analysis Launch the mixing PC daemon and select H2O as the working fluid. Set up the anchor states and the overall device (device-A) as described in the TEST-codes posted in *TEST.Examples*. Super-Calculate to obtain the net power and the entropy generation rate for the system's universe.

What-if Scenario Change p1 to 1.5 MPa, press the Enter key, and Super-Calculate. The turbine power is updated in Device-A as Wdot_ext = 570.8 kW.

Discussion The solution illustrates the advantages of treating a network of multiple devices as a single composite system. Of course, the same solution could be obtained by analyzing each device separately.

4.3 Closure

This chapter presents a comprehensive mass, energy, and entropy (*mEE*) analysis of open steady systems based on the governing equations developed in chapter 2. Open steady systems are divided into three separate categories: single-flow, non-mixing multi-flow, and mixing multi-flow systems. Analysis of single flow devices – pipes, nozzles, diffusers, turbines, compressors, pumps and throttling valves – have been emphasized because of their importance. Energetic and isentropic device efficiencies and reversible heat and work transfer are discussed for specific devices. Every manual solution presented in this chapter is verified by TEST solution, which is then used for what-if studies in selected examples.

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